**CS727: Convex Analysis** **Scribe:** Tien-Lung Fu

**Title:** hw8

## 0.1 Exercise 5.4.19

(a)
First, suppose that such a solution does not exist, then we have $< c^*, x >= +\infty$, which contradicts the description of the question. Thus, there must exist a feasible solution to the system.
Secondly, if the infimum is finite and is C. We can pick a sequence $\{x_n\}_{n=1}^{\infty}$ s.t. $\{x_n\} \rightarrow x^*$ and $< c^*, x_n >$ approach C. We know that $x^*$ is in the feasible region since the feasible region is closed.
We have $< c^*, x^* >= C$

(b)
According to (a), we know that there exists a $x_0 \geqslant 0$ s.t. $Dx_0 \geqslant d_0$, $Fx_0 = f$, and $< c^*, x_0 >$ approach the infimum $c_0$.
According to Hoffman's theorem, there exists a constant $\gamma > 0$ and there exists a $x' \geqslant 0$ s.t. $Dx' \geqslant d$, $Fx' = f$, and $||x' - x_0|| \leqslant \gamma||(d - Dx_0)_+||$. (Notice that $Fx_0 - f = 0$)
Since $d - Dx_0 \leqslant d - d_0$, we have $||x' - x_0|| \leqslant \gamma||(d - Dx_0)_+|| \leqslant \gamma||(d - d_0)_+||$
Assuming that $x^*$ is the point that satisfies the demand d, we have:

$$\begin{aligned}
< c^*, x^* > &\leqslant < c^*, x' > \\
&= < c^*, x_0 > + < c^*, x' - x_0 > \\
&\leqslant c_0 + ||c^*||||x' - x_0|| \\
&\leqslant c_0 + \gamma||c^*||(d - d_0)_+||
\end{aligned}$$

We choose $\alpha = \gamma||c^*||$ to complete the proof.