MATH 345 Exam 2

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1 Problems

11. Let X be a topological space, let Y be a subset of X, and let A be a subset of Y. If A is closed in Y and Y closed in X, show that A is closed in X.

Proof. Let X be a topological space, let Y be a subset of X, and let A be a subset of Y. Assume A is closed in Y and Y is closed in X. Since A is closed in the subspace topology on Y, $A = D \cap Y$ for some closed set D in X, by Theorem 3.4. Because D and Y are both closed sets in X, it follows by Theorem 1.17 that the intersection of closed sets $D \cap Y$ is closed in X. Thus, A is closed in X. Therefore, if A is closed in Y and Y closed in X, then A is closed in X.