By def 1.1.18 we know: if X non-empty, $\operatorname{conv}(X,Y) := \operatorname{conv}X + \operatorname{pos}Y$ is convex.

first we show that, for any convex set C, if $X \subset C$, set of recession directions of C contains Y, then $\operatorname{conv}X + \operatorname{pos}Y \subset C$

$\forall z \in \operatorname{conv}X + \operatorname{pos}Y$, $\Leftrightarrow$ $z = x + y$ where $x \in \operatorname{conv}X$, $y \in \operatorname{pos}Y$. and C is convex. $X \subset C$

$\Rightarrow \operatorname{conv}X \subset C$.

$x \in C$ and $y \in \operatorname{pos}Y \Rightarrow y = \sum_1^n \alpha_i y_i$ $\alpha_i \geq 0$, $y_i \in Y$.

since set of recession direction of C contains Y,

for $\forall x \in X \subset C$, we have $x + \alpha_1 y_1 \in C \Rightarrow x + \alpha_1 y_1 + \alpha_2 y_2 \in C$ (by $\alpha_1 \geq 0, y_1 \in Y$)

by induction: $x + \sum_1^n \alpha_i y_i \in C$.

$\Rightarrow x + \sum_1^n \alpha_i y_i \in C \Rightarrow z \in C$

so $\operatorname{Conv}X + \operatorname{Pos}Y \subseteq C$

and then we show: $X \subset \operatorname{conv}X + \operatorname{Pos}Y$, and the recession direction of $\operatorname{Conv}X + \operatorname{Pos}Y$ contains Y.

first statement is trivial since $X \subset \operatorname{conv}X$. Second statement we have: $\forall z \in \operatorname{conv}X + \operatorname{Pos}Y$, $z = x + \sum_1^n \alpha_i y_i$

for some $x \in \text{conv} X$, some n, $\alpha_i \geq 0$ $y_i \in Y$

$\forall \alpha > 0$, $y \in Y$, $z = x + \sum_i^n \alpha_i y_i + \alpha y \in \text{conv} X + \text{Pos} Y$

since $\sum_i^n \alpha_i y_i + \alpha y \in \text{Pos} Y$. $\Rightarrow$ the recession direction of $\text{conv} X + \text{Pos} Y$ contains Y.

So $\text{Conv} X + \text{Pos} Y$ is the smallest convex set C that contains X and recession direction of C contains X

because every such C contains $\text{Conv} X + \text{Pos} Y$.,
and $\text{Conv} X + \text{Pos} Y$ is one of such C. so it's the - smallest