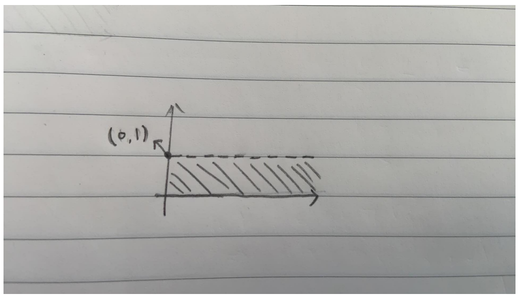
CS727: Convex Analysis	**Scribe:** Tien-Lung Fu
Title: hw5	

0.1 Exercise 4.1.23



Consider the convex set C without top boundary line and (0,1) is included.
Given d = $(d_1, d_2) \in rcC$, we want to discuss the property of d:
1.

$$d_2 \; should \; be \; 0, \; sicne \; C \; is \; bounded \; at \; top \; and \; bottom$$

2.

$$d_1 \; should \; be \; \geqslant 0, \; sicne \; C \; is \; bounded \; at \; left \; side$$

3.

$$d_1 > 0 \; does \; not \; hold \; at \; (0,1) \; since \; C \; is \; without \; top \; boundary \; line$$

Thus, the only possible situation is rcC = $\{(d_1, d_2)\} = \{(0,0)\}$

0.2 Exercise 4.1.24

We want to show $rc\prod_{i=1}^n C_i \subset \prod_{i=1}^n rcC_i$

$$\forall d \in rc\prod_{i=1}^n C_i, \; we \; have \; \forall x \in \prod_{i=1}^n C_i, \; (x + \lambda d) \in \prod_{i=1}^n C_i, \; \forall \lambda \geqslant 0$$

$$=> \; (x_1 + \lambda d_1, ..., x_n + \lambda d_n) \in \prod_{i=1}^n C_i$$

$$=> \; (d_1, ..., d_n) \in \prod_{i=1}^n rcC_i$$

$$=> \; d \in \prod_{i=1}^n rcC_i$$

We want to show $\prod_{i=1}^{n} rcC_i \subset rc\prod_{i=1}^{n} C_i$

$$\forall d = (d_1, ..., d_n) \in \prod_{i=1}^{n} rcC_i$$

$$We\ have\ (x_1 + \lambda d_1, ..., x_n + \lambda d_n) \in \prod_{i=1}^{n} C_i,\ \ \forall \lambda \geqslant 0$$

$$=>\ \ (d_1, ..., d_n) \in rc\prod_{i=1}^{n} C_i$$

$$=>\ \ d \in rc\prod_{i=1}^{n} C_i$$

0.3 Exercise 4.1.25

$$\begin{aligned} rcP &= \{z|A(x+\lambda z) = a, D(x+\lambda z) \leqslant d, \forall \lambda \geqslant 0, \forall x \in P\} \\ &= \{z|Ax + \lambda Az = a, Dx + \lambda Dz \leqslant d, \forall \lambda \geqslant 0, \forall x \in P\} \\ &= \{z|Az = 0, Dz \leqslant 0\} \end{aligned}$$

$$\begin{aligned} linP &= rcP \cap (-rcP)\ (By\ Proposition\ 4.1.6) \\ &= \{z|Az = 0, Dz \leqslant 0\} \cap \{z|Az = 0, Dz \geqslant 0\} \\ &= \{z|Az = 0, Dz = 0\} \end{aligned}$$