

# Project Report

## SAT-based Sudoku Solving

CSC 320

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# Summary

In this project, our goal is to solve partially solved Sudoku puzzles by formulating Sudoku puzzles as SAT problems. In order to accomplish this, we implemented two program modules *sud2sat* and *sat2sud* in Java. *sud2sat.java* takes an partially solved Sudoku puzzle as its input and converts to a CNF formula suitable for *miniSAT* SAT solver. The output of *sud2sat.java* is a DIMACS encoding of the puzzle which is then given as input to *miniSAT* solver. *sat2sud.java* reads the output of *miniSAT* and then converts it back into a solved Sudoku puzzle that is suitable for printing.

## Solved Puzzles Results

50 Sudoku puzzles provided at [projecteuler.net/project/resources/p096\\_sudoku.txt](http://projecteuler.net/project/resources/p096_sudoku.txt) are solved using our SAT-based Sudoku solver. The results of each solved puzzle are shown below along with the CPU time taken and the memory used.

Grid01

Memory used: 13.4 MB

CPU time: 0 s

4 8 3 | 9 2 1 | 6 5 7

9 6 7 | 3 4 5 | 8 2 1

2 5 1 | 8 7 6 | 4 9 3

-----

5 4 8 | 1 3 2 | 9 7 6

7 2 9 | 5 6 4 | 1 3 8

1 3 6 | 7 9 8 | 2 4 5

-----

3 7 2 | 6 8 9 | 5 1 4

8 1 4 | 2 5 3 | 7 6 9

6 9 5 | 4 1 7 | 3 8 2

Grid02

Memory used: 13.4 MB

CPU time: 0.015625 s

2 4 5 | 9 8 1 | 3 7 6

1 6 9 | 2 7 3 | 5 8 4

8 3 7 | 5 6 4 | 2 1 9

-----

9 7 6 | 1 2 5 | 4 3 8

5 1 3 | 4 9 8 | 6 2 7

4 8 2 | 7 3 6 | 9 5 1

-----

3 9 1 | 6 5 7 | 8 4 2

7 2 8 | 3 4 9 | 1 6 5  
6 5 4 | 8 1 2 | 7 9 3

#### Grid03

Memory used: 13.4 MB

CPU time: 0 s

4 6 2 | 8 3 1 | 9 5 7  
7 9 5 | 4 2 6 | 1 8 3  
3 8 1 | 7 9 5 | 4 2 6

-----  
1 7 3 | 9 8 4 | 2 6 5  
6 5 9 | 3 1 2 | 7 4 8  
2 4 8 | 5 6 7 | 3 1 9

-----  
9 2 6 | 1 7 8 | 5 3 4  
8 3 4 | 2 5 9 | 6 7 1  
5 1 7 | 6 4 3 | 8 9 2

#### Grid04

Memory used: 13.4 MB

CPU time: 0 s

1 3 7 | 2 5 6 | 8 4 9  
9 2 8 | 3 1 4 | 5 6 7  
4 6 5 | 8 9 7 | 3 1 2

-----  
6 7 3 | 5 4 2 | 9 8 1  
8 1 9 | 6 7 3 | 2 5 4  
5 4 2 | 1 8 9 | 7 3 6

-----  
2 5 6 | 7 3 1 | 4 9 8  
3 9 1 | 4 2 8 | 6 7 5  
7 8 4 | 9 6 5 | 1 2 3

#### Grid05

Memory used: 13.4 MB

CPU time: 0 s

5 2 3 | 8 1 6 | 7 4 9  
7 8 4 | 5 9 3 | 1 2 6  
6 9 1 | 4 7 2 | 8 3 5

-----  
2 3 9 | 1 4 5 | 6 8 7  
4 5 7 | 2 6 8 | 9 1 3  
1 6 8 | 9 3 7 | 2 5 4

-----  
3 4 2 | 7 8 9 | 5 6 1  
9 1 5 | 6 2 4 | 3 7 8  
8 7 6 | 3 5 1 | 4 9 2

Grid06

Memory used: 13.53 MB

CPU time: 0 s

1 7 6 | 9 2 3 | 5 8 4  
5 2 4 | 8 1 7 | 6 3 9  
8 9 3 | 6 5 4 | 2 7 1

-----  
9 5 7 | 3 4 8 | 1 6 2  
6 3 8 | 1 9 2 | 4 5 7  
4 1 2 | 7 6 5 | 3 9 8

-----  
2 6 5 | 4 8 9 | 7 1 3  
7 8 1 | 2 3 6 | 9 4 5  
3 4 9 | 5 7 1 | 8 2 6

Grid07

Memory used: 13.4 MB

CPU time: 0 s

1 4 3 | 9 8 6 | 2 5 7  
6 7 9 | 4 2 5 | 3 8 1  
2 8 5 | 7 3 1 | 6 9 4

-----  
9 6 2 | 3 5 4 | 1 7 8  
3 5 7 | 6 1 8 | 9 4 2  
4 1 8 | 2 7 9 | 5 6 3

-----  
8 2 1 | 5 6 7 | 4 3 9  
7 9 6 | 1 4 3 | 8 2 5  
5 3 4 | 8 9 2 | 7 1 6

Grid08

Memory used: 13.4 MB

CPU time: 0 s

4 8 7 | 1 5 6 | 9 3 2  
3 6 2 | 4 9 8 | 7 5 1  
9 1 5 | 3 7 2 | 8 6 4

-----  
8 4 6 | 5 1 9 | 2 7 3

5 9 3 | 7 2 4 | 1 8 6  
2 7 1 | 8 6 3 | 5 4 9

-----  
1 2 4 | 6 8 5 | 3 9 7  
7 3 8 | 9 4 1 | 6 2 5  
6 5 9 | 2 3 7 | 4 1 8

Grid09

Memory used: 13.53 MB

CPU time: 0.015625

8 1 4 | 9 7 6 | 5 3 2  
6 5 9 | 1 2 3 | 4 7 8  
7 3 2 | 8 5 4 | 1 6 9

-----  
9 4 8 | 2 6 5 | 3 1 7  
2 7 5 | 3 4 1 | 8 9 6  
1 6 3 | 7 9 8 | 2 4 5

-----  
3 9 1 | 6 8 2 | 7 5 4  
5 8 7 | 4 3 9 | 6 2 1  
4 2 6 | 5 1 7 | 9 8 3

Grid10

Memory used: 13.4 MB

CPU time: 0 s

7 6 1 | 9 2 8 | 4 5 3  
9 2 5 | 7 4 3 | 1 6 8  
4 3 8 | 6 1 5 | 9 2 7

-----  
3 5 7 | 4 6 1 | 2 8 9  
8 9 4 | 3 7 2 | 6 1 5  
2 1 6 | 5 8 9 | 3 7 4

-----  
6 8 9 | 1 5 4 | 7 3 2  
1 4 2 | 8 3 7 | 5 9 6  
5 7 3 | 2 9 6 | 8 4 1

Grid11

Memory used: 13.4 MB

CPU time: 0 s

9 7 6 | 1 2 5 | 4 3 8  
1 5 8 | 4 3 6 | 9 2 7  
4 2 3 | 8 7 9 | 1 5 6

-----  
2 3 4 | 7 6 1 | 8 9 5  
8 6 7 | 9 5 2 | 3 1 4  
5 1 9 | 3 8 4 | 7 6 2  
-----

7 8 2 | 5 1 3 | 6 4 9  
3 9 5 | 6 4 7 | 2 8 1  
6 4 1 | 2 9 8 | 5 7 3

Grid12

Memory used: 13.4 MB

CPU time: 0 s

9 6 2 | 3 4 1 | 7 5 8  
1 4 8 | 9 7 5 | 6 2 3  
5 7 3 | 2 6 8 | 1 4 9  
-----

3 2 1 | 6 9 4 | 8 7 5  
4 8 7 | 5 1 2 | 9 3 6  
6 9 5 | 8 3 7 | 4 1 2  
-----

8 3 4 | 7 2 6 | 5 9 1  
2 1 6 | 4 5 9 | 3 8 7  
7 5 9 | 1 8 3 | 2 6 4

Grid13

Memory used: 13.4 MB

CPU time: 0 s

3 9 7 | 6 8 1 | 5 2 4  
6 4 5 | 2 7 9 | 8 1 3  
2 1 8 | 5 3 4 | 9 7 6  
-----

8 2 3 | 9 5 6 | 7 4 1  
1 6 9 | 7 4 2 | 3 5 8  
7 5 4 | 3 1 8 | 6 9 2  
-----

4 7 2 | 8 9 3 | 1 6 5  
5 3 1 | 4 6 7 | 2 8 9  
9 8 6 | 1 2 5 | 4 3 7

Grid14

Memory used: 13.4 MB

CPU time: 0 s

6 3 9 | 2 1 8 | 4 5 7

4 7 1 | 5 3 9 | 2 6 8  
8 2 5 | 6 7 4 | 1 3 9

-----  
5 6 4 | 8 2 3 | 7 9 1  
7 9 3 | 4 5 1 | 8 2 6  
2 1 8 | 7 9 6 | 3 4 5

-----  
3 5 2 | 9 8 7 | 6 1 4  
1 8 6 | 3 4 5 | 9 7 2  
9 4 7 | 1 6 2 | 5 8 3

Grid15

Memory used: 13.4 MB

CPU time: 0 s

6 9 7 | 1 2 8 | 3 4 5  
4 2 8 | 6 3 5 | 1 9 7  
3 1 5 | 4 7 9 | 6 8 2

-----  
5 3 1 | 2 4 6 | 9 7 8  
2 8 6 | 3 9 7 | 4 5 1  
9 7 4 | 5 8 1 | 2 6 3

-----  
1 4 9 | 8 5 2 | 7 3 6  
7 5 2 | 9 6 3 | 8 1 4  
8 6 3 | 7 1 4 | 5 2 9

Grid16

Memory used: 13.4 MB

CPU time: 0 s

3 6 1 | 7 2 5 | 9 4 8  
5 8 7 | 9 6 4 | 2 1 3  
4 9 2 | 8 3 1 | 6 5 7

-----  
6 3 8 | 2 5 9 | 4 7 1  
1 7 4 | 6 8 3 | 5 9 2  
2 5 9 | 1 4 7 | 8 3 6

-----  
7 4 6 | 3 9 2 | 1 8 5  
9 2 3 | 5 1 8 | 7 6 4  
8 1 5 | 4 7 6 | 3 2 9

Grid17

Memory used: 13.4 MB

CPU time: 0 s

3 5 9 | 8 6 7 | 1 2 4

6 4 8 | 3 1 2 | 5 9 7

7 1 2 | 5 4 9 | 8 3 6

-----

8 7 6 | 9 2 4 | 3 5 1

5 2 4 | 7 3 1 | 9 6 8

1 9 3 | 6 8 5 | 4 7 2

-----

9 3 1 | 4 7 6 | 2 8 5

4 6 5 | 2 9 8 | 7 1 3

2 8 7 | 1 5 3 | 6 4 9

Grid18

Memory used: 13.4 MB

CPU time: 0 s

7 8 6 | 9 4 5 | 3 1 2

2 1 9 | 8 6 3 | 4 5 7

5 3 4 | 2 7 1 | 8 6 9

-----

1 6 5 | 4 8 2 | 9 7 3

3 2 7 | 6 1 9 | 5 4 8

4 9 8 | 5 3 7 | 1 2 6

-----

9 5 1 | 7 2 8 | 6 3 4

8 4 2 | 3 5 6 | 7 9 1

6 7 3 | 1 9 4 | 2 8 5

Grid19

Memory used: 13.4 MB

CPU time: 0 s

7 4 3 | 5 1 2 | 9 8 6

5 8 9 | 3 4 6 | 2 1 7

1 2 6 | 9 8 7 | 3 4 5

-----

9 3 4 | 2 5 1 | 7 6 8

6 7 1 | 4 9 8 | 5 3 2

8 5 2 | 7 6 3 | 4 9 1

-----

3 9 8 | 6 7 5 | 1 2 4

4 1 7 | 8 2 9 | 6 5 3

2 6 5 | 1 3 4 | 8 7 9



Grid20

Memory used: 13.39 MB

CPU time: 0 s

7 8 2 | 6 1 4 | 3 5 9

4 3 9 | 8 2 5 | 1 7 6

6 5 1 | 9 3 7 | 4 2 8

-----

2 9 3 | 4 7 1 | 8 6 5

5 6 8 | 3 9 2 | 7 1 4

1 4 7 | 5 6 8 | 2 9 3

-----

3 2 6 | 7 4 9 | 5 8 1

9 7 5 | 1 8 3 | 6 4 2

8 1 4 | 2 5 6 | 9 3 7

Grid21

Memory used: 13.4 MB

CPU time: 0 s

4 2 8 | 5 3 1 | 7 9 6

3 6 5 | 9 4 7 | 1 8 2

9 7 1 | 2 6 8 | 4 3 5

-----

2 1 4 | 8 9 6 | 5 7 3

6 9 7 | 4 5 3 | 2 1 8

5 8 3 | 1 7 2 | 9 6 4

-----

8 4 9 | 6 1 5 | 3 2 7

7 5 2 | 3 8 9 | 6 4 1

1 3 6 | 7 2 4 | 8 5 9

Grid22

Memory used: 13.4 MB

CPU time: 0.015625 s

4 2 5 | 7 8 1 | 9 3 6

1 7 8 | 3 6 9 | 5 2 4

3 6 9 | 5 2 4 | 1 8 7

-----

8 9 4 | 1 5 7 | 3 6 2

6 5 2 | 8 4 3 | 7 9 1

7 1 3 | 6 9 2 | 8 4 5

-----

9 8 7 | 2 1 6 | 4 5 3

5 3 6 | 4 7 8 | 2 1 9

2 4 1 | 9 3 5 | 6 7 8

Grid23

Memory used: 13.4 MB

CPU time: 0.015625 s

3 4 8 | 2 6 7 | 9 5 1

5 7 1 | 9 4 3 | 6 2 8

2 6 9 | 1 8 5 | 3 7 4

-----

6 9 7 | 3 5 1 | 4 8 2

1 2 3 | 8 7 4 | 5 9 6

8 5 4 | 6 2 9 | 1 3 7

-----

4 1 5 | 7 9 8 | 2 6 3

9 8 2 | 4 3 6 | 7 1 5

7 3 6 | 5 1 2 | 8 4 9

Grid24

Memory used: 13.4 MB

CPU time: 0 s

1 2 4 | 9 8 6 | 7 3 5

8 6 7 | 4 3 5 | 9 1 2

3 9 5 | 7 1 2 | 6 8 4

-----

4 7 8 | 3 5 9 | 2 6 1

2 5 9 | 8 6 1 | 3 4 7

6 3 1 | 2 7 4 | 5 9 8

-----

7 1 2 | 6 9 8 | 4 5 3

9 8 3 | 5 4 7 | 1 2 6

5 4 6 | 1 2 3 | 8 7 9

Grid25

Memory used: 13.4 MB

CPU time: 0.015625 s

3 6 1 | 5 2 4 | 7 8 9

7 8 9 | 3 6 1 | 4 2 5

5 2 4 | 8 7 9 | 3 6 1

-----

8 9 3 | 1 5 7 | 6 4 2

4 1 2 | 6 8 3 | 5 9 7

6 5 7 | 9 4 2 | 1 3 8

-----

1 4 8 | 7 9 6 | 2 5 3

2 3 5 | 4 1 8 | 9 7 6  
9 7 6 | 2 3 5 | 8 1 4

Grid26

Memory used: 13.54 MB

CPU time: 0 s

5 8 1 | 4 7 9 | 2 6 3  
3 2 9 | 1 5 6 | 8 4 7  
6 4 7 | 3 2 8 | 1 5 9

-----  
9 5 6 | 7 3 1 | 4 2 8  
2 3 8 | 9 6 4 | 5 7 1  
7 1 4 | 5 8 2 | 9 3 6

-----  
1 7 2 | 6 9 5 | 3 8 4  
8 9 3 | 2 4 7 | 6 1 5  
4 6 5 | 8 1 3 | 7 9 2

Grid27

Memory used: 13.4 MB

CPU time: 0.015625 s

3 8 7 | 2 5 6 | 4 1 9  
4 6 9 | 7 8 1 | 3 2 5  
5 1 2 | 4 3 9 | 8 6 7

-----  
1 2 3 | 5 4 8 | 9 7 6  
7 5 8 | 9 6 3 | 2 4 1  
6 9 4 | 1 2 7 | 5 8 3

-----  
8 3 5 | 6 7 4 | 1 9 2  
2 7 1 | 8 9 5 | 6 3 4  
9 4 6 | 3 1 2 | 7 5 8

Grid28

Memory used: 13.4 MB

CPU time: 0 s

3 4 5 | 8 7 1 | 2 6 9  
2 7 9 | 6 5 3 | 1 8 4  
8 6 1 | 4 2 9 | 5 3 7

-----  
1 9 7 | 3 4 6 | 8 5 2  
4 5 2 | 7 1 8 | 3 9 6  
6 8 3 | 5 9 2 | 7 4 1

-----  
7 3 8 | 2 6 4 | 9 1 5  
5 1 6 | 9 3 7 | 4 2 8  
9 2 4 | 1 8 5 | 6 7 3

Grid29

Memory used: 13.4 MB

CPU time: 0 s

2 3 5 | 7 6 1 | 4 8 9  
4 1 9 | 3 2 8 | 5 7 6  
8 6 7 | 5 4 9 | 2 1 3

-----  
7 4 6 | 1 3 5 | 9 2 8  
5 2 1 | 8 9 6 | 7 3 4  
9 8 3 | 4 7 2 | 6 5 1

-----  
3 9 4 | 2 8 7 | 1 6 5  
6 5 2 | 9 1 3 | 8 4 7  
1 7 8 | 6 5 4 | 3 9 2

Grid30

Memory used: 13.4 MB

CPU time: 0.015625 s

2 9 8 | 1 7 5 | 6 4 3  
6 5 7 | 3 9 4 | 1 2 8  
1 3 4 | 2 8 6 | 5 7 9

-----  
8 2 1 | 6 4 9 | 7 3 5  
5 7 3 | 8 2 1 | 4 9 6  
4 6 9 | 7 5 3 | 2 8 1

-----  
3 1 2 | 4 6 8 | 9 5 7  
7 8 5 | 9 1 2 | 3 6 4  
9 4 6 | 5 3 7 | 8 1 2

Grid31

Memory used: 13.4 MB

CPU time: 0 s

7 6 1 | 5 4 3 | 2 8 9  
8 3 2 | 7 9 1 | 6 4 5  
5 4 9 | 6 2 8 | 1 3 7

-----  
3 7 4 | 2 1 5 | 9 6 8

1 2 8 | 9 3 6 | 5 7 4  
6 9 5 | 4 8 7 | 3 2 1

-----  
4 1 7 | 3 6 9 | 8 5 2  
9 5 3 | 8 7 2 | 4 1 6  
2 8 6 | 1 5 4 | 7 9 3

Grid32

Memory used: 13.4 MB

CPU time: 0 s

1 3 2 | 6 4 9 | 7 8 5  
7 5 8 | 2 1 3 | 6 4 9  
9 6 4 | 7 8 5 | 1 2 3

-----  
5 4 3 | 8 9 7 | 2 1 6  
2 7 6 | 5 3 1 | 8 9 4  
8 9 1 | 4 2 6 | 5 3 7

-----  
6 1 9 | 3 7 8 | 4 5 2  
3 2 7 | 1 5 4 | 9 6 8  
4 8 5 | 9 6 2 | 3 7 1

Grid33

Memory used: 13.4 MB

CPU time: 0 s

6 9 8 | 1 7 3 | 5 4 2  
3 5 4 | 6 2 8 | 1 7 9  
1 7 2 | 5 4 9 | 3 6 8

-----  
5 3 1 | 8 9 7 | 4 2 6  
9 4 6 | 3 1 2 | 8 5 7  
8 2 7 | 4 5 6 | 9 1 3

-----  
7 6 5 | 9 3 1 | 2 8 4  
2 1 3 | 7 8 4 | 6 9 5  
4 8 9 | 2 6 5 | 7 3 1

Grid34

Memory used: 13.4 MB

CPU time: 0.015625 s

8 5 2 | 7 1 6 | 9 4 3  
1 9 7 | 8 4 3 | 6 5 2  
4 6 3 | 9 2 5 | 1 8 7

-----  
2 7 8 | 6 3 4 | 5 9 1  
6 4 5 | 1 7 9 | 3 2 8  
9 3 1 | 5 8 2 | 4 7 6

-----  
7 8 6 | 4 9 1 | 2 3 5  
3 1 4 | 2 5 8 | 7 6 9  
5 2 9 | 3 6 7 | 8 1 4

Grid35

Memory used: 13.4 MB

CPU time: 0 s

4 5 3 | 2 1 8 | 7 9 6  
6 2 9 | 7 5 3 | 4 8 1  
1 7 8 | 4 9 6 | 5 3 2

-----  
7 9 6 | 5 8 2 | 3 1 4  
3 1 4 | 9 6 7 | 8 2 5  
2 8 5 | 1 3 4 | 6 7 9

-----  
5 4 2 | 8 7 9 | 1 6 3  
9 3 7 | 6 4 1 | 2 5 8  
8 6 1 | 3 2 5 | 9 4 7

Grid36

Memory used: 13.4 MB

CPU time: 0 s

5 1 6 | 2 8 9 | 3 4 7  
8 4 9 | 1 7 3 | 2 5 6  
7 3 2 | 4 6 5 | 9 1 8

-----  
6 9 8 | 3 1 7 | 5 2 4  
3 2 7 | 9 5 4 | 8 6 1  
1 5 4 | 8 2 6 | 7 3 9

-----  
9 6 1 | 7 3 2 | 4 8 5  
2 7 5 | 6 4 8 | 1 9 3  
4 8 3 | 5 9 1 | 6 7 2

Grid37

Memory used: 13.4 MB

CPU time: 0 s

9 4 5 | 6 8 1 | 7 2 3

7 8 1 | 2 3 4 | 9 6 5  
3 2 6 | 7 5 9 | 1 8 4

-----  
2 6 9 | 1 7 5 | 3 4 8  
1 3 8 | 9 4 2 | 5 7 6  
5 7 4 | 8 6 3 | 2 1 9

-----  
4 5 7 | 3 2 6 | 8 9 1  
6 1 2 | 5 9 8 | 4 3 7  
8 9 3 | 4 1 7 | 6 5 2

Grid38

Memory used: 13.4 MB

CPU time: 0 s

3 6 5 | 9 4 2 | 8 7 1  
1 2 8 | 7 5 6 | 4 9 3  
9 7 4 | 8 1 3 | 5 6 2

-----  
8 1 9 | 4 3 5 | 6 2 7  
5 3 7 | 2 6 8 | 1 4 9  
6 4 2 | 1 7 9 | 3 5 8

-----  
2 9 6 | 3 8 4 | 7 1 5  
7 5 3 | 6 9 1 | 2 8 4  
4 8 1 | 5 2 7 | 9 3 6

Grid39

Memory used: 13.4 MB

CPU time: 0 s

1 3 4 | 5 8 7 | 2 9 6  
2 7 8 | 1 6 9 | 3 5 4  
6 9 5 | 2 3 4 | 8 1 7

-----  
3 5 9 | 8 1 6 | 4 7 2  
8 2 1 | 4 7 3 | 5 6 9  
7 4 6 | 9 2 5 | 1 8 3

-----  
9 1 7 | 3 4 8 | 6 2 5  
4 6 2 | 7 5 1 | 9 3 8  
5 8 3 | 6 9 2 | 7 4 1

Grid40

Memory used: 13.4 MB

CPU time: 0.015625 s

1 9 3 | 6 7 2 | 4 8 5

4 6 2 | 3 5 8 | 9 7 1

7 8 5 | 9 1 4 | 6 2 3

-----  
5 3 8 | 2 9 6 | 7 1 4

6 7 4 | 1 3 5 | 2 9 8

2 1 9 | 4 8 7 | 3 5 6

-----  
8 2 6 | 7 4 1 | 5 3 9

9 4 1 | 5 2 3 | 8 6 7

3 5 7 | 8 6 9 | 1 4 2

Grid41

Memory used: 13.53 MB

CPU time: 0 s

8 1 4 | 9 7 6 | 5 3 2

6 5 9 | 1 2 3 | 4 7 8

7 3 2 | 8 5 4 | 1 6 9

-----  
9 4 8 | 2 6 5 | 3 1 7

2 7 5 | 3 4 1 | 8 9 6

1 6 3 | 7 9 8 | 2 4 5

-----  
3 9 1 | 6 8 2 | 7 5 4

5 8 7 | 4 3 9 | 6 2 1

4 2 6 | 5 1 7 | 9 8 3

Grid42

Memory used: 13.4 MB

CPU time: 0 s

3 8 4 | 5 6 7 | 9 2 1

1 2 6 | 4 3 9 | 7 8 5

7 5 9 | 8 2 1 | 3 4 6

-----  
5 6 3 | 7 9 8 | 2 1 4

8 4 7 | 3 1 2 | 6 5 9

9 1 2 | 6 4 5 | 8 7 3

-----  
2 3 1 | 9 7 4 | 5 6 8

4 9 5 | 2 8 6 | 1 3 7

6 7 8 | 1 5 3 | 4 9 2



Grid43

Memory used: 13.4 MB

CPU time: 0 s

4 6 9 | 1 5 8 | 3 7 2

7 1 2 | 4 6 3 | 8 5 9

5 3 8 | 2 9 7 | 6 4 1

-----

9 2 7 | 6 3 4 | 5 1 8

3 8 5 | 7 1 9 | 4 2 6

1 4 6 | 5 8 2 | 7 9 3

-----

6 5 3 | 9 4 1 | 2 8 7

2 9 4 | 8 7 6 | 1 3 5

8 7 1 | 3 2 5 | 9 6 4

Grid44

Memory used: 13.54 MB

CPU time: 0 s

3 1 6 | 5 4 9 | 2 7 8

9 8 7 | 3 2 1 | 6 4 5

4 5 2 | 6 7 8 | 9 3 1

-----

5 9 4 | 2 3 6 | 8 1 7

2 3 8 | 4 1 7 | 5 6 9

6 7 1 | 9 8 5 | 3 2 4

-----

8 4 5 | 1 6 2 | 7 9 3

1 2 9 | 7 5 3 | 4 8 6

7 6 3 | 8 9 4 | 1 5 2

Grid45

Memory used: 13.4 MB

CPU time: 0 s

5 8 6 | 1 2 7 | 9 4 3

7 2 3 | 4 6 9 | 8 5 1

4 9 1 | 8 5 3 | 2 6 7

-----

1 3 5 | 9 7 4 | 6 2 8

2 7 9 | 6 1 8 | 5 3 4

6 4 8 | 5 3 2 | 1 7 9

-----

9 1 7 | 2 4 6 | 3 8 5

3 5 2 | 7 8 1 | 4 9 6

8 6 4 | 3 9 5 | 7 1 2

Grid46

Memory used: 13.54 MB

CPU time: 0 s

9 5 4 | 2 1 3 | 6 8 7

6 1 7 | 5 4 8 | 9 2 3

8 3 2 | 7 9 6 | 5 4 1

-----

7 6 3 | 8 5 1 | 2 9 4

1 2 8 | 9 7 4 | 3 6 5

5 4 9 | 3 6 2 | 1 7 8

-----

2 8 1 | 6 3 7 | 4 5 9

4 7 5 | 1 2 9 | 8 3 6

3 9 6 | 4 8 5 | 7 1 2

Grid47

Memory used: 13.55 MB

CPU time: 0 s

1 5 9 | 7 4 3 | 8 6 2

2 7 6 | 5 8 9 | 4 3 1

3 4 8 | 6 1 2 | 7 5 9

-----

6 2 4 | 9 7 8 | 3 1 5

9 1 7 | 2 3 5 | 6 8 4

5 8 3 | 1 6 4 | 2 9 7

-----

4 3 5 | 8 2 1 | 9 7 6

8 6 1 | 4 9 7 | 5 2 3

7 9 2 | 3 5 6 | 1 4 8

Grid48

Memory used: 13.53 MB

CPU time: 0 s

8 6 1 | 3 5 7 | 2 9 4

5 9 7 | 4 8 2 | 3 6 1

4 3 2 | 6 1 9 | 7 8 5

-----

9 1 6 | 2 7 5 | 8 4 3

3 5 8 | 9 6 4 | 1 2 7

2 7 4 | 1 3 8 | 9 5 6

-----

7 8 9 | 5 4 1 | 6 3 2

```
1 4 3 | 8 2 6 | 5 7 9
6 2 5 | 7 9 3 | 4 1 8
```

Grid49

Memory used: 13.55 MB

CPU time: 0 s

```
2 9 4 | 8 6 3 | 5 1 7
7 1 5 | 4 2 9 | 6 3 8
8 6 3 | 7 5 1 | 4 9 2
```

```
-----
1 5 2 | 9 4 7 | 8 6 3
4 7 9 | 3 8 6 | 2 5 1
6 3 8 | 5 1 2 | 9 7 4
```

```
-----
9 8 6 | 1 3 4 | 7 2 5
5 2 1 | 6 7 8 | 3 4 9
3 4 7 | 2 9 5 | 1 8 6
```

Grid50

Memory used: 13.4 MB

CPU time: 0 s

```
3 5 1 | 2 8 6 | 4 9 7
4 9 2 | 1 5 7 | 6 3 8
7 8 6 | 9 3 4 | 5 1 2
```

```
-----
2 7 5 | 4 6 9 | 1 8 3
9 3 8 | 5 2 1 | 7 6 4
6 1 4 | 8 7 3 | 2 5 9
```

```
-----
8 2 9 | 6 4 5 | 3 7 1
1 6 3 | 7 9 2 | 8 4 5
5 4 7 | 3 1 8 | 9 2 6
```

For the 50 solved puzzles above, the average time taken to solve a puzzle is 0.0028125s, and the average memory used to solve a puzzle is approximately 13.4MB.

## Extended Task

### Exploring General N x N Size Puzzles

The encoding method used in *sud2sat.java* can be easily modified to encode more general NxN size Sudoku puzzles. The original minimal encoding contains  $9^3 = 729$  variables representing the row, column, and cell logic required to solve a 9x9 Sudoku puzzle. As the size of the grid

$N \times N$  increases, more variables in the encoding are needed in order to solve the Sudoku puzzle. Hence, the runtime increases significantly as the size of the grid increases. The Sudoku problem is an NP-complete problem. Therefore, no known algorithm can solve it in polynomial time in terms of the input size, that is, the size of the grid. When given an  $N \times N$  Sudoku puzzle with very large  $N$ , the solution to the puzzle becomes intractable.