CS727: Convex Analysis	**Scribe:** Tien-Lung Fu
Title: hw6	

0.1 Exercise 4.3.7

First, we want to show for any convex set C, if $X \subset C$ and the set of recession directions of C contains Y, then $convX + posY \subset C$

$$\forall z \in convX + posY,\ \forall x \in convX,\ \forall y \in posY,\ we\ have\ z = x + y$$
$$C\ is\ convex\ and\ X \subset C\ \ => \ convX \subset C$$
$$=> \ we\ have\ x \in C\ and\ y = \sum_{i=1}^{n} \alpha_i y_i, \forall \alpha_i \geqslant 0\ and\ y_i \geqslant 0\ since\ y \in posY$$
$$Since\ set\ of\ recession\ directions\ of\ C\ contains\ Y\ and\ x \in X \subset C,$$
$$we\ have\ x + \alpha_1 y_1 \in C\ \ => \ (x + \alpha_1 y_1) + \alpha_2 y_2 \in C$$
$$By\ induction,\ x + \sum_{i=1}^{n} \alpha_i y_i = z \in C$$
$$Thus,\ we\ have\ convX + posY \subset C$$

Secondly, we want to show $X \subset convX + posY$ and the recession directions of convX + posY contains Y

$$Since\ X \subset convX\ and\ posY\ contains\ \{0\}\ \ => \ X \subset convX + posY$$
$$\forall z \in convX + posY,\ z = x + \sum_{i=1}^{n} \alpha_i y_i,\ \forall x \in convX,\ \forall \alpha_i \geqslant 0,\ and\ \forall y_i \in Y$$
$$\forall \alpha \geqslant 0,\ \forall y \in Y,\ z + \alpha y = x + \sum_{i=1}^{n} \alpha_i y_i + \alpha y \in convX + posY$$
$$=> \ the\ recession\ directions\ of\ convX + posY\ contains\ Y$$

Thus, we conclude that convX + posY is the smallest among all convex sets C that contains X and the set of recession directions of C contains Y since convX + posY is always contained by other such C.

0.2 Exercise 4.3.8

We want to find coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2$ s.t.
$\alpha_1(-1,-1) + \alpha_2(1,-1) + \beta_1(-1,1) + \beta_2(1,1) = (0,0)$,
$0 \leqslant \alpha_1, \alpha_2 \leqslant 1,\ \alpha_1 + \alpha_2 = 1,\ \beta_1 \geqslant 0, \beta_2 \geqslant 0$
$(\alpha_1, \alpha_2, \beta_1, \beta_2)\ could\ be\ (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})\ or\ (1,0,0,1)$
$Thus,\ the\ representation\ need\ not\ be\ unique$