More Recursion

Tiziana Ligorio
tligorio@hunter.cuny.edu

Today's Plan

Recursion Review

8 Qeens Problem

Permutations

Combinations

Announcements

Midterm Exam postponed to **Friday March 22**

It will cover everything up to and including Recursion

Review requires your active participation

Types of Recursion

Reverse String:

- single recursive call
- Base case: stop => no return value

Dictionary:

- split problem into halves but solve only 1
- Base case: stop => no return value

Fractal Tree:

- split problem into halves and solve both
- Base case: stop => no return value

Factorial:

- single recursive call
- Base case: return a value for computation in each recursive call

Why/When use recursion

Usually less efficient than iterative counterparts (we will see example later in the course)

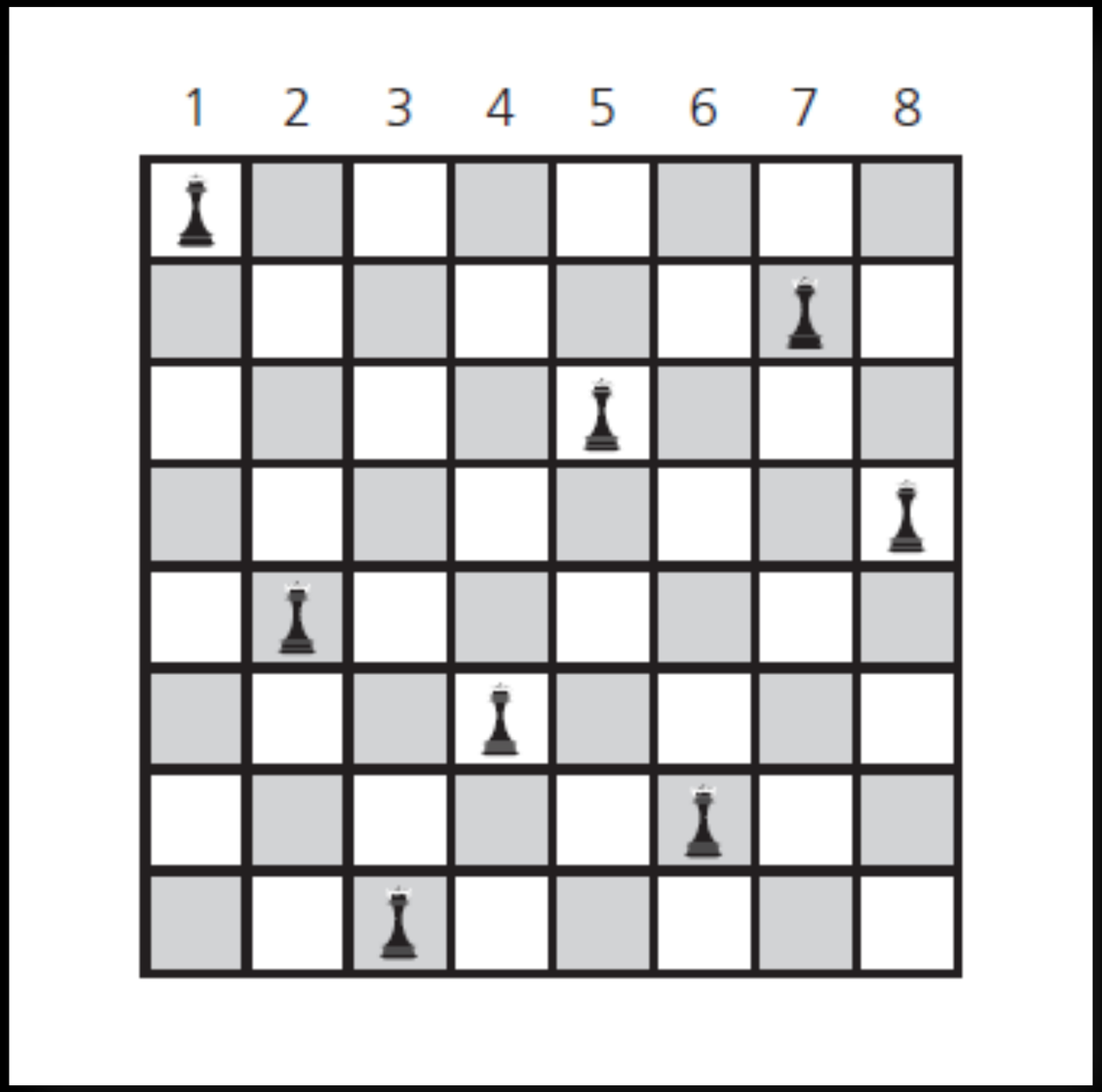
Inherent overhead associated with function calls

Repeated recursive calls with same parameters

Compilers can optimize tail-recursive (recursive call is the last statement in the function) functions to be iterative

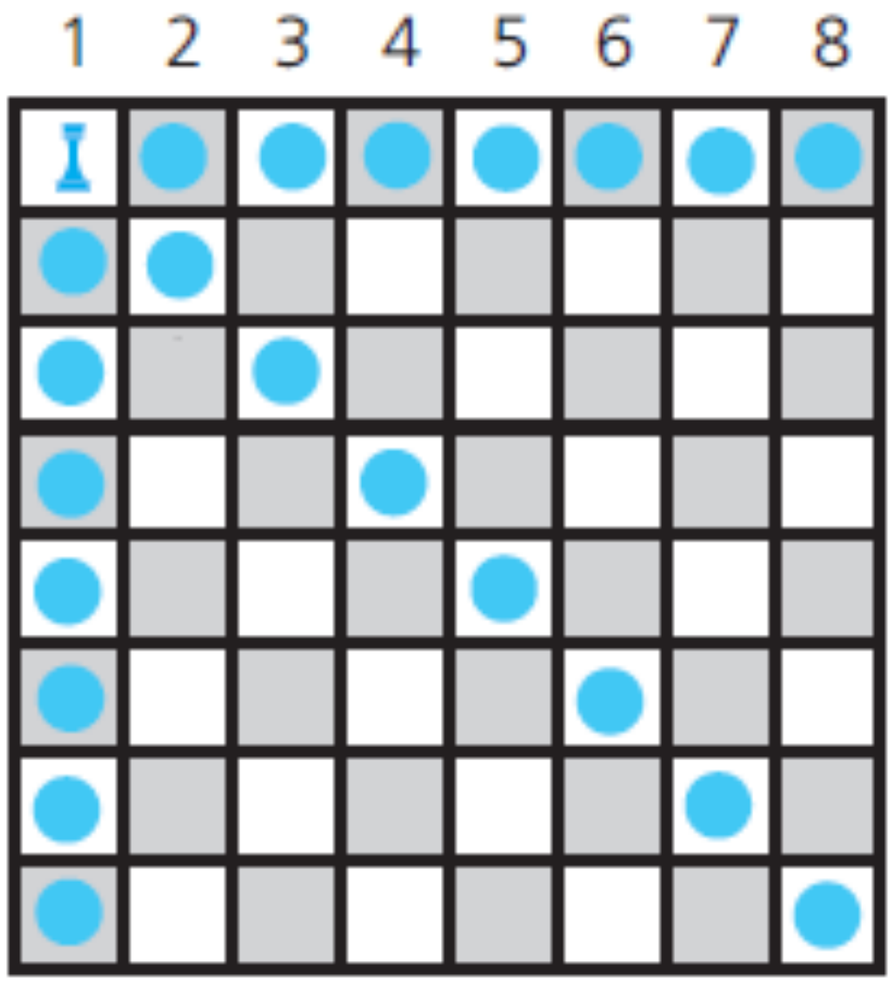
Sometimes logic of iterative solution can be very complex in comparison to recursive solution

The Eight Queens Problem



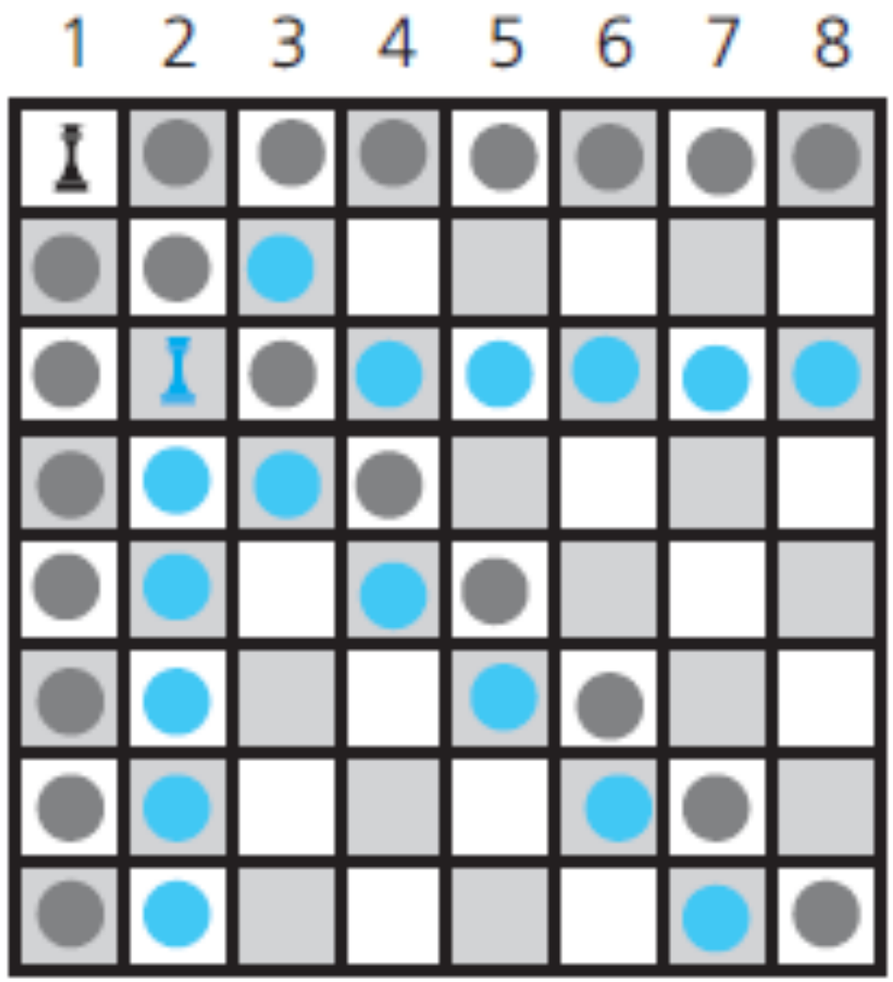
Place 8 Queens on the board s.t. no queen is on the same row, column or diagonal

The Eight Queens Problem



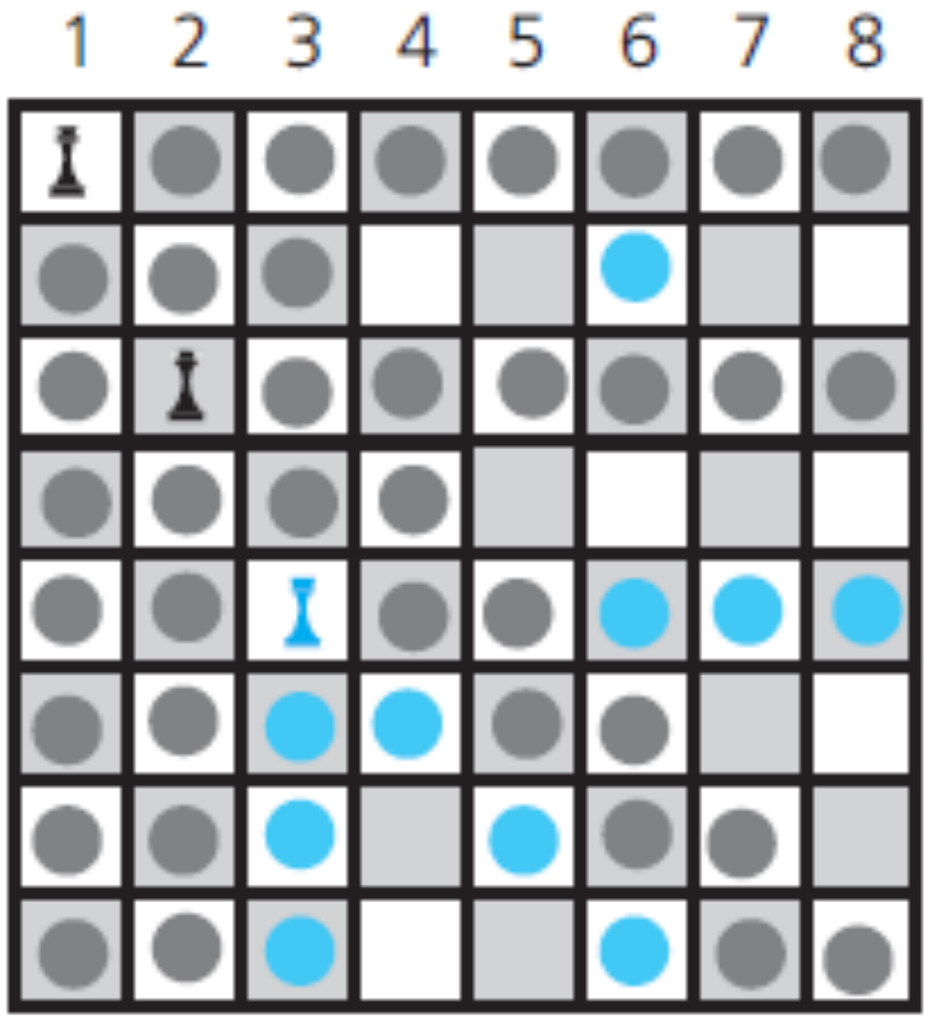
(a) The first queen in column 1

The Eight Queens Problem



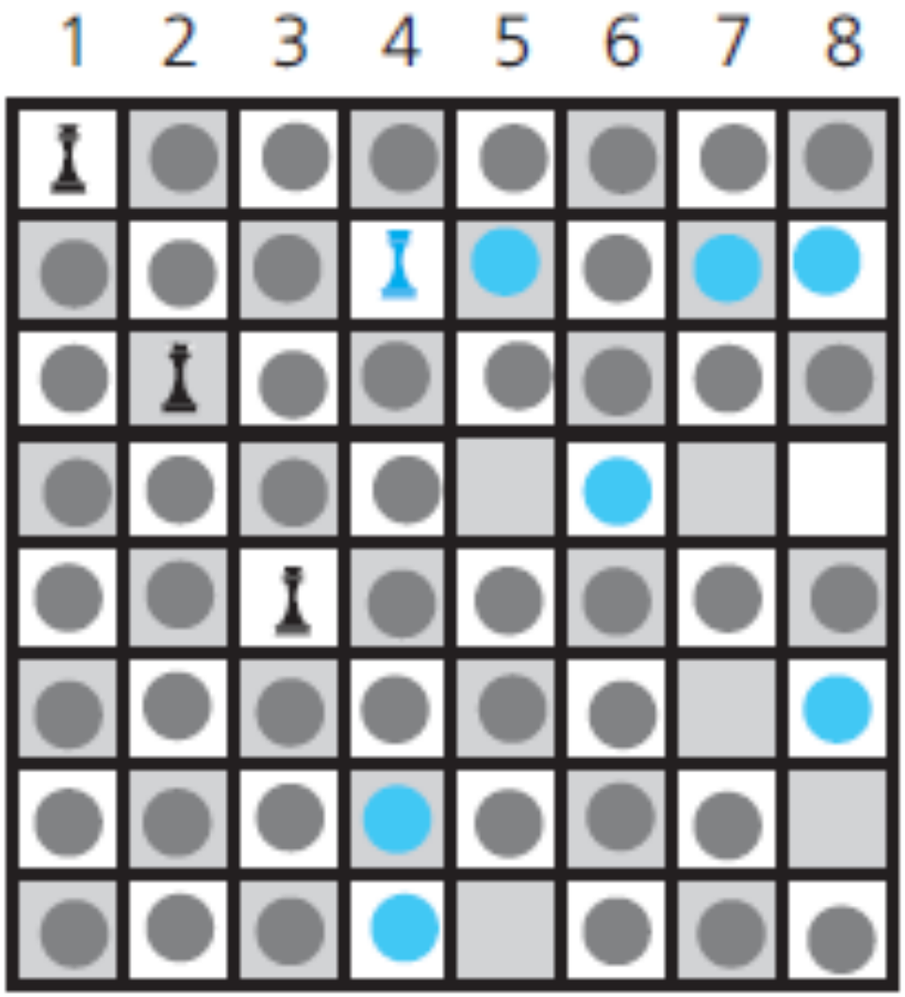
(b) The second queen in column 2

The Eight Queens Problem



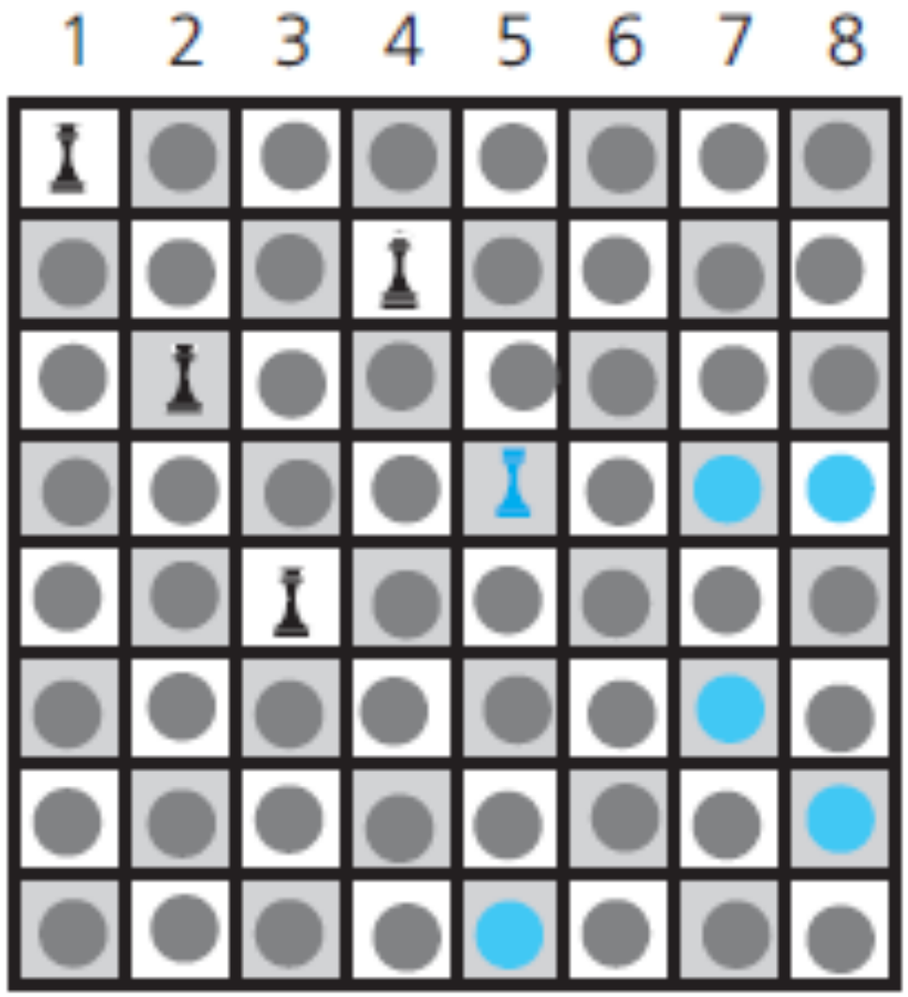
(c) The third queen in column 3

The Eight Queens Problem



(d) The fourth queen in column 4

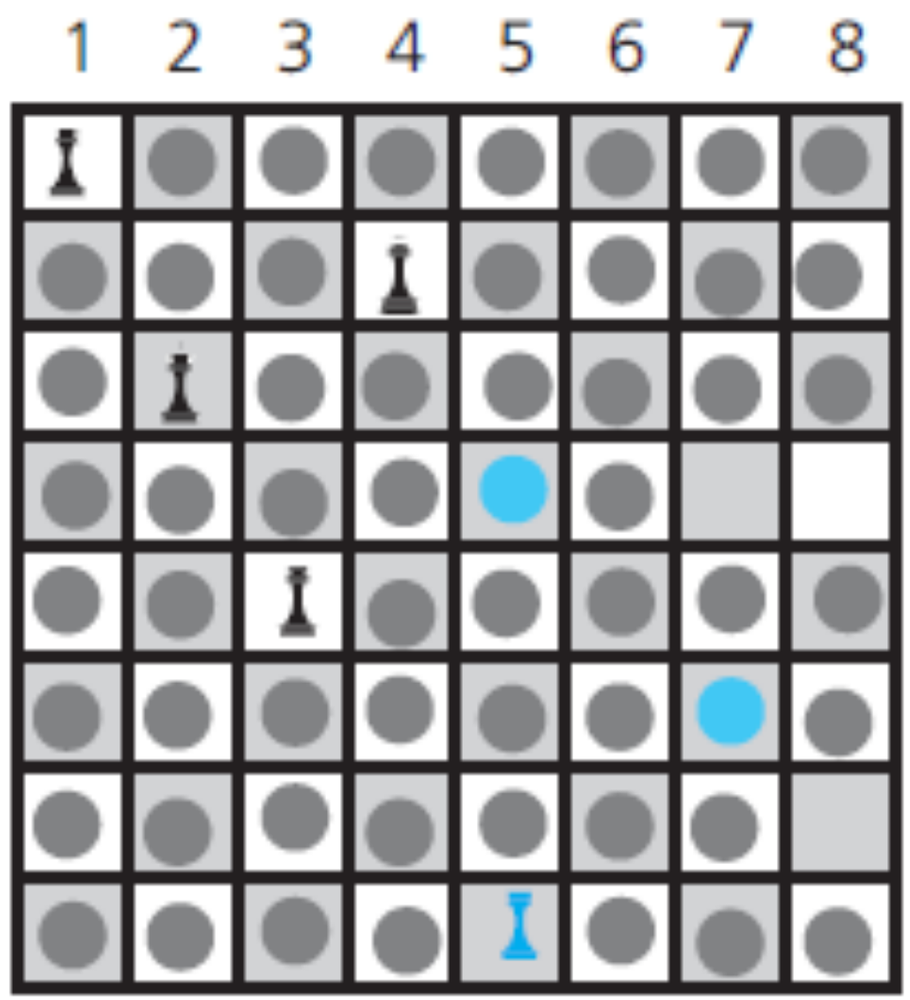
The Eight Queens Problem



(e) The five queens
can attack all of column 6

The Eight Queens Problem

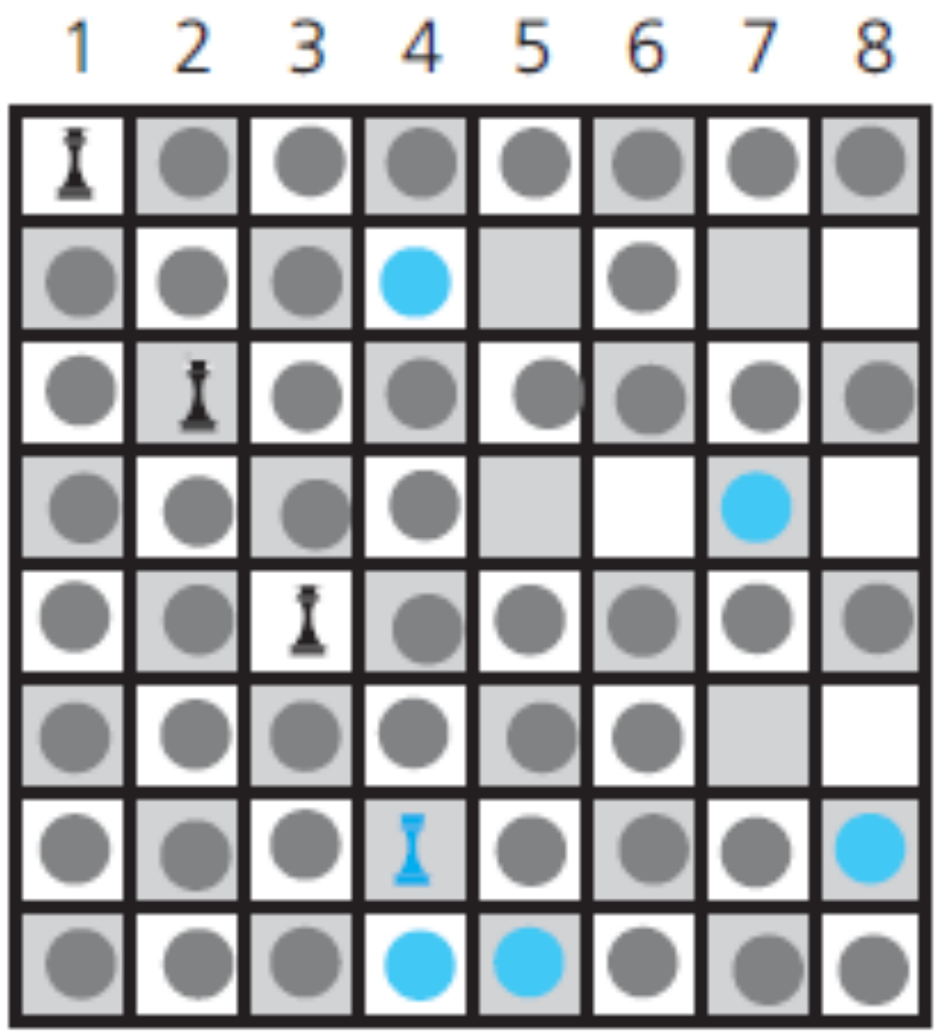
Recursive Backtracking!



(f) Backtracking to column 5 to try another square for the queen

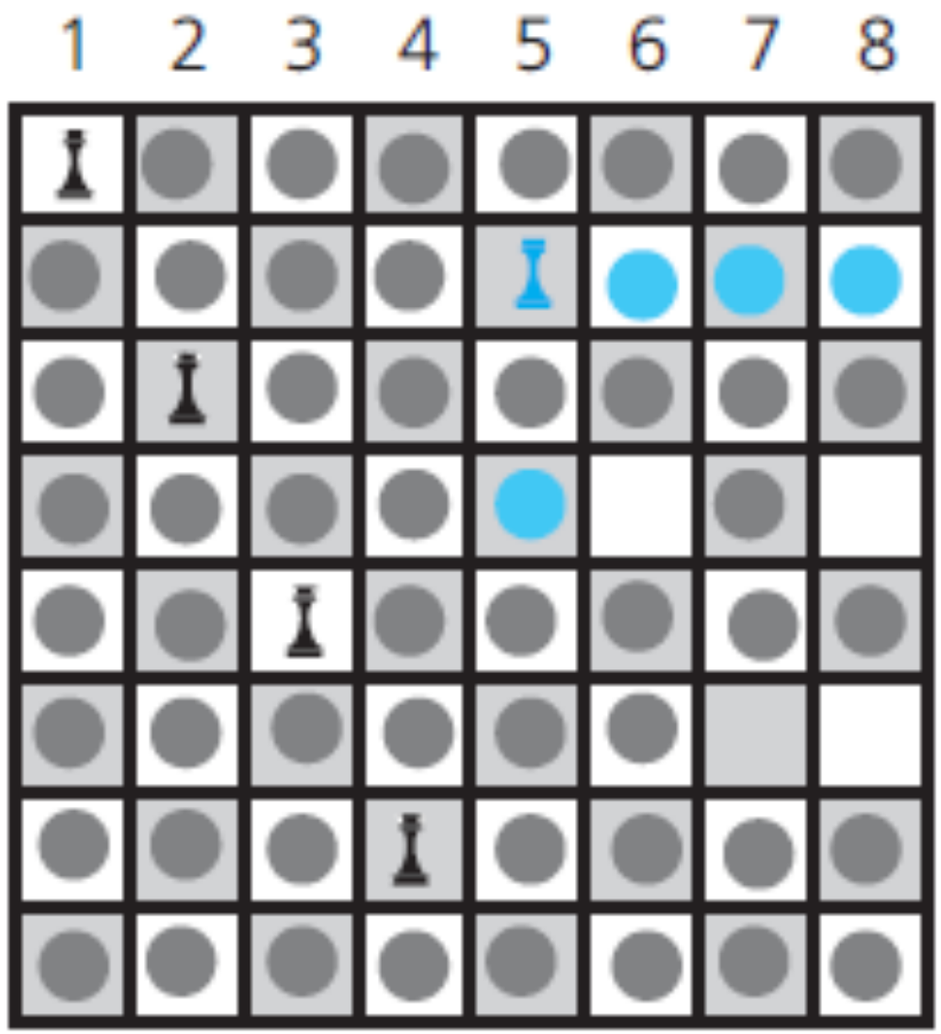
The Eight Queens Problem

Recursive Backtracking!



(g) Backtracking to column 4 to try another square for the queen

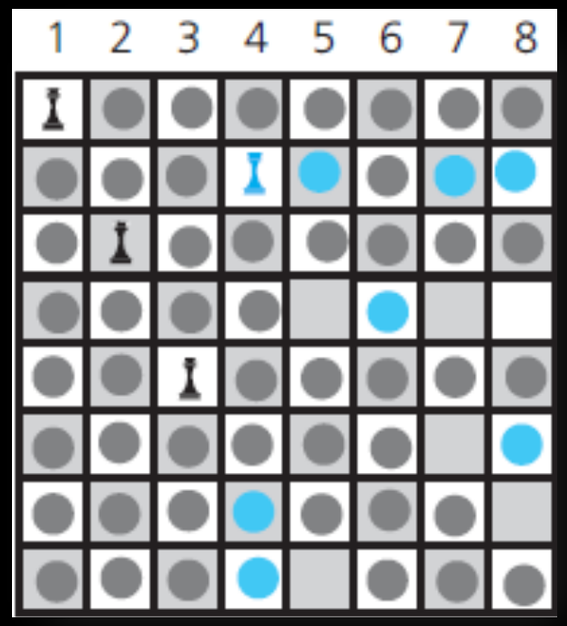
The Eight Queens Problem

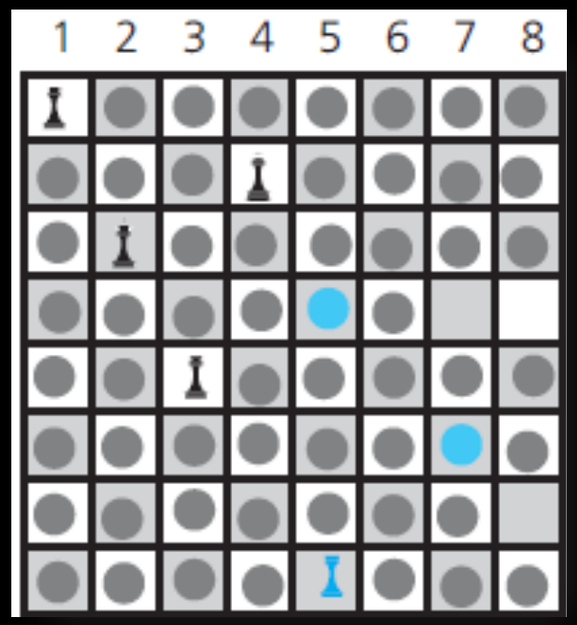


(h) Considering column 5 again

The Eight Queens Problem

```
bool placeQueens(board, column)
{
    if(column > BOARD_SIZE)
        return true; //Problem is solved!
    else
    {
        while(there are safe squares in this column)
        {
            place queen in next safe  square;
            if(placeQueen(board, column+1)) //recursively look forward
                return true; //queen safely placed
        }
        return false; //recursive backtracking
    }
}
```



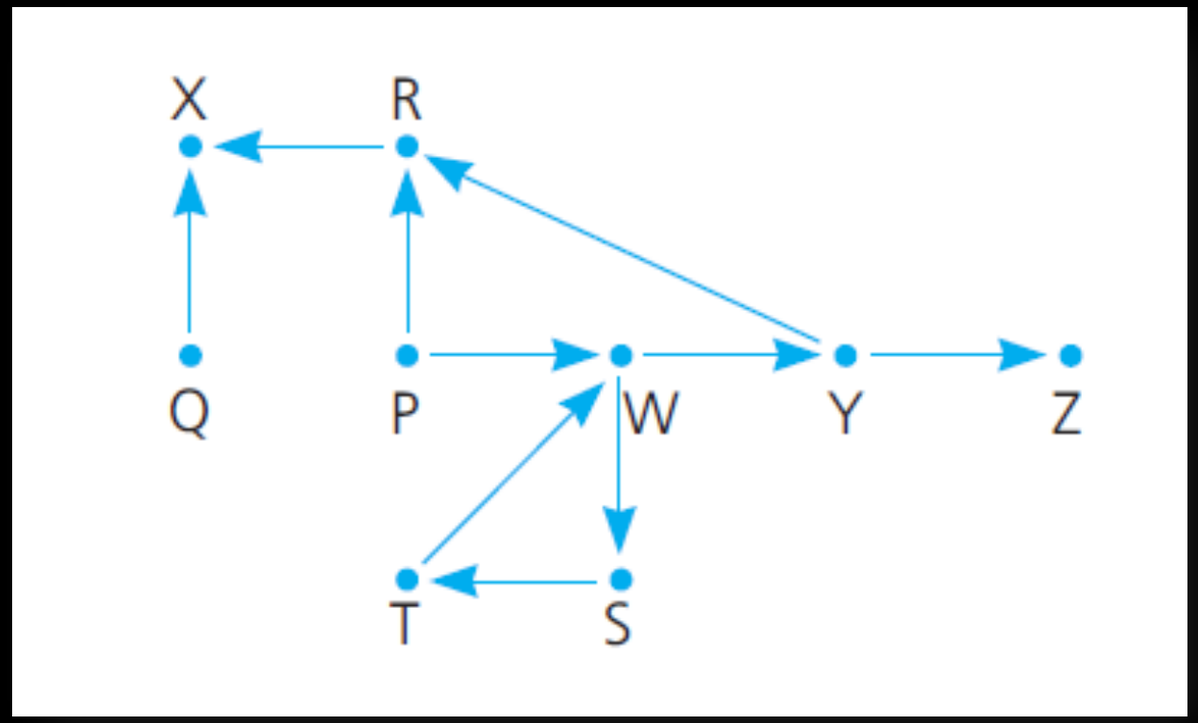


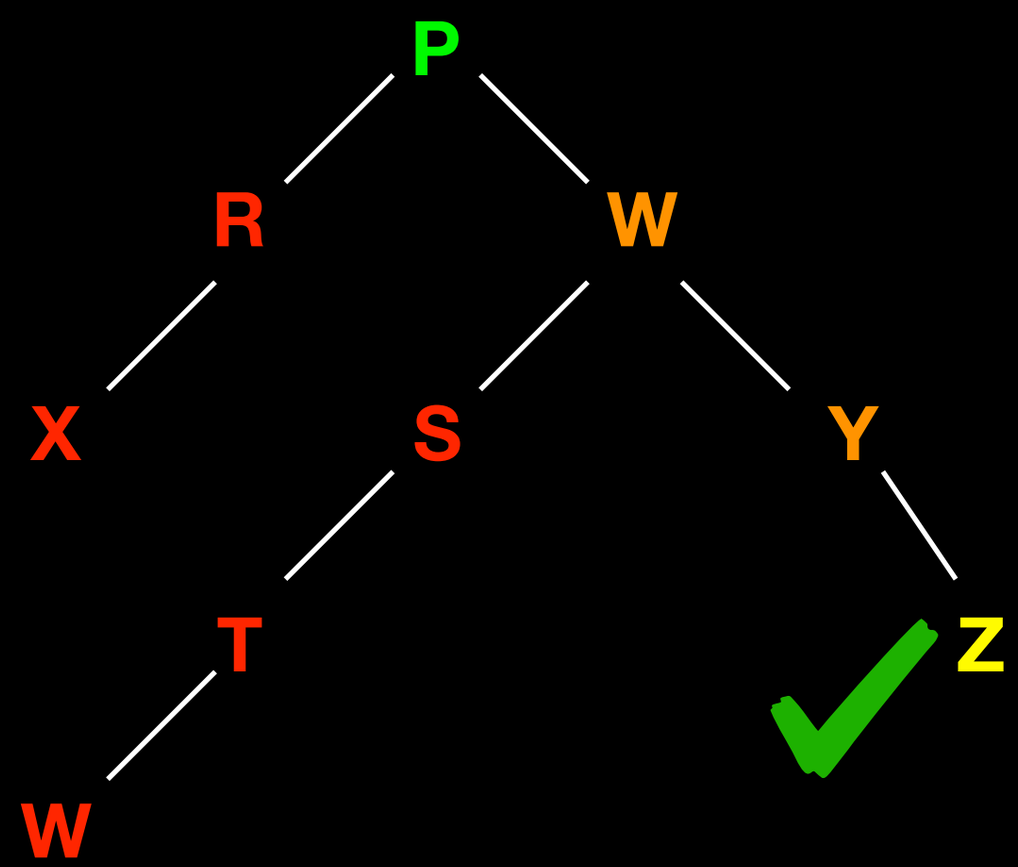
Lecture Activity

Write **PSEUDOCODE** for a **RECURSIVE** function that finds a path from origin to destination

```
bool findPath(map, origin, destination)
```

Origin = P , Destination = Z

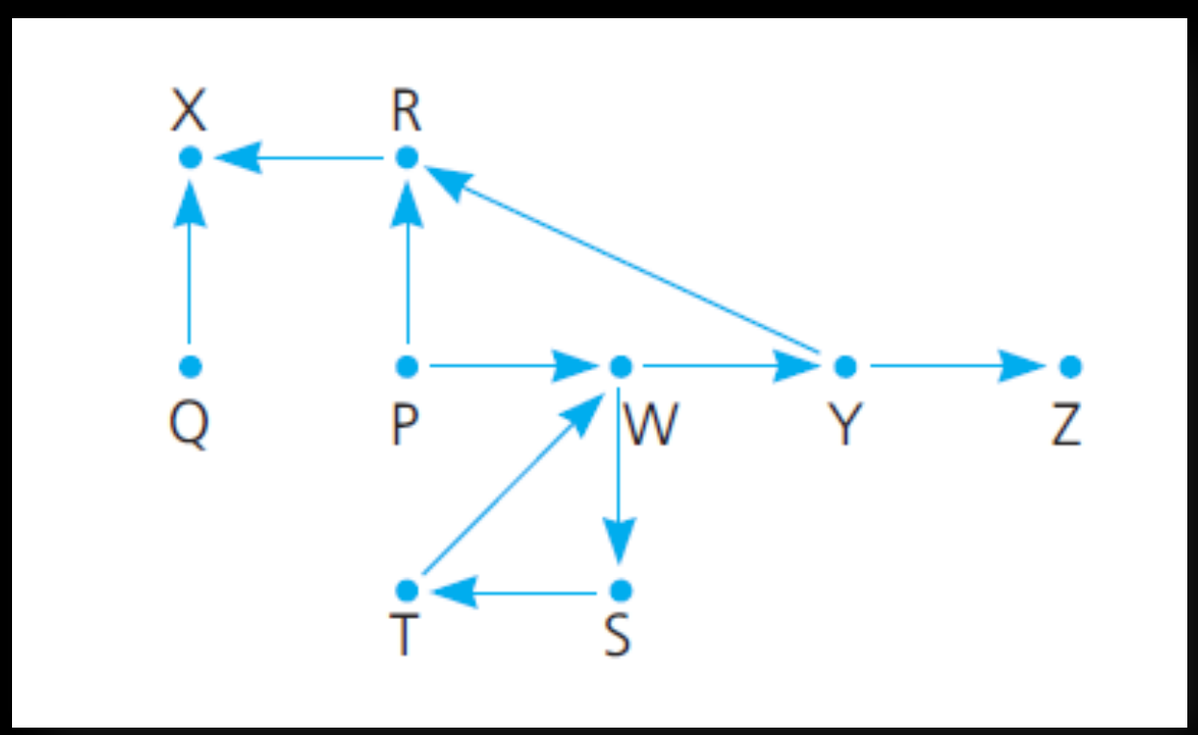


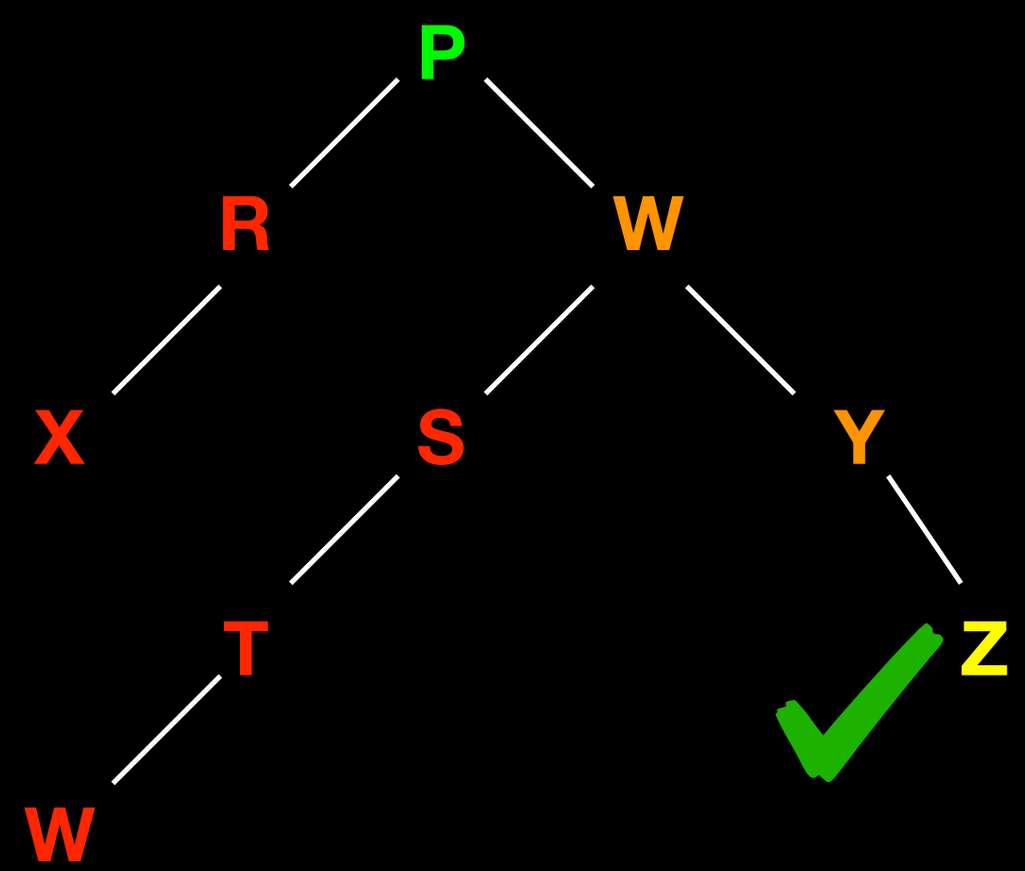


Lecture Activity

```
bool findPath(map, origin, destination)
{
    mark origin as visited in map
    if origin == destination
        return true
    else
        for each unvisited city C reachable from origin
            if findPath(map, C, destination)
                return true
        return false //recursive backtracking
}
```

Origin = P , Destination = Z





Find Permutations

Order Matters!

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D	B	A	C	D	C	A	B	D	D	A	B	C
A	B	D	C	B	A	D	C	C	A	D	B	D	A	C	B
A	C	B	D	B	C	A	D	C	B	A	D	D	B	A	C
A	C	D	B	B	C	D	A	C	B	D	A	D	B	C	A
A	D	B	C	B	D	A	C	C	D	A	B	D	C	A	B
A	D	C	B	B	D	C	A	C	D	B	A	D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D	B	A	C	D	C	A	B	D	D	A	B	C
A	B	D	C	B	A	D	C	C	A	D	B	D	A	C	B
A	C	B	D	B	C	A	D	C	B	A	D	D	B	A	C
A	C	D	B	B	C	D	A	C	B	D	A	D	B	C	A
A	D	B	C	B	D	A	C	C	D	A	B	D	C	A	B
A	D	C	B	B	D	C	A	C	D	B	A	D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations

A	B	C	D

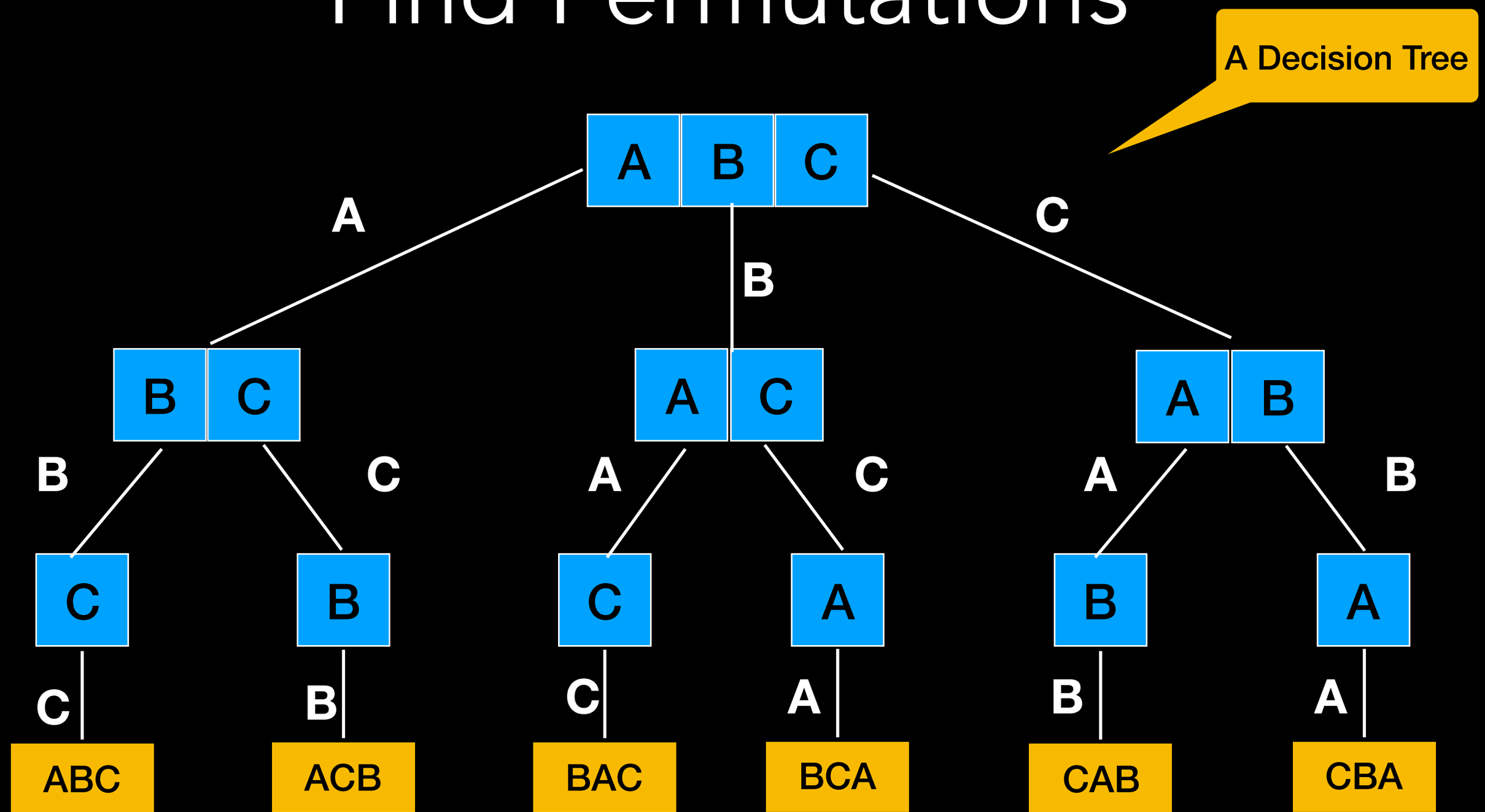
A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B

B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

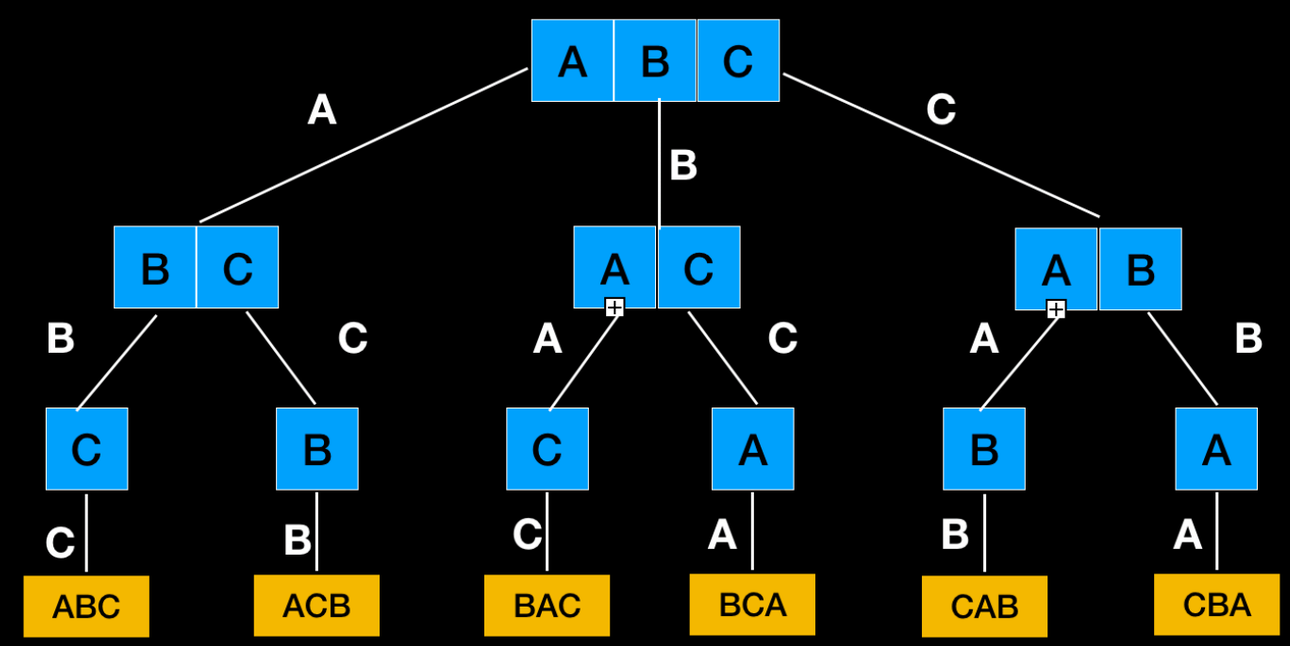
D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

Find Permutations



```
/**
 Prints permutations of a string
 @param str the string to be permuted
 @param l the index of the leftmost character in str substring to be permuted
 @param r the index of the rightmost character in str substring to be permuted
 */
void permuteStr(std::string str, int l, int r)
{
    if (l == r)
        std::cout << str << std::endl; //obtained one permutation to print
    else
    {
        for (int i = l; i <= r; i++)
        {
            std::swap(str[l],str[i]);//swap other characters with current first
            permuteStr(str, l+1, r);
            std::swap(str[l],str[i]); //restore first char
        }
    }
}
```

ABCD
BACD
CBAD
DBCA

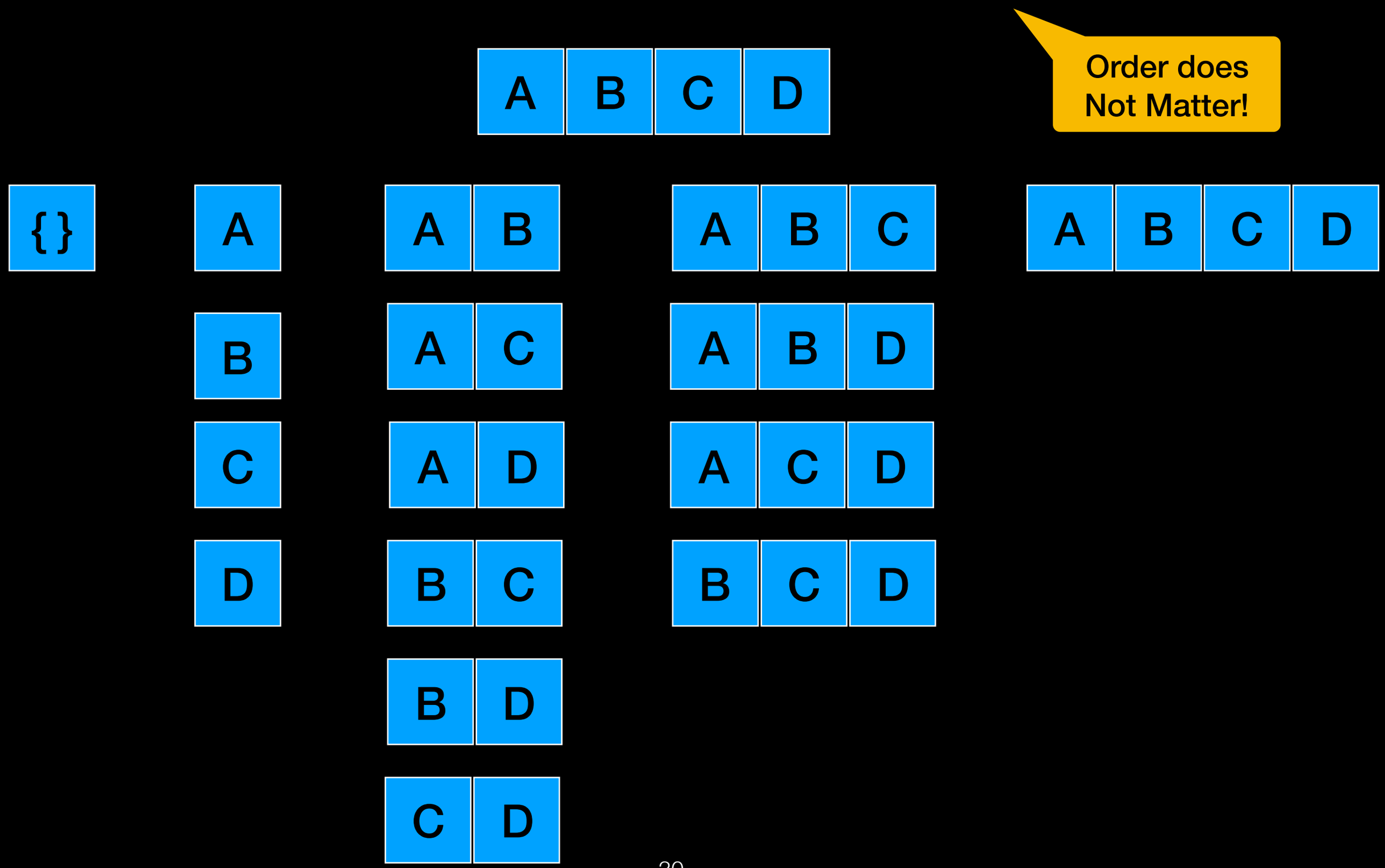


Recursive Decision Tree

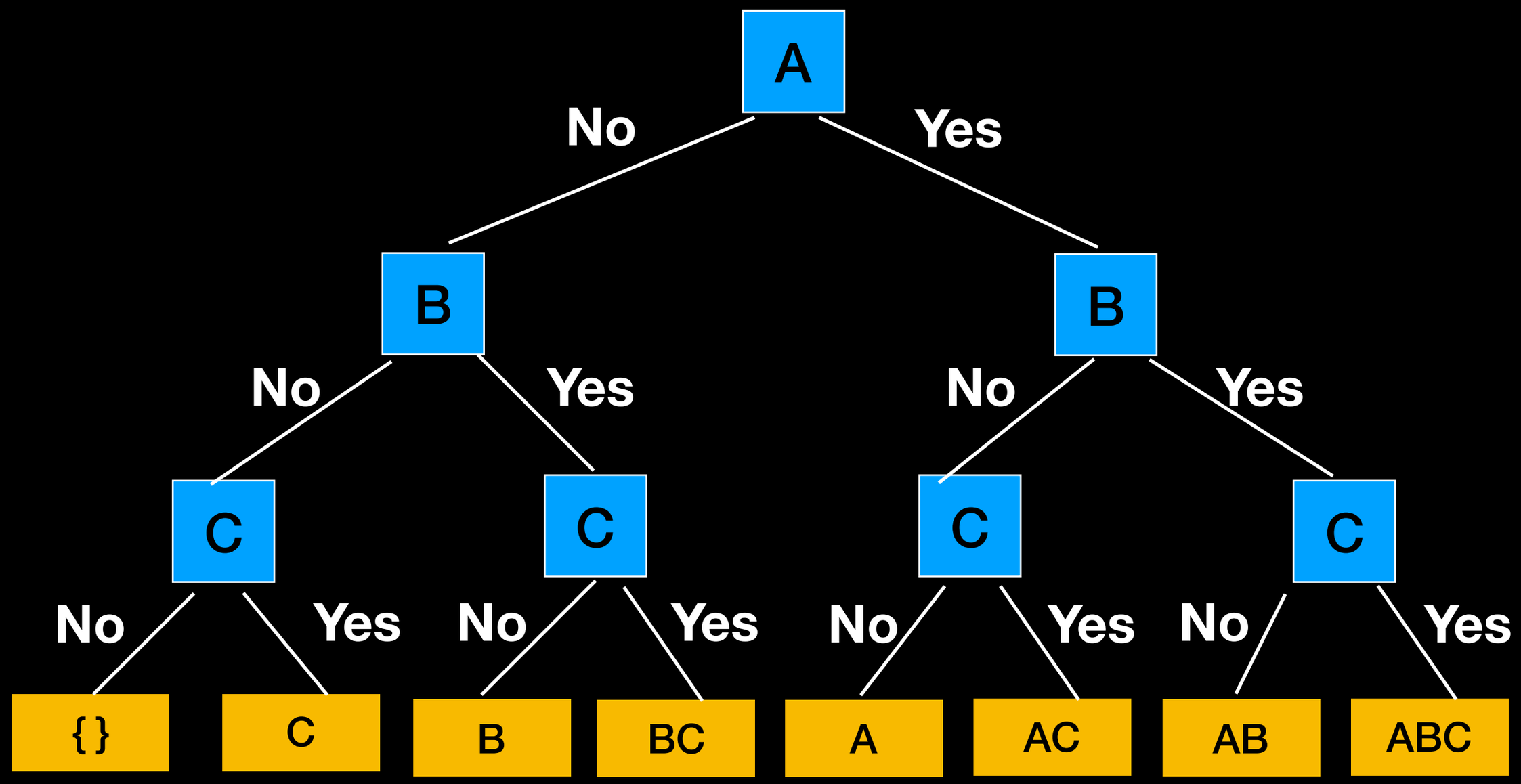
```
void exploreFrom(current_state, decisions_made) {
    if (all decisions have been made) { //base case
        output the result of the decisions we've made;
    } else {
        for (each decision we can make) {
            exploreFrom(result of making that decision,
                        decisions_made + this_decision);
        }
    }
}
```

Generally, if you can express a problem solution with a decision tree you can translate it into a recursive algorithm

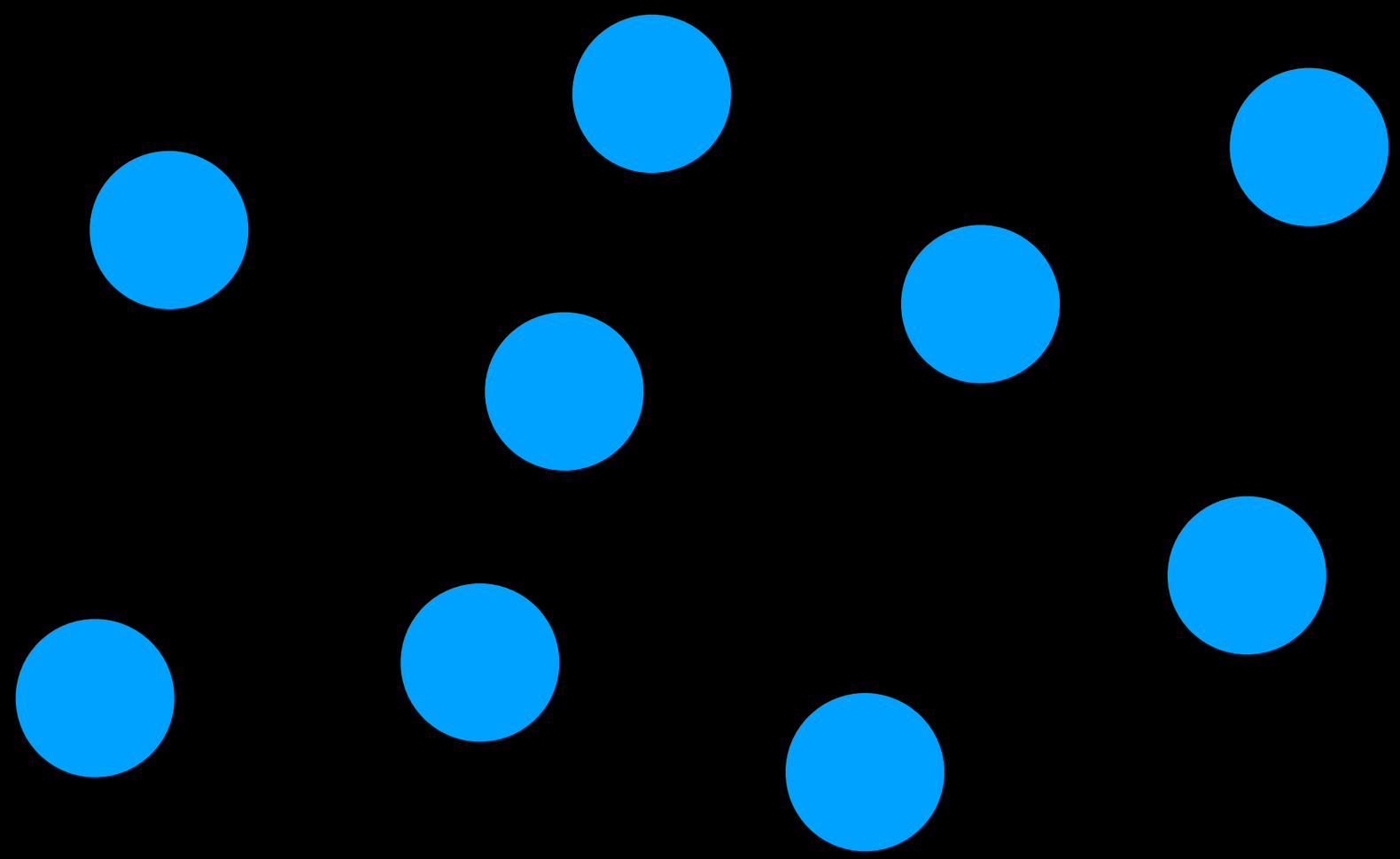
Find Combinations



Find All Combinations

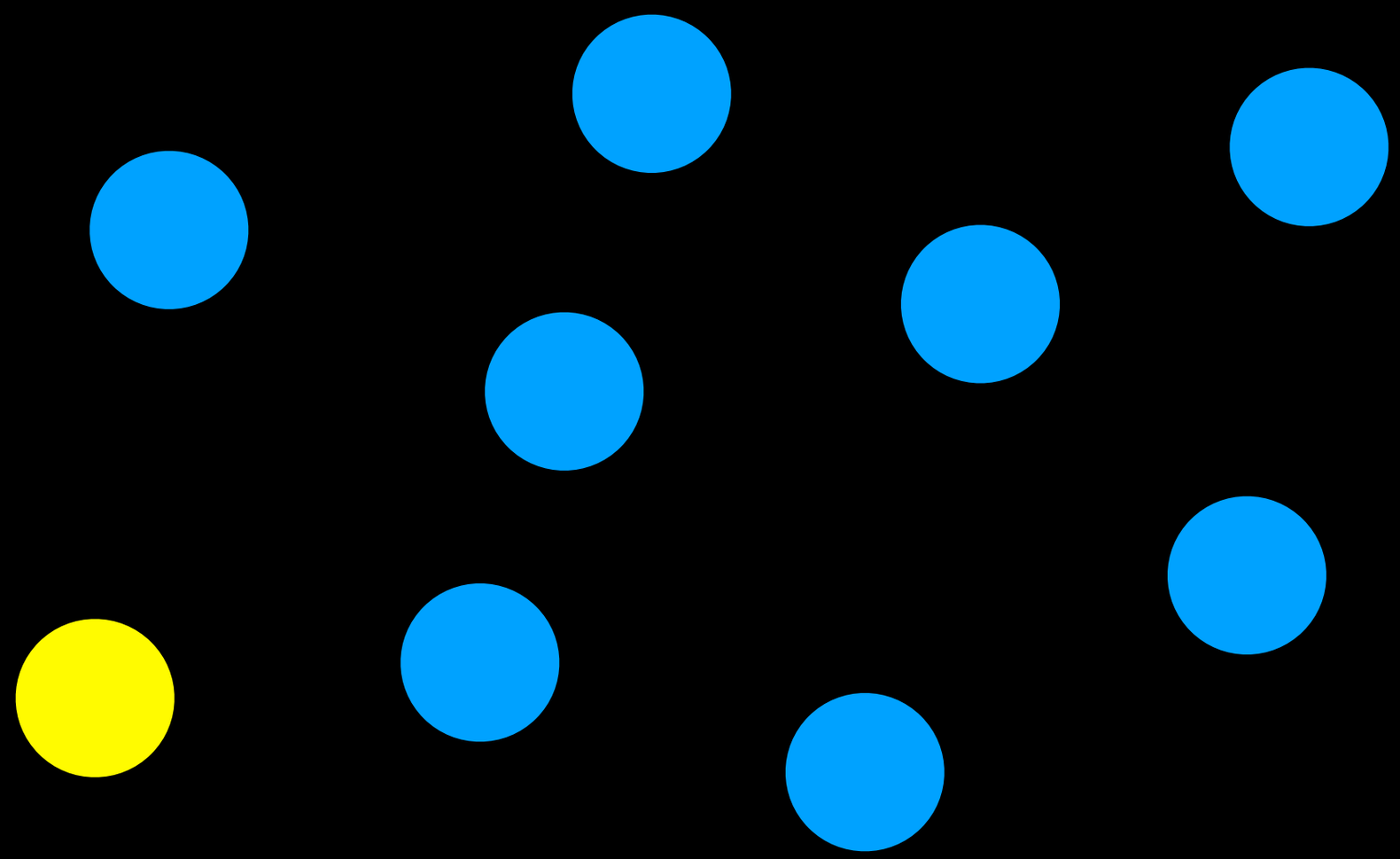


Find Combinations (n choose k)



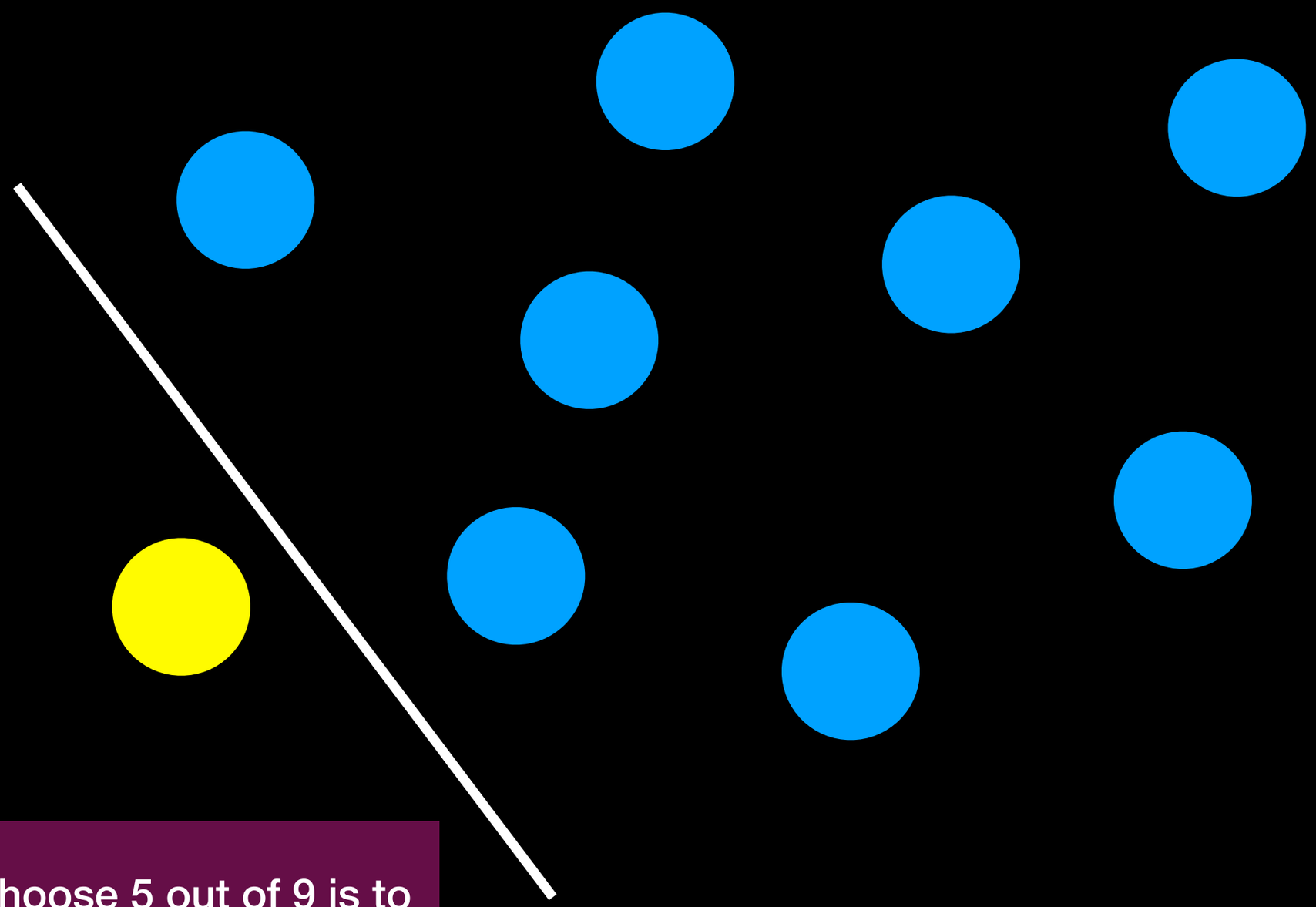
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



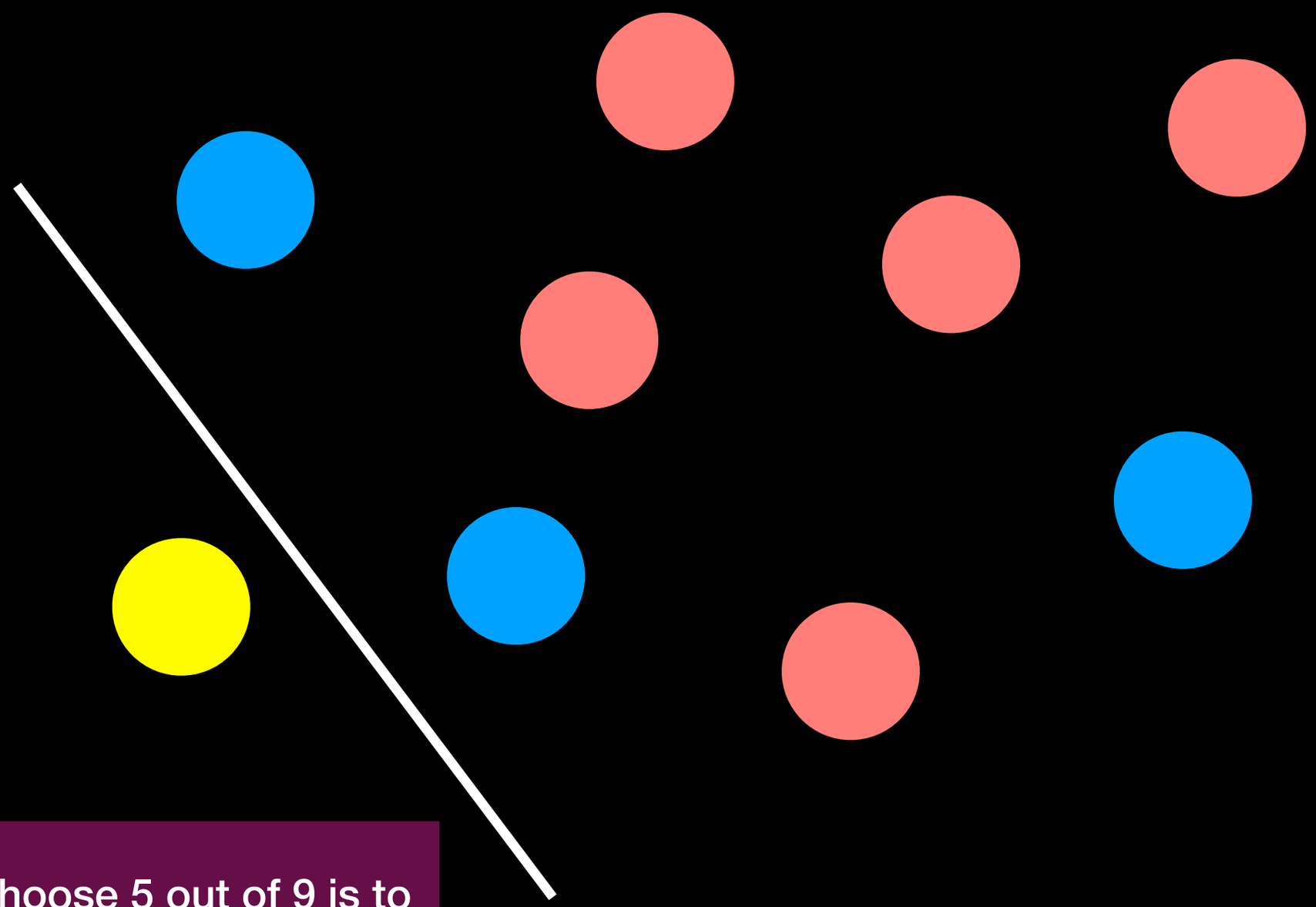
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



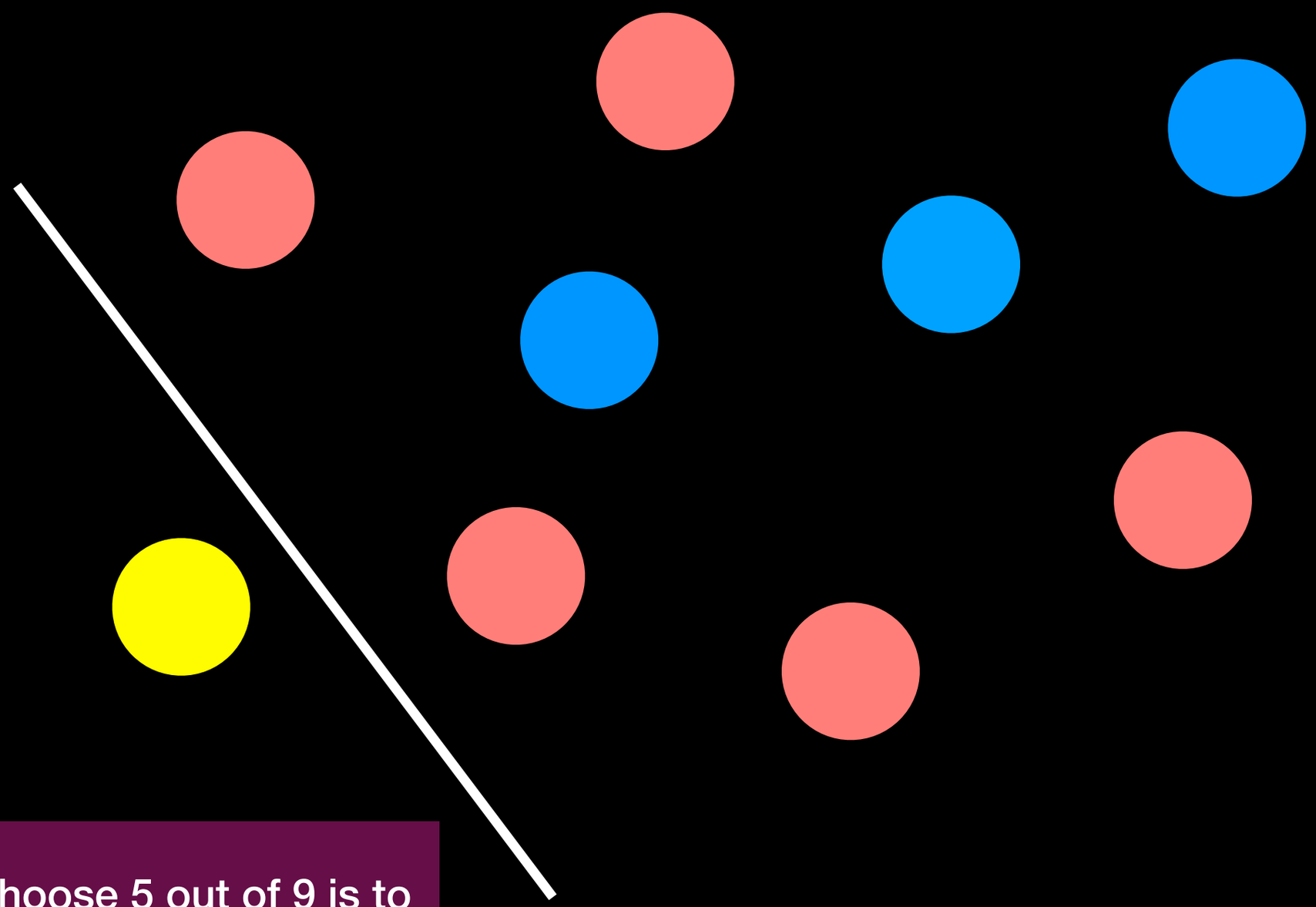
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



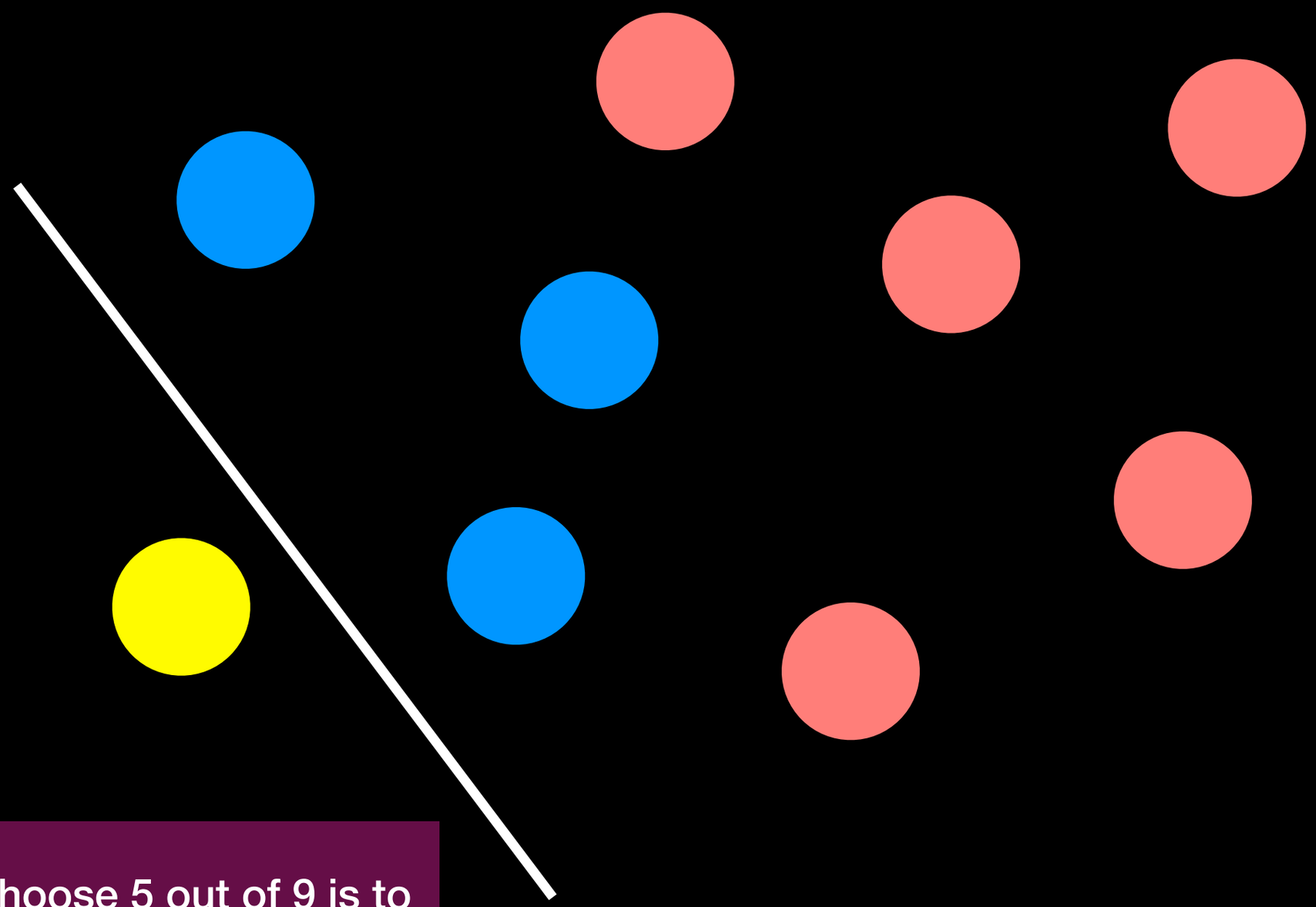
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



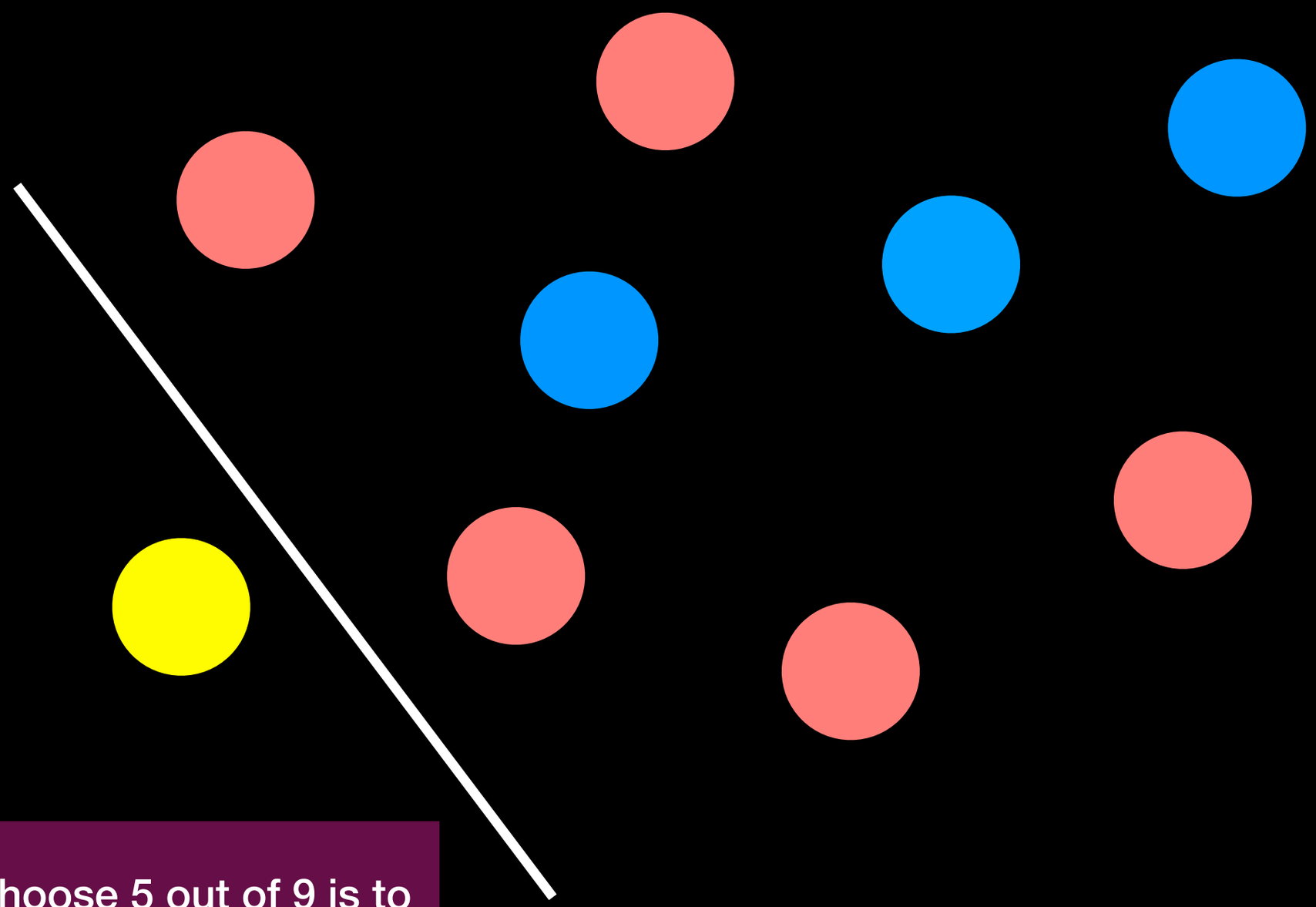
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



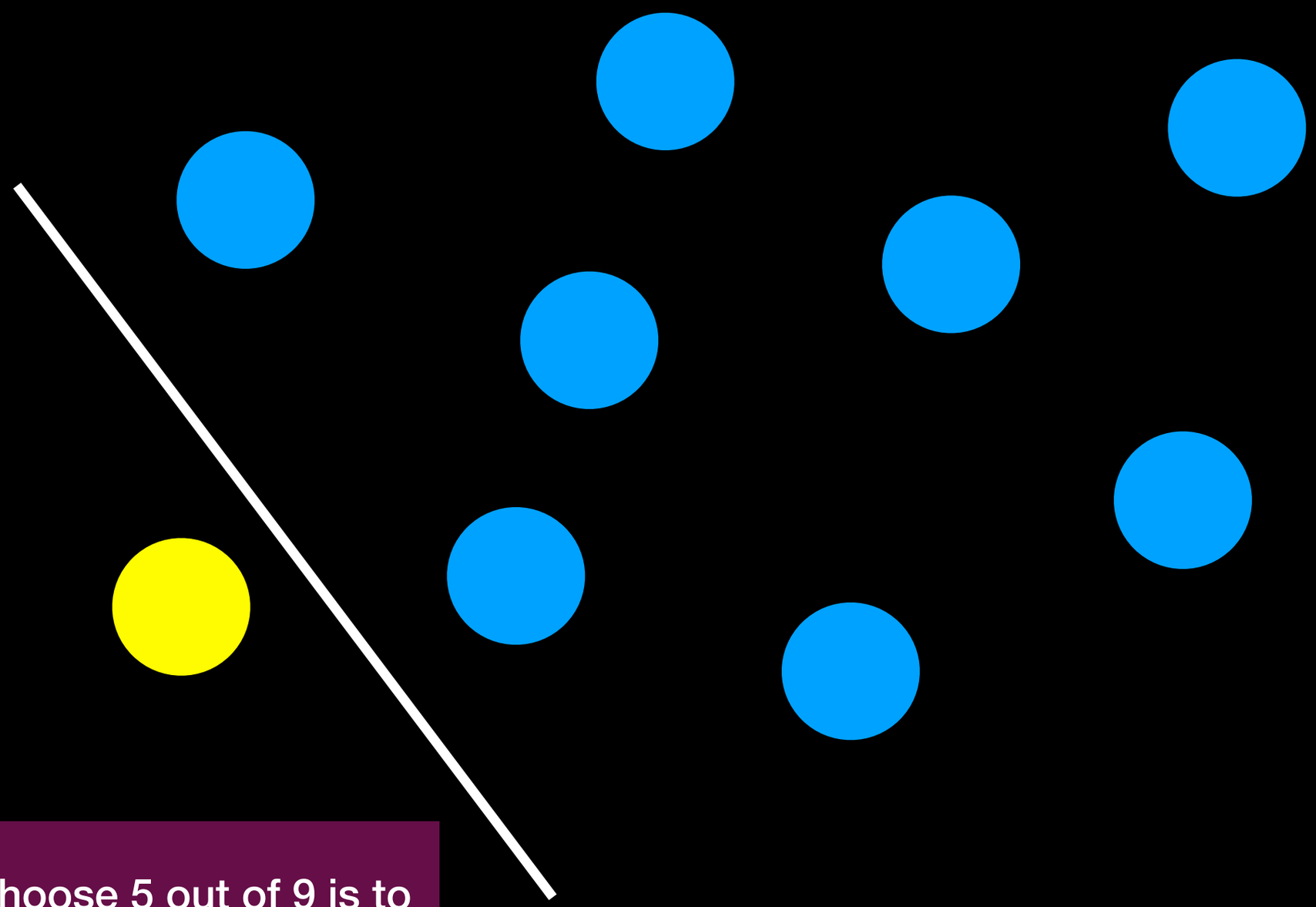
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



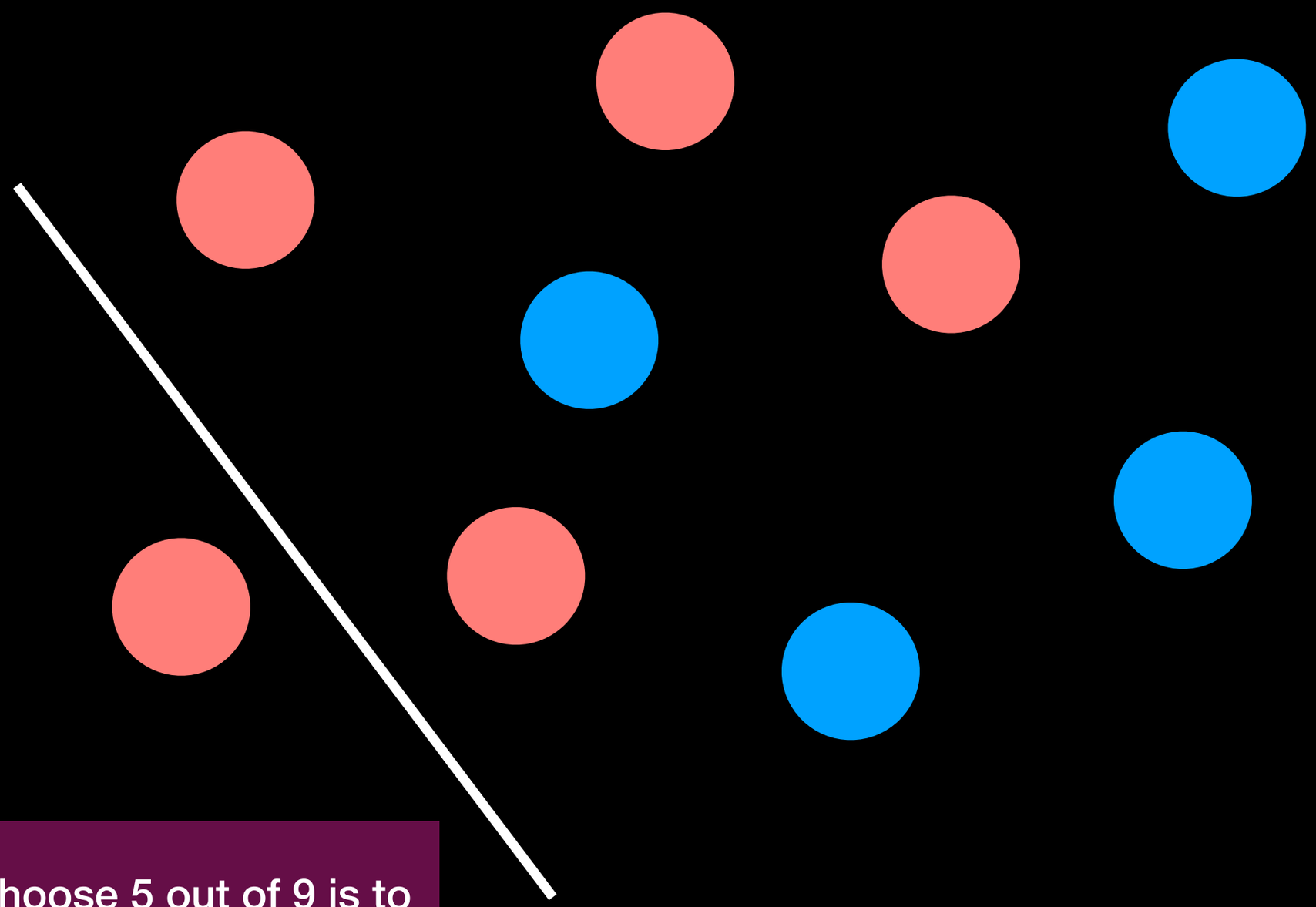
One way to choose 5 out of 9 is to
Exclude 1 and choose 5 out of 8

Find Combinations (n choose k)



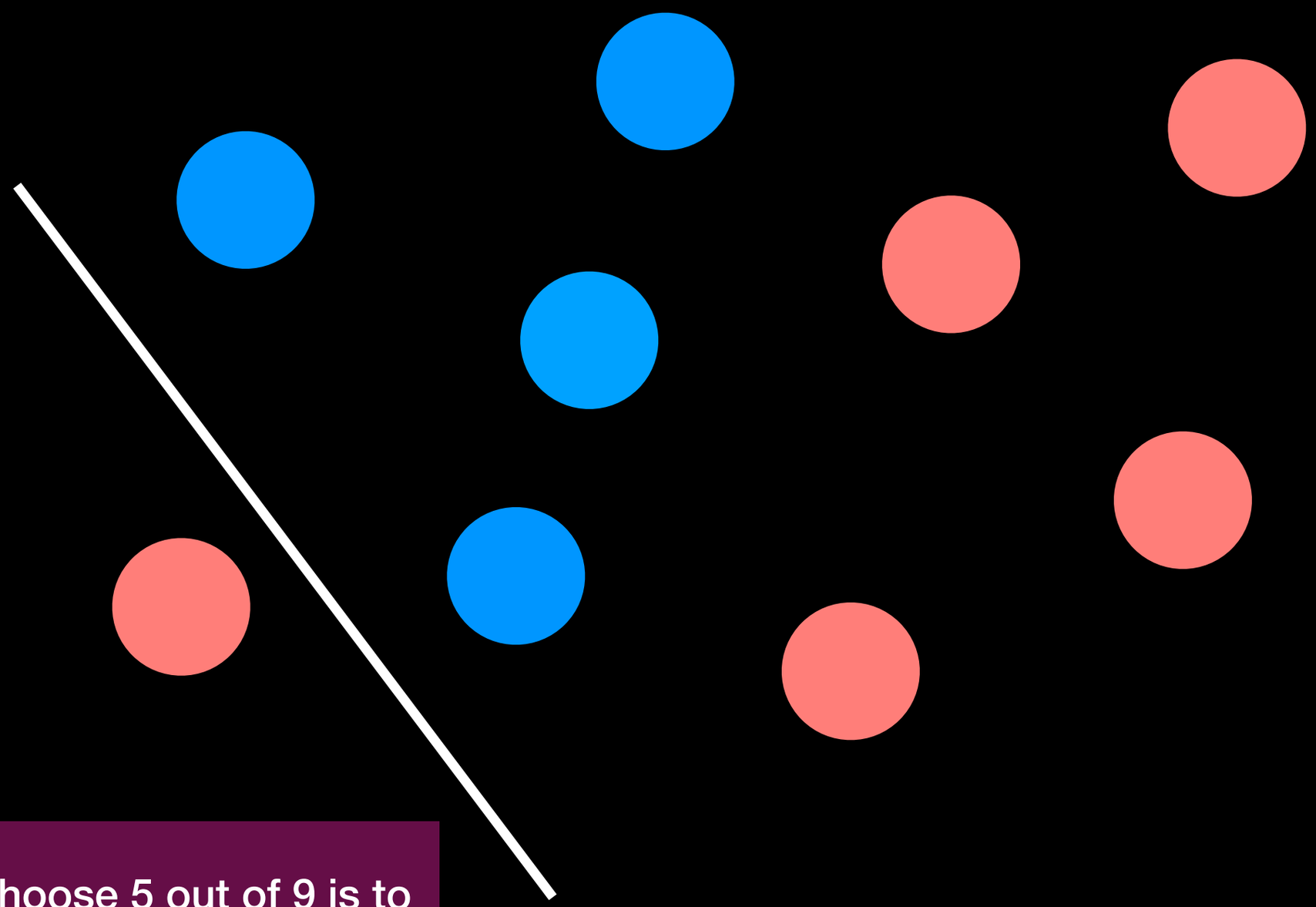
One way to choose 5 out of 9 is to
Include 1 and choose 4 out of 8

Find Combinations (n choose k)



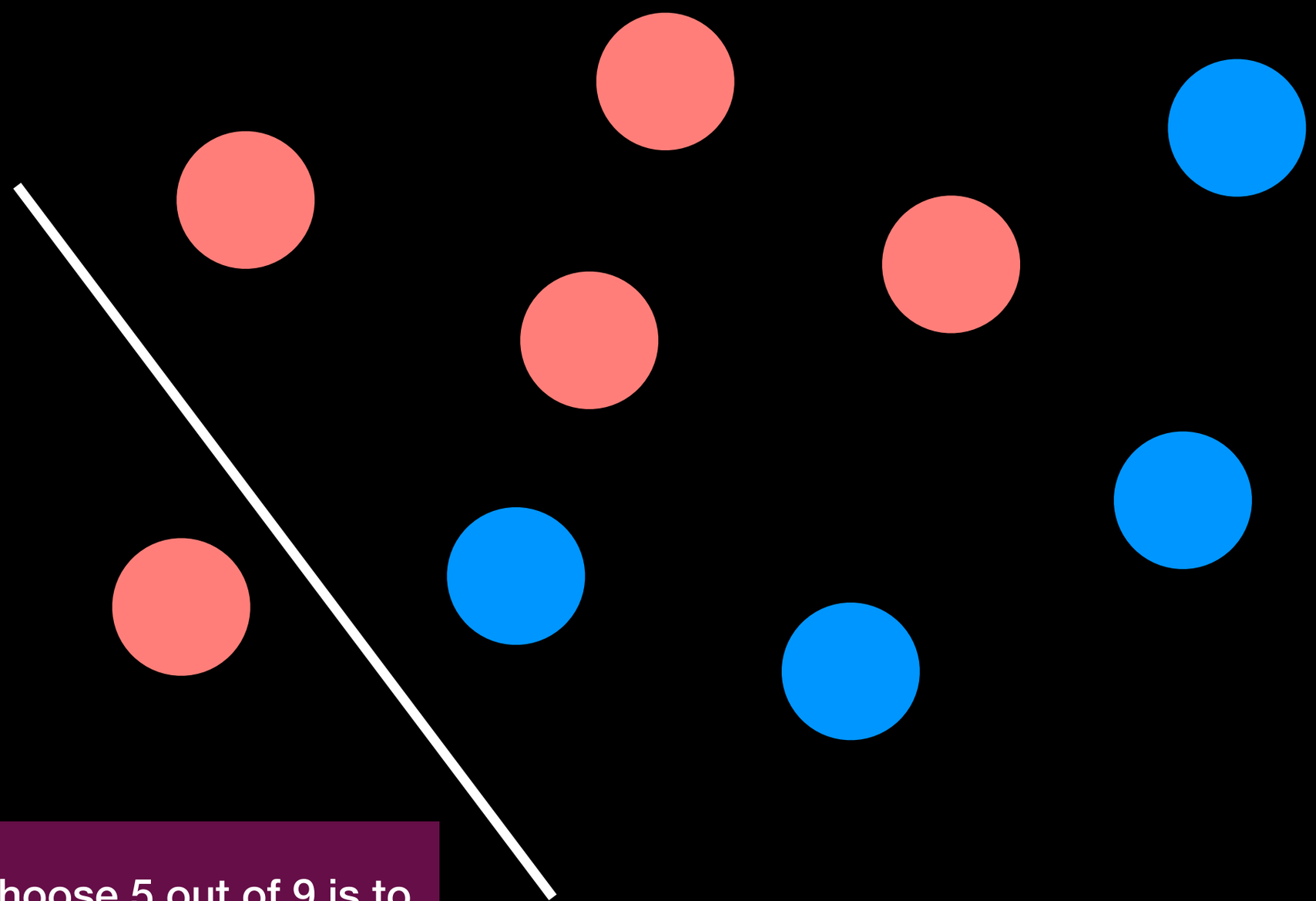
One way to choose 5 out of 9 is to
Include 1 and choose 4 out of 8

Find Combinations (n choose k)



One way to choose 5 out of 9 is to
Include 1 and choose 4 out of 8

Find Combinations (n choose k)



One way to choose 5 out of 9 is to
Include 1 and choose 4 out of 8

Count Combinations

```
int countCombinations(int n, int k)
{
    if ( (k == 0) || (k == n))
        return 1;
    else
        return countCombinations(n-1, k-1) +
                    countCombinations((n-1), k);
}
```

Exam Drill

Write a recursive function that returns true if the input string is a palindrome (same when reversed)

Exam Drill

Write a recursive function that returns true if the input string is a palindrome (same when reversed)

```
bool isPalindrome(std::string s)
{
    if(s.length() == 0 || s.length() == 1) //base case
        return true; //empty string or string of size 1 are palindrome
    if(s[0] == s[s.length()-1]) //if first and last char are same
        //check substring leaving out first and last character
        return isPalindrome(s.substr(1, s.length()-2));

    return false; //not palindrome
}
```

Exam Drill

Write a recursive function for the fibonacci numbers where f(n) = f(n-1) + f(n-2)

Exam Drill

Write a recursive function for the fibonacci numbers where f(n) = f(n-1) + f(n-2)

```
int fib(int n)
{
    if (n <= 1)//base case
        return n;
    return fib(n-1) + fib(n-2);
}
```

Exam Drill

Write a recursive function to find the max value in an array of integers

Exam Drill

Write a recursive function to find the max value in an array of integers

```
int findMax(int* a, int index) {
    if (index > 0)
        return std::max(a[index], findMax(a, index-1));
    else
        return a[0];
}
```