CS 784 Homework 1 Tien-Lung Fu

A: Query Containment

1. (a) There is a homomorphism from q' to q: $y \to x$, $z \to z$, $t \to z$, $w \to y$
But there is no homomorphism from q to q'. Hence, $q \subseteq q'$

(b) There is a homomorphism from q' to q: $x \to x$, $y \to y$, $z \to z$, $u \to x$, $v \to y$
But there is no homomorphism q to q'. Hence, $q \subseteq q'$

(c) There is no homomorphism from q to q' nor q' to q. Hence, None

2. Given $q(x) :- R(x, z_1)$, we want to prove $S_K \equiv q$
$1°$ There is a homomorphism from S_K to q: $x \to x$, $z_n \to z_1$ $(n \in [1, K])$
Hence, $q \subseteq S_K$

$2°$ There is a homomorphism from q to S_K: $x \to x$, $z_1 \to z_1$
Hence, $S_K \subseteq q$

In conclusion, $S_K \equiv q$ and q is the minimal CQ since it only has one atom.

3. We want $q \subset q_2$ and $q \neq q_2$, q could only be the three cases:
$1°$ $q = R(x, y) R(z) \ldots$
$2°$ $q = R(x, y) R(z, 8) \ldots$
$\Rightarrow$ there is no homomorphism from q to q_1
$3°$ $q = R(x, y) R(y, t) \Rightarrow$ implying $q_1 = q$ which violates $q_1 \neq q$
Thus, such q does not exist.

4. Let $q_1 = q = R(x, 10)$, $x < 10$
$q_2 = q = R(x, 10)$, $x < 10$
$\Rightarrow q = q_1 \cup q_2$ ①
Let $q_1' = R(y, 10) R(9, y)$, $y < 10$
$q_2' = R(y, 10) R(9, y)$, $y = 10$
$\Rightarrow q' = q_1' \cup q_2'$ ②
There is a homomorphism from q_1 to q_1': $x \to y \Rightarrow q_1' \subseteq q_1$ ③
There is a homomorphism from q_2 to q_2': $x \to 9 \Rightarrow q_2' \subseteq q_2$ ④

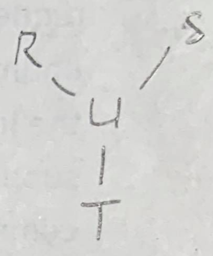
By ①, ②, ③, ④, $q' \subseteq q$

B. Query Complexity

1. (a) Given $|q|$ is the size of query, $|I|$ is the size of dataset.
&
(b) Since we can join the variables and atoms one by one $\Rightarrow$ $O(|q| \cdot |I|)$
the combined time complexity is polynomial

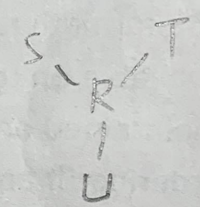
2. We use GYO to check

(a) choose ear R, remaining $\{\{y, t_2, z\}, \{z, t_3, x\}, \{x, y, z\}\}$
choose ear S, remaining $\{\{z, t_3, x\}, \{x, y, z\}\}$
choose ear T, remainig $\{\{x, y, z\}\}$
choose ear U, remaining $\{\}$ $\Rightarrow$ acyclic



(b) Not acyclic since we can not choose ears for reduction

(c) choose ear S, remaining $\{\{x, y, z\}, \{y, z\}, \{z, x\}\}$
choose ear T, remaining $\{\{x, y, z\}, \{z, x\}\}$
choose ear U, remaining $\{\{x, y, z\}\}$
choose ear R, remaining $\{\}$ $\Rightarrow$ acyclic

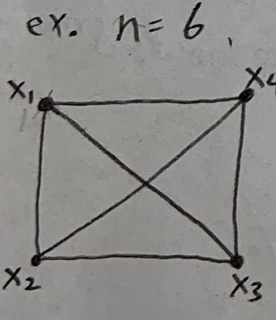


3.
(a) Chooce $\chi(v_i) = \{x, z_1, z_i, z_{i+1}\}$, $i \in [1, K]$ $\Rightarrow$ ghw $= 3$

(b) Choose $\chi(v_i) = \{x_1, x_i, x_{i+1}\}$ or $\{x_1, z_i, z_{i+1}\}$, $i \in [1, k]$ $\Rightarrow$ ghw $= 2$

4. When n atoms serve the edges of a complete graph

ex. $n = 6$, $q = R(x_1, x_2)\, R(x_1, x_3)\, R(x_1, x_4)\, R(x_2, x_3)\, R(x_2, x_4)\, R(x_3, x_4)$



then there is only one way to choose χ as $\chi(v) = \{x_1, x_2, x_3, x_4\}$
then ghw $=$ # vertices $- 1 = 4 - 1 = 3$

Generally, x vertices and n edges to form a complete graph

$$\frac{x(x-1)}{2} = n \Rightarrow x = \frac{1 + \sqrt{1+8n}}{2}$$

Thus, $\text{ghw} = \frac{1+\sqrt{1+8n}}{2} - 1$