MATH 200, Fall Semester, 2017 - Final - Gregg

Name:______________________________ Score:______________

Instructions: Write answers carefully and in complete sentences, making sure that all work and answers are legible. Be sure to sign the honor code on the last page.

(1) (3 points) Give the definition of the **intersection of sets X and Y**.

(2) (6 points) Let $A = \{n \in \mathbb{N} \mid n < 8\}$, $B = \{n \in \mathbb{N} \mid n > 4\}$, and $C = \{n \in \mathbb{N} \mid 4 \leq n \leq 10\}$. For the universal set, take $U = \mathbb{N}$. Complete the following:

(a) $A \cup B =$ ____________________

(b) $A \cap B =$ ____________________

(c) $A \setminus B =$ ____________________

(d) $B^C =$ ____________________

(e) $|\mathcal{P}(A)| =$ ______________.

(f) $A \times C$ contains ______________ ordered pairs.

(3) (10 points) Show that if $A \cap B = A$, then $B \subseteq A$.

(4) (3 points) Give the definition of a **function**.

(5) (6 points) Each set of ordered pairs is a relation on $\mathbb{N}^2$. **(A)** Match each with its description. Then **(B)** create an item (e) for the description in the right-hand column which is left blank.

(a) $\{(m, n) \in \mathbb{N}^2 \mid n \leq m\}$

(b) $\{(m, n) \in \mathbb{N}^2 \mid n - m = 5\}$

(c) $\{(m, n) \in \mathbb{N}^2 \mid n = |m - 5|\}$

(d) $\{(m, n) \in \mathbb{N}^2 \mid n = m\}$

(e)

(i) _____ a relation which is not a function

(ii) _____ a function which is one-to-one and onto $\mathbb{N}$

(iii) _____ a function which is neither one-to-one nor onto $\mathbb{N}$

(iv) _____ a function which is onto $\mathbb{N}$, but not one-to-one $\mathbb{N}$

(v) _____ a function which is one-to-one, but not onto

(6) (3 points) What are the domain and range of the relation $R = \{(m, n) \in \mathbb{N}^2 \mid m < 4, n = m^2 + 3\}$

(7) (10 points) Prove that the sets $S = \{5, 6, 7, \ldots\}$ and $T = \{7, 14, 21, 28, \ldots\}$ have the same cardinality.

(8) (3 points) Give the definition of the **converse** of a relation R.

(9) (3 points) Complete the following definition: A set S is **countably infinite** if

(10) (3 points) Give an example of a set which is

(a) finite: __________________________

(b) countably infinite: __________________________

(c) uncountably infinite: __________________________

(11) (6 points) Recall that we defined a relation $\sim$ on the set $\mathbb{N}^2$ by $(m, n) \sim (p, q)$ if and only if $m + q = n + p$. Complete the following:

(a) $(7, 13) \sim (13, \underline{\hspace{2cm}})$

(b) $[(1, 4)] \times [(2, 8)] = \underline{\hspace{3cm}}$

(c) The $\sim$ equivalence class $\underline{\hspace{3cm}}$ acts as the additive identity element.

(12) (3 points) Give the definition of an **equivalence relation**.

(13) (3 points) Consider the relation $R = \{(a, b) \in \mathbb{N}^2 \mid a \leq b\}$. Complete the following by filling in the blanks:

R is **not** an equivalence relation because, while R is ______________________ and ______________________, it fails to be ______________________ because ______________________

(14) (10 points) Recall that we defined a relation $\sim$ on the set $\mathbb{N}^2$ by $(m, n) \sim (p, q)$ if and only if $m + q = n + p$. Prove that $\sim$ is an equivalence relation.

(15) (3 points) Give the definition of a **group**.

(16) (3 points) Give an example of a set which is a semi-group, but not a group. Explain briefly why it fails to be a group.

(17) (3 points) Recall that we defined an equivalence relation $\approx$ on $\mathbb{Z} \times \mathbb{Z} \setminus \{0\} =: \mathbb{Z}^{II}$ by $(m, n) \approx (p, q)$ if and only if $mq = np$. We subsequently defined addition of these $\approx$ equivalence classes by $[(m, n)] + [(p, q)] = [(mq + np, nq)]$. Complete the following:

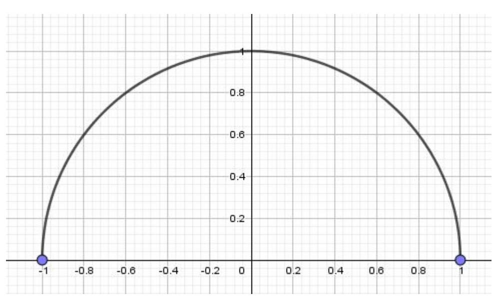
(a) $(a, a + 2) = (3a, \underline{\hspace{2cm}})$

(b) $[(2, 3)] + \underline{\hspace{3cm}} = [(1, 1)]$

(c) $[(3, 6)] < [(9, \underline{\hspace{2cm}})]$

(18) (10 points) Prove (a) there is an identity element for addition on the $\approx$ equivalence classes, and that (b) each $\approx$ equivalence class has an additive inverse.

(19) (6 points) Sketch the line through the point $(-1, 0)$ which has slope $m = \dfrac{1}{5}$. Find the equation of the line and the point of intersection of the line with the unit circle which lies in the first quadrant.



(20) (6 points) Use the interval bisection to find an approximation of $\sqrt{10}$ with error less than $\dfrac{1}{8}$.

(21) (6 points) Calculate the first two approximations of $\sqrt{10}$ by Newton's method. That is, let $x_0 = 4$, then compute x_1 and x_2 by setting $x_{n+1} = x_n - f(x_n)/f'(x_n)$. Is this approximation more or less accurate than the one you found in Problem 20.