

# Stack Implementations

Tiziana Ligorio

[tligorio@hunter.cuny.edu](mailto:tligorio@hunter.cuny.edu)

# Today's Plan

Announcements

Recap

Stack Implementations:

Array

Vector

Linked Chain



# Announcements and Syllabus Check

Queens College Hackathon

# Stack ADT

```
#ifndef STACK_H_
#define STACK_H_

template<class ItemType>
class Stack
{
public:
    Stack();
    void push(const ItemType& newEntry); // adds an element to top of stack
    void pop(); // removes element from top of stack
    ItemType top() const; // returns a copy of element at top of stack
    int size() const; // returns the number of elements in the stack
    bool isEmpty() const; // returns true if no elements on stack false otherwise

private:
    //implementation details here

}; //end Stack

#include "Stack.cpp"
#endif // STACK_H_`
```

# Choose a Data Structure

Array?

Vector?

Linked chain?

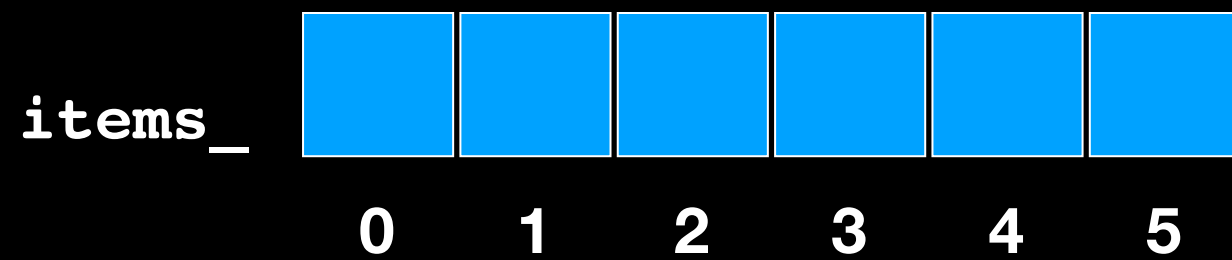
# Choose a Data Structure

Inserting and removing from same end (**LIFO**)

Goal: minimize work (operations)

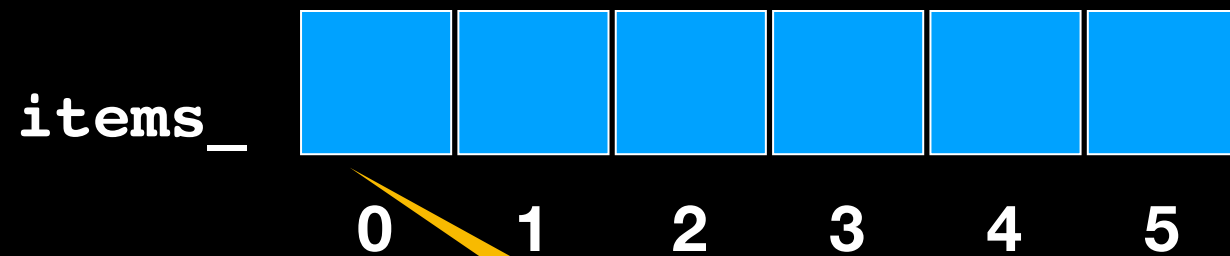
What would you suggest?

# Array



Where is the top  
of the stack?

# Array



```
item_count_ = 0
```

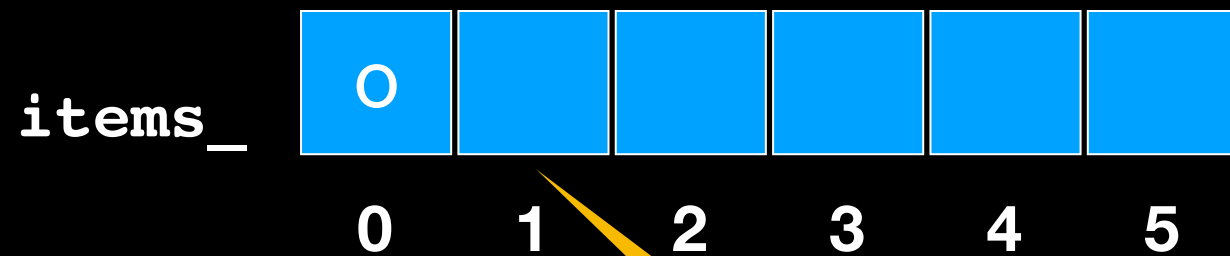
```
max_items_ = 6
```

Top of the stack:  
`items_[item_count_]`



# Array

`push('0')`



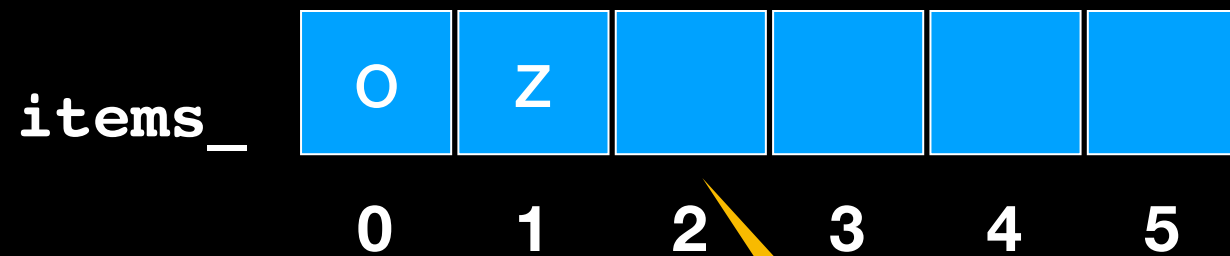
`item_count_ = 1`

`max_items_ = 6`

Top of the stack:  
`items_[item_count_]`

# Array

`push( 'Z' )`



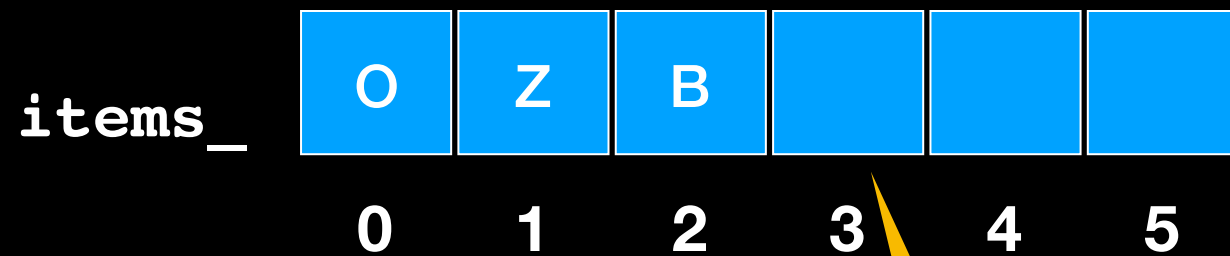
`item_count_ = 2`

`max_items_ = 6`

Top of the stack:  
`items_[item_count_]`

# Array

`push( 'B' )`

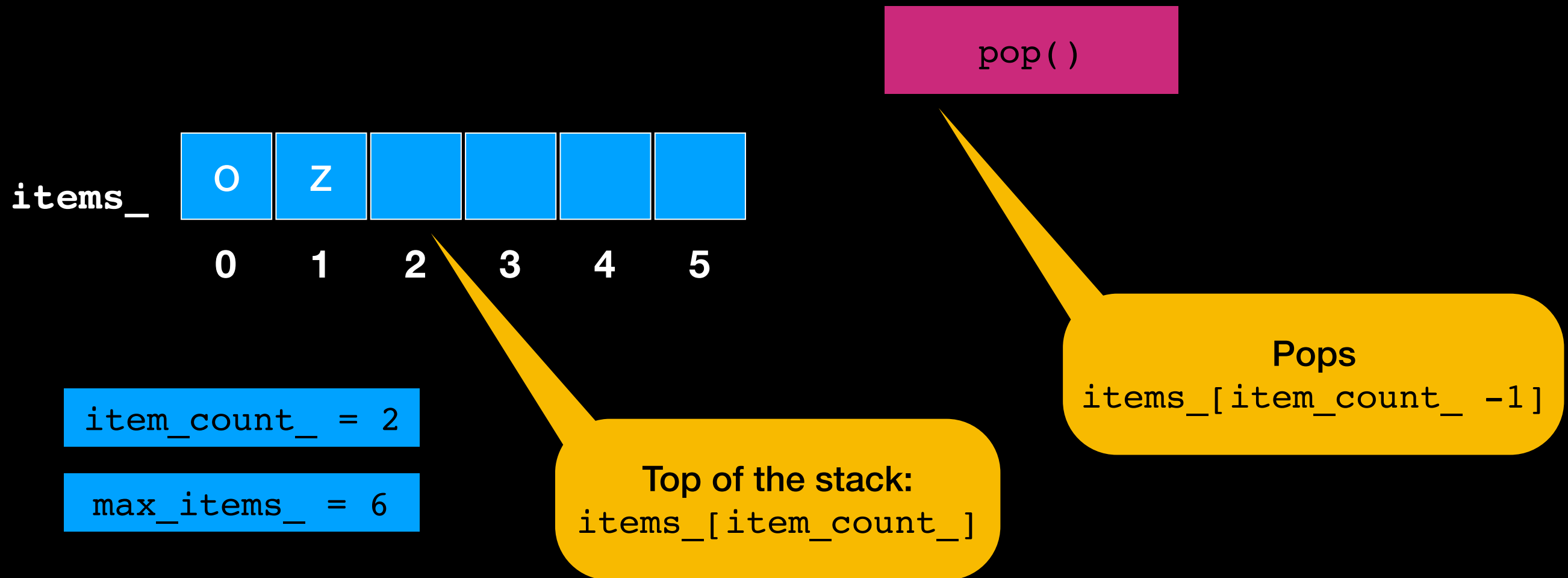


`item_count_ = 3`

`max_items_ = 6`

Top of the stack:  
`items_[item_count_]`

# Array



# Array Analysis

1 assignment + 1 increment/decrement = 1 "step"

*size* : 1 "step"

*isEmpty*: 1 "step"

*push*: 1 "step"

*pop* : 1 "step"

*top* : 1 "step"

Fixed amount of work

GREAT!!!!

# Array Analysis

1 assignment + 1 increment/decrement = 1 "step"

*size* : 1 "step"

*isEmpty*: 1 "step"

*push*: 1 "step"

*pop* : 1 "step"

*top* : 1 "step"

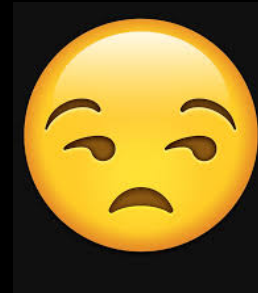
Fixed amount of work

GREAT???

# Array

`push( 'T' )`

<code>items_</code>	O	Z	B	Y	L	P
	0	1	2	3	4	5



Sorry Stack is Full!!!

`item_count_ = 6`

`max_items_ = 6`

Top of the stack:  
`items_[item_count_]`

# Vector

```
std::vector<ItemType> some_vector;
```

So what is a vector really?



# Vector

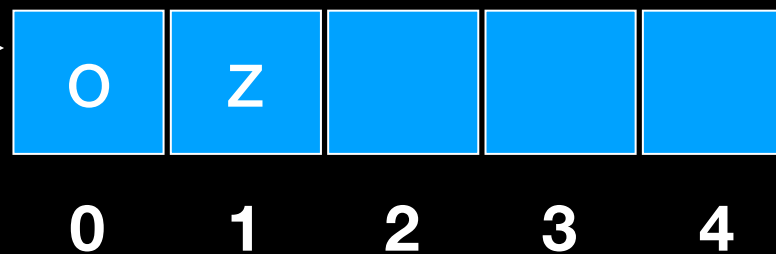
```
std::vector<ItemType> some_vector;
```

So what is a vector really?

Push and pop same as  
with arrays

**Vector (simplified)**

buffer\_ =  
len\_ = 0  
capacity\_ = 5



# Vector

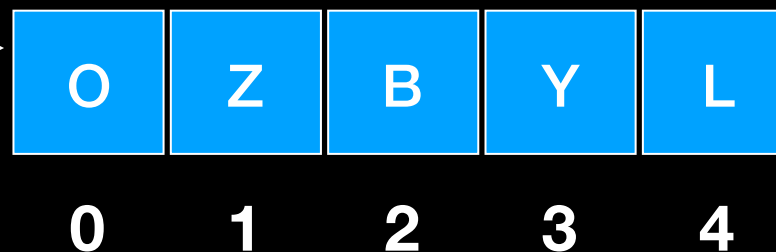
```
std::vector<ItemType> some_vector;
```

So what is a vector really?

Stack is Full?

**Vector (simplified)**

buffer\_ =  
len\_ = 0  
capacity\_ = 5



# Vector

```
std::vector<ItemType> some_vector;
```

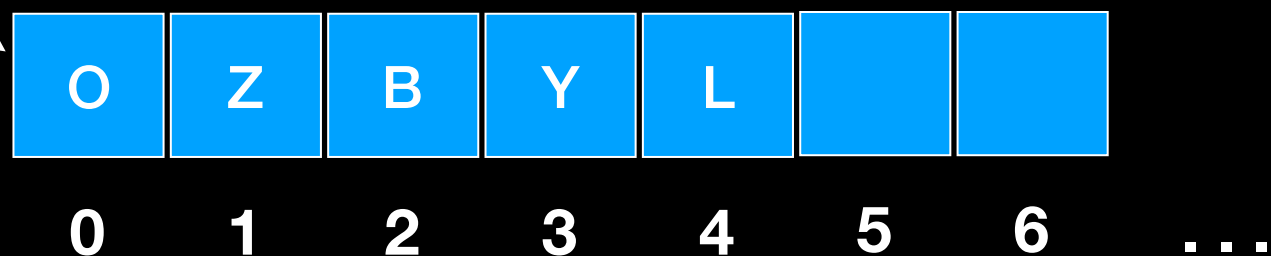
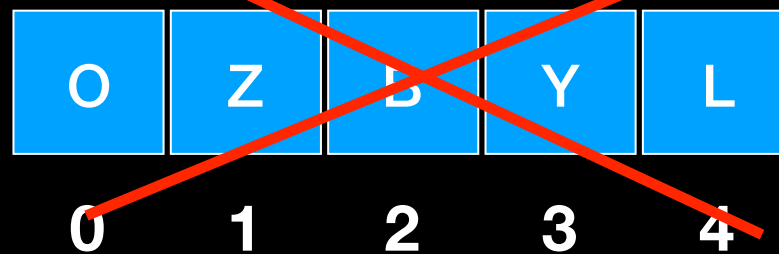
So what is a vector really?



No, I'll Grow!!!

**Vector (simplified)**

buffer\_  
len\_ = 0  
capacity\_ = 5



# In-Class Task

How much should it grow?

Write a short paragraph arguing the **pros** and **cons** of growing by the amount you propose

# Vector Analysis

1 assignment + 1 increment/decrement = 1 "step"

*size* : 1 "step"

*isEmpty*: 1 "step"

*push*: 1 "step"

*pop* : 1 "step"

*top* : 1 "step"

Fixed amount of work

GREAT!!!!

# Vector Analysis

1 assignment + 1 increment/decrement = 1 "step"

*size* : 1 "step"

*isEmpty*: 1 "step"

*push*: 1 "step"

*pop* : 1 "step"

*top* : 1 "step"

Fixed amount of work

Except when stack is full must:

- allocate new array
- copy elements in new array
- delete old array

GREAT!!!!



# Vector Analysis

1 assignment + 1 increment/decrement = 1 "step"

*size* : 1 "step"

*isEmpty*: 1 "step"

*push*: 1 "step" or **sometimes  $n$  "steps"**

*pop* : 1 "step"

*top* : 1 "step"

Fixed amount of work

**Except** when stack is full must:

- allocate new array (**assume 1 step**)
- copy elements in new array ( **$n$  steps**)
- delete old array (**assume 1 step**)

**GREAT!!!!** 

# How should Vector grow?

Sometimes 1 "*step*"

Sometimes  $n$  "*steps*"

Consider behavior **over several pushes (on average)**



# Vector Growth: a naive approach

```
std::vector<ItemType> some_vector;
```

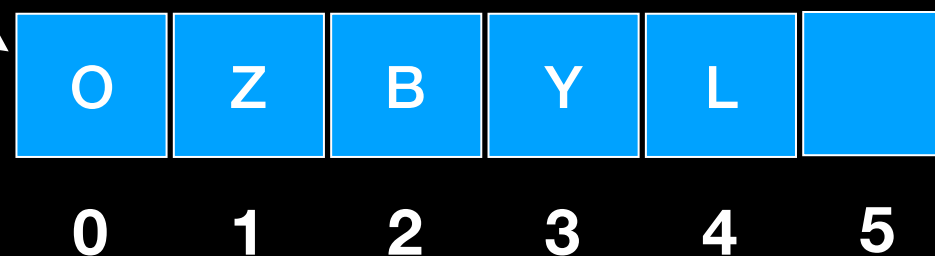
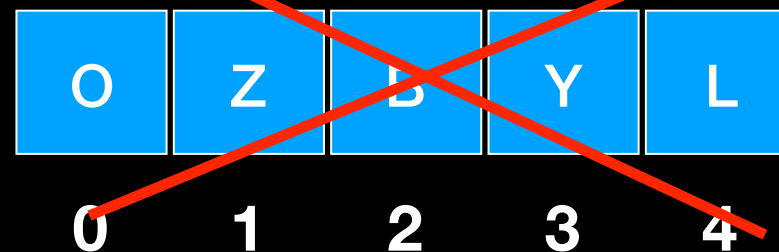
So what is a vector really?



I'll Grow!!!  
I will add space for the  
item to be added

**Vector (simplified)**

buffer\_ =  
len\_ = 0  
capacity\_ = 5



# Vector Growth: a naive approach

If vector grows by 1 each time, every push costs  $n$  “steps”

Cost of pushes:

$$1 + 2 + 3 + 4 + 5 + \dots + n$$
$$= n(n+1)/2$$

# Vector Growth: a naive approach

If vector grows by 1 each time, every push costs  $n$  “steps”

Cost of pushes:

$$1 + 2 + 3 + 4 + 5 + \dots + n$$

$$= n(n+1)/2$$

$$= n^2 / 2 + n / 2$$

$$= n^2 / \text{something} + \text{something} / \text{something}$$

**$n^2$  highest degree**

# Vector Growth: a naive approach

If vector grows by 1 each time, every push costs  $n$  “steps”

Cost of pushes:

$$\begin{aligned} &1 + 2 + 3 + 4 + 5 + \dots + n \\ &= n(n+1)/2 \\ &= n^2 + n / 2 \\ &= n^2 + \text{something} / \text{something} \end{aligned}$$

$n^2$  highest degree

Same cost of pop:

$$\begin{aligned} &1 + 1 + 1 + \dots + 1 \\ &= n \\ &= n + \text{nothing} / \text{nothing} \end{aligned}$$

$n$  highest degree

# Vector Growth: a better approach

```
std::vector<ItemType> some_vector;
```

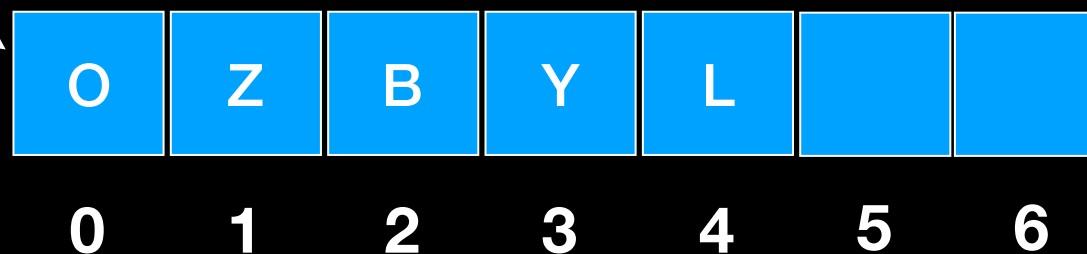
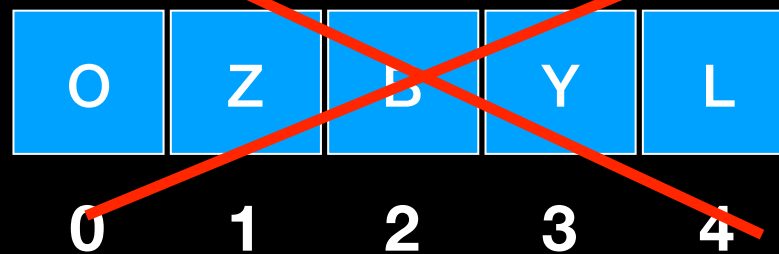
So what is a vector really?



I'll Grow!!!  
I will add two more slots!

**Vector (simplified)**

buffer\_ =  
len\_ = 0  
capacity\_ = 5



# Vector Growth: a better approach

Let a “hard push” be one where the whole vector needs to be copied

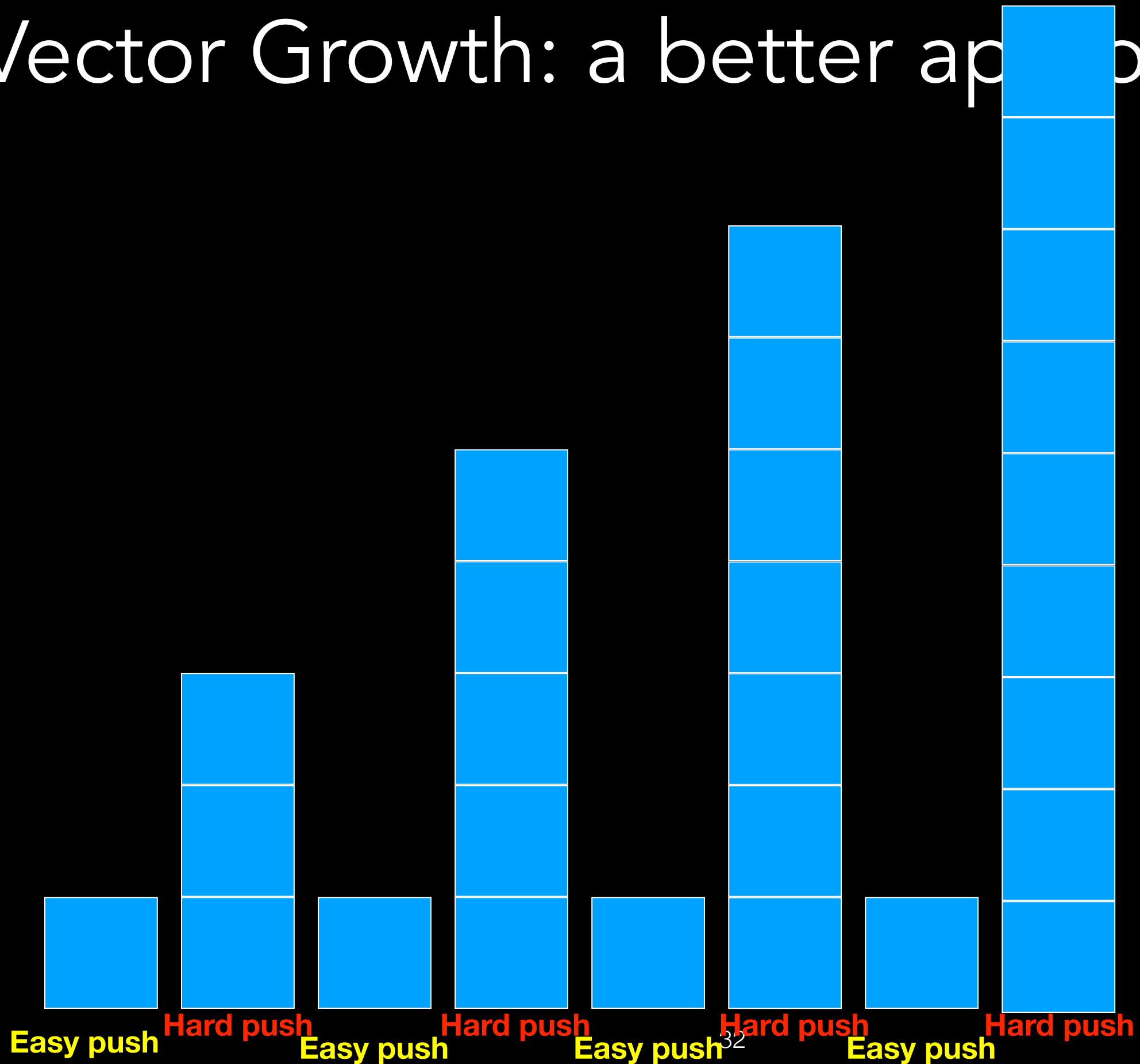
When vector is not copied we have an “easy push”

Now half our pushes will be easy (1 step) and half will be hard (n steps)

So if reconsider the work over several pushes? (On Average?)

Analysis visualization adapted from Keith Schwarz

# Vector Growth: a better approach





# Vector Growth: a better approach

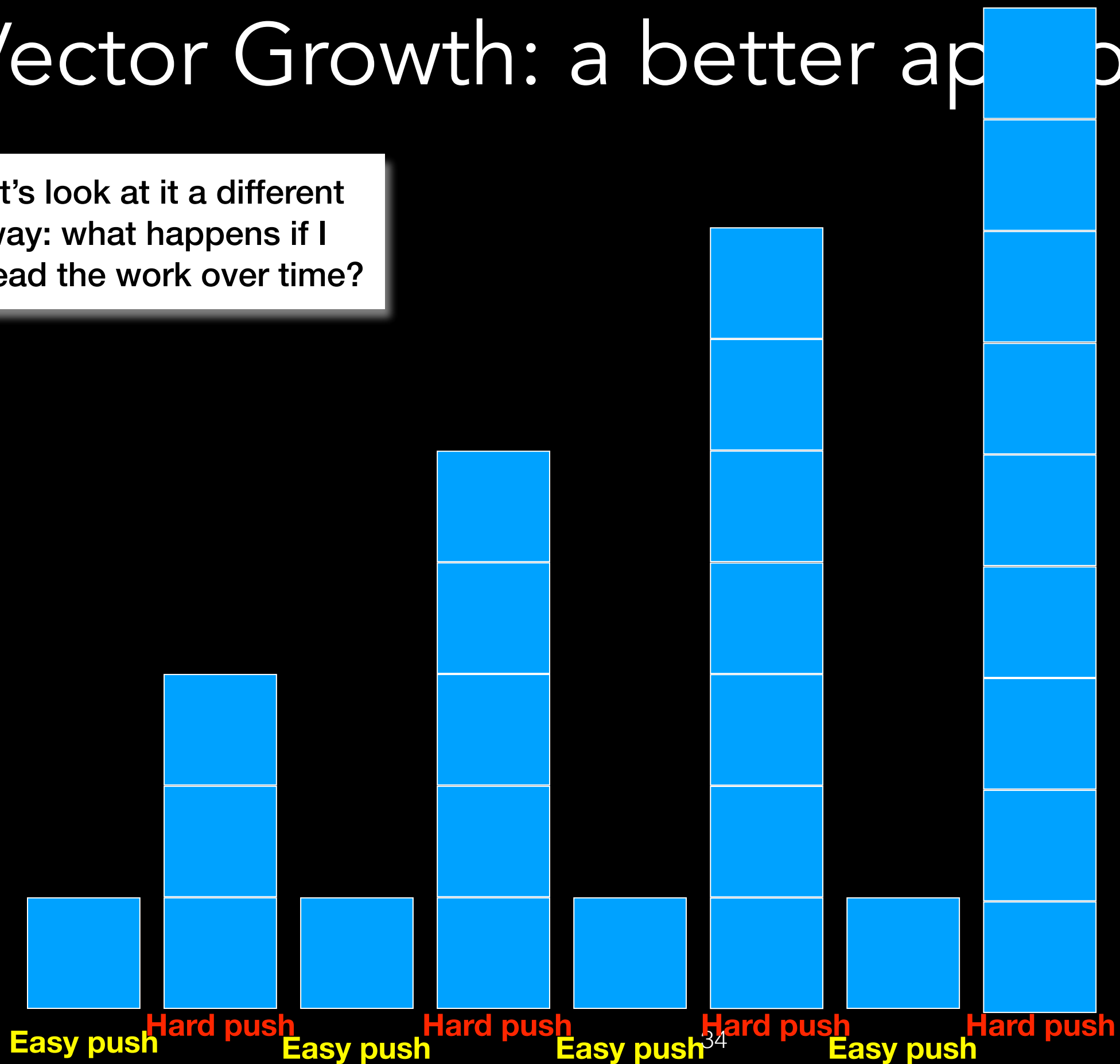
Work Saved

By simply adding one extra “slot” we roughly cut down the work by half on average (over several pushes)

Easy push Hard push Easy push Hard push Easy push Hard push Easy push Hard push

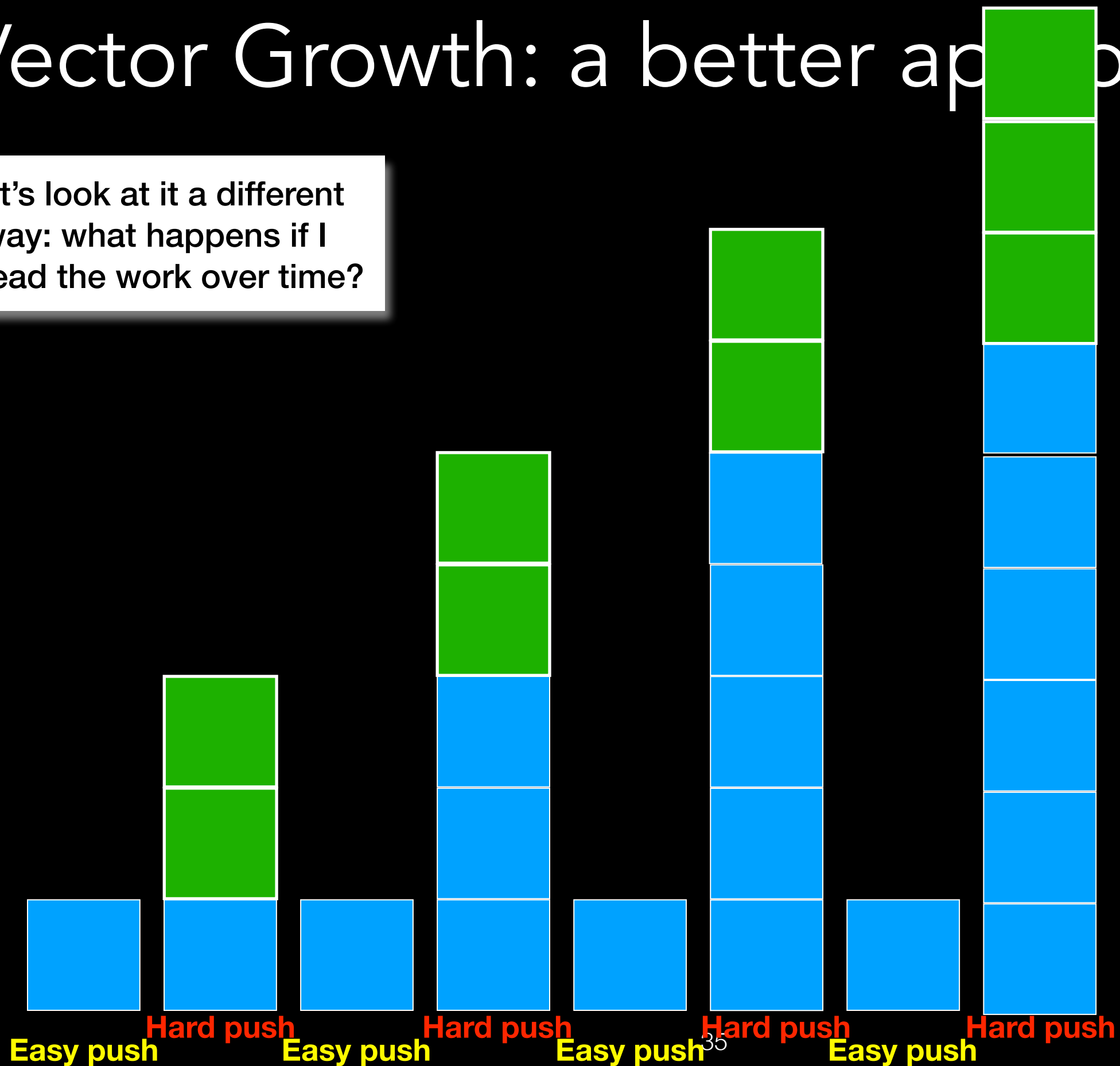
# Vector Growth: a better approach

Let's look at it a different way: what happens if I spread the work over time?



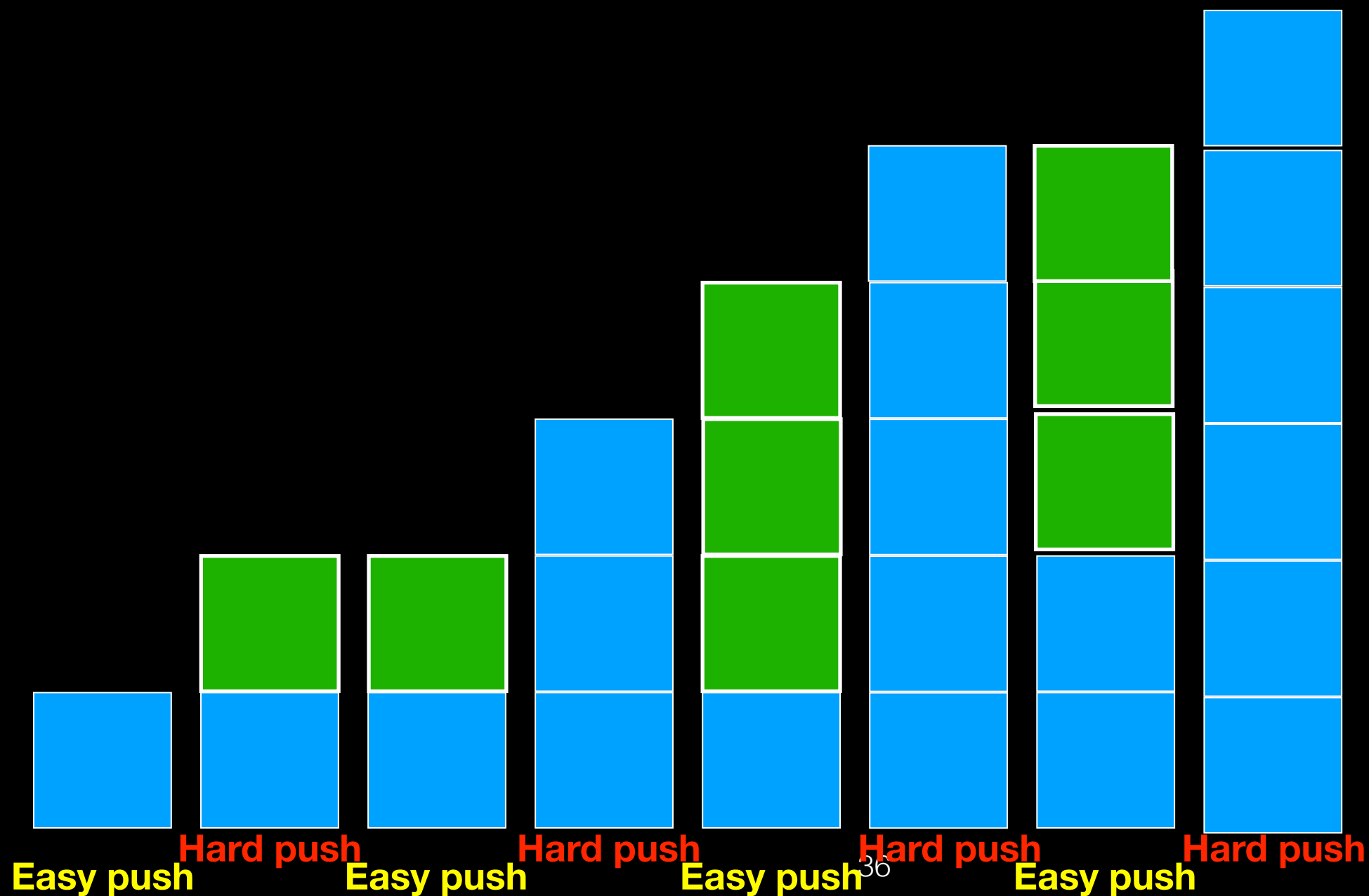
# Vector Growth: a better approach

Let's look at it a different way: what happens if I spread the work over time?



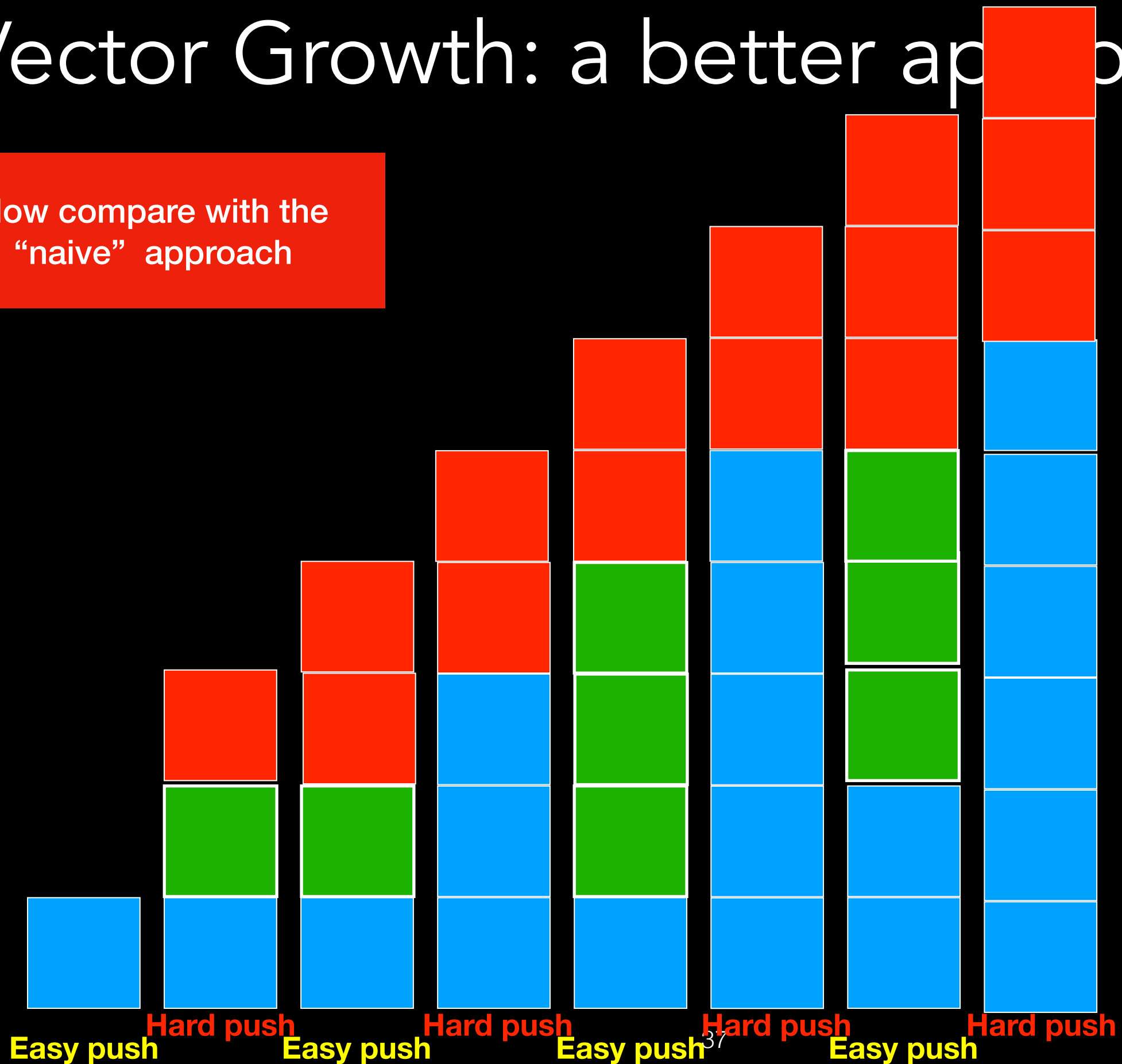
# Vector Growth: a better approach

Let's look at it a different way: what happens if I spread the work over time?



# Vector Growth: a better approach

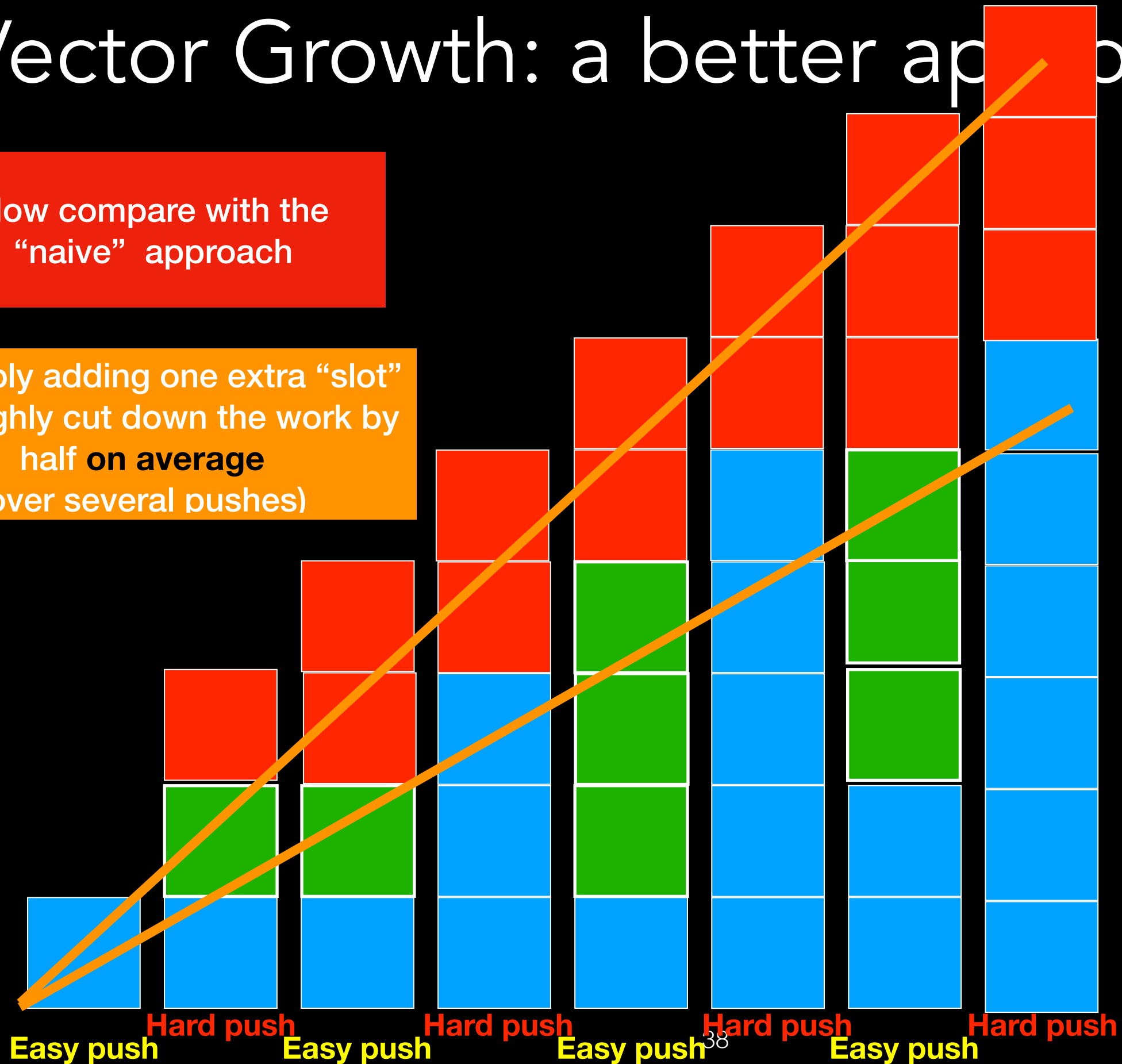
Now compare with the  
“naive” approach



# Vector Growth: a better approach

Now compare with the  
“naive” approach

By simply adding one extra “slot”  
we roughly cut down the work by  
half **on average**  
(over several pushes)



Can we do better?

# Vector Growth: a much better approach

```
std::vector<ItemType> some_vector;
```

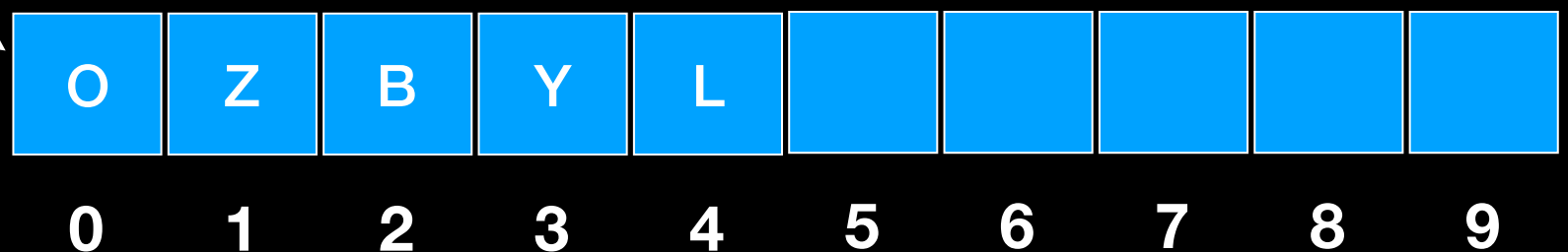
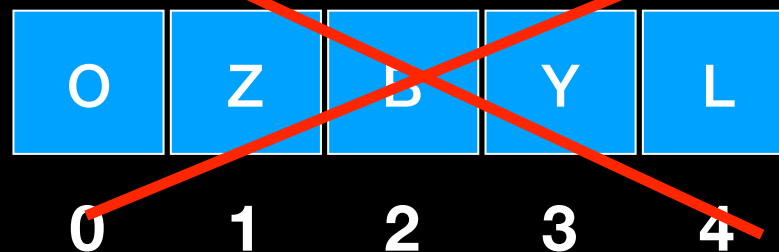
So what is a vector really?



I'll Grow!!!  
I'll double my size!

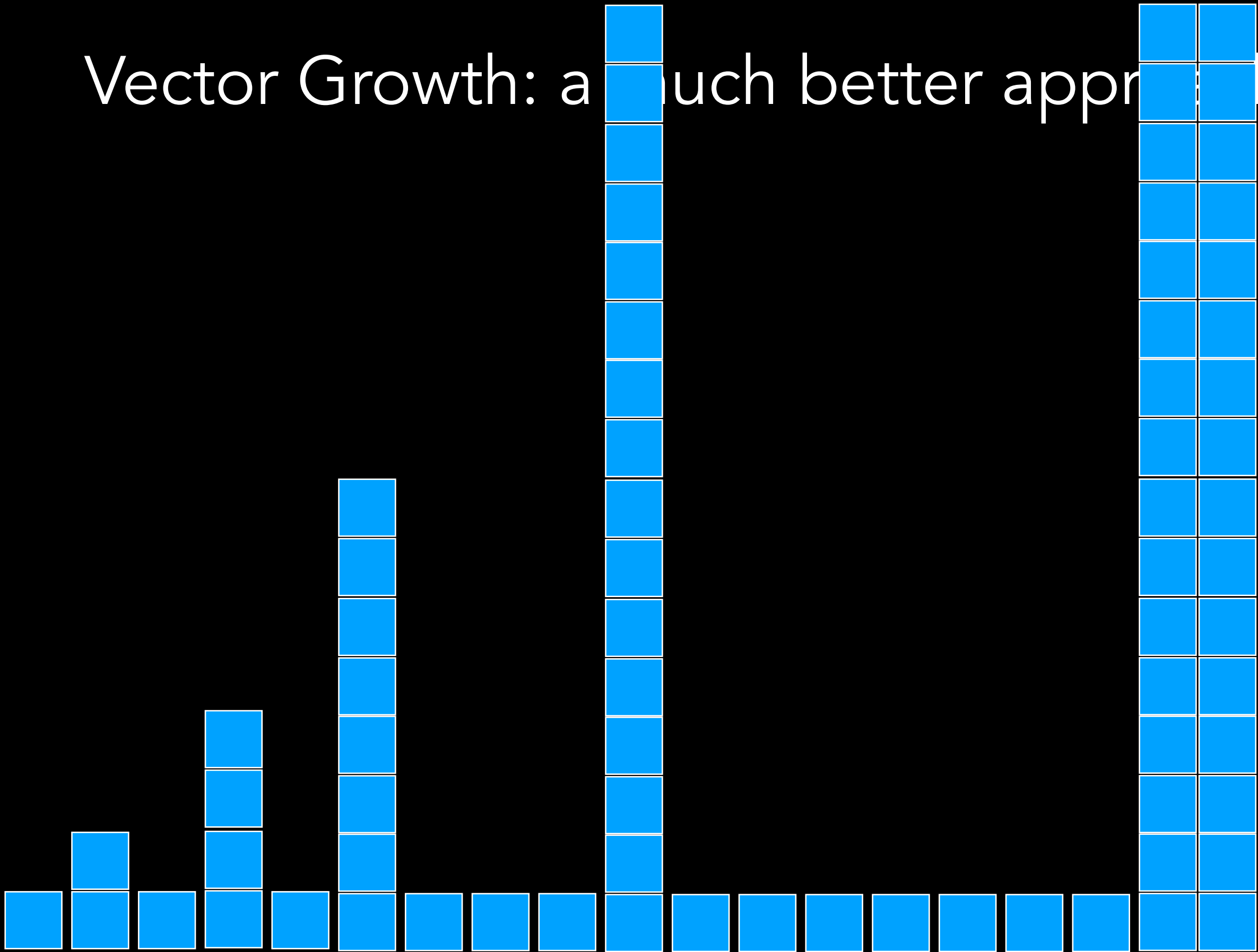
**Vector (simplified)**

buffer\_ =  
len\_ = 0  
capacity\_ = 5



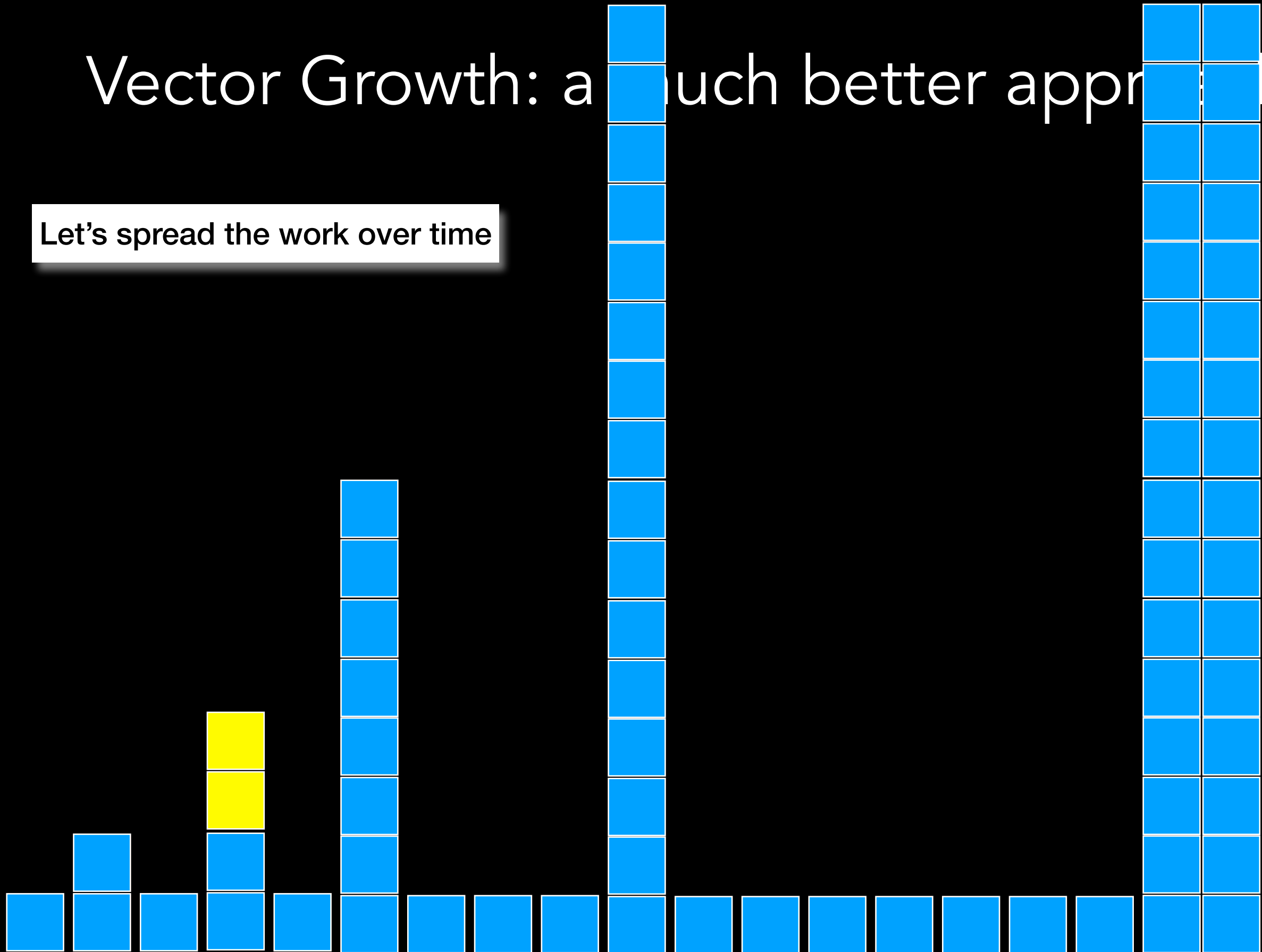


Vector Growth: a much better approach



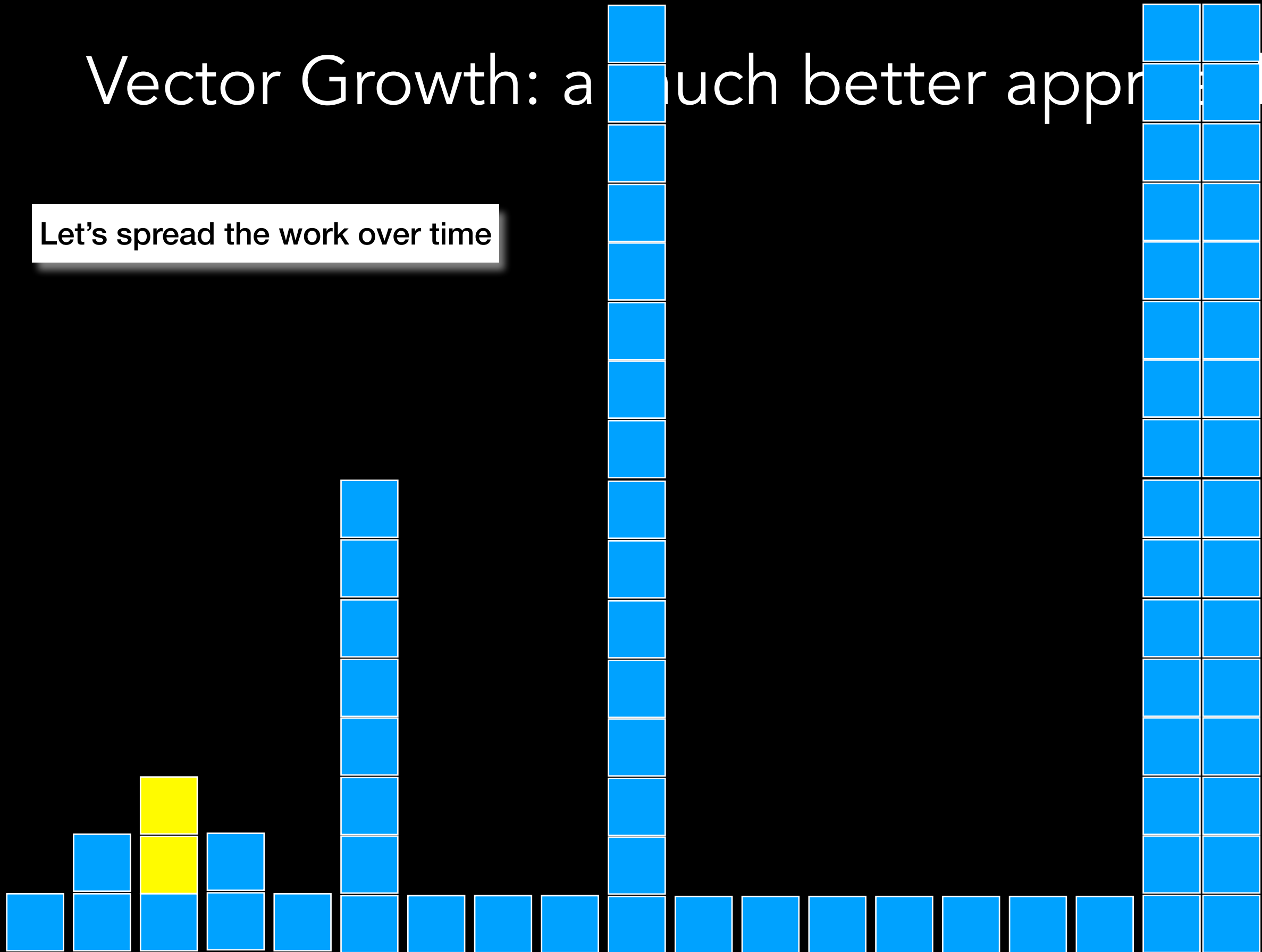
# Vector Growth: a much better approach

Let's spread the work over time



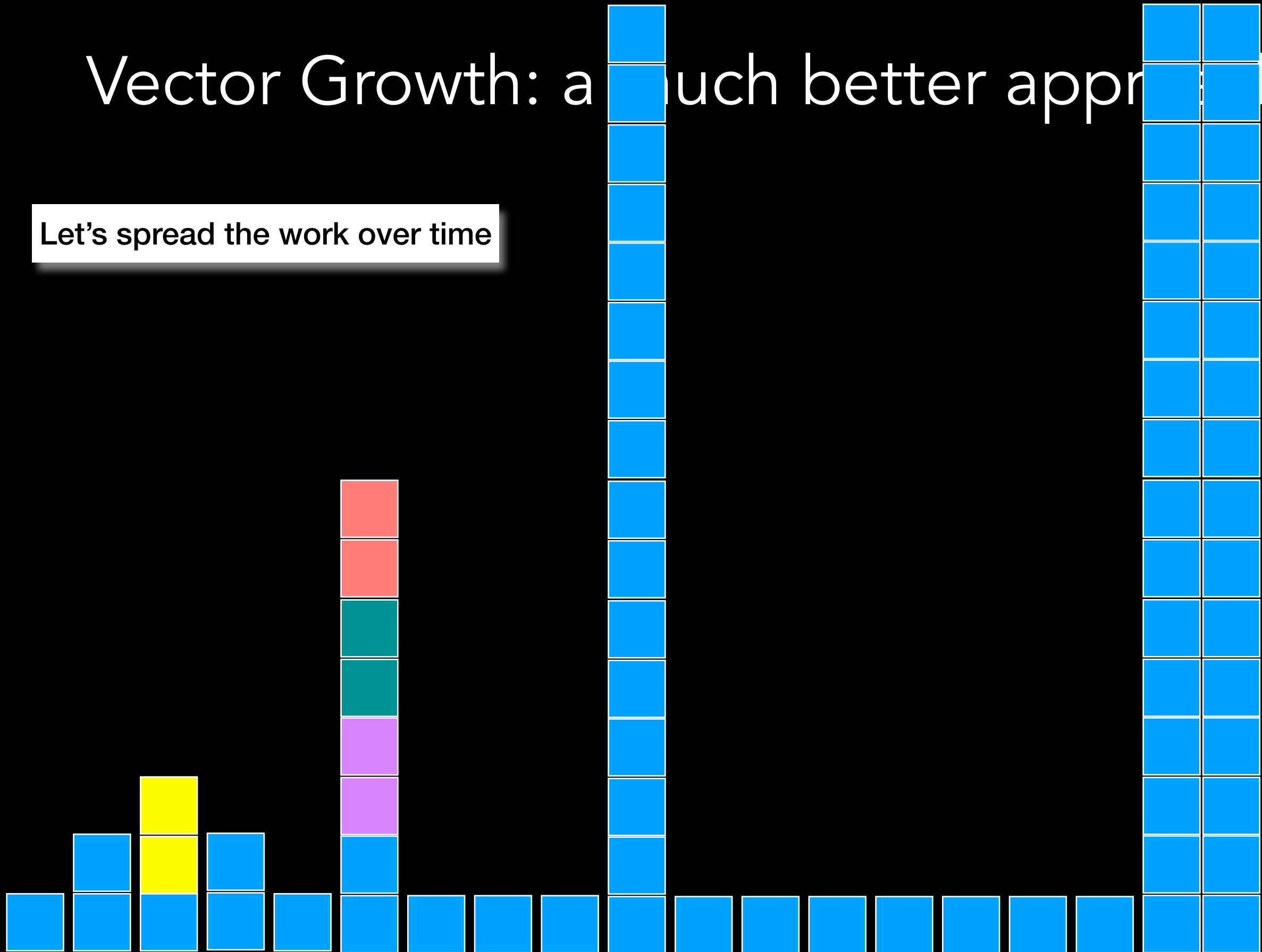
# Vector Growth: a much better approach

Let's spread the work over time



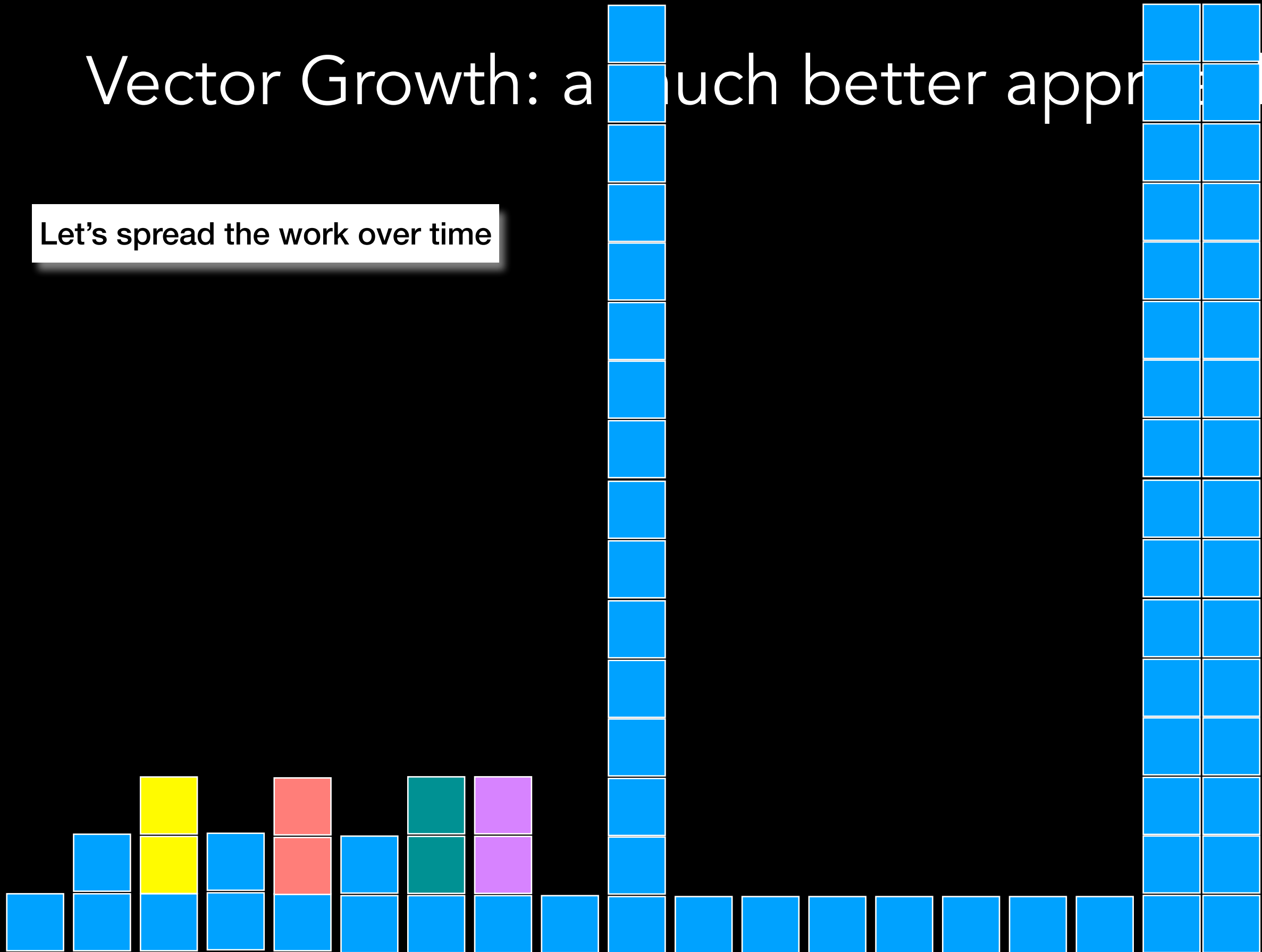
# Vector Growth: a much better approach

Let's spread the work over time



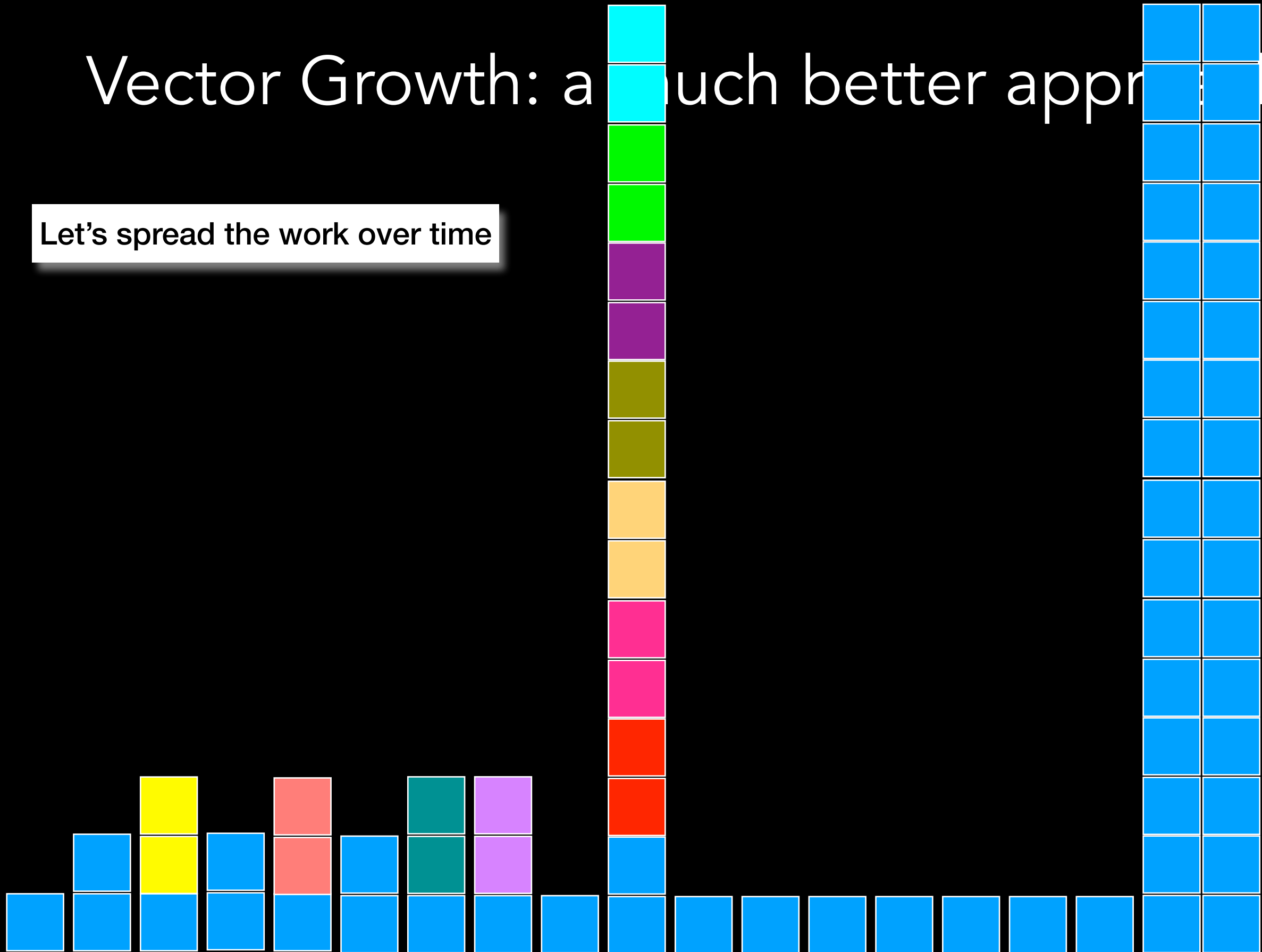
# Vector Growth: a much better approach

Let's spread the work over time



# Vector Growth: a much better approach

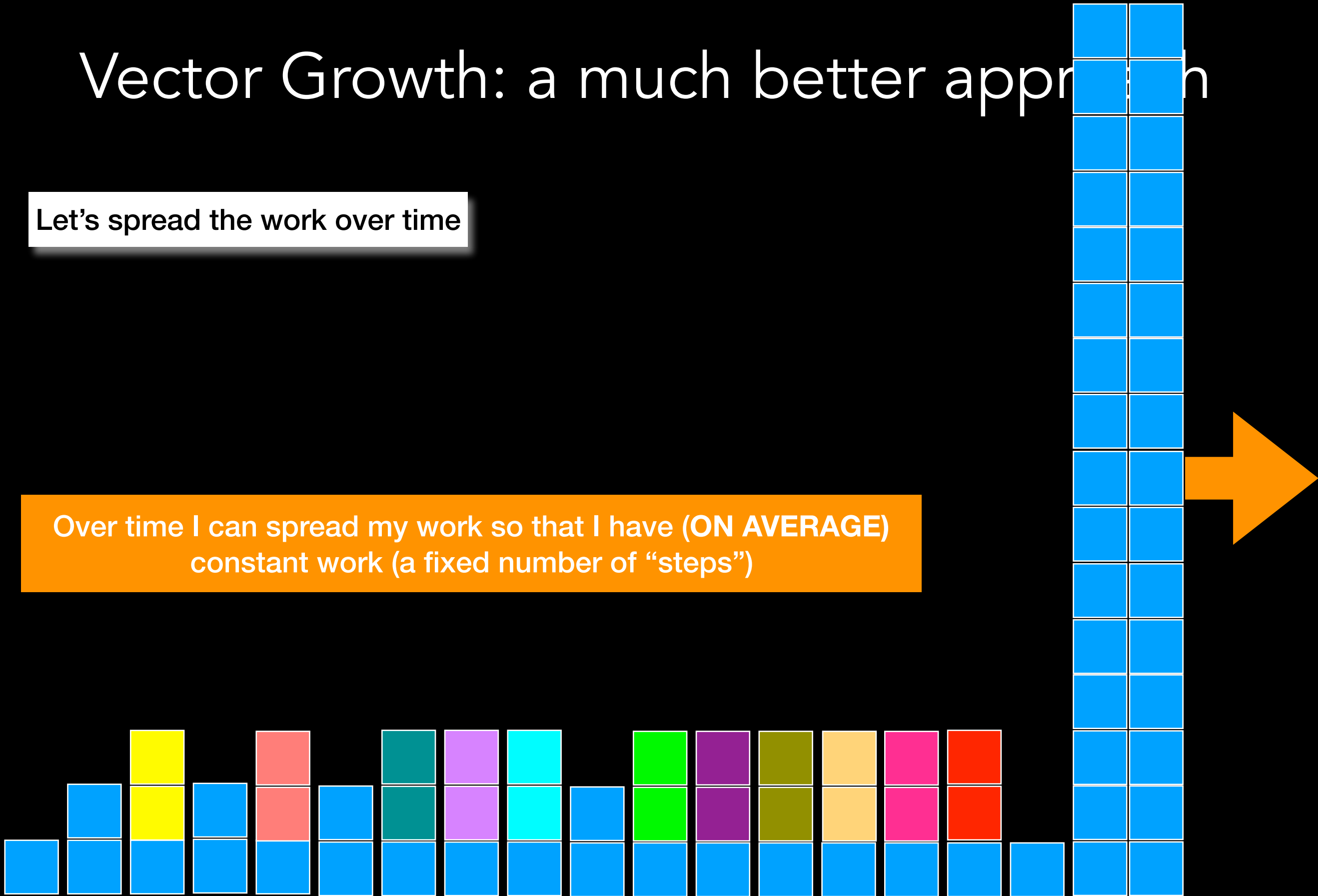
Let's spread the work over time



# Vector Growth: a much better approach

Let's spread the work over time

Over time I can spread my work so that I have (ON AVERAGE) constant work (a fixed number of "steps")

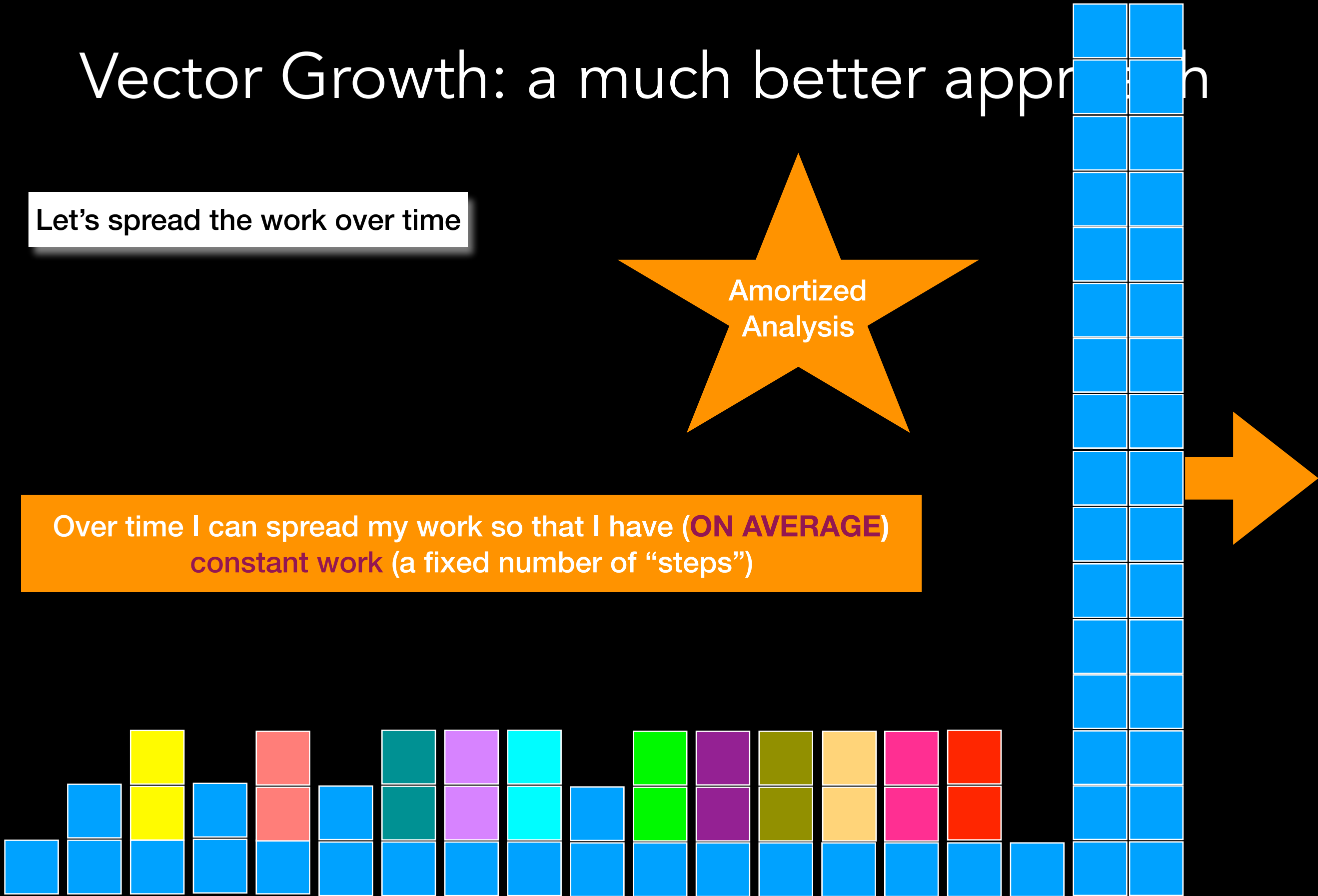


# Vector Growth: a much better approach

Let's spread the work over time

Amortized  
Analysis

Over time I can spread my work so that I have **(ON AVERAGE)**  
**constant work** (a fixed number of "steps")





# Vector Growth summarized

If it grows by 1, push takes  $n^2$  "steps"

If it grows by 2, push takes roughly half the "steps" over time (AMORTIZED ANALYSIS)

If it doubles its size, push takes constant work over time (AMORTIZED ANALYSIS)

# A steadily shrinking Stack


Let's consider this application:

- Push the 524,288<sup>th</sup> element onto Stack which causes it to double it's size to 1,048,576
- Reading an input file
  - pop those that match
  - manipulate input record accordingly
  - repeat

# A steadily shrinking Stack

Let's consider this application:

- Push the 524,288th element onto Stack which causes it to double it's size to 1,048,576
- Reading an input file
  - pop those that match
  - manipulate input record accordingly
  - repeat



How much I pop will depend on input

# A steadily shrinking Stack

Let's consider this application:

- Push the 524,288th element onto stack which causes it to double it's size to 1,048,576
- Reading an input file
  - pop those that match
  - manipulate input record accordingly
  - repeat

Assume a few matches at each iteration => mostly empty stack but it will be around for a long time!



I will not shrink!

Useless  
memory  
waste

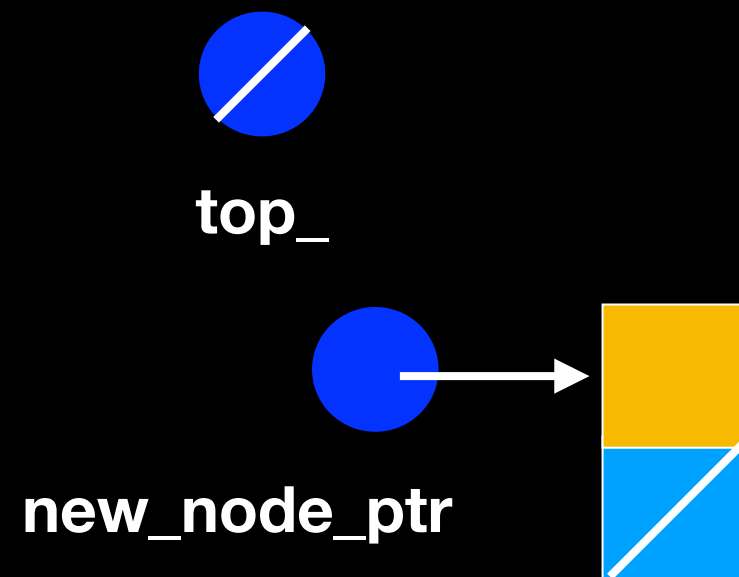
# Linked Chain



top\_

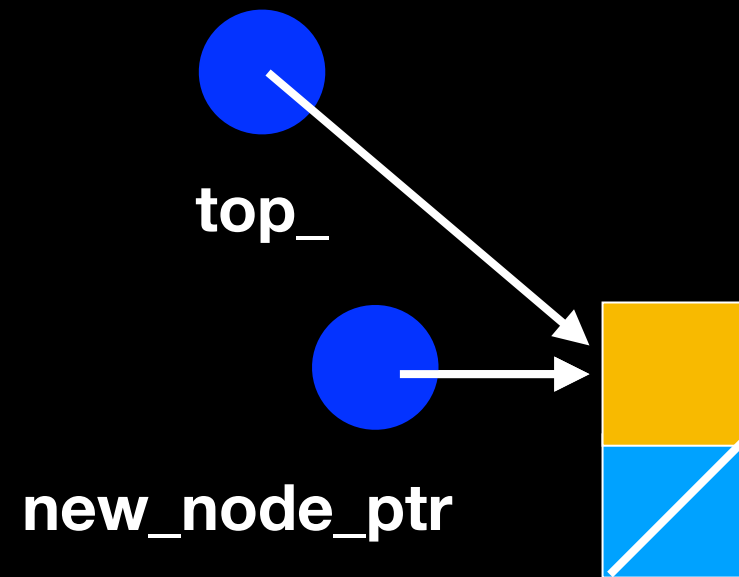
# Linked Chain

push



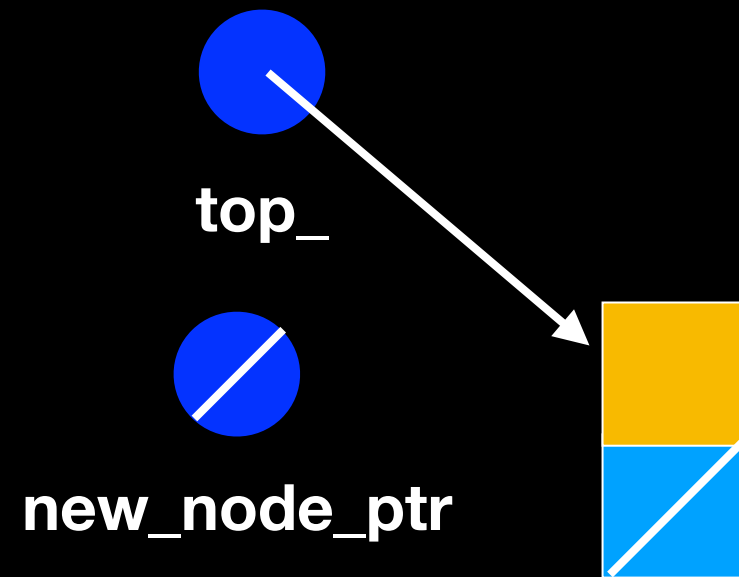
# Linked Chain

push



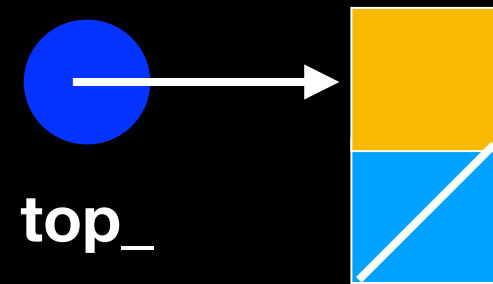
# Linked Chain

push



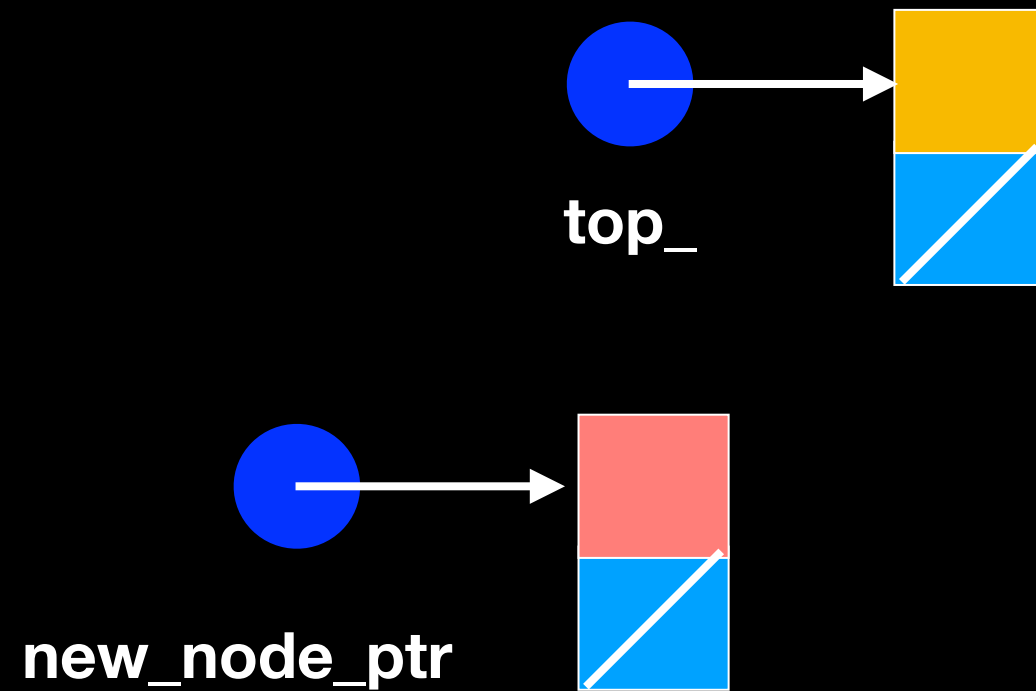


# Linked Chain



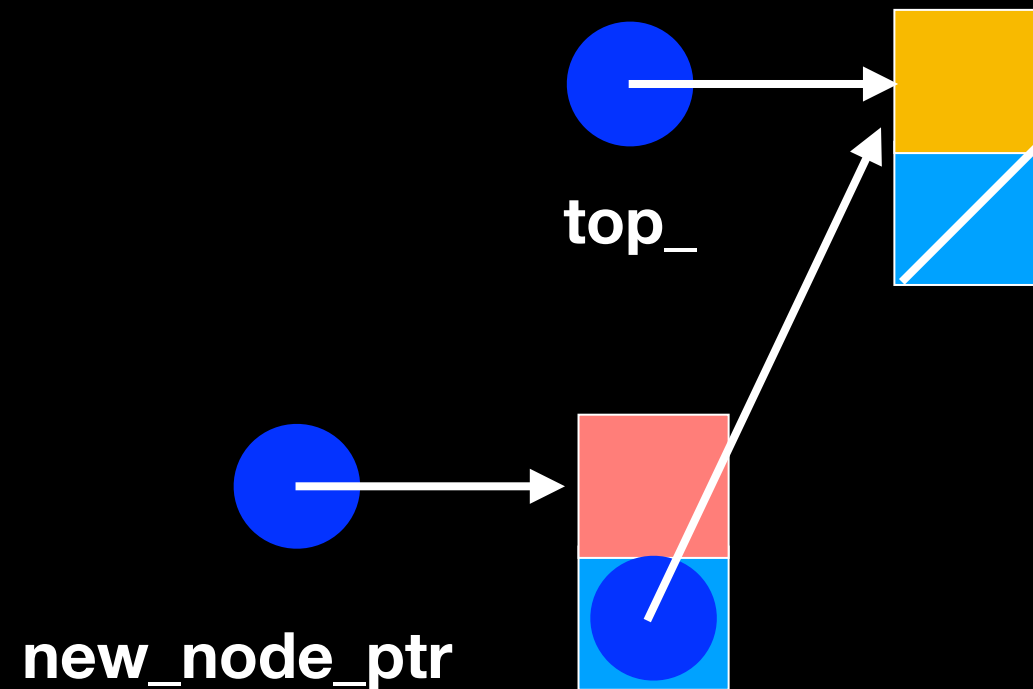
# Linked Chain

push



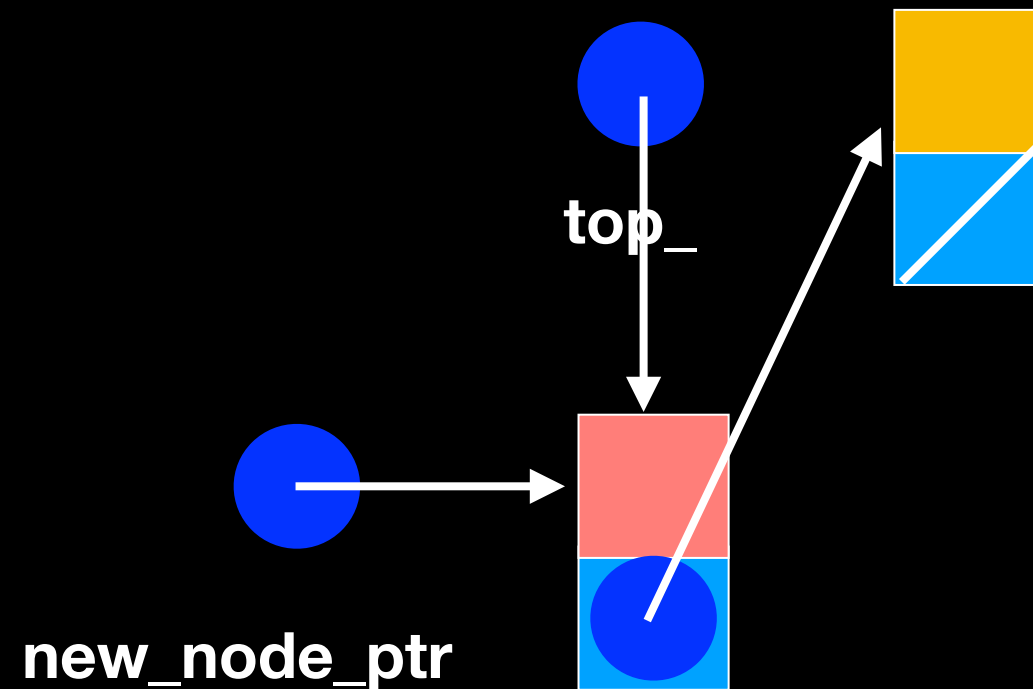
# Linked Chain

push



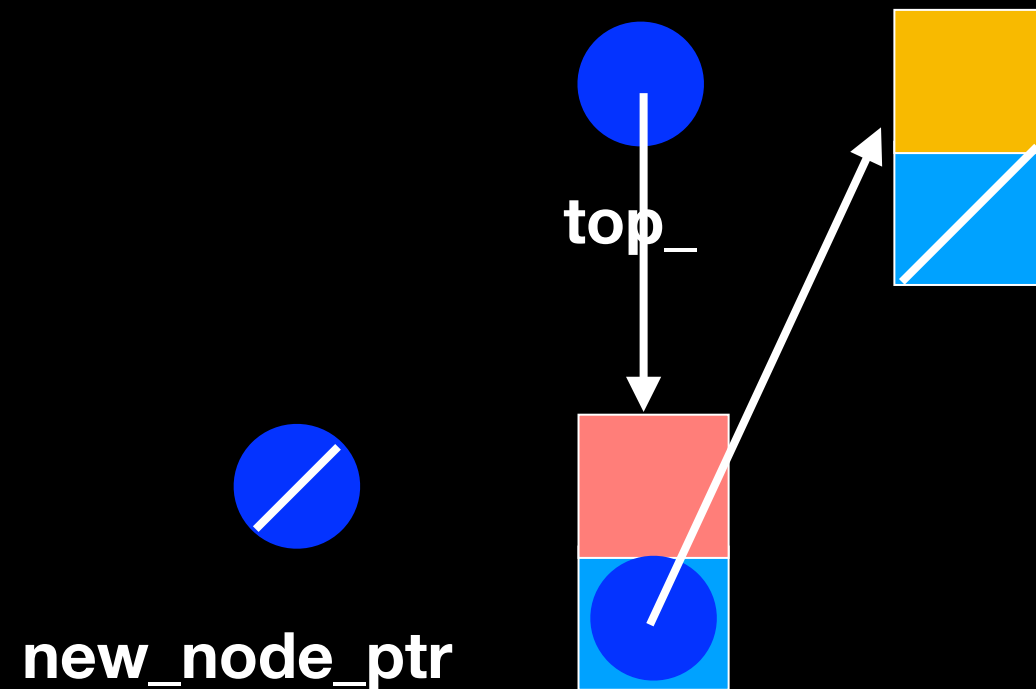
# Linked Chain

push

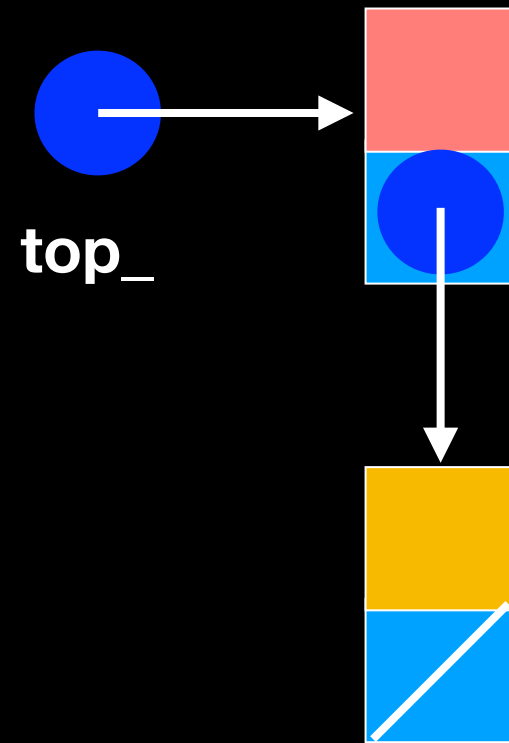


# Linked Chain

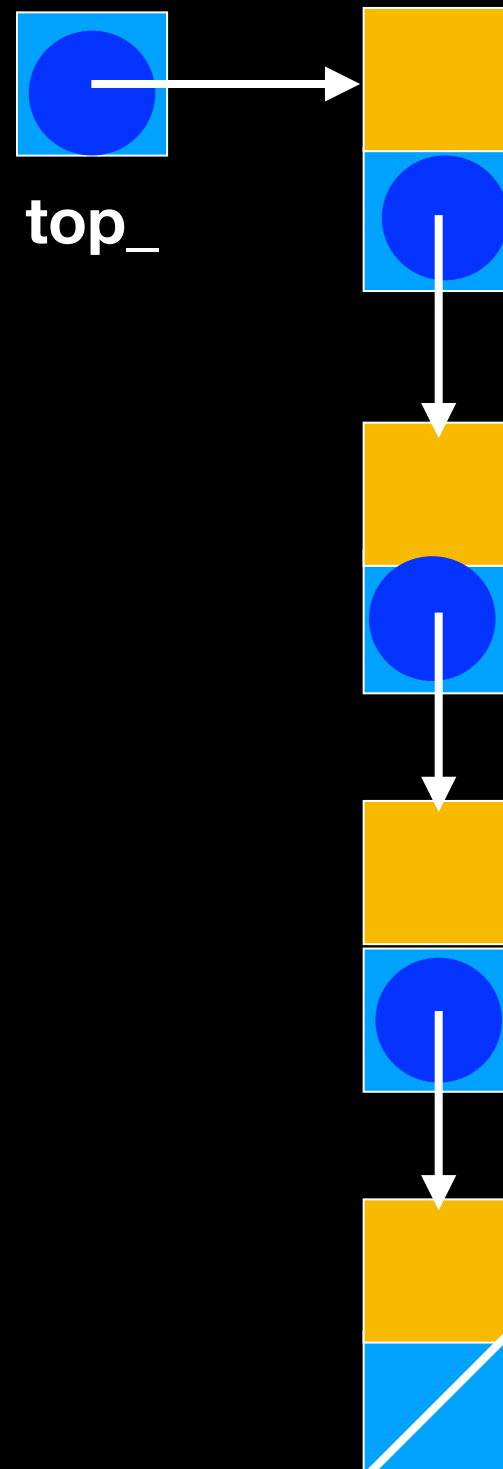
push



# Linked Chain

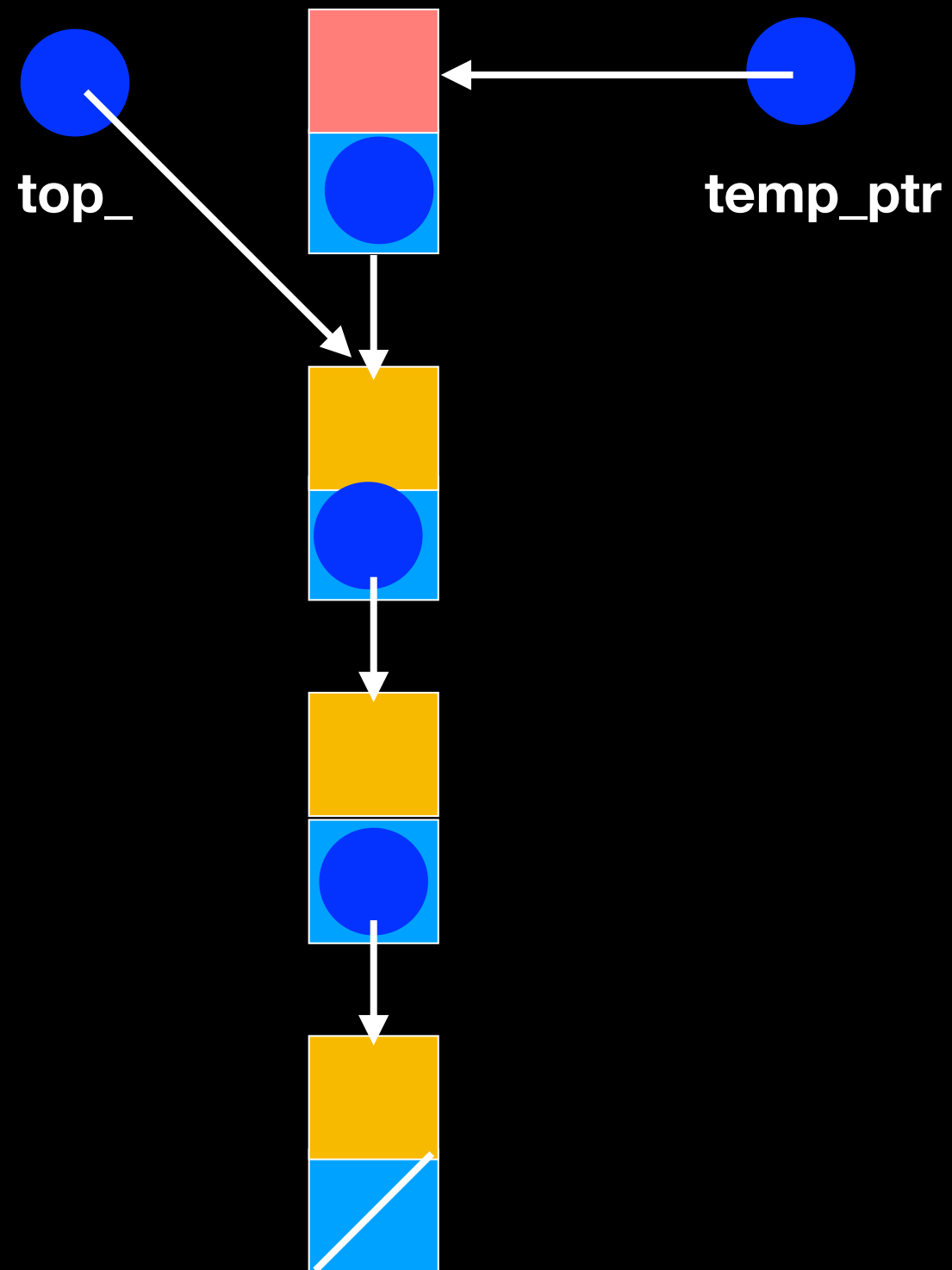


# Linked Chain



# Linked Chain

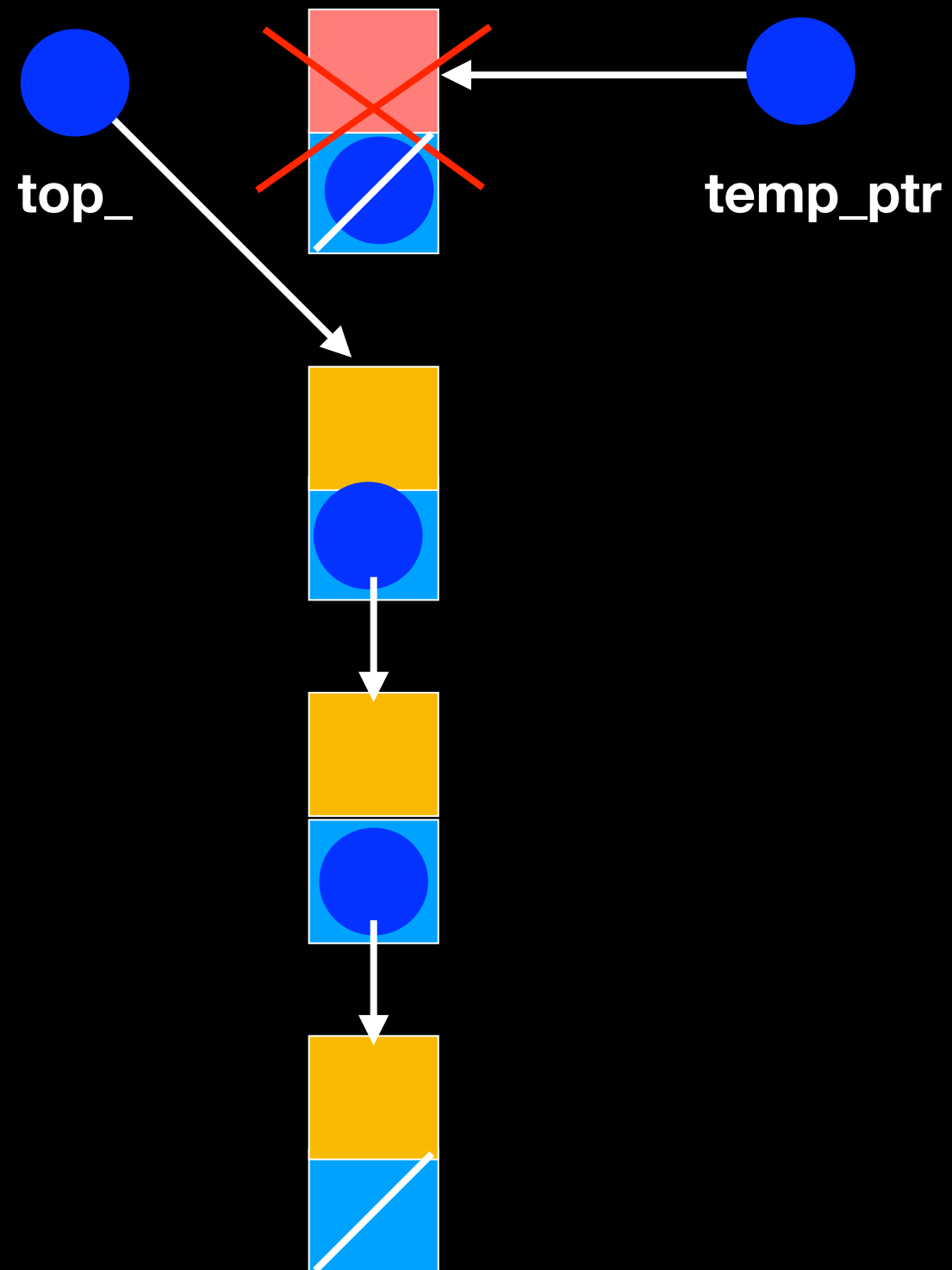
pop





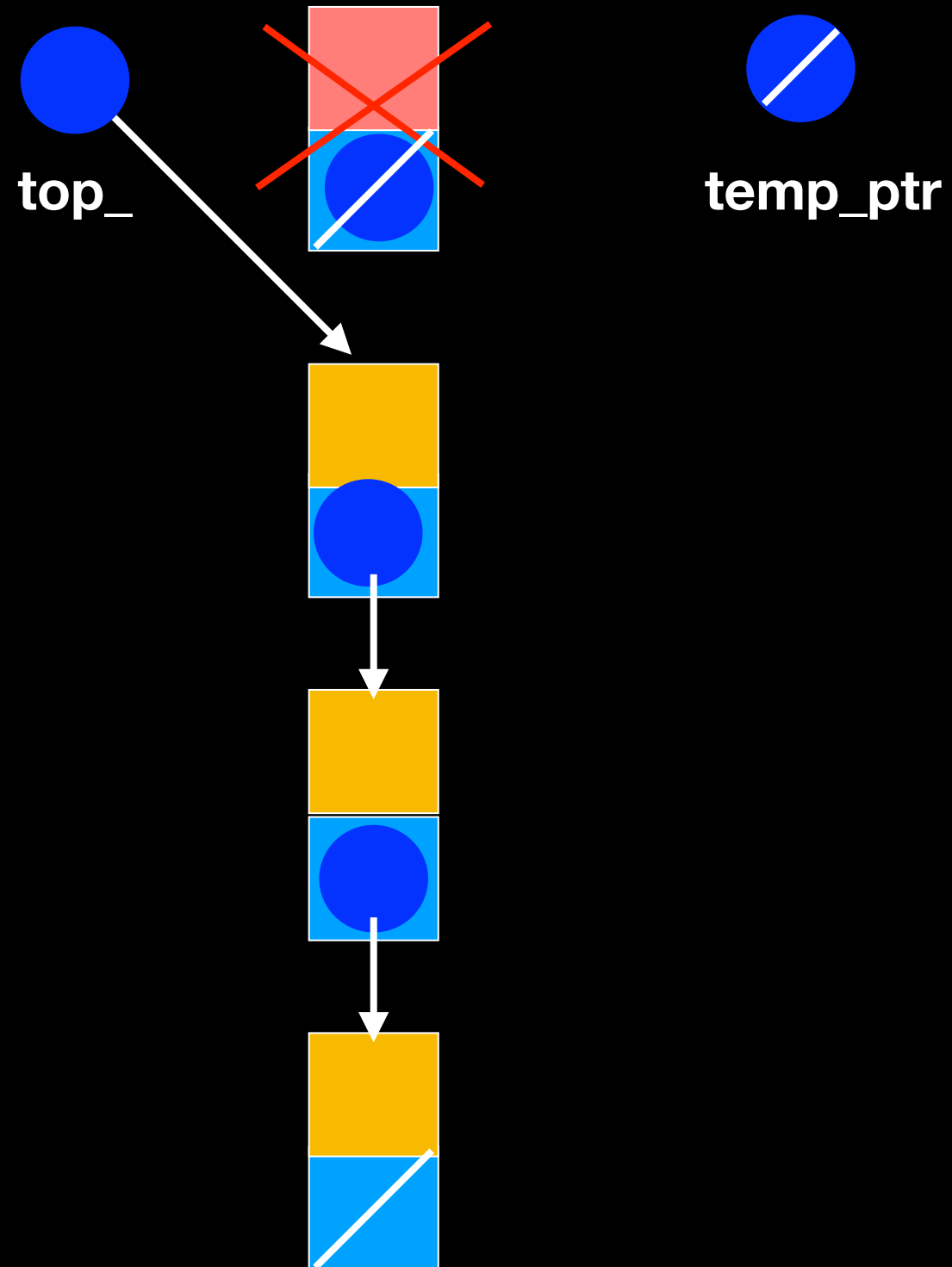
# Linked Chain

pop

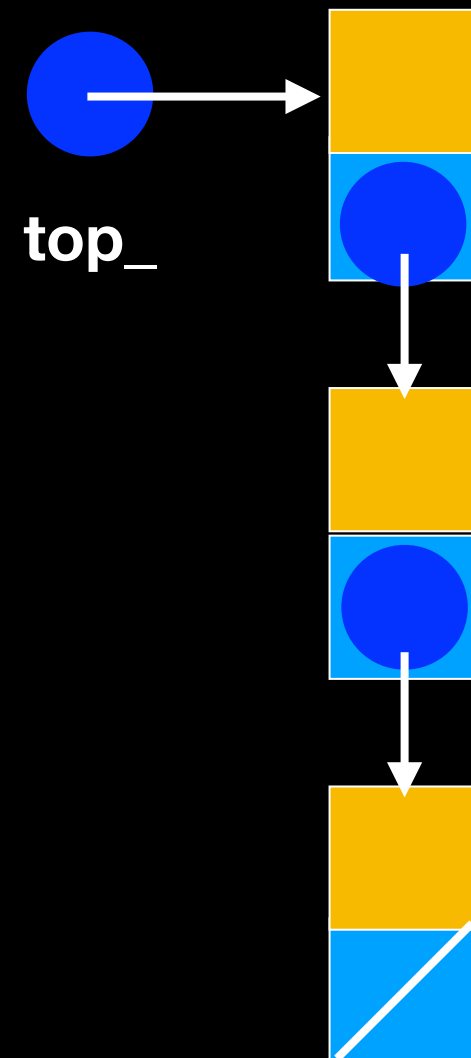


# Linked Chain

pop



# Linked Chain



# Linked-Chain Analysis

Create new node + 1 data-item assignment + a few  
pointer assignment = 1 "step"

*size* : 1 "step"  
*isEmpty*: 1 "step"  
*push*: 1 "step"  
*pop* : 1 "step"  
*top* : 1 "step"



Fixed amount of work

GREAT!!!! And there is no "Except" case here, always  
guaranteed fixed amount of work!

# To summarize

Array: constant amount of work for push and pop, but size is bounded

Vector: size is unbounded but

- If it grows by 1, push takes  $n^2$  "steps"
- If it grows by 2, push roughly half the "steps" over time (AMORTIZED ANALYSIS)
- If it grows doubles, push takes constant work over time (AMORTIZED ANALYSIS)

Linked-Chain: constant amount of work for push and pop and size is unbounded

# Implement Stack ADT

```
#ifndef STACK_H_
#define STACK_H_

template<class ItemType>
class Stack
{
public:
    Stack();
    void push(const ItemType& newEntry); // adds an element to top of stack
    void pop(); // removes element from top of stack
    ItemType top() const; // returns a copy of element at top of stack
    int size() const; // returns the number of elements in the stack
    bool isEmpty() const; // returns true if no elements on stack false otherwise

private:
    Node<ItemType>* top_; // Pointer to top of stack
    int itemCount; // number of items currently on the stack

}; //end Stack

#include "Stack.cpp"
#endif // STACK_H_
```