CS727: Convex Analysis	**Scribe:** Tien-Lung Fu
Title: hw1	

0.1 Exercise 1.1.24

Let S be $\{(x, y)|y \geq \frac{1}{1+x^2}\}$ which is a close set. S has its convex hull convS: $\{(x, y)|y > 0\}$.
Let (x,y) $\in$ convS be given. We want to show that the open ball B((x,y),y) is contained entirely within convS.

$$\begin{aligned} Given\ (a, b) \in B((x, y), y),\ we\ have\ (x-a)^2 + (y-b)^2 &< y^2 \\ => (y-b)^2 &< y^2 \\ => b(b-2y) &< 0 \\ => 0 < b &< 2y \\ => (a, b) &\in convS \end{aligned}$$

So convS is open in R^2

0.2 Exercise 1.1.25

Part1:
Let w, x $\in$ C and y, z $\in$ D. Given a = μw + vx, b = μy + vz, and $0 \leq t \leq 1$. We can derive:

$$\begin{aligned} ta + (1-t)b &= t(\mu w + vx) + (1-t)(\mu y + vz) \\ &= (t(\mu w) + (1-t)(\mu y)) + (t(vx) + (1-t)(vz)) \\ &\in \mu C + vD \end{aligned}$$

So $\mu C + vD$ is convex.

Part2:
First, when μ=v=0, the equality holds.
Second, when at least one of $\mu, v \neq 0$, we want to show $(\mu + v)C \subset \mu C + vC$

$$\begin{aligned} \forall x \in (\mu + v)C,\ we\ have\ \frac{x}{\mu + v} &\in C \\ => x = \frac{\mu x}{\mu + v} + \frac{vx}{\mu + v} &\in \mu C + vC \\ => (\mu + v)C &\subset \mu C + vC \end{aligned}$$

Third,

$$\forall x \in \mu C + vC, \exists c_1, c_2 \in C \ s.t.\ x = \mu c_1 + vc_2.$$
$$Since\ C\ is\ convex\ and\ \mu, v\ are\ nonnegative(\mu + v \geq 0)$$
$$=> \ \frac{\mu c_1}{\mu + v} + \frac{vc_2}{\mu + v} \in C$$
$$=> \ x = \mu c_1 + vc_2 \in (\mu + v)C$$
$$=> \ \mu C + vC \subset (\mu + v)C$$

Hence, we complete the proof.

Part3:

$$Let\ C = \{0, 1\} \subset R^1\ and\ a, b\ are\ constant$$
$$We\ have\ aC = \{0, a\}\ and\ bC = \{0, b\}$$
$$=> \ (a + b)C = \{0, a + b\}\ and\ aC + bC = \{0, a, b, a + b\}$$
$$=> \ (a + b)C \neq aC + bC$$

0.3 Exercise 1.1.27

First, we want to show $E(ConvU) \subset ConvE(U)$

$$\forall u \in ConvU,\ we\ have\ u = \sum t_j u_j\ where\ u_j \in U,\ t_j \geq 0,\ and\ \sum_j t_j = 1$$
$$Since\ E\ is\ an\ affine\ transformation,\ we\ have$$
$$E(\sum_j t_j u_j) = \sum_j t_j E(u_j) \in ConvE(U)$$

Second, we want to show $ConvE(U) \subset E(ConvU)$

$$An\ affine\ transformation\ is\ a\ map\ of\ the\ form\ E(u) = b + A(u)$$
$$where\ b\ is\ some\ fixed\ vector\ and\ A\ is\ an\ invertible\ linear\ transformation.$$
$$a\ point\ in\ ConvE(U)\ has\ the\ form\ \sum_j t_j E(u_j)$$
$$\sum_j t_j E(u_j) = \sum_j t_j (Au_j + b) = A(\sum_j t_j u_j) + \sum_j t_j b = A(\sum_j t_j u_j) + b$$
$$= E(\sum_j t_j u_j)$$

Hence, we complete the proof.