Foundations Homework 7

TJ Liggett

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Chapter Three

Exercise 1 *Let the universal set* $U = \{1, 2, 3, \ldots, 14, 15\}$*. Define* $S = \{1, 2, \ldots, 10\}$*,* $A = \{2, 4, 6, 8, 10\}$*,* $B = \{1, 3, 5, 7, 9\}$*,* $C = \{3, 6, 9\}$*,* $D = \{8, 9, 10, 11, 12\}$*. Define each of the following:*

(a) $A \cup B = \{x | x \varepsilon \mathbb{N}, x < 11\}$

(b) $B \cup C = \{1, 3, 5, 6, 7, 9\}$

(c) $A \cap B = \varnothing$

(d) $B \cap C = \{3, 9\}$

(e) $S \setminus A = \{1, 3, 5, 7, 9\}$

(f) $A \setminus B = A$

(g) $B \setminus D = \{1, 3, 5, 7\}$

(h) $D^C = \{1, 2, 3, 4, 5, 6, 7, 13, 14, 15\}$

(i) $S^C = \{11, 12, 13, 14, 15\}$

Assignment 2 *Prove each of the following:*

(a) $A \cup (B \cup C) = (A \cup B) \cup C$:
Let $x \varepsilon A \cup (B \cup C)$. Then x is an element of either set A or the union of set B and C. Hence, by definition of set unions, x is an element of set A, B, or C. It follows that x is an element of either the union of set A and B or set C. Thus, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$. Similarly, let $x \varepsilon (A \cup B) \cup C$. Then x is an element of either the union of set A and B or set C, and by set unions, x is an element of A, B, or C. Thus x is an element of either set A or the union of set B and C, and $(A \cup B) \cup C \subseteq A \cup (B \cup C)$. Since both $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ and $(A \cup B) \cup C \subseteq A \cup (B \cup C)$, $A \cup (B \cup C) = (A \cup B) \cup C$.

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$:

Let $x \varepsilon A \cup (B \cap C)$. Then either x is an element of A or both B and C. Assume x is an element of A. Then x is an element in both $A \cup B$ and $A \cup C$, and thus $(A \cup B) \cap (A \cup C)$. Likewise, if x is and element of both B and C, x is an element in both $A \cup B$ and $A \cup C$, and thus $(A \cup B) \cap (A \cup C)$. Hence, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Likewise, let $x \varepsilon (A \cup B) \cap (A \cup C)$. Then x is an element of both $A \cup B$ and $A \cup C$, and x is either in A or not. If $x \varepsilon A$, then it is trivial that $x \varepsilon A \cup (B \cap C)$. If not, then x must be in both B and C, and thus $x \varepsilon A \cup (B \cap C)$. Then $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(c) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$:

Let $x \varepsilon A \cap (B \setminus C)$. Then x is in both set A and $(B \setminus C)$. Thus $x \varepsilon A, B$ but not $A \cap C$. Hence $x \varepsilon (A \cap B) \setminus (A \cap C)$, and $A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$. Likewise, let $x \varepsilon (A \cap B) \setminus (A \cap C)$. Then $x \varepsilon A, B$, but x is not in $(A \cap C)$, and thus not in C. It follows that $x \varepsilon A \cap (B \setminus C)$. Thus, $(A \cap B) \setminus (A \cap C) \subseteq A \cap (B \setminus C)$. Therefore, $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$.

(d) $(A \setminus B) \cup B = A$ if and only if $B \subseteq A$:

Let $(A \setminus B) \cup B = A$, $x \varepsilon B$. Then $x \varepsilon (A \setminus B) \cup B$, and thus $x \varepsilon A$. Hence, if $(A \setminus B) \cup B = A$, then $B \subseteq A$.

Let $B \subseteq A$, $x \varepsilon (A \setminus B) \cup B$. Then either $x \varepsilon (A \setminus B)$ or $x \varepsilon B$. Since $B \subseteq A$, in either case $x \varepsilon A$, and thus $(A \setminus B) \cup B \subseteq A$. Now let $x \varepsilon A$. Since $B \subseteq A$, then either x is in B or not. If $x \varepsilon B$, then it is trivial that $x \varepsilon (A \setminus B) \cup B$. If not, by definition $x \varepsilon (A \setminus B)$, and $x \varepsilon (A \setminus B) \cup B$. Hence, $A \subseteq (A \setminus B) \cup B$. Thus if $B \subseteq A$, then $(A \setminus B) \cup B = A$.

Therefore, $(A \setminus B) \cup B = A$ if and only if $B \subseteq A$.