Convexity of Trend following & Variance arbitrage

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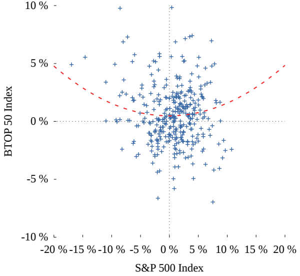
Metori Capital Management, December 2017

Convexity of CTA

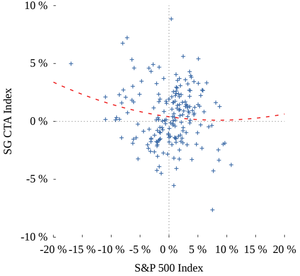
- W. Fung and D. A. Hsieh: Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds, *Review of Financial studies*, 10, 275-302 (1997)
- W. Fung and D. A. Hsieh: The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers, Review of Financial studies, *Review of Financial studies*, 14, 313-341 (2001)
- M. Potters and J.-P. Bouchaud: Trend followers lose more often than they gain, *Wilmott Magazine*, 26, 58-63 (January 2006).
- B. Bruder and N. Gaussel: Risk-Return Analysis of Dynamical investment Strategies, *Lyxor White Paper*, Issue 7 (June 2011).
- **T.-L. Dao et al: Tail Protection for Long Investors: Trend Convexity at Work *Journal of Investment Strategies* (May 2016)**
- P. Jusselin et al: Understanding the Momentum Risk Premium: An In-Depth Journey Through Trend-Following Strategies, *Amundi White Paper* (Oct 2017).

Outlook

How to observe the convexity of CTA?



- Monthly returns of the Barclays BTOP 50 Index vs. monthly returns of the S&P 500 Index (symbols) and parabolic fit



- Monthly returns of the SG CTA Index vs. monthly returns of the S&P 500 Index (symbols) and parabolic fit

Approach: Volatility at different timescale

- Price model

$$S_t = S_0 + \sum_{t'=1}^{t} D_{t'} \ .$$

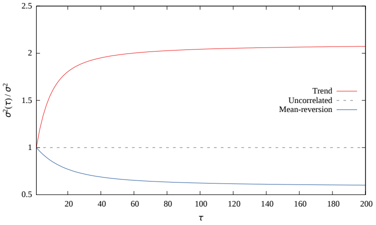
- Price changes $D_{t'<t}$ are stationary random variables with zero mean and covariance given by:

$$\mathbb{E}[D_u D_v] = \mathcal{C}(|u-v|) \ ,$$

- Uncorrelated random walks corresponds to $\mathcal{C}(u) = \sigma^2 \delta_{u,0}$. Trending random walks are such that $\mathcal{C}(u) > 0$, while mean-reverting random walks are such that $\mathcal{C}(u) < 0$
- How does this translate in terms of the volatility of the walk? We define the volatility of scale τ:

$$\sigma^2(\tau) := \frac{1}{\tau}\mathbb{E}\left[(S_{t+\tau} - S_t)^2\right] = \sigma^2 + \frac{2}{\tau}\sum_{u=1}^{\tau}(\tau - u)\ \mathcal{C}(u) \ ,$$

with single step volatility $\sigma^2(1) = \sigma^2$.



Simple Trend = long variance - short variance

- Consider a simple strategy such that the position Π_t is proportional to the price difference between t and 0:

$$\Pi_t := (S_t - S_0) ,$$

- P&L from $t-1$ to t is given by:

$$G_t := \Pi_{t-1} D_t = D_t \sum_{t'=1}^{t-1} D_{t'} , \quad G_1 := 0 .$$

- Cummulative performance of from day 0 to day T:

$$\begin{aligned} \mathcal{G}_T &= \sum_{t=1}^{T} G_t = \sum_{t=2}^{T} \sum_{t'=1}^{t-1} D_t D_{t'} \\ &= \frac{1}{2} \left(\sum_{t=1}^{T} D_t \right)^2 - \frac{1}{2} \sum_{t=1}^{T} D_t^2 \end{aligned}$$

$$\Rightarrow \quad \boxed{\mathcal{G}_T = \underbrace{\frac{1}{2} \left(S_T - S_0 \right)^2}_{\text{Long-term Variance}} - \underbrace{\frac{1}{2} \sum_{t=1}^{T} D_t^2}_{\text{Short-term Variance}}}$$

Convexity with general trend

- Trend estimator defined with EMA filter:

$$d\pi_t = -\frac{2}{\tau}\pi_t dt + \frac{2}{\tau} dS_t.$$

The solution of this SDE given by the following expression:

$$\pi_t = \mathcal{L}_\tau(S_t) = \frac{2}{\tau}\int_{-\infty}^{t} e^{2(t-s)/\tau} dS_s.$$

- Daily profit and loss: $dG_t = \phi(\pi_t) \times dS_t$

$$dG_t = \phi(\pi_t)\pi_t dt + \frac{\tau}{2}\phi(\pi_t)d\pi_t.$$

Let $F(x)$ be such that $F'(x) = \phi(x)$, then using Ito's lemma we have:

$$dF(\pi_t) = \phi(\pi_t)dP_t + \frac{2\phi'(\pi_t)}{\tau^2} dS_t^2.$$

Inserting this expression in the equation of the P&L, we obtain:

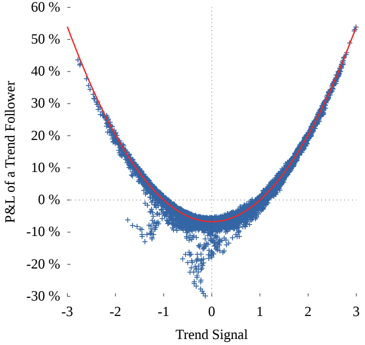
$$dG_t = \underbrace{\phi(\pi_t)\pi_t dt - \frac{\phi'(\pi_t)}{\tau} dS_t^2}_{\text{Drift term}} + \underbrace{\frac{\tau}{2} dF(\pi_t)}_{\text{Risk term}}$$

Re-arrange the different terms and introduce the filter $\mathcal{L}_T$:

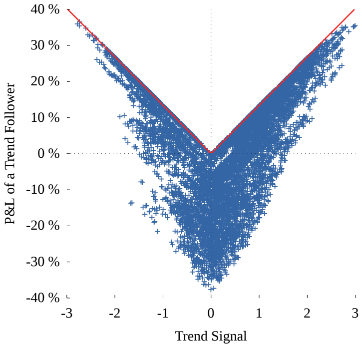
$$\mathcal{L}_T[dG_t] = \underbrace{\frac{\tau}{T}F(\pi_t)}_{\text{Payoff}} - \underbrace{\mathcal{L}_T\left[\frac{\phi'(\pi_t)}{\tau} dS_t^2\right]}_{\text{Volatility Cost}} + \underbrace{\mathcal{L}_T\left[\phi(\pi_t)\pi_t - \frac{\tau}{T}F(\pi_t)\right] dt}_{\text{Error term}}$$

Illustration of convexity

- Linear trend: $\mathcal{L}_{\tau/2}[dG_t] = \pi_t^2 - \frac{1}{\tau}\mathcal{L}_{\tau/2}[dS_t^2]$.



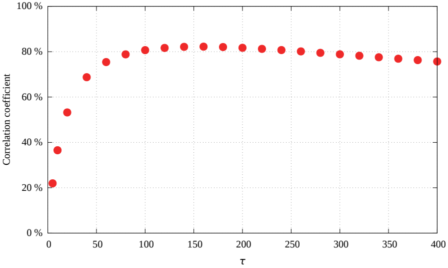
- Sign of the trend: $\mathbb{E}[\mathcal{L}_\tau[dG_t]|\pi_t] = |\pi_t| - \sqrt{\frac{2}{\pi\tau}}\sigma$.



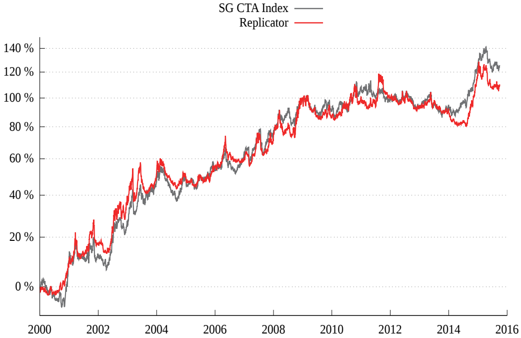
Replication of Risk Parity and CTA

Commodities	Stock Indices	Foreign Exchange Rates	Short term interest rates	Government bonds
WTI Crude Oil (CME)	S&P 500 (CME)	EUR/USD (CME)	Euribor (ICE)	10Y U.S. Treasury Note (CME)
Gold (CME)	EuroStoxx 50 (Eurex)	JPY/USD (CME)	Eurodollar (CME)	Bund (Eurex)
Copper (CME)	FTSE 100 (ICE)	GBP/USD (CME)	Short Sterling (ICE)	Long Gilts (ICE)
Soybean (CME)	Nikkei 225 (JPX)	AUD/USD (CME)		JGB (JPX)
		CHF/USD (CME)		

- Employ the most liquid futures in each sector.
- This selection is stable across time.
- The time series are considered between January 2000 and October 2015.



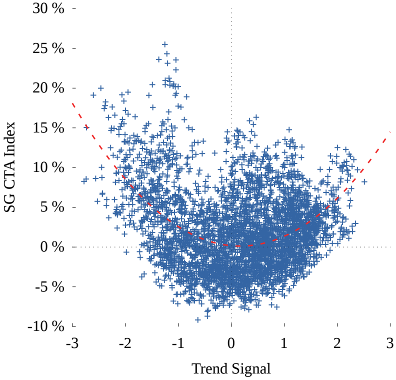
Correlation between CTA replicator and the SG CTA Index as a function of the time-scale of the trend. maximum is around $\tau = 180$ days.



Cumulated returns of trend replicator and the SG CTA Index. We seem to capture all the alpha contained in the SG CTA Index.

Convexity of CTA versus S&P500

- Plot aggregated performance over τ' days the SG CTA Index as a function of a τ-day trend on S&P 500 index.
- we need a careful choice of timescale to observe the convexity. Trend on S&P 500 is computed with $\tau = 180$ while trend on SG CTA Index is computed with $\tau' \approx 90$.
- Convexity feauture is much more significantly than the first monthly plot.

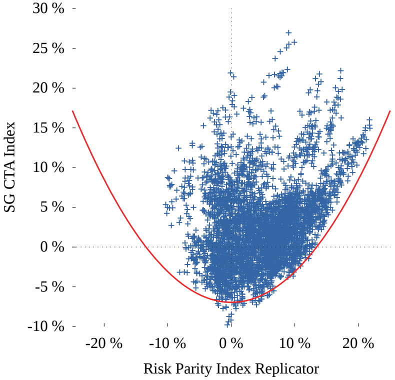


Convexity of CTA versus Risk Parity

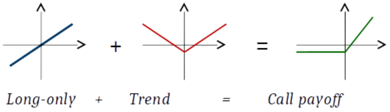
- A simple trend following strategy applied on Risk Parity provides the exact quadratic behavior of convexity.
- CTA index provides also convexity to Riks Parity strategy
- We can derive an inequality for lower bound of convexity:

$$\mathbb{E}[\mathcal{G}^{CTA}|\mathcal{T}^{\mathrm{RP}}] \geq \Upsilon(\tau)\left(\left(\mathcal{T}^{\mathrm{RP}}\right)^2 - 1\right)$$

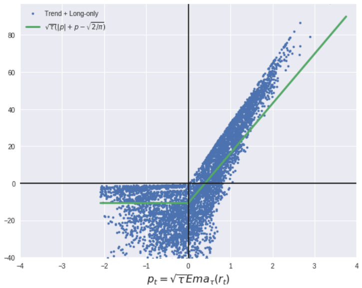
where $\mathcal{T}^{\mathrm{RP}}$ is the trend on Risk Parity index.



Conclusion: trend as a hedge of Risk Parity



- Trend following is a natural way to do stop-loss for long risk parity.
- The hidden cost of for doing this hedge is the realized volatility.
- Tuning the mixing between Trend and Long-only, one may obtain the desired the convexity for the portfolio.
- Both global trend and diversified trend can be used to hedge long risk parity.



Straddle and collection of strangles

- ATM straddle payoff:

$$\mathcal{G}_T^{\text{straddle}} := |S_T - S_0| - (C_{0,T}^{S_0} + P_{0,T}^{S_0}) \\ = |S_T - S_0| - \sqrt{\frac{2T}{\pi}} S_0 \sigma_0^a$$

- For a collection of strangles:

$$\mathcal{G}_T^{\text{strangles}} := \underbrace{\int_0^{S_0} (K - S_T)_+ dK + \int_{S_0}^{\infty} (S_T - K)_+ dK}_{\frac{1}{2}(S_T - S_0)^2} - \underbrace{\int_0^{S_0} P_{0,T}^K + \int_{S_0}^{\infty} C_{0,T}^K}_{\frac{1}{2} T \bar{\sigma}_{0,T}^2}$$

We obtain the payoff:

$$\boxed{\mathcal{G}_T^{\text{strangles}} = \frac{1}{2}(S_T - S_0)^2 - \frac{1}{2} T \bar{\sigma}_{0,T}^2}$$

Buying straddle or strangles is a simple way to have long exposure on long-term variance by paying fixed implied volatility

Trend as Delta-hegde: Model-free approach

- When the price move, we need to trade a quantity of underlying to keep neutral Delta. The Delta-hedge position is given simply by:

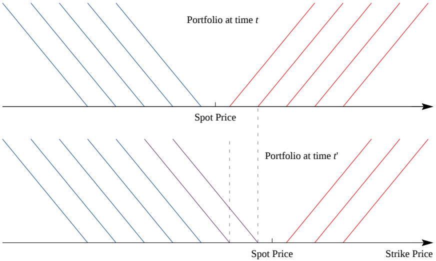
$$\Delta_{hdg} = -(S_t - S_0)$$

- Total P&L of the hedge from $t = 0$ to the maturity is given by:

$$\mathcal{G}_T^{\text{hedge}} := \frac{1}{2}\sum_{t=1}^{T} D_t^2 - \frac{1}{2}(S_T - S_0)^2$$

- Add the pay-off of strangles to this hedge P&L, we find:

$$\mathcal{G}_T^{\text{strangles}} + \mathcal{G}_T^{\text{hedge}} = \underbrace{\frac{1}{2}\sum_{t=1}^{T} D_t^2 - \frac{T}{2}\bar{\sigma}_{0,T}^2}_{\text{Variance Swap payoff}}$$



Variance arbitrage

- Selling option will allow a *risk premium*. In fact, $\mathbb{E}[\mathcal{G}_T^{\text{strangles}}] < 0$ because the implied volatility is usually *overpriced* and more expensive than the realized volatility.

$$\boxed{\mathcal{G}_T^{\text{strangles}} = \frac{1}{2}(S_T - S_0)^2 - \frac{1}{2}T\bar{\sigma}_{0,T}^2}$$

- Doing simple trend is also a simple way to exposure to long-variance. Indeed, *trend anomaly* has been proved empirically $\mathbb{E}[\mathcal{G}_T^{\text{trend}}] > 0$.

$$\boxed{\mathcal{G}_T^{\text{trend}} = \frac{1}{2}\left(S_T - S_0\right)^2 - \frac{1}{2}\sum_{t=1}^{T} D_t^2}$$

- Trading volatility is interesting because we profit from both effects. Selling long-variance with high cost *"implied variance"* and buying long-variance with smaller cost *"realized variance"* $\Rightarrow$ *Variance arbitrage*

$$\boxed{\mathcal{G}_T^{\text{trend}} - \mathcal{G}_T^{\text{strangles}} = \frac{1}{2}T\bar{\sigma}_{0,T}^2 - \frac{1}{2}\sum_{t=1}^{T} D_t^2}$$

Conclusions

Trend following and convexity:

- Trend is an arbitrage between long-term variance and short-term variance
- Convexity of trend is observed in association with a timescale (investment horizon or trend timescale)
- Timescale of trend defines a maturity (like call/put). Trend behaves as a hedge only for this maturity.

Trend following as a hedge of Risk Parity:

- Trend can be used as a hedging tool of long risk parity
- Both diversified trend and global trend can be used for hedging the long risk

Variance arbitrage:

- Delta hedge of options is a trend. To harvest option premium, hedging frequency is important.
- Choosing hedging frequency is choosing timescale of volatility that one wants to be exposed.
- Best choice for hedging is to employ the *trend anomaly* with *low frequency*.