

0.1 Exercise 1.2.15

First, we want to prove $ri(C \times D) \subseteq riC \times riD$

Let $x = (c, d) \in ri(C \times D)$

we have $\forall y = (\bar{c}, \bar{d}) \in C \times D, \exists \epsilon > 0$ s.t. $x + \epsilon(x - y) \in C \times D$ (Collollary 1.2.5 c)

$(c, d) + \epsilon((c, d) - (\bar{c}, \bar{d})) \in C \times D$

we have $c + \epsilon(c - \bar{c}) \in C$ and $d + \epsilon(d - \bar{d}) \in D$

then we have $c \in riC, d \in riD$ (Collollary 1.2.5 a)

and then $(c, d) \in riC \times riD$. (Q.E.D)

Second, we want to prove $riC \times riD \subseteq ri(C \times D)$

Let $x = (c, d) \in riC \times riD$, then $c \in riC, d \in riD$

$\exists \epsilon > 0, \forall \bar{c} \in C, s.t. c + \epsilon(c - \bar{c}) \in C$ and $\exists \epsilon > 0, \forall \bar{d} \in D, s.t. d + \epsilon(d - \bar{d}) \in D$ (Collollary 1.2.5 c) implying

$\forall y = (\bar{c}, \bar{d}), \exists \epsilon > 0$ s.t. $x + \epsilon(x - y) \in C \times D$

therefore, $x \in ri(C \times D)$ (Collollary 1.2.5 a) (Q.E.D)

0.2 Exercise 1.2.18

Let $C_n = [\frac{-1}{n^2}, 1]$, then $riC_n = (\frac{-1}{n^2}, 1)$, then $\bigcap_{n=1}^{\infty} riC_n = [0, 1)$

also, we have $\bigcap_{n=1}^{\infty} C_n = [0, 1]$, then $ri \bigcap_{n=1}^{\infty} C_n = (0, 1)$

$\bigcap_{n=1}^{\infty} riC_n \neq ri \bigcap_{n=1}^{\infty} C_n$

0.3 Exercise 1.2.20

Since C is subset of R^n , we have $\text{ri}clC = \text{ri}C$ (Proposition 1.2.6)

Since clC is affine, we have $\text{ri}clC = clC$

Hence, we have $clC = \text{ri}C$

Originally, we have $\text{ri}C \subseteq C \subseteq clC$

Hence, we have $C = clC$

Since clC is affine, C is also affine