

Introduction to Mathematical Theology

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1 Introduction

This is my attempt to formulate the Catholic theology of the Trinity in mathematical language. If you find any errors in the math or the theology, please email me at mail@tobilehman.com, I care deeply about getting this right. To my mathematician's eyes, the Catechism of the Catholic Church (CCC) has parts that remind me of a mathematical text. The axiomatic truths asserted early on, the precise and consistent use of language, the fidelity to reason. I don't know if the entire Catechism could be written to look like a math text, or if it even should, but I think the doctrine of trinity can. This is my best attempt to do so. God bless.

For background on category theory or Catholic theology, see my sources section.

2 Definitions

$$\mathcal{G} = \text{God} \tag{1}$$

$$F = \text{The Father} \tag{2}$$

$$S = \text{The Son} \tag{3}$$

$$H = \text{The Holy Spirit} \tag{4}$$

$$(X \stackrel{\dagger}{=} Y) = (X \text{ is consubstantial with } Y) \tag{5}$$

2.1 The category \mathcal{G}

We will represent the triune God \mathcal{G} as a category where the set of objects is:

$$\text{ob}(\mathcal{G}) = \{F, S, H\}$$

$$\begin{array}{ccc} & F & \\ \stackrel{\dagger}{\leftarrow} & & \stackrel{\dagger}{\leftarrow} \\ H & \stackrel{\dagger}{\longleftrightarrow} & S \end{array}$$

And all the sets of morphisms are isomorphisms, they are the $\stackrel{\dagger}{=}$ relation, which is read "is consubstantial with".

$$\text{Hom}(F, S) = \{\stackrel{\dagger}{=}: F \rightarrow S\} \quad (6)$$

$$\text{Hom}(S, H) = \{\stackrel{\dagger}{=}: S \rightarrow H\} \quad (7)$$

$$\text{Hom}(H, F) = \{\stackrel{\dagger}{=}: H \rightarrow F\} \quad (8)$$

$$\text{Hom}(S, F) = \{\stackrel{\dagger}{=}: S \rightarrow F\} \quad (9)$$

$$\text{Hom}(F, H) = \{\stackrel{\dagger}{=}: F \rightarrow H\} \quad (10)$$

$$\text{Hom}(H, S) = \{\stackrel{\dagger}{=}: H \rightarrow S\} \quad (11)$$

2.2 Interpreting "God is the Father"

Bill Clinton once reminded us that it depends on what your definition of "is" is. We have formalized the sentences like "The Father is consubstantial with the Son", but then defined God as the category that contains the Father, Son and Holy Spirit. What does it mean to say that "God is the Father" (In our notation: \mathcal{G} "is" F)?

2.2.1 The Is Functors

We interpret the sentence "God is the Father" using the $\text{Is}_F : \mathcal{G} \rightarrow \mathcal{G} \downarrow F$ Functor.

$\mathcal{G} \downarrow F$ is the slice category construction, which makes a category out of an object that isomorphic to \mathbb{K} .

Functors Given a category \mathcal{C} and a category \mathcal{D} , a **functor** is a mapping from the objects of \mathcal{C} to the objects of \mathcal{D} , such that morphism composition is preserved. So if $f : A \rightarrow B$ is a morphism in \mathcal{C} , then $F(f) : F(A) \rightarrow F(B)$ is a morphism in \mathcal{D} .

Slice Category Given a category \mathcal{C} and an object $c \in \text{ob}(\mathcal{C})$, the **slice category** $\mathcal{C} \downarrow c$ is a category where:

- Objects are morphisms $f : x \rightarrow c$ in \mathcal{C}
- Morphisms from $f : x \rightarrow c$ to $g : y \rightarrow c$ are morphisms $h : x \rightarrow y$ in \mathcal{C} such that $g \circ h = f$

2.3 Lemma 1.1: \mathcal{G} is a groupoid

A groupoid is a category in which every morphism is an isomorphism. That's another way of saying every morphism has an inverse.

2.3.1 Proof:

If you look at the hom-sets above, you will see that for every $\text{Hom}(X, Y)$ there's a $\text{Hom}(Y, X)$. Without loss of generality, consider $\text{Hom}(F, S)$, which asserts that "The Father is consubstantial with The Son".

3 Axioms

1. There is one God (CCC:200)
2. There are three Divine Persons, The Father, The Son and The Holy Spirit. All are fully God. (CCC:253).
3. The Father is consubstantial with The Son, who is consubstantial with the Holy Spirit. (CCC:689)

$$F \stackrel{\dagger}{=} S \stackrel{\dagger}{=} H$$

4. All things come from The Father (CCC:258)

$$\forall x : \text{from}_F(x)$$

5. All things are through The Son (CCC:258)

$$\forall x : \text{through}_S(x)$$

6. All things are in The Holy Spirit (CCC:258)

$$\forall x : \text{in}_H(x)$$

4 The Most Holy Trinity

God is the Father, the Son, and the Holy Spirit. The use of the word "is" doesn't follow the same transitive rules as $=$ in mathematics. There's another branch of math that deals with this, category theory. Category theory goes beyond the idea of "strict equality" and focuses on relationships

and mappings between objects. This distinction will be useful in trying to understand the Trinity.

Let $G = \text{God}$, $F = \text{The Father}$, $S = \text{The Son}$ and $H = \text{The Holy Spirit}$.
If "is" was the normal $=$ sign, then we could prove something false.

False reasoning we will avoid when we make the definition of "is" precise:

If Father = God and Son = God, then Father = Son.
(We will call this the transitivity error).

To avoid false arguments like the above, we have to define "Is" in a precise way.

4.1 Definitions of "Is":

4.1.1 Definition of "Divine Person is God"

We define the following functors:

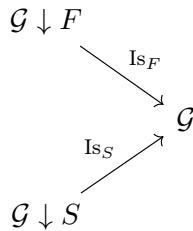
- $\text{Is}_F : \mathcal{G} \downarrow F \rightarrow \mathcal{G}$
- $\text{Is}_S : \mathcal{G} \downarrow S \rightarrow \mathcal{G}$
- $\text{Is}_H : \mathcal{G} \downarrow H \rightarrow \mathcal{G}$

The way we defined \mathcal{G} means that **the whole category** is God. The objects, like F are not equal to the whole, but they are equivalent in the above precise sense.

4.1.2 Proof that Is_x avoids the transitivity error

The Is_F functor asserts that the Father is equivalent to God.

The Is_S functor asserts that the Son is equivalent to God.



The diagram above says that "The Father is God" and that "The Son is God", but it does not say that "The Father is the Son".

The Father F is *consubstantial with* ($\stackrel{\dagger}{=}$) The Son S :

$$\stackrel{\dagger}{=}: F \rightarrow S$$

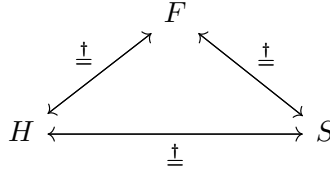
But the $\stackrel{\dagger}{=}: F \rightarrow S$ and $\text{Is}_S: \mathcal{G} \downarrow S \rightarrow \mathcal{G}$ arrows don't compose. Graphically, you cannot find a path from F to S that goes through \mathcal{G} , this is how the above definition avoids the transitivity error.

5 The Filioque and the Great Schism of 1054

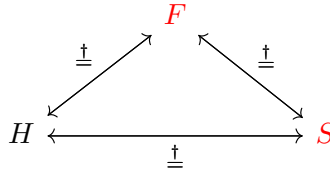
The **Filioque** (Latin for "and the Son") is a theological doctrine concerning the procession of the Holy Spirit. This doctrine became a major point of contention between the Eastern and Western churches, ultimately contributing to the Great Schism of 1054 that divided Christianity into Roman Catholicism and Eastern Orthodoxy.

5.1 The Western Position: Filioque

The Western Church (Roman Catholic) teaches that the Holy Spirit proceeds from both the Father and the Son. In our categorical framework, this can be represented as:



To emphasize the Filioque doctrine, we highlight the joint source of the Holy Spirit's procession:

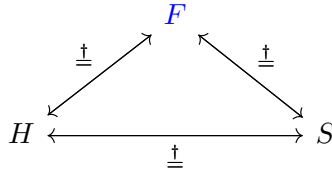


Both the Father (F) and the Son (S) are highlighted in red, indicating that according to Western theology, the Holy Spirit proceeds from both persons of the Trinity.

5.2 The Eastern Position: No Filioque

The Eastern Orthodox Church maintains that the Holy Spirit proceeds from the Father alone, though *through* the Son. This distinction preserves what they see as the Father's role as the sole source (Greek: *monarchia*) within the Trinity.

In categorical terms, the Eastern position would emphasize only the morphism from F to H :



Only the Father (F) is highlighted in blue, emphasizing the Eastern teaching that procession originates solely from the Father.

5.3 The Great Schism of 1054

The theological disagreement over the Filioque was one of several factors that led to the mutual excommunications between Pope Leo IX and Patriarch Michael I Cerularius in 1054. This event formally split the Christian Church into:

- The **Roman Catholic Church** (West): Affirming the Filioque
- The **Eastern Orthodox Church** (East): Rejecting the Filioque

From a categorical perspective, both churches agree on the fundamental structure of \mathcal{G} as a groupoid with objects $\{F, S, H\}$ and consubstantial relations. The disagreement lies in the *interpretation* of the arrows and their directional significance regarding procession.

The Western emphasis on the joint procession from Father and Son can be seen as highlighting the symmetric nature of the groupoid structure, while the Eastern emphasis on procession from the Father alone maintains a hierarchical reading of the morphisms that preserves the Father's unique role as *principium sine principio* (principle without principle).

6 Sources

6.1 BCT

Basic Category Theory by Tom Leinster

6.2 CTIC

Category Theory in Context by Emily Riehl

6.3 CCC

Catechism of the Catholic Church