Computation and Robustness in Spatially Irregular Cellular Automata

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Abstract

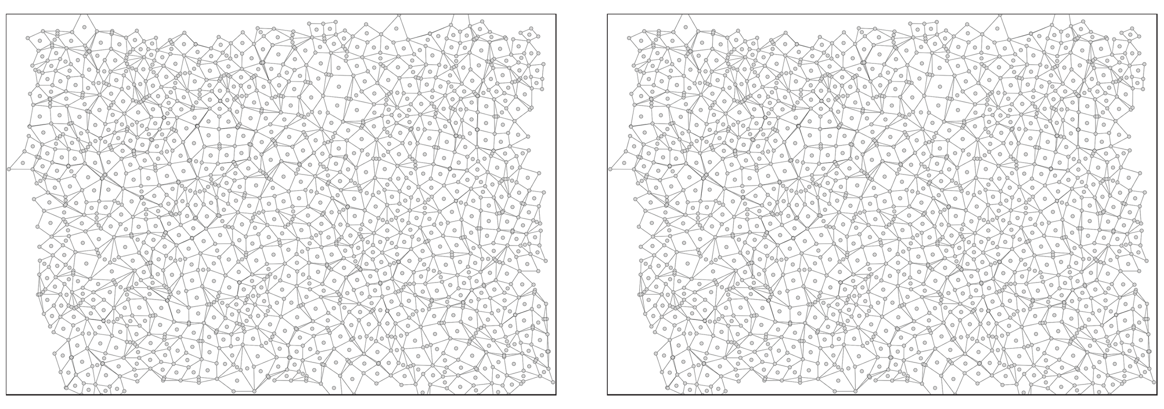
Cellular Automata (CA) can be used to effectively model natural dynamical systems even though they typically assume spatial regularity and structure. Some natural systems appear to follow the CA computational metaphor directly. For example, Peak and others[2] observed that plant leaf stoma control gas exchange in a way that is statistically indistinguishable from Local Majority CA computations, suggesting that the stomatal arrays can engage in complex behavior without uniformly structured connections. We wonder, here, if regularly connected CA meshes play an important role in the convergence and robustness of natural CA computation.

The experiments described here seek to identify qualitative differences between computations performed on square grids (Messinger [4]) and analogous computations performed on derived irregular meshes. We also develop strategies for progressively isolating regions of these networks by decreasing communication quality.

With these methods, we investigated the qualitative structure across the entire irregular CA space, comparing it with studies of regular networks by Langton and Wootters[5]. We observe the impact that isolation has on criticality in automata, as well as the graceful degradation of complex behavior at varying levels of isolation.

Testing Robustness

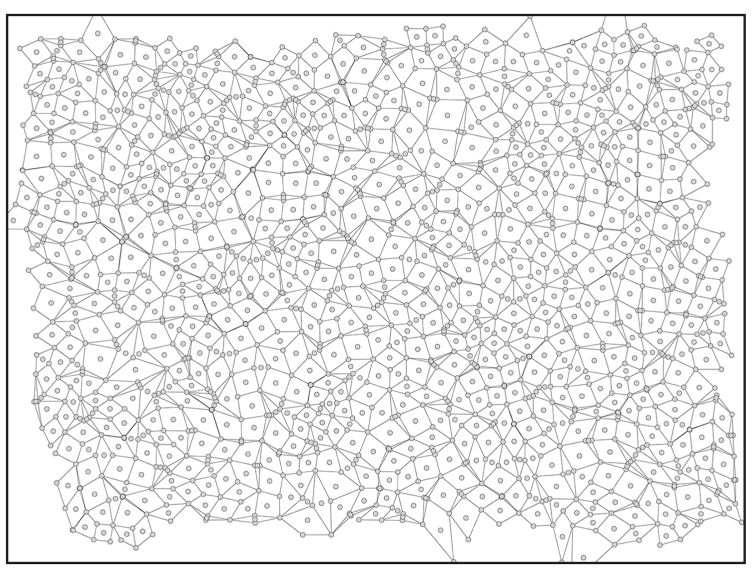
How important is communication in these networks? We developed a crosshatching degradation strategy that, in a controlled manner, isolates regions of cells by progressively degrading communication between network patches.



We overlayed our networks with a mesh of width *w*. Links crossing mesh lines, with probability *p*, were removed (shown as red). When *p* is low, network computation is relatively unaffected. As *p* increases quality of computation degrades, but when mesh width is sufficiently large the local patch-based computation lends a certain robustness to the computation, even for high levels of *p*.

Network Generation

We constructed naturalistic networks to compare with existing results from regular networks. It was not clear that CA embedded in irregular networks would perform similarly to those embedded in traditional meshes.



a. Our naturalistic networks were developed from micrographs of stomatal networks[3]. Here, stoma appear as red dots.
b. We performed a Delaunay triangulation to identify areas of stomatal influence.
c. We computed the Voronoi diagram, the dual of the triangulation
d. Here, 4-connected grid generated by merging Voronoi cells.

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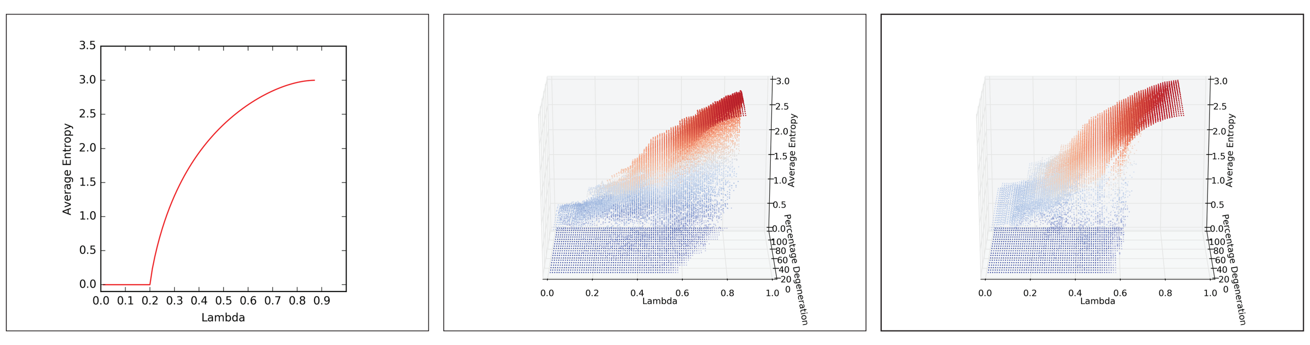
We also considered random grids generated using a Poisson disk generator and networks induced by aperiodic tilings.

Information Profile

Langton's λ parameter—a measure of the relative quiescent potential of a rule—is an effective way to parameterize the CA rule space. Langton and Wootters[5] explored relations between λ and Shannon entropy (left). As λ increases, CA behavior transitions to disorder, with a sudden jump in entropy marking the critical transition point at "the edge of chaos." This profile appeared as a feature of irregular networks, and held up remarkably well to degradation in communication (middle: neighborhood width is 10; right: 30). As might be expected, the critical point is less defined for smaller neighborhoods where communication is heavily restricted.

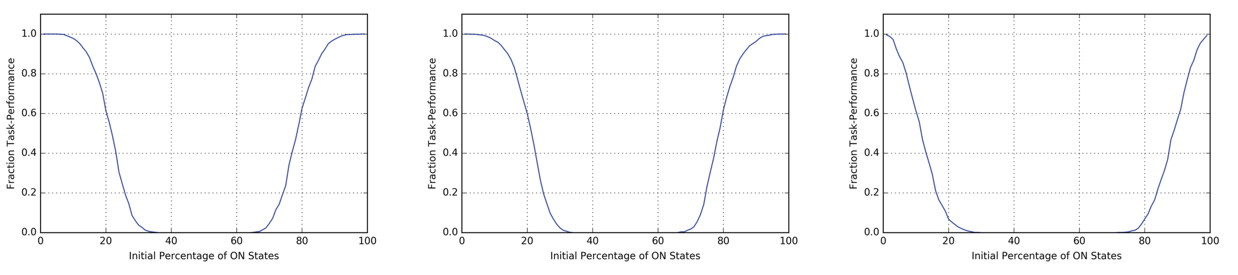


Idealized λ profile | Degradation of λ profile, w=10 | Degradation of λ profile, w=30

Local Majority

Work by Messinger et al.[4] studied the profile of convergence for CA-simulated stomatal meshes to develop consensus. We computed task performance profiles for majority computation using stomatal meshes (above). Figures below illustrate the success of networks to reach consensus. On the left, we show the probability a regular toroidal network develops the correct decision. On the right, the same computation on a regular network without end-around connections. In the middle, the task performance of naturalistic, irregular stomatal network developed above. Stomatal networks appear as computationally effective as square-mesh networks without the benefit of continuous boundaries.

regular network (toroidal) | irregular stomatal network | regular network (nontoroidal)



Conclusions

Meshes of computation that appear in nature—and are likely to appear in the future by design—demonstrate various irregularities in their structure and imperfections in their coordination. This work suggests that CA-based computation weathers these irregularities.

In particular, (1) these networks demonstrate similar performance profiles for consensus-building problems and (2) the automata space defined on these networks have similar Langton-Wootters entropy profiles that gracefully decay as communication failures isolate patches of computation. Similar results appear for networks of Poisson-disk random points and networks with aperiodic structure. These observations are important since traditional CA simulation often imposes strict grid structuring where none exists in the problem domain.

Ultimately, we are trying to understand how natural dynamic environments can support inherent computation with such rigid restrictions. This work takes a first step in explaining how decentralized computational systems handle and embrace imperfections in the wild.

Tony Liu works at IBM. This work represents his honors thesis completed while at Williams College. Tony is interested in natural adaptive systems as it pertains to the emergence of cognition, as well as the application of machine learning and NLP techniques to dark data in healthcare.

Duane Bailey teaches Computer Science at Williams College. He studies the inefficiencies of interactions between hardware and software. He is also interested in the identification of paradigms for robustness and resilience in our understanding of computation in nature.

References

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2. David Peak, Jevin D West, Susanna M Messinger, and Keith A Mott. *Evidence for complex, collective dynamics and emergent, distributed computation in plants.* Proceedings of the National Academy of Sciences, 101(4):918–922, 2004.
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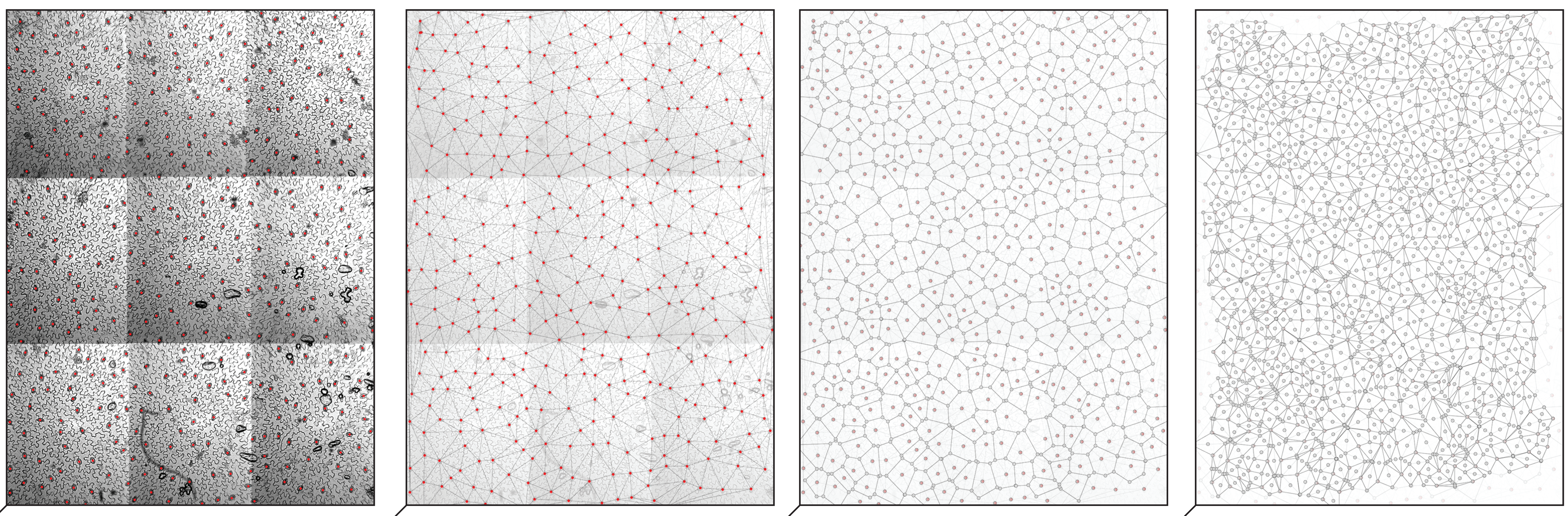

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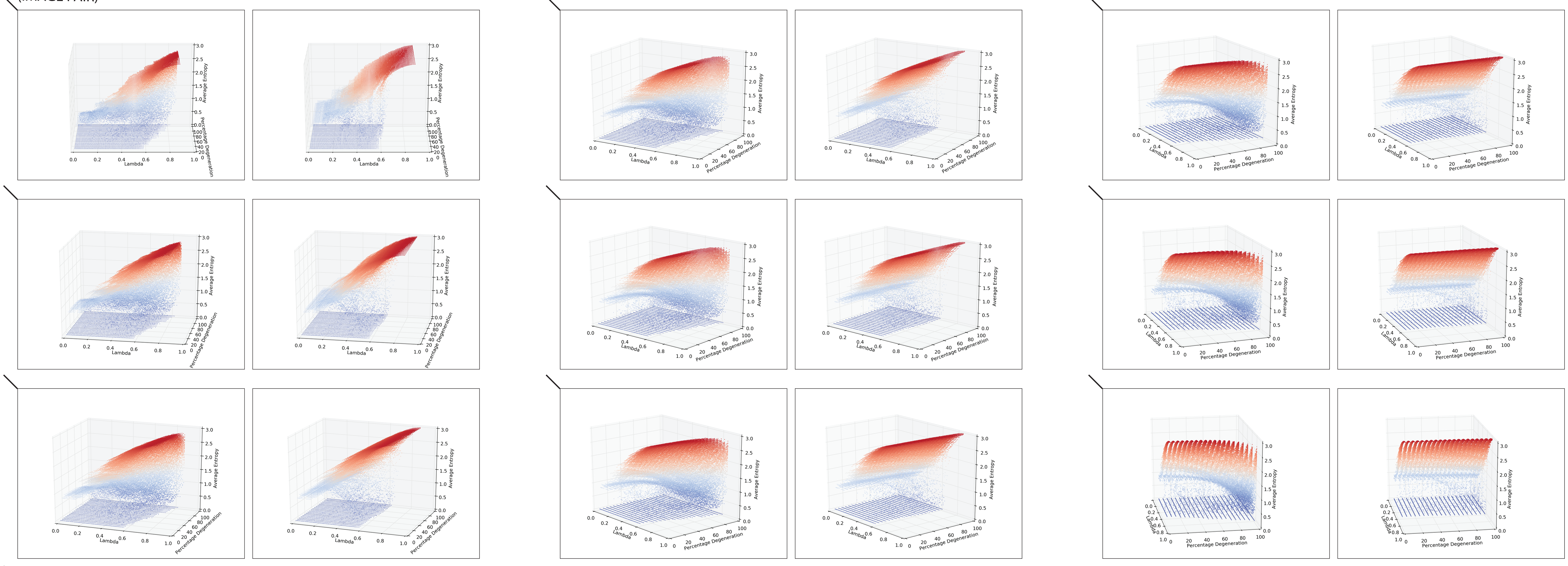

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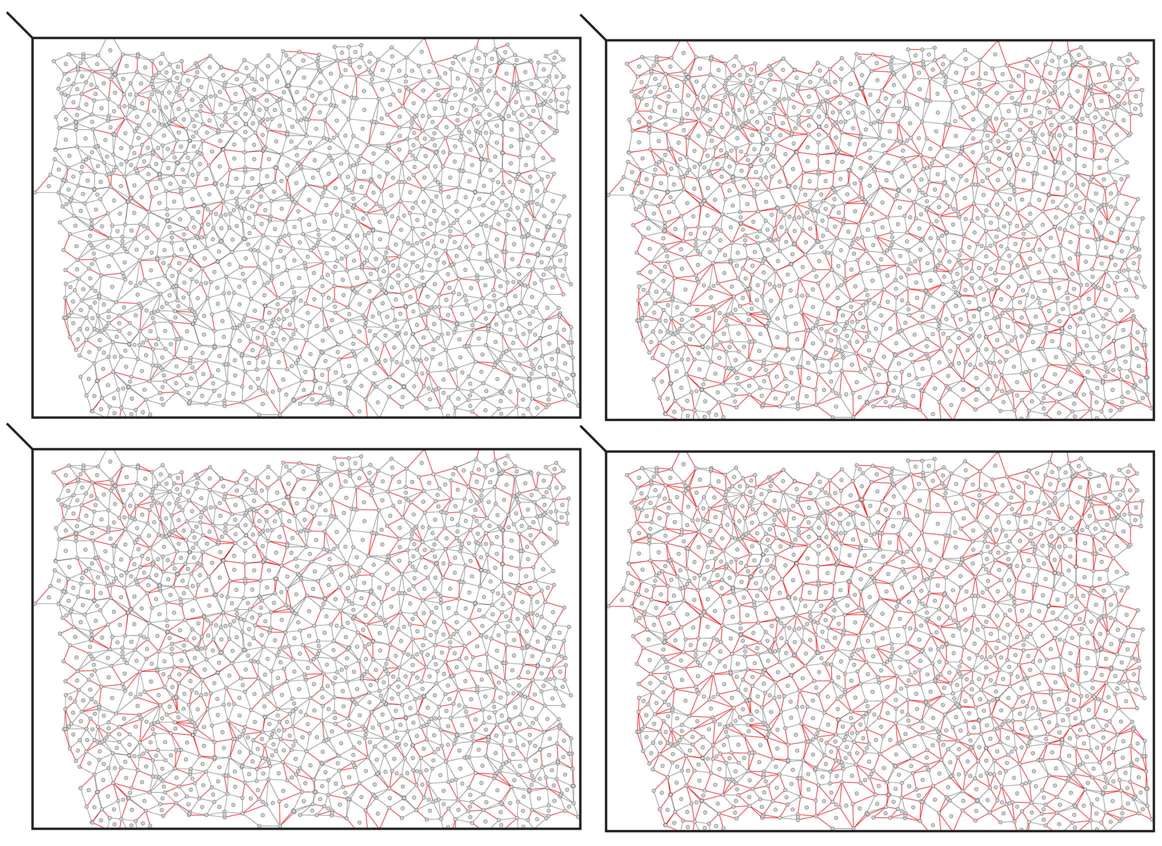

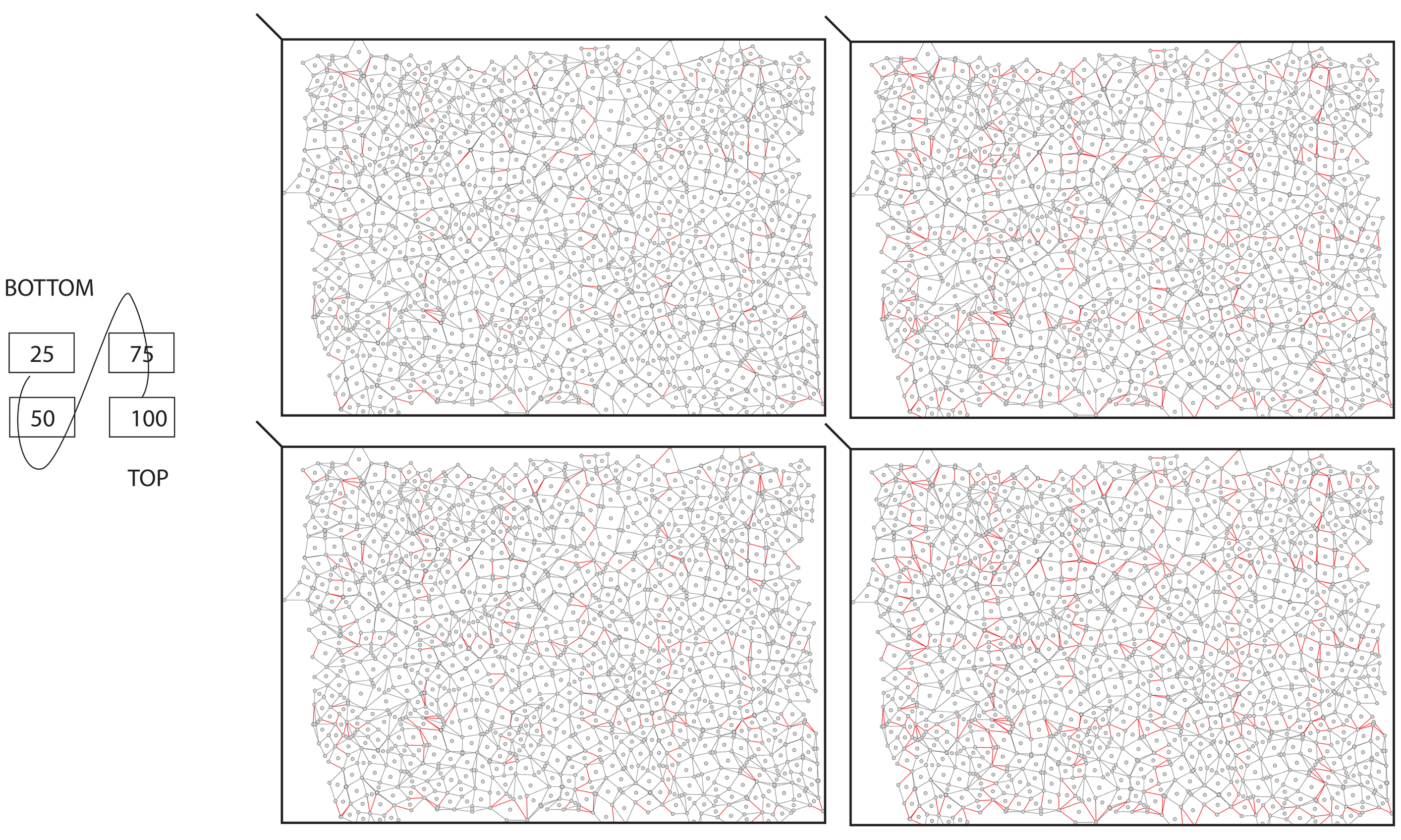

W10

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