Shiny Application and Reproducible Pitch

Volume of an n-ball and area of an n-sphere

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n-ball and n-sphere

Let n be a positive integer and r be a positive real number.

The Euclidean n-ball of radius r is defined as the set of all points $x \in \mathbb{R}^n$ such that $\|x\|_2^2 \leq r^2$ where $\|\cdot\|_2$ refers to the usual Euclidean norm in $\mathbb{R}^n$.

(`https://en.wikipedia.org/wiki/Euclidean_distance`)

The Euclidean n-sphere of radius r is defined as the set of all points $x \in \mathbb{R}^{n+1}$ such that $\|x\|_2^2 = r^2$. Please note that an n-sphere lives in a space of dimension $n+1$ (for example, a classical sphere is an object of dimension 2 embedded in the Euclidean space of dimension 3).

We refer the reader to the two following articles on Wikipedia for more informations regarding n-ball and n-sphere:

`https://en.wikipedia.org/wiki/Ball_(mathematics)`

`https://en.wikipedia.org/wiki/N-sphere`

Volume of an n-ball and surface area of an n-sphere

The volume of an n-ball of radius r can be calculated thanks to the following formula:

$$V_n(r) = \frac{\pi^{\frac{n}{2}}}{\Gamma(1 + \frac{n}{2})} r^n$$

where Γ is the gamma function (`https://en.wikipedia.org/wiki/Gamma_function`).

The surface area of an n-sphere is given by the formula below:

$$S_n(r) = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} r^{n-1}$$

See the following Wikipedia articles for a formal proof of each formula:

`https://en.wikipedia.org/wiki/Volume_of_an_n-ball`

`https://en.wikipedia.org/wiki/N-sphere`

Example

For $n = 3$ and $r = 1$ the volume of the unit ball is:

```
pi^(3/2) / gamma(1+3/2)
```

```
## [1] 4.18879
```

while the surface area of the associated sphere is:

```
2*pi^(3/2) / gamma(3/2)
```

```
## [1] 12.56637
```

Please note that those (rather simple) results could also have been derived from the well-known expressions $V_3(r) = \frac{4}{3}\pi r^3$ and $S_3(r) = 4\pi r^2$ (which are actually special cases of the formulas given on the previous slide).

Web application functionality

The application can be found at:

- Pick the dimension n (positive integer)
- Pick the radius r (positive real number)

The application computes the volume $V_n(r)$ of the n-ball and the surface area $S_n(r)$ of the n-sphere according to the formulas presented in the previous slides.