ISYE 727 Homework 8

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a)

We first show that for each d such for which the infimum in (5.50) is finite, a solution x exists. Now we suppose that such a solution doesn't exist, then we have $< c^*, x >= +\infty$, which gives a contradiction. So there exists a feasible solution x to the system.

Furthermore, if the infimum is finite we can suppose that it is C. Then we pick a sequence $\{x_n\}_{n=1}^{\infty}$ such that $< c^*, x_n >$ approach C, then suppose that $\{x_n\} \to x^*$ since the feasible reigon is closed, we know that x^* is in the feasible region. And $< c^*, x^* >= C$.

b)

According to a) we know that there exists an $x_0 \geq 0$ such that $Dx_0 \geq d_0$, $Fx_0 = f$ and $< c^*, x_0 >$ achieve the infimum. That is to say $< c^*, x_0 >= c_0$.

According to Hoffman's theorem, there exists a constant $\gamma > 0$ and there exists an $x' \geq 0$ such that $Dx' \geq d$, $Fx' = f$ and $\|x' - x_0\| \leq \gamma \|(d - Dx_0)_+\|$. Here, don't need to talk about F, because $Fx_0 - f = 0$.

Notice that $d - Dx_0 \leq d - d_0$. So we have $\|x' - x_0\| \leq \gamma \|(d - Dx_0)_+\| \leq \gamma \|(d - d_0)_+\|$.

Thus, assume that x^* is the point satisfying the new demand d, then we have

$$< c^*, x^* > \leq < c^*, x' > = < c^*, x_0 > + < c^*, x' - x_0 > \leq c_0 + \|c^*\| \|x' - x_0\| \leq c_0 + \gamma \|c^*\| \|(d - d_0)_+\|.$$

We choose $\alpha = \gamma \|c^*\|$ and finish the proof.