Category Theory in Haskell

Tobi Lehman

September 8, 2024

Contents

1 Introduction

Category Theory is the mathematical study of other mathematical theories. I first really appreciated it while studying topology. Topology is hard. Group Theory is easier. A lot of topology is about simplifying topology problems by reducing them to group theory problems.

1.1 Motivating Example

Suppose you have a continuous function $f : X \to Y$ where X and Y are topological spaces. You want to know if X is homeomorphic to Y (topology-speak for "the same").

However, proving a homeomorphism can be really hard, since the spaces X and Y could have a uncountable (transfinite) number of points. The shape might be really weird. So topologists developed a way to compute a group called the fundamental group of a space. The fundamental group $\pi_1(X)$ has elements which are loops in the space X, the group operation is just following one loop and then following the other loop.

There's a very important theorem we proved about the fundamental group. Before we state it, let's look at this diagram:

```
\begin{CD}
  X @>{f}>> Y \\
  @V{\pi_1(-)}VV @V{\pi_1(-)}VV \\
```

```
  \Pi_1(X) @>>> \Pi_1(Y)
\end{CD}
```