

COSC 330 Homework 2*Homework 0.4*

1. Is the following Bare Bones program self-terminating? Explain your answer.

```
incr X           // X++  
decr Y           // Y--
```

This program is self-terminating. A Bare Bones program will self-terminate without any while loops. This program will increment X by one and decrement Y by one, assuming X and Y have been initialized, and then terminate.

2. Is the following Bare Bones program self-terminating? Explain your answer.

```
Y=X  
incr Y  
incr Y           // Y = X + 2  
while X not 0:  
    decr X  
    decr X  
    decr Y  
  
    decr Y  
decr Y           // Y = At end of loop, Y will be 2 if X was even, 1 if Y was odd  
while Y not 0:   // If X was even, Y = 1 and loop will run forever. Otherwise program terminates
```

If x is odd, this program will terminate. If x is even, this program will not terminate. $Y = X + 2$ at the beginning of the first while loop. The first loop will terminate when $X = 0$, and Y will be decremented each time X is. However, if X is odd, there will be an extra decrement of Y , leaving Y to be 1 in this case. In this case, Y will be decremented again and $Y = 0$ for the second while loop, allowing the program to terminate.

In the case X is even, Y will be equal to 2 at the end of the first loop, and thus equal to 1 at the beginning of the second loop, and the program will not terminate.

Homework 0.5

1. Suppose a problem can be solved by an algorithm in $O(n^2)$ as well as another algorithm in $O(2^n)$.

Will one algorithm always outperform the other?

One algorithm will not always outperform the other. With small values of n , an algorithm in $O(2^n)$

can outperform an algorithm in $O(n^2)$. However, for some large n , eventually the $O(n^2)$

algorithm will outperform the $O(2^n)$ algorithm for all values greater than n .

2. Give an example of a polynomial problem. Give an example of a nonpolynomial problem. Give an example of an NP problem that has yet has not been shown to be a polynomial problem.

An example of a polynomial problem is sorting an integer array. For example, if we use an integer array using quick sort the time complexity is $O(n \log n)$ (radix sort could perform linearly but is worse in practice).

An example of an NP problem is the Knapsack Problem: Given a set of items, each with weight and value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible. The decision problem here is NP-complete. That is, can a value of at least V be achieved without exceeding the weight W .

3. If the time complexity of algorithm X is greater than that of algorithm Y , is algorithm X necessarily harder to understand than algorithm Y ? Explain your answer.

Not necessarily. The understandability of a program is its intellectual complexity. Time complexity and intellectual complexity are related, but not the same. Understandability by humans is viewed differently than complexity from a machine's point of view. It is possible that a program with a greater time complexity than another has a lower time complexity. For example, binary search and quick search can both be understood at the same complexity for a human, but their time complexity is not the same.

Homework 0.6

1. Find the factors of 66,043.

$$66043 = 1 * ____ * ____ * ____ * \dots$$

$$66043 = 1 * 66043$$

$$66043 = 211 * 313$$

2. Using the public keys $n = 91$ and $e = 5$, encrypt the binary bit string 101. Give your answer in the form of a binary bit string.

Encrypt the binary bit string 101

101 binary \rightarrow 5 decimal

$$5^5 \% 91 = 3125 \% 91 = 31$$

31 decimal \rightarrow 11111 binary

3. Using the private keys $n = 91$ and $d = 29$, decrypt the binary bit string 10. Give your answer in the form of a binary bit string.

Decrypt the binary bit string 10

10 binary \Rightarrow 2 decimal

$$2^{29} \% 91 = 32$$

32 decimal \Rightarrow 100000

4. Find the appropriate value for the decrypting keys n and d in an RSA public-key cryptography system based on the primes $p = 7$ and $q = 19$ and the encryption key $e = 5$.

$$n = pq = 133$$

$$ed = k(p-1)(q-1) + 1 \text{ for some } k$$

$$5d = k(6)(18) + 1$$

$$d = \frac{108k+1}{5}$$

$$d = 65 \text{ when } k = 3$$

5. Write a program to read in an integer (long) and display all pairs of factors of the input integer.

That is, the output must be given as a list of pairs of factors such that the product of each pair is the input integer. Then, print which of these pairs contains only prime numbers (there can be either 1 such pair or none). Note that

1. the Scanner class supports a nextLong method to read a long integer, nextBigInteger to read a BigInteger, and nextBigDecimal to read a BigDecimal.
2. the printf method uses %d for int, long, BigInteger; and %f for float, double, and BigDecimal.