## Parallel join algorithms

Join q(x,y,z): -R(x,y), S(y,z) where R has  $N_R$  tuples and S has  $N_S$  tuples Parallel hash join: partition both partitions per join variable. Requires no skew in the hash function

for load  $\frac{N_R}{p} + \frac{N_S}{p} = O\left(\frac{N}{p}\right)$ 

Broadcast join: broadcast smallest relation, partition the larger using both attributes. Load  $N_R + \frac{N_S}{p}$ .

Requires  $N_R \ll N_S$  for  $O\left(\frac{N}{p}\right)$ 

Cartesian product: general (applies to  $\theta$  joins). Reshape the p machines into  $p_x \times p_z$ .

Hash the values of x and y into  $\{1, \dots, p_x\}$  and  $\{1, \dots, p_y\}$  respectively; distribute to machine  $p(x, p_j)$  he

tuples hashed with 
$$p(p_i, p_j)$$
 and  $\frac{N_R}{p_x} + \frac{N_S}{p_z}$ . Then  $p_x = \sqrt{p \frac{N_R}{N_S}}$ ,  $p_z = \sqrt{p \frac{N_S}{N_R}}$ , thus load is  $2\sqrt{\frac{N_R N_S}{p}}$ . If  $N_R = \sqrt{p \frac{N_S}{N_S}}$ .

 $N_S$ , then load is  $\frac{N}{\sqrt{p}}$ . In the limit where  $N_R \ll N_S$ , then  $p_\chi = 1$ , and we achieve the broadcast join

algorithm. Therefore the load is 
$$O\left(\max\left\{\frac{N_R}{p},\frac{N_S}{p},\sqrt{\frac{N_RN_S}{p}}\right\}\right)$$

Generalises to multiple dimensions. Can be used for cyclic queries, e.g. Triangle query:

$$\Delta(x, y, z) : -R(x, y), S(y, z), T(z, x)$$

$$L = \frac{N_R}{p_x p_y} + \frac{N_S}{p_y p_z} + \frac{N_T}{p_x p_z} = O\left(\frac{N}{\frac{2}{p^3}}\right)$$

Join algorithms with skew

Heavy hitter: value of a join variable y that appears more than  $\frac{N}{n}$  times

Skew join algorithm:

For light hitter values of y: use parallel hash join algorithm

For every heavy hitter value a of y with frequencies  $d_{a,R}$ ,  $d_{a,S}$ : use Cartesian product algorithm with  $p_a$  machines, where  $\sum_a p_a = p$  and  $p_a$  is proportional to  $d_{a,R}$ ,  $d_{a,S}$ 

$$L = \max\left\{\sqrt{\frac{\sum_{a} d_{a,R} d_{a,S}}{p}}, \frac{N_R}{p}, \frac{N_S}{p}\right\} = \max\left\{\sqrt{\frac{|R \bowtie S|}{p}}, \frac{N_R}{p}, \frac{N_S}{p}\right\}$$