Image alignment

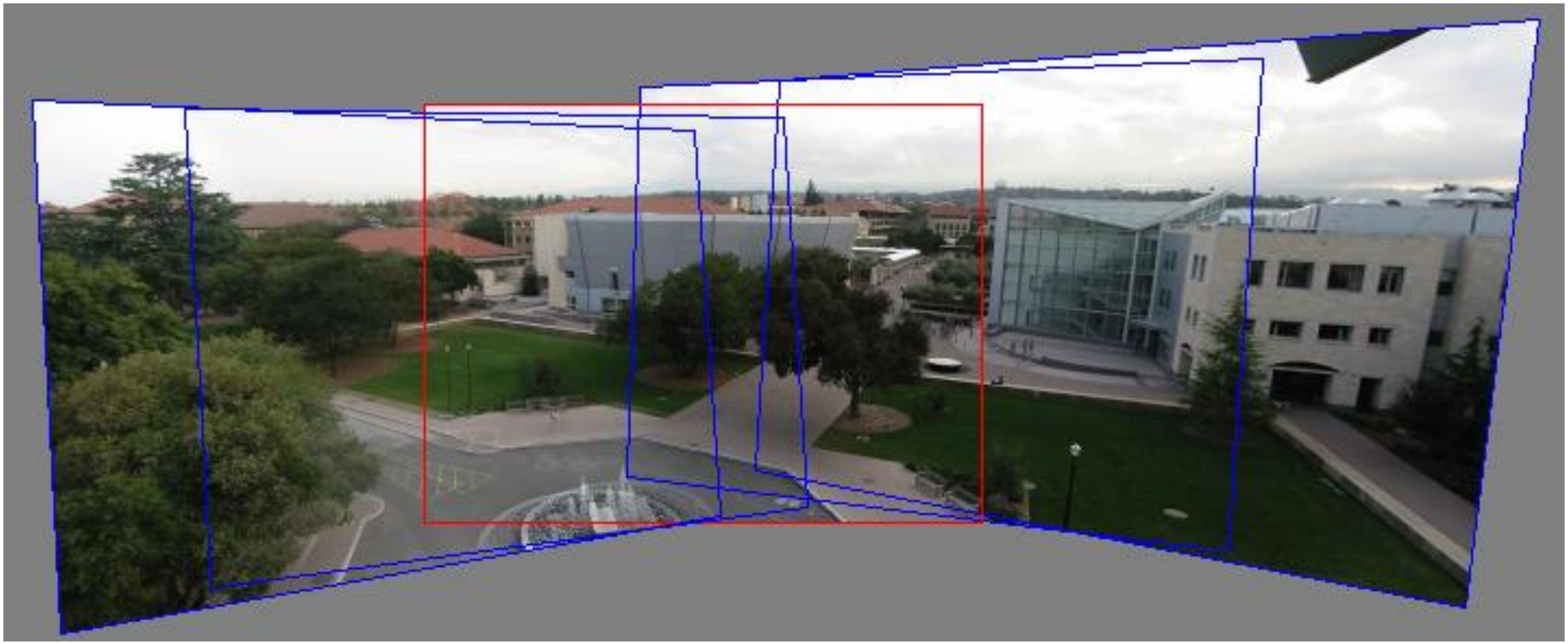
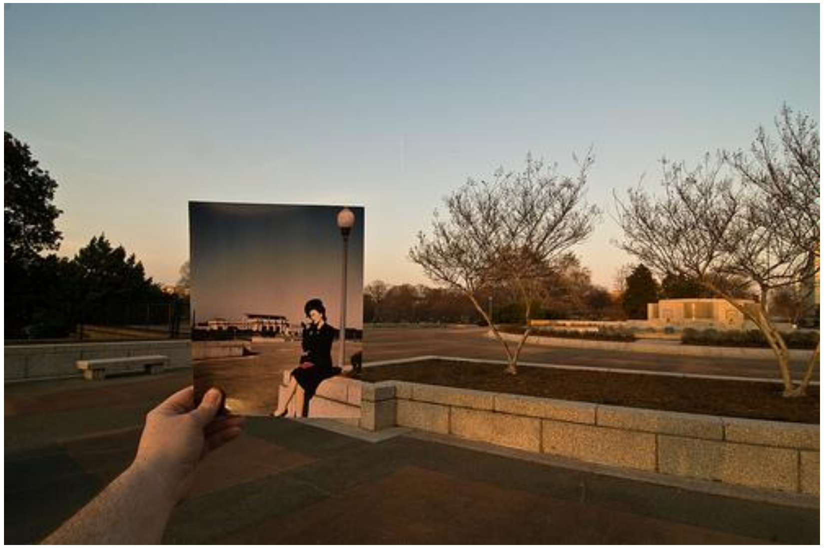
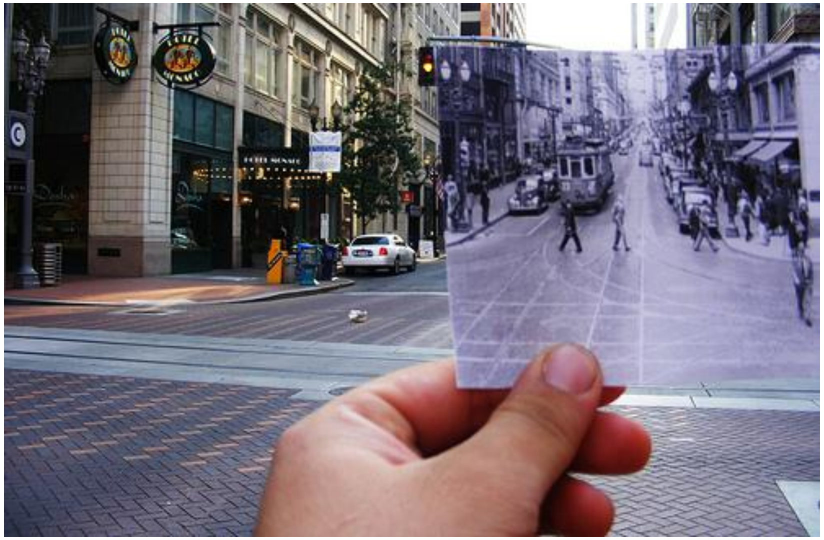
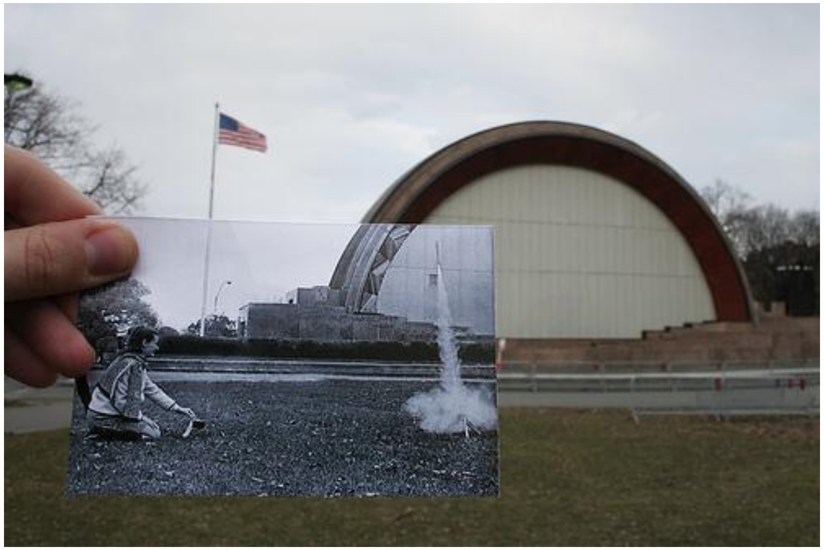


Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

A look into the past



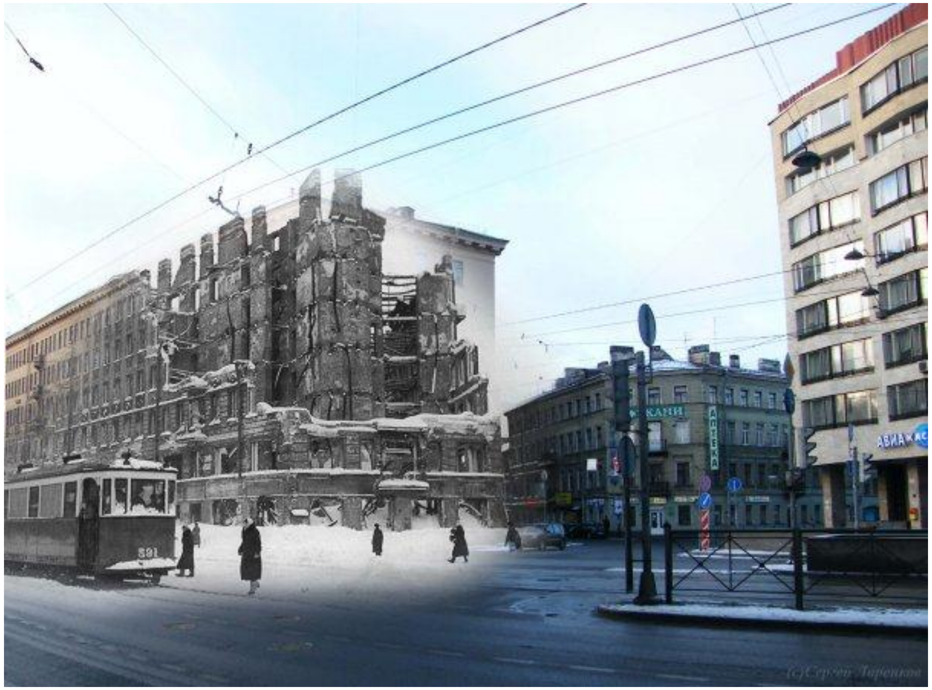




http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/

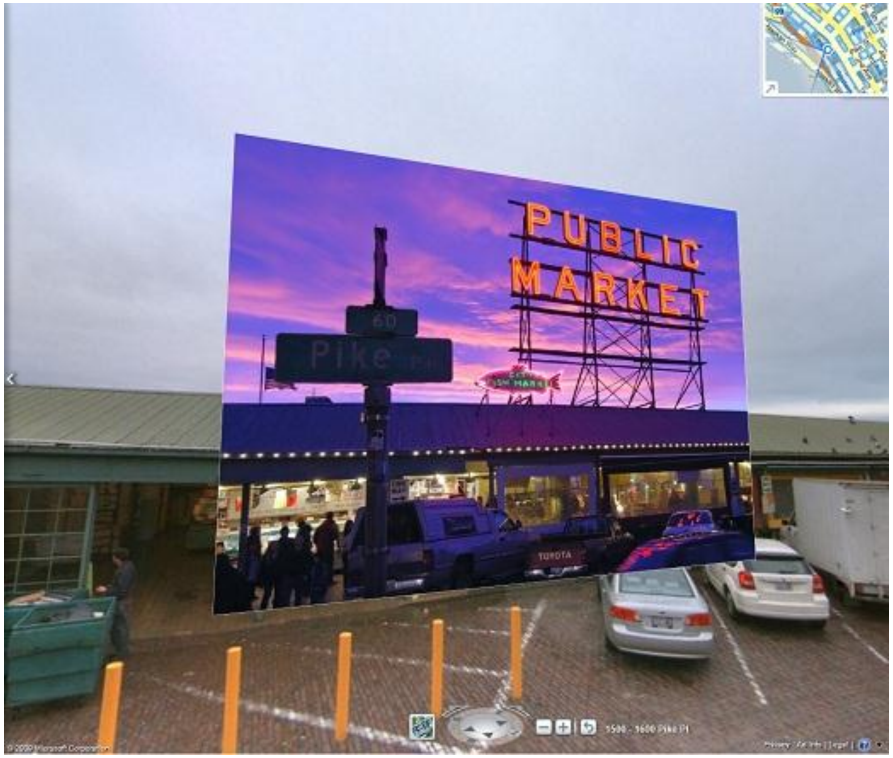
A look into the past

Leningrad during the blockade



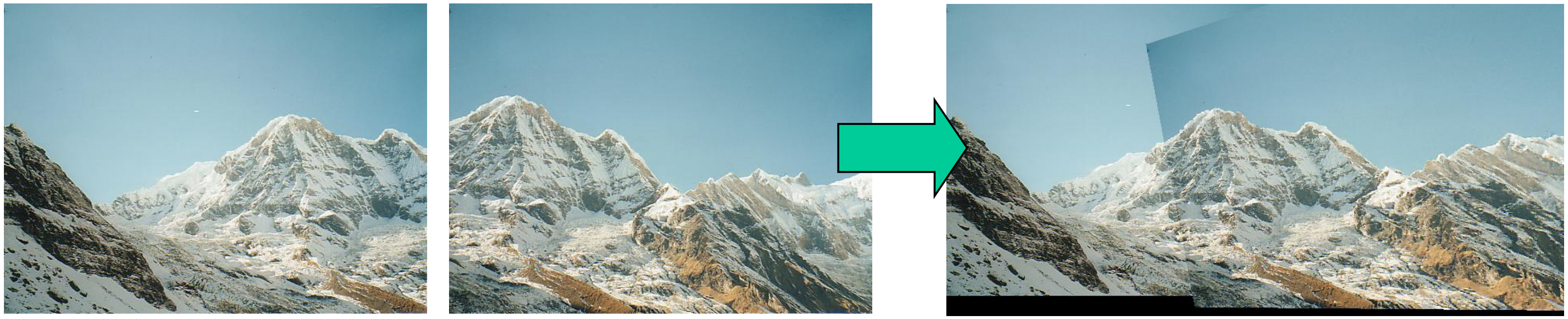
http://komen-dant.livejournal.com/345684.html

Bing streetside images

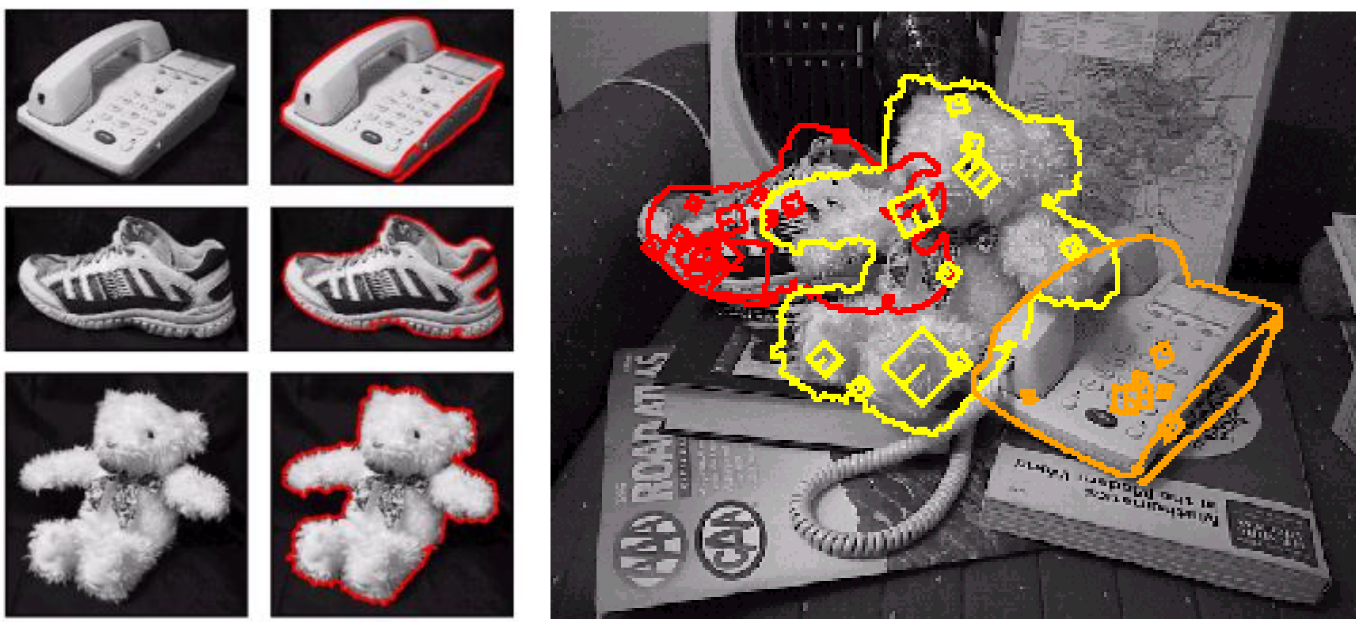


http://www.bing.com/community/blogs/maps/archive/2010/01/12/new-bing-maps-application-streetside-photos.aspx

Image alignment: Applications

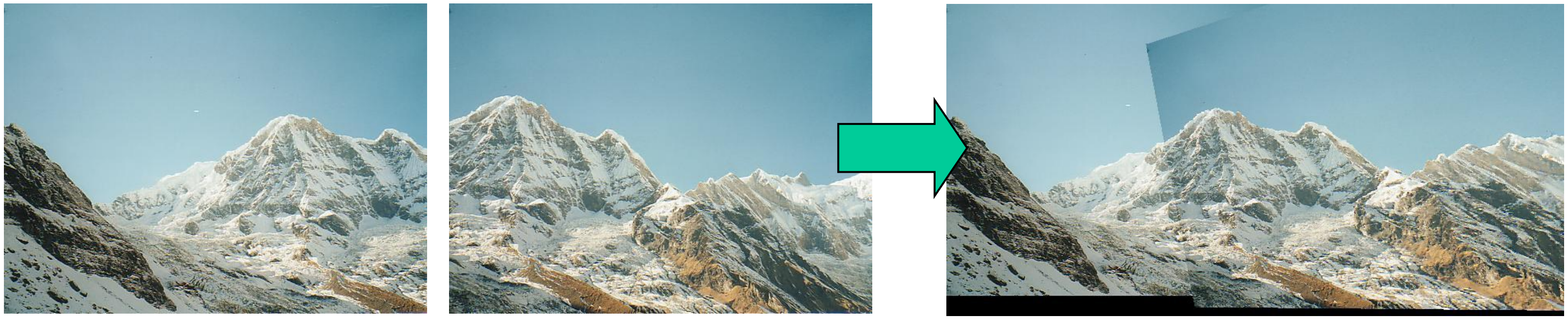


Panorama stitching

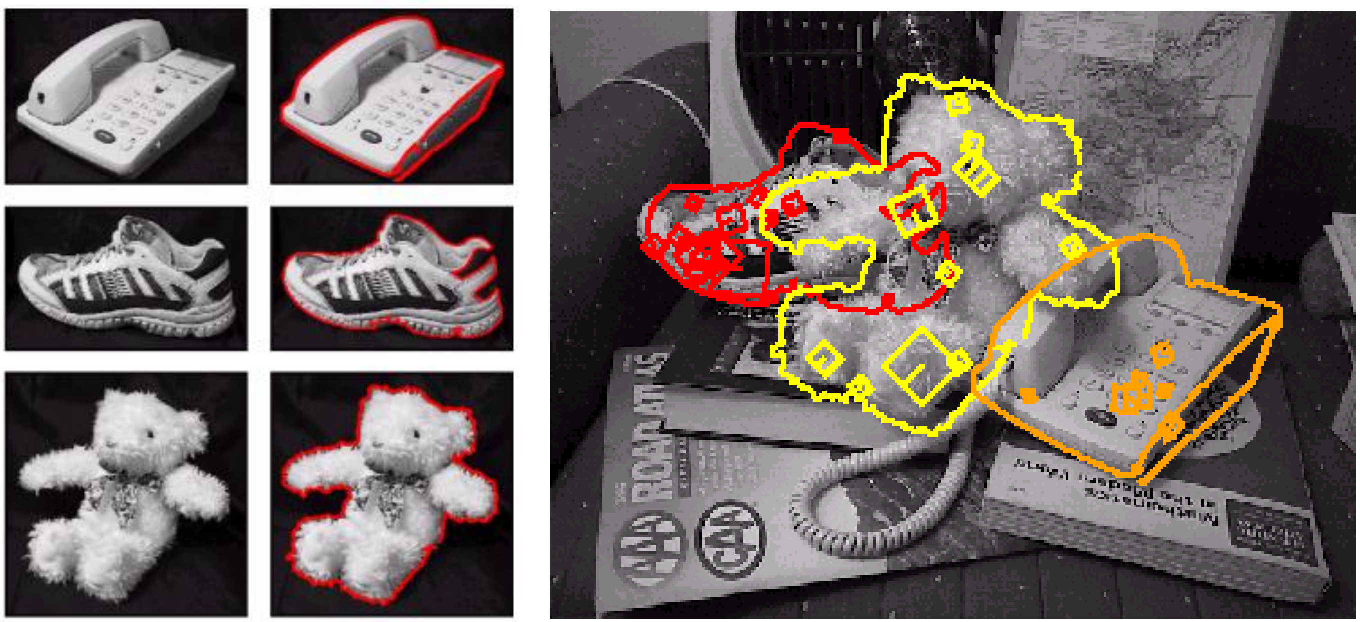


Recognition of object instances

Image alignment: Challenges

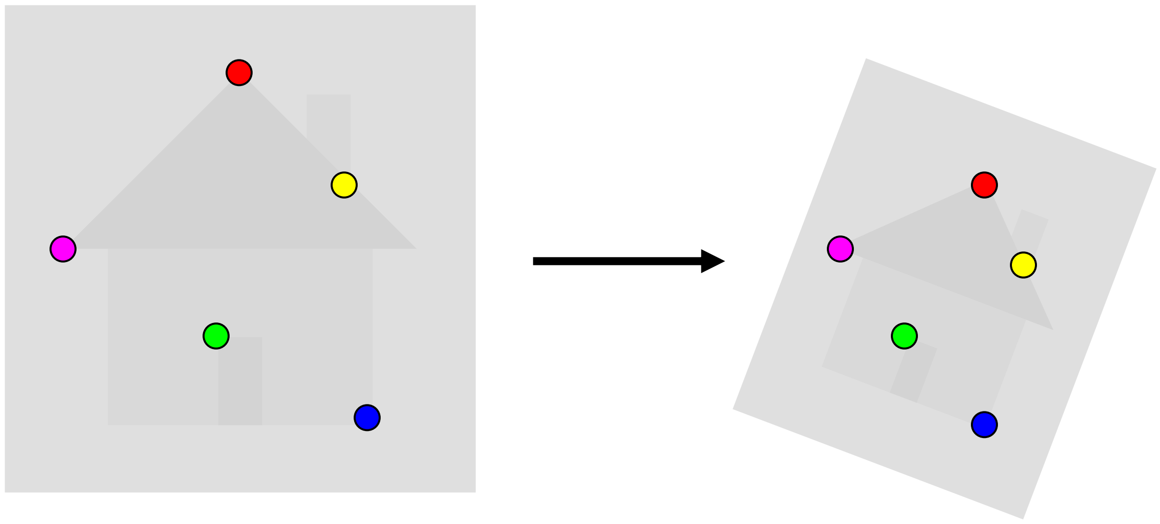


Small degree of overlap
Intensity changes



Occlusion,
clutter

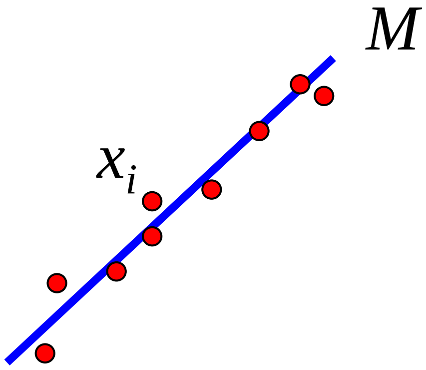
Image alignment



- Two families of approaches:
  - **Direct (pixel-based) alignment**
    - – Search for alignment where most pixels agree
  - **Feature-based alignment**
    - – Search for alignment where *extracted features* agree
    - – Can be verified using pixel-based alignment

Alignment as fitting

- Previous lectures: fitting a model to features in one image

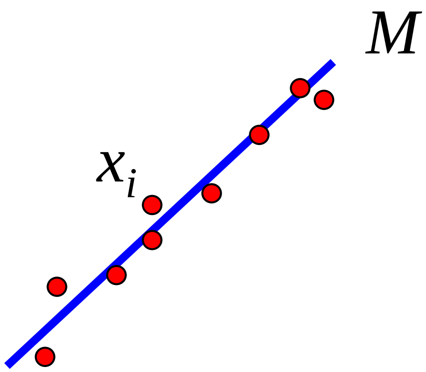


Find model *M* that minimizes

$$\sum_i \text{residual}(x_i, M)$$

Alignment as fitting

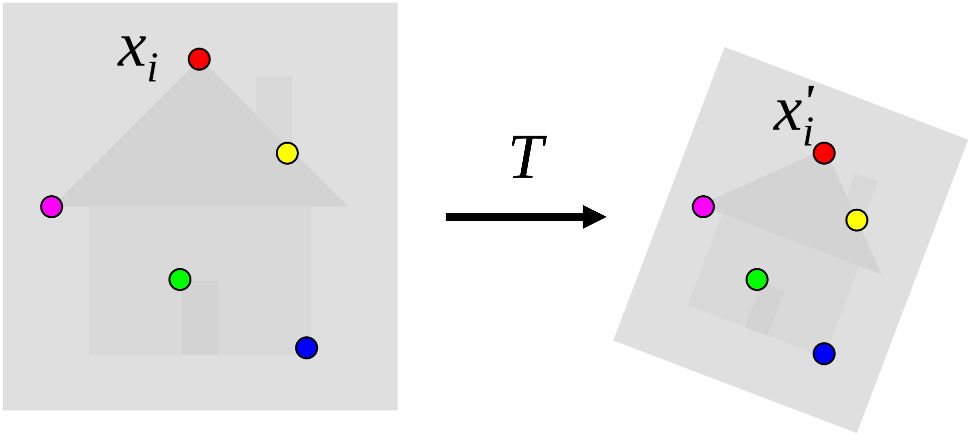
- Previous lectures: fitting a model to features in one image



Find model M that minimizes

$$\sum_i \text{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images

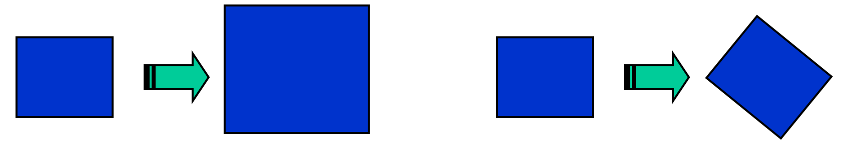


Find transformation T that minimizes

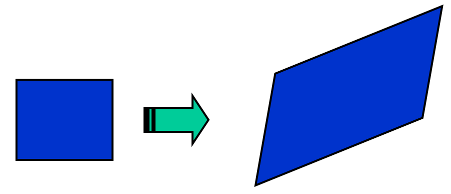
$$\sum_i \text{residual}(T(x_i), x'_i)$$

2D transformation models

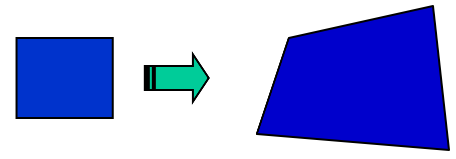
- Similarity (translation, scale, rotation)



- Affine

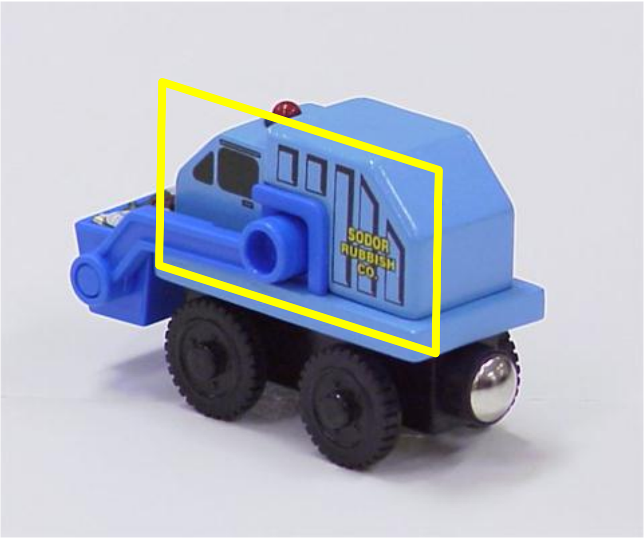


- Projective (homography)



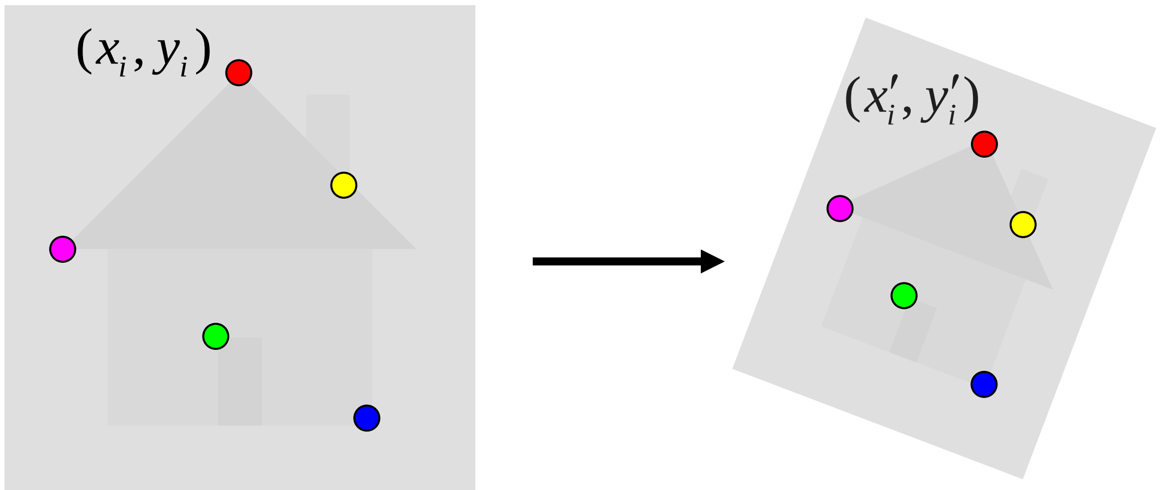
Let's start with affine transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models



Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

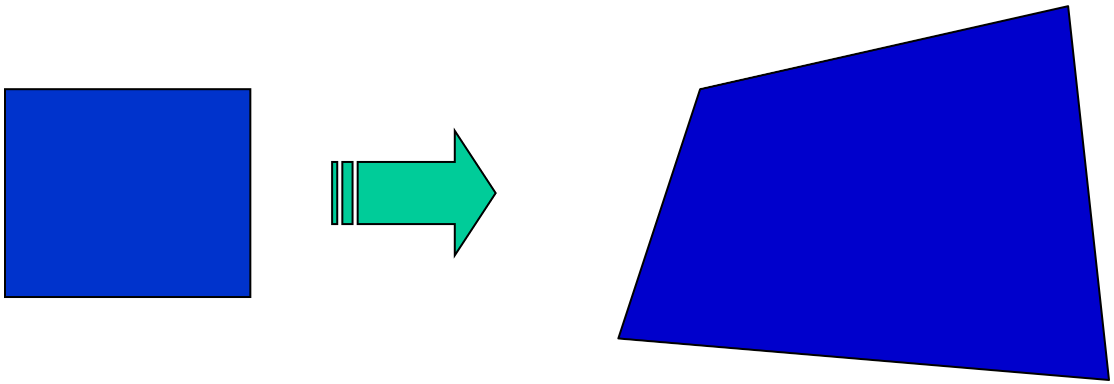
Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters

Fitting a plane projective transformation

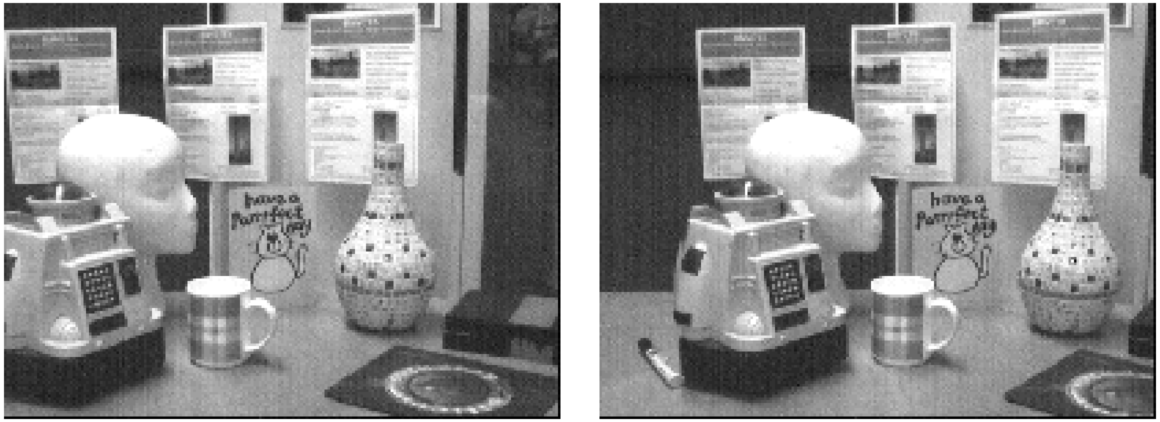
- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)



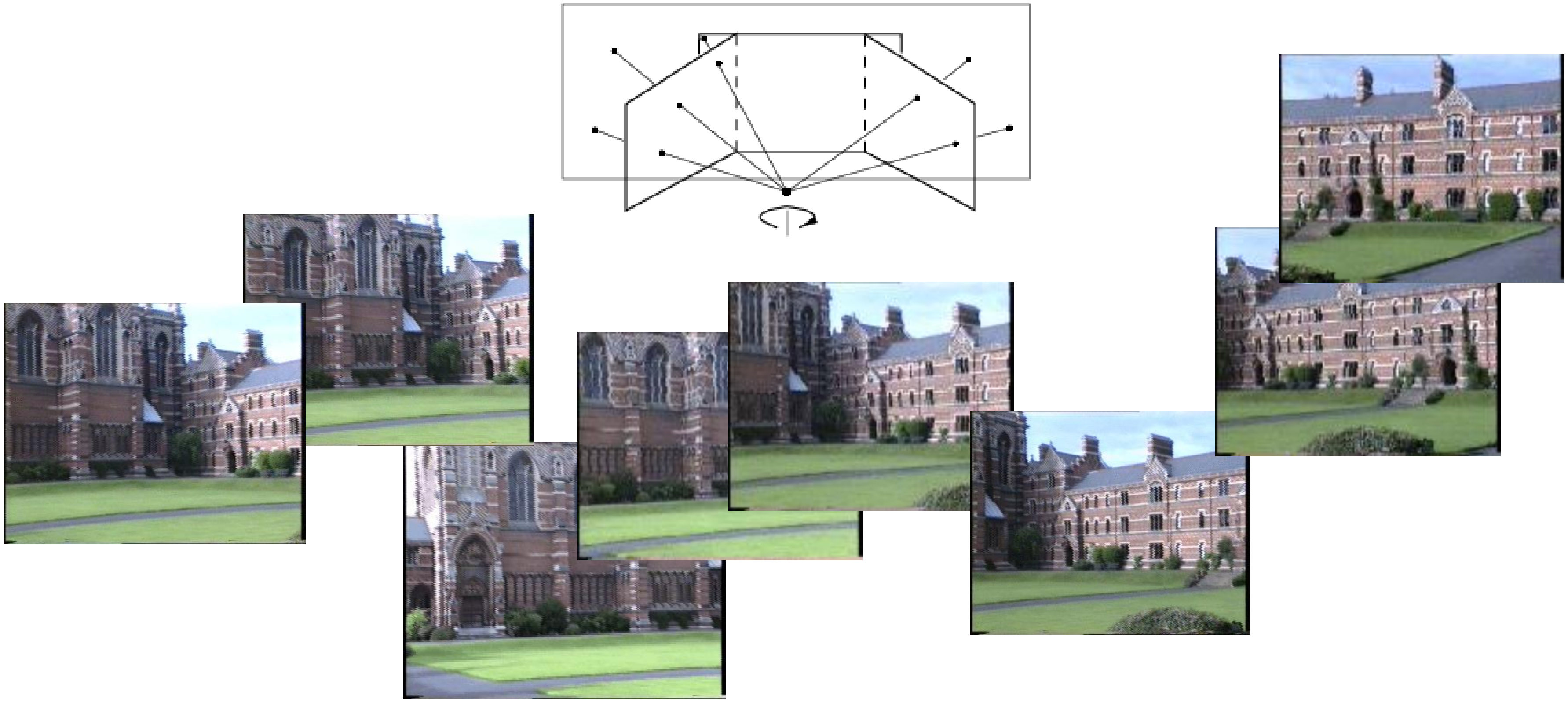
Homography

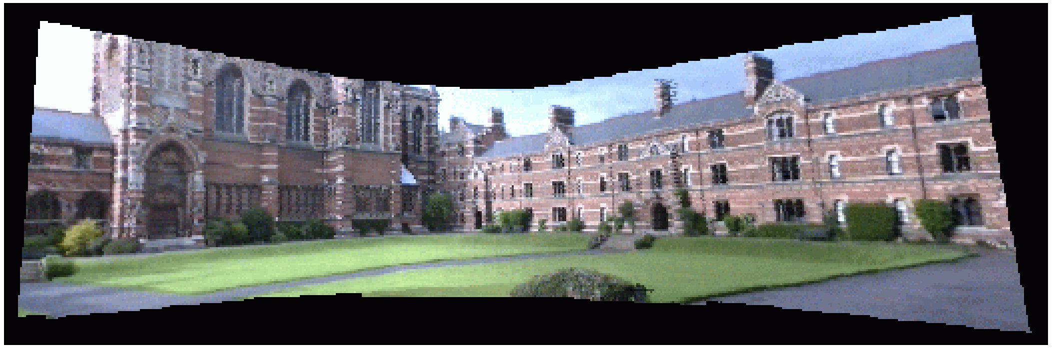
- The transformation between two views of a planar surface

- The transformation between images from two cameras that share the same center



Application: Panorama stitching





Source: Hartley & Zisserman

Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \left[\begin{array}{c} x \\ y \\ 1 \end{array} \right]$$

Converting *to* homogeneous image coordinates

$$\left[\begin{array}{c} x \\ y \\ w \end{array} \right] \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

- Equation for homography:

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Fitting a homography

- Equation for homography:

$$\lambda \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{x}_i' = \mathbf{H}\,\mathbf{x}_i$$

$$\mathbf{x}_i' \times \mathbf{H}\,\mathbf{x}_i = 0$$

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{h}_1^T \mathbf{x}_i \\ \mathbf{h}_2^T \mathbf{x}_i \\ \mathbf{h}_3^T \mathbf{x}_i \end{bmatrix} = \begin{bmatrix} y_i'\,\mathbf{h}_3^T \mathbf{x}_i - \mathbf{h}_2^T \mathbf{x}_i \\ \mathbf{h}_1^T \mathbf{x}_i - x_i'\,\mathbf{h}_3^T \mathbf{x}_i \\ x_i'\,\mathbf{h}_2^T \mathbf{x}_i - y_i'\,\mathbf{h}_1^T \mathbf{x}_i \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -\mathbf{x}_i^T & y_i'\,\mathbf{x}_i^T \\ \mathbf{x}_i^T & 0^T & -x_i'\,\mathbf{x}_i^T \\ -y_i'\,\mathbf{x}_i^T & x_i'\,\mathbf{x}_i^T & 0^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

3 equations, only 2 linearly independent

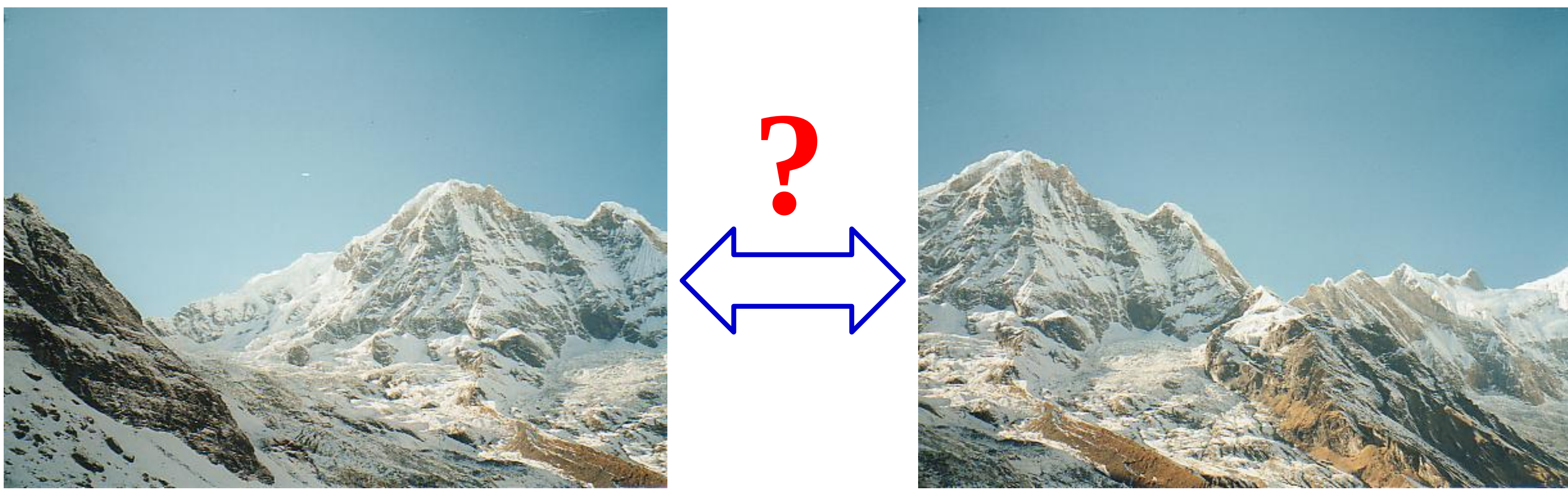
Direct linear transform

$$\begin{bmatrix} 0^T & \mathbf{x}_1^T & -y_1'\,\mathbf{x}_1^T \\ \mathbf{x}_1^T & 0^T & -x_1'\,\mathbf{x}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{x}_n^T & -y_n'\,\mathbf{x}_n^T \\ \mathbf{x}_n^T & 0^T & -x_n'\,\mathbf{x}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0 \qquad \mathbf{A}\,\mathbf{h} = 0$$

- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Four matches needed for a minimal solution (null space of 8x9 matrix)
- More than four: homogeneous least squares

Robust feature-based alignment

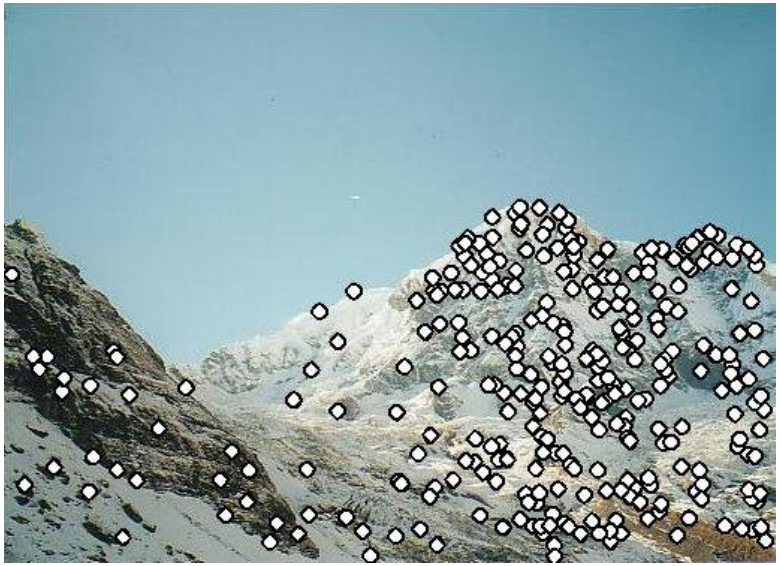
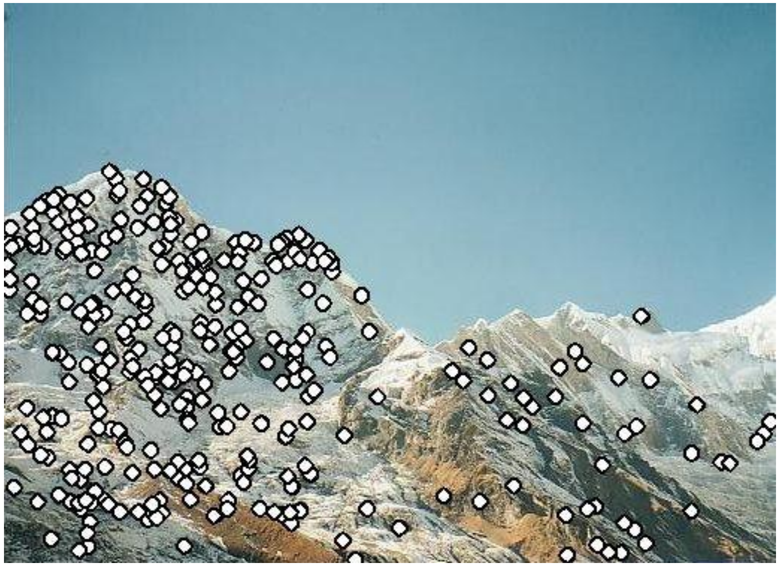
- So far, we've assumed that we are given a set of "ground-truth" correspondences between the two images we want to align
- What if we don't know the correspondences?



Robust feature-based alignment

Robust feature-based alignment

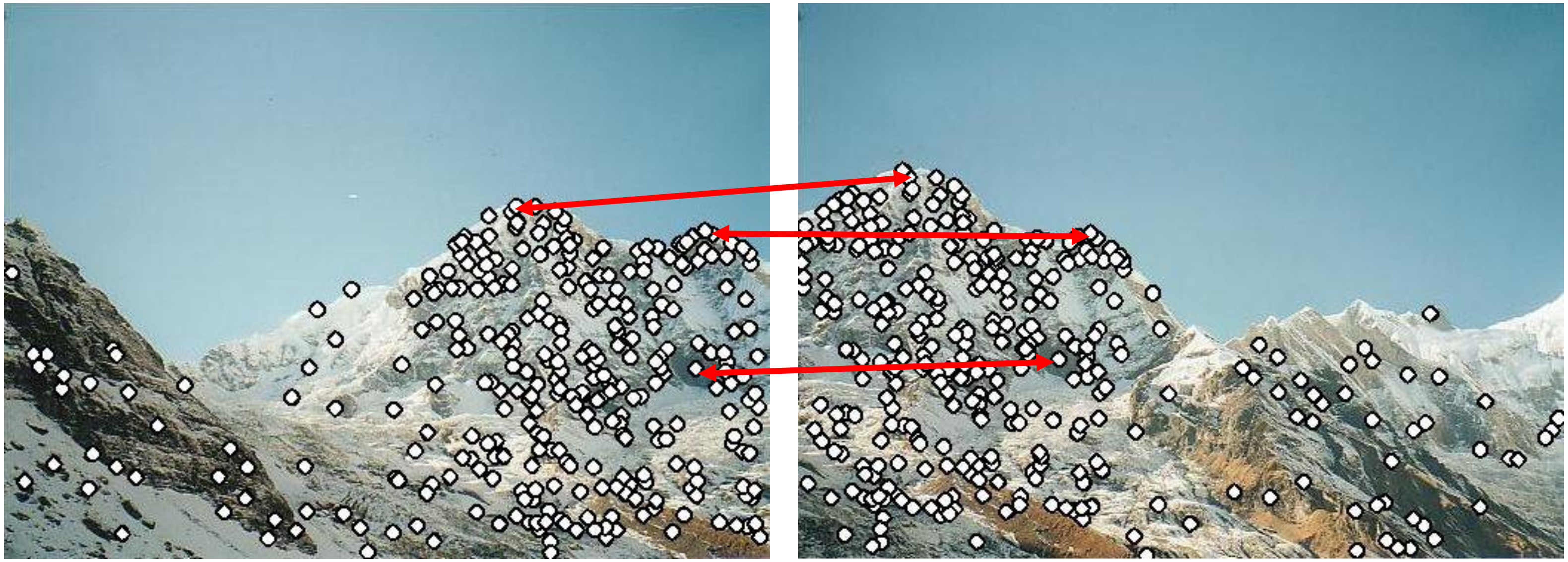



- Extract features

Robust feature-based alignment

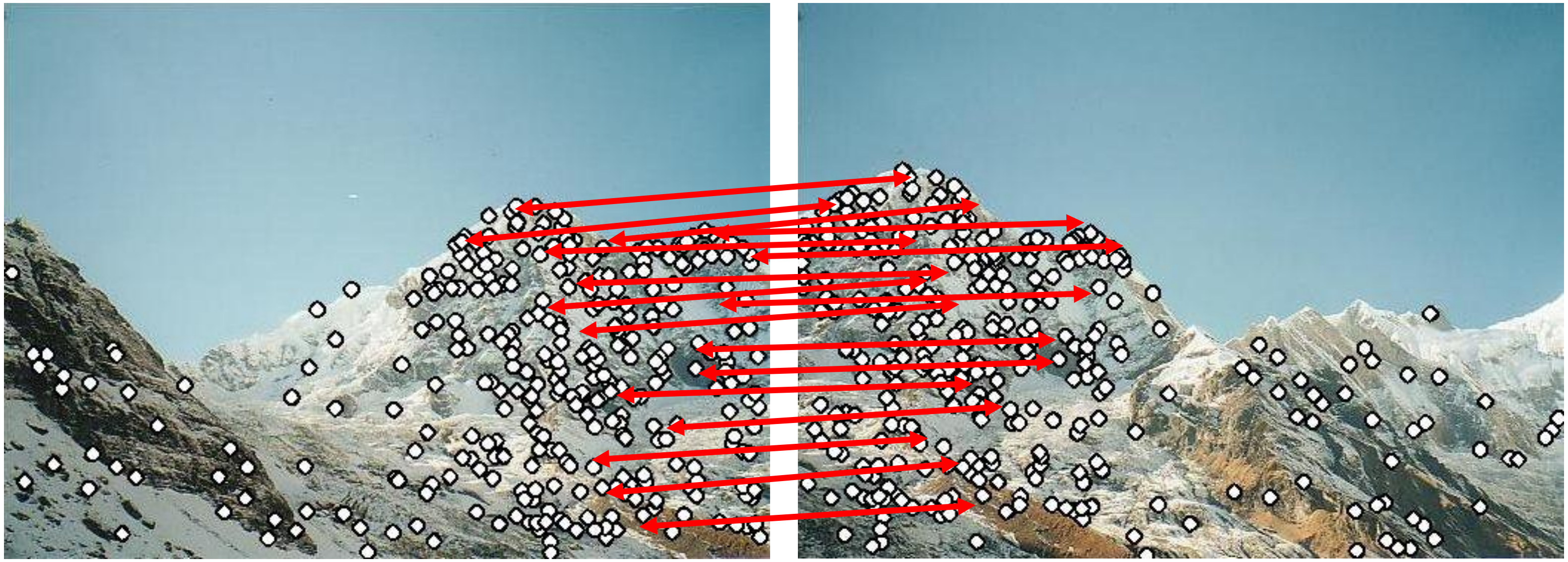
- Extract features
- Compute *putative matches*

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
    - *Hypothesize* transformation *T*

Robust feature-based alignment

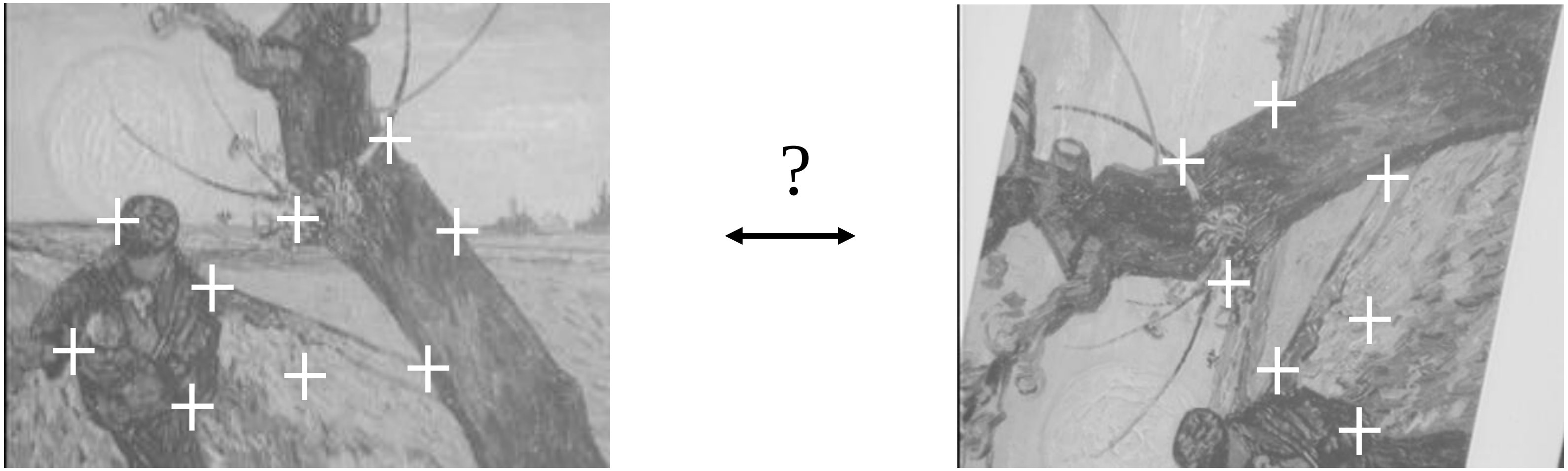


- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

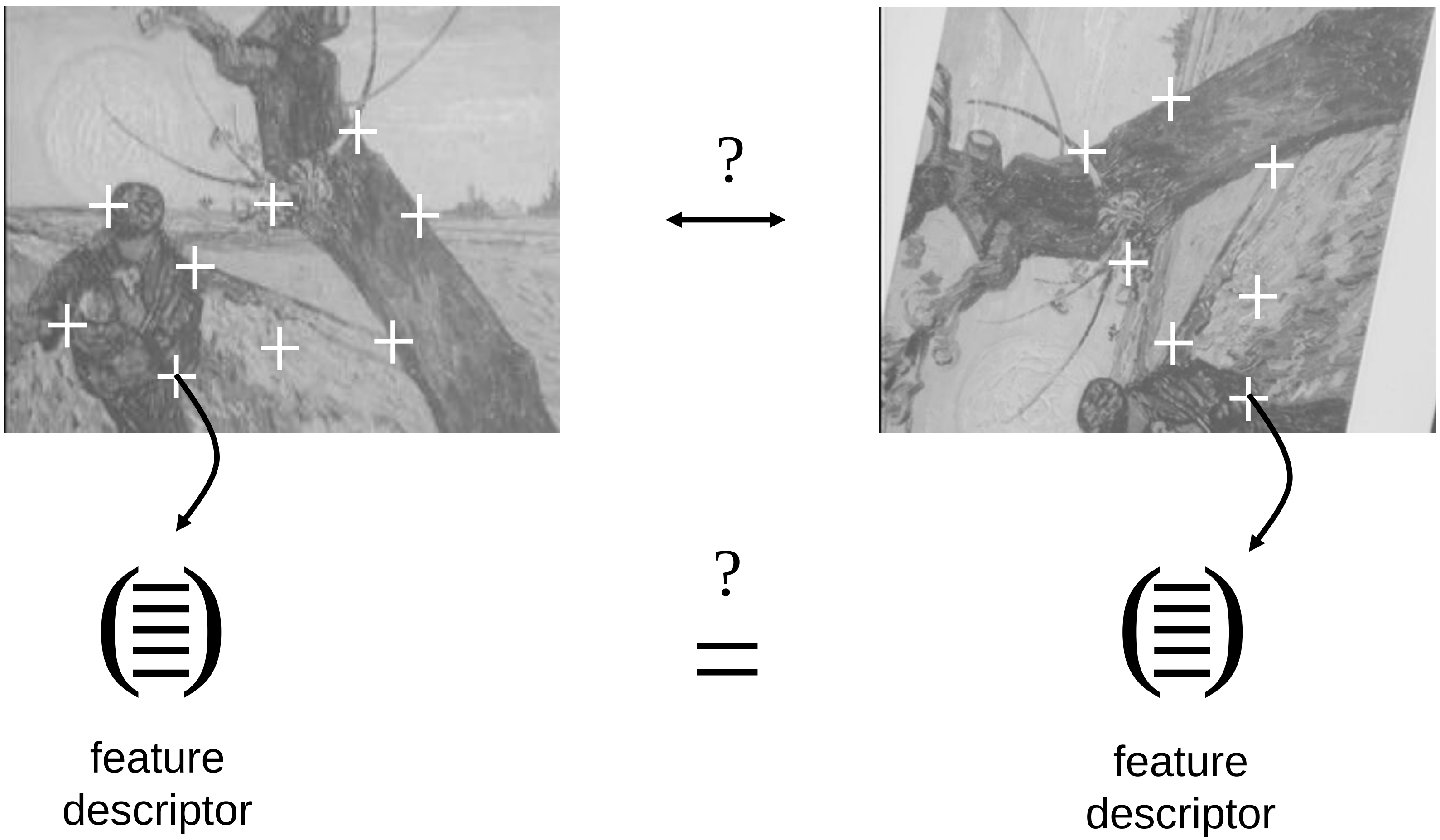
Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation T
  - *Verify* transformation (search for other matches consistent with T)

Generating putative correspondences



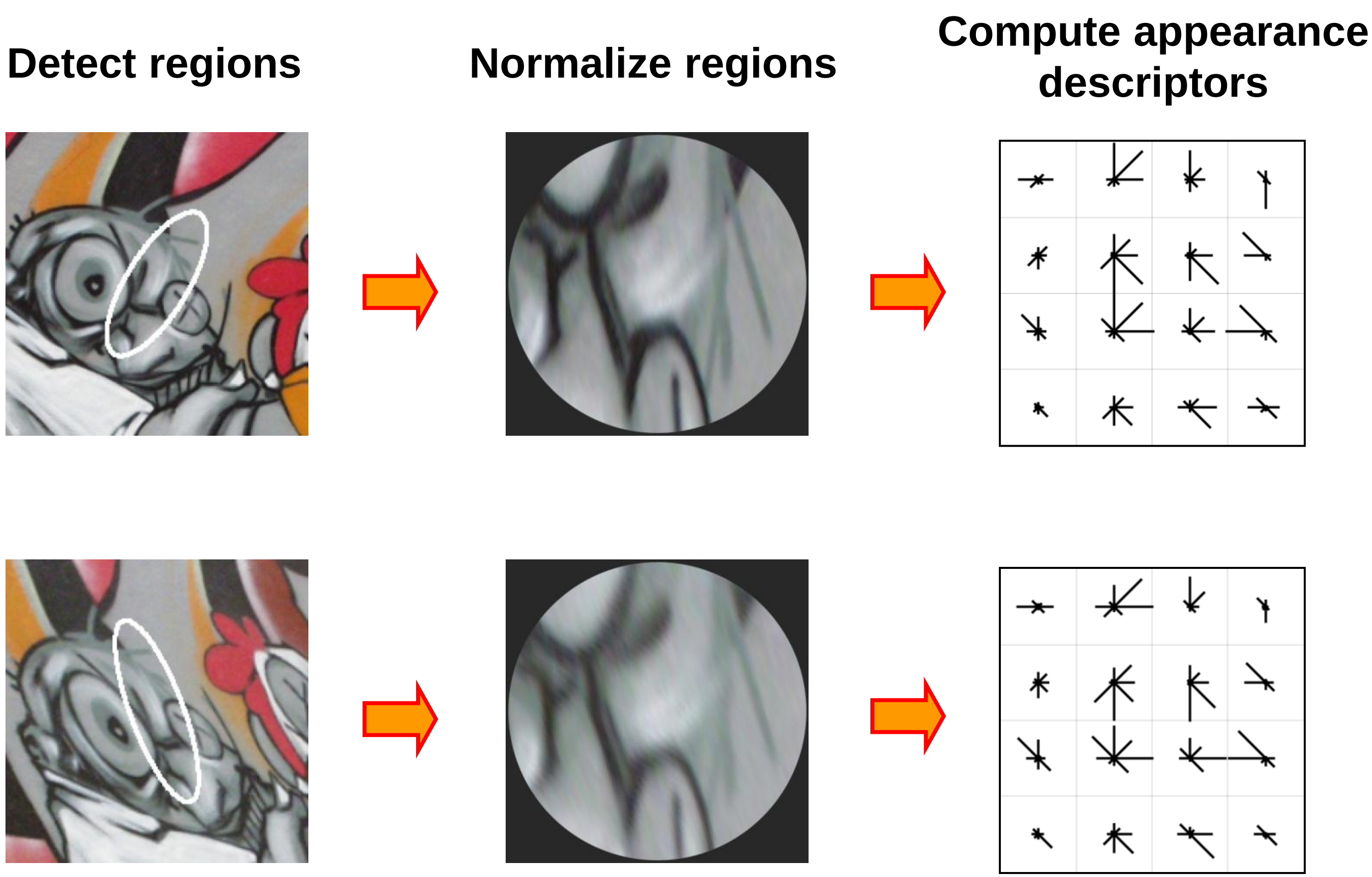
Generating putative correspondences



- Need to compare *feature descriptors* of local patches surrounding interest points

Feature descriptors

- Recall: covariant detectors => invariant descriptors



Feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors?
  - Sum of squared differences (SSD)

$$\mathrm{SSD}(u,v) = \sum_i \left(u_i - v_i\right)^2$$

– Not invariant to intensity change

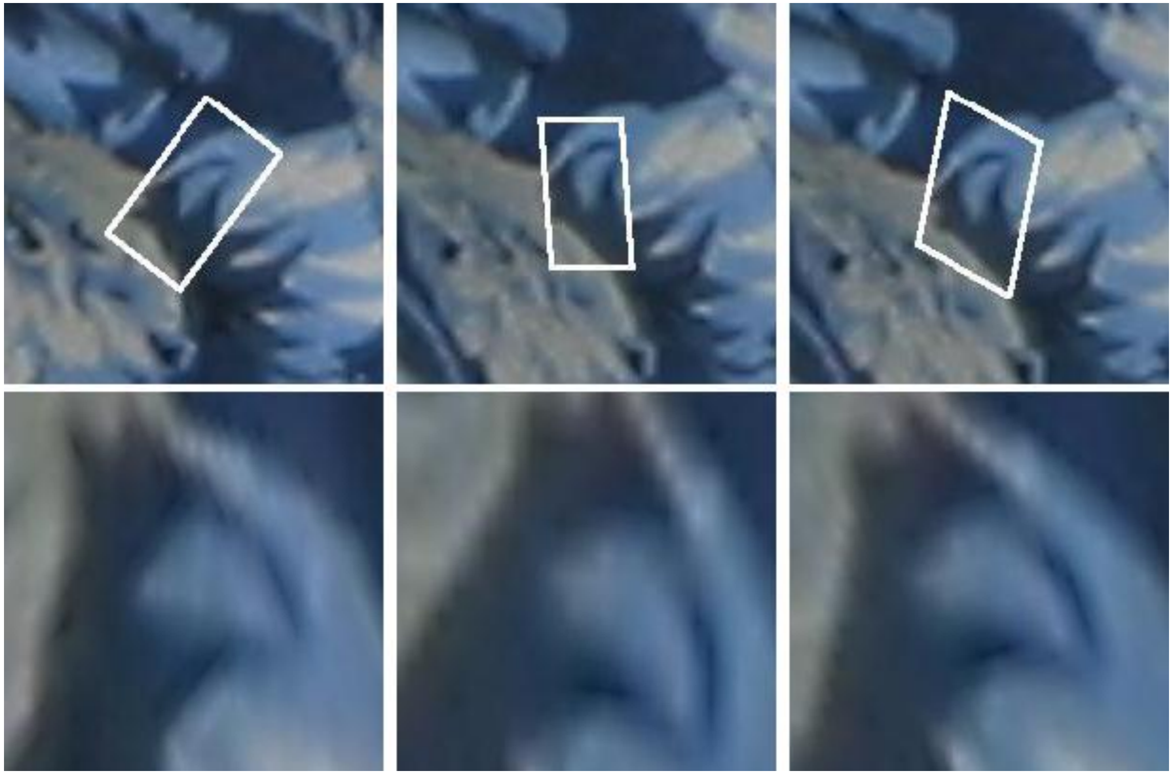
  - Normalized correlation

$$\rho(u,v) = \frac{\sum_i (u_i - \overline{u})(v_i - \overline{v})}{\sqrt{\left(\sum_j (u_j - \overline{u})^2\right)\left(\sum_j (v_j - \overline{v})^2\right)}}$$

– Invariant to affine intensity change

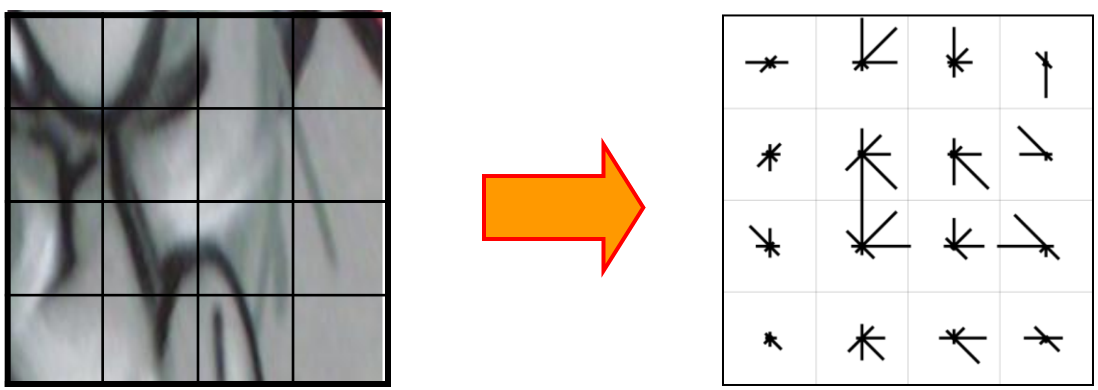
Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot



Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions



David G. Lowe. ["Distinctive image features from scale-invariant keypoints."]() *IJCV* 60 (2), pp. 91-110, 2004.

Feature descriptors: SIFT

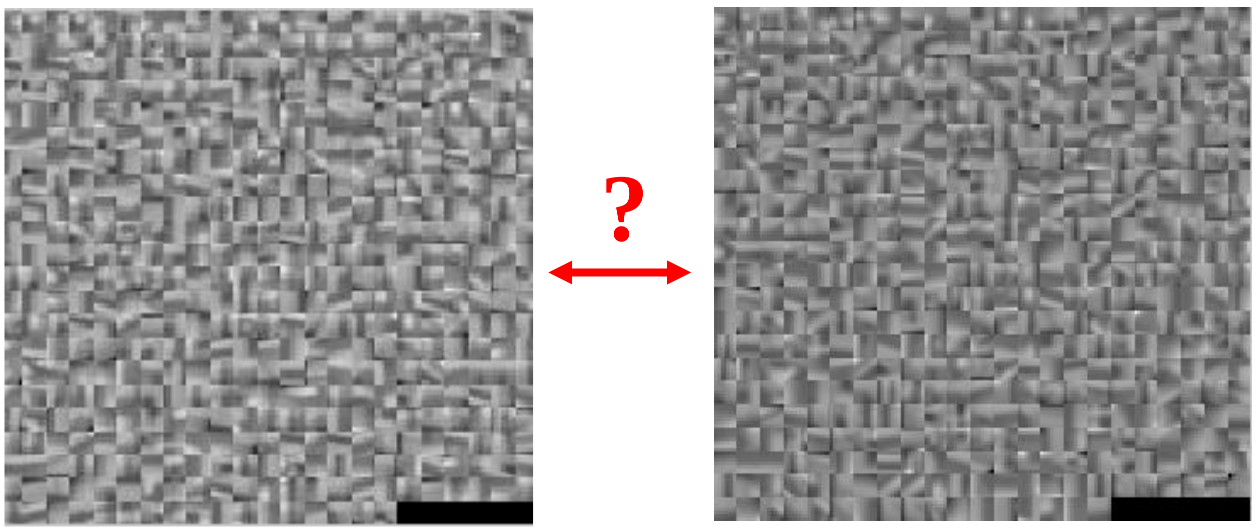
- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions

- Advantage over raw vectors of pixel values
  - Gradients less sensitive to illumination change
  - Pooling of gradients over the sub-patches achieves robustness to small shifts, but still preserves some spatial information

David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

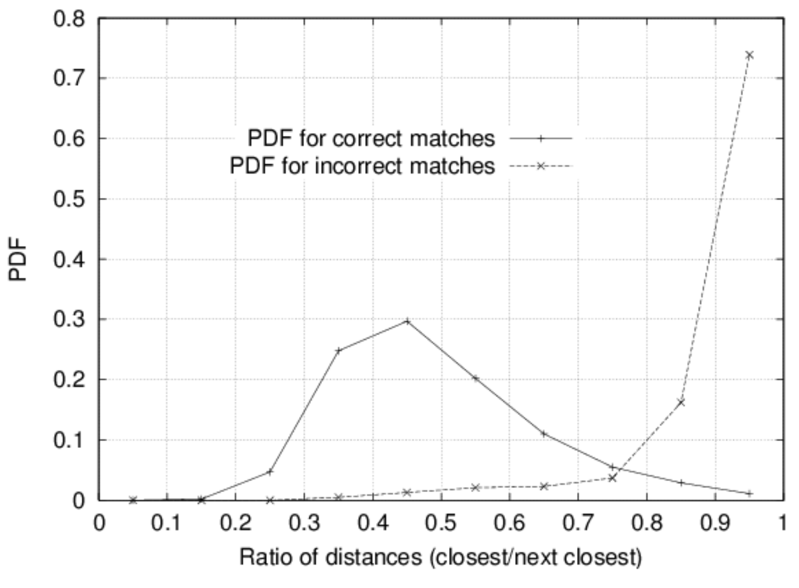
Feature matching

- Generating *putative matches*: for each patch in one image, find a short list of patches in the other image that could match it based solely on appearance



Feature space outlier rejection

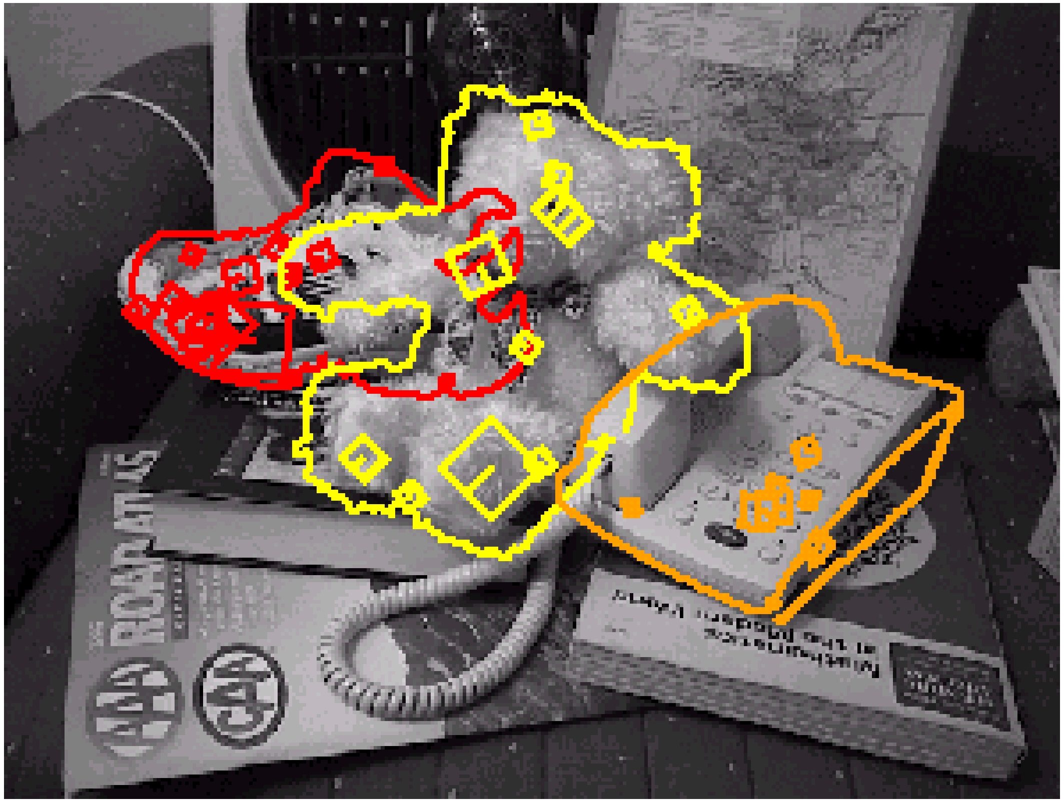
- How can we tell which putative matches are more reliable?
- Heuristic: compare distance of **nearest** neighbor to that of **second** nearest neighbor
  - Ratio of closest distance to second-closest distance will be *high* for features that are *not* distinctive



Threshold of 0.8 provides good separation

David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Reading



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

RANSAC

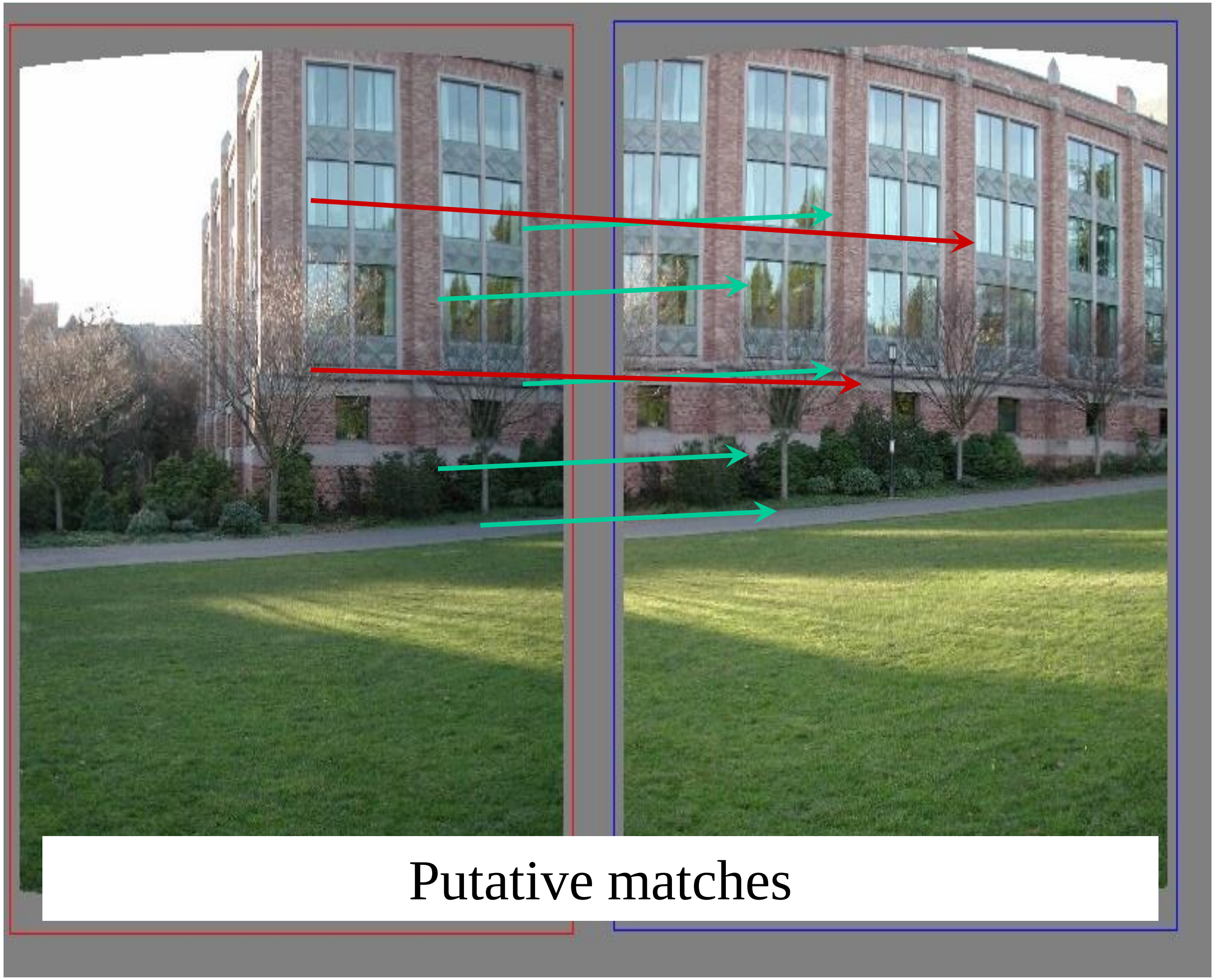
- The set of putative matches contains a very high percentage of outliers

RANSAC loop:

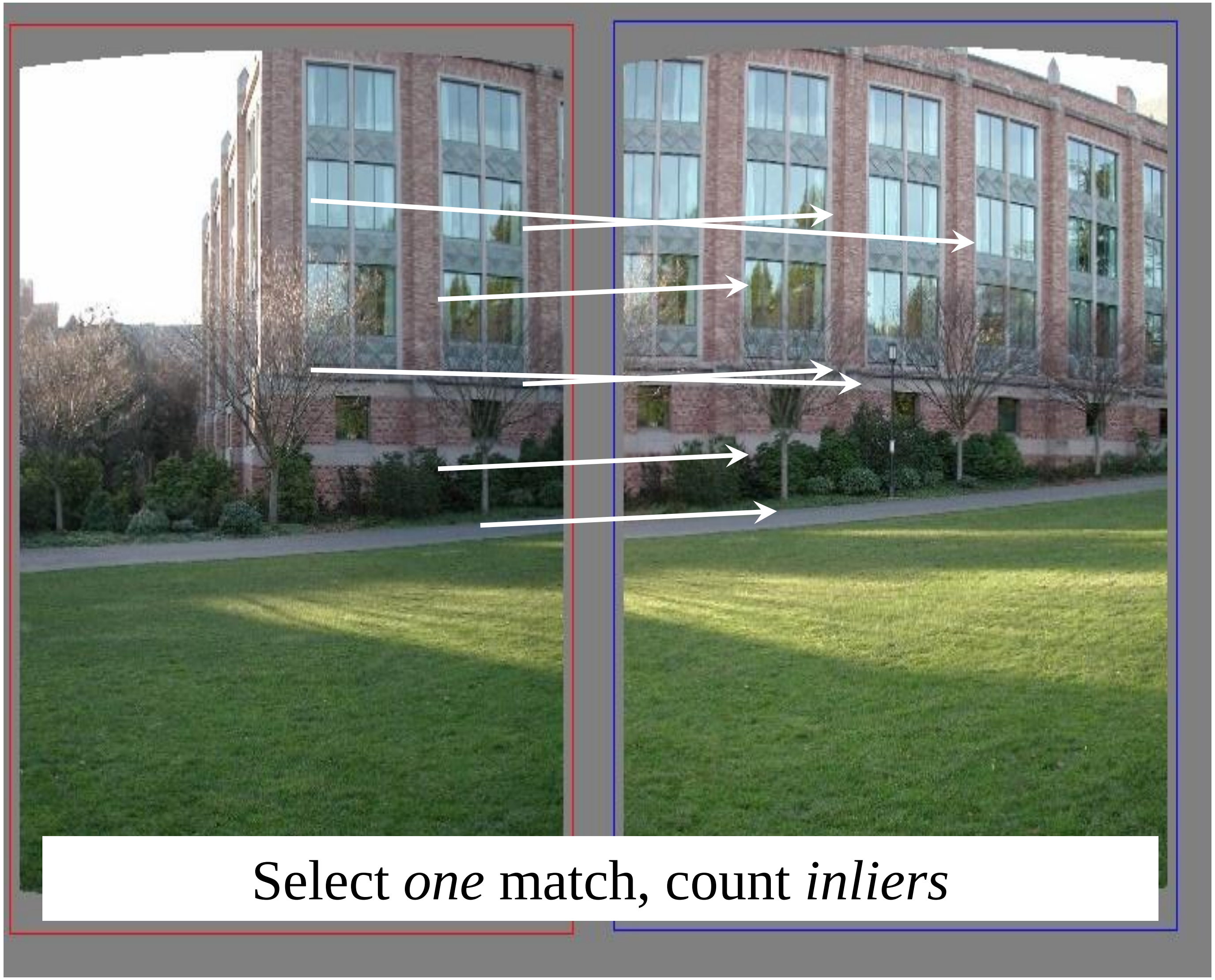
1. Randomly select a *seed group* of matches
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

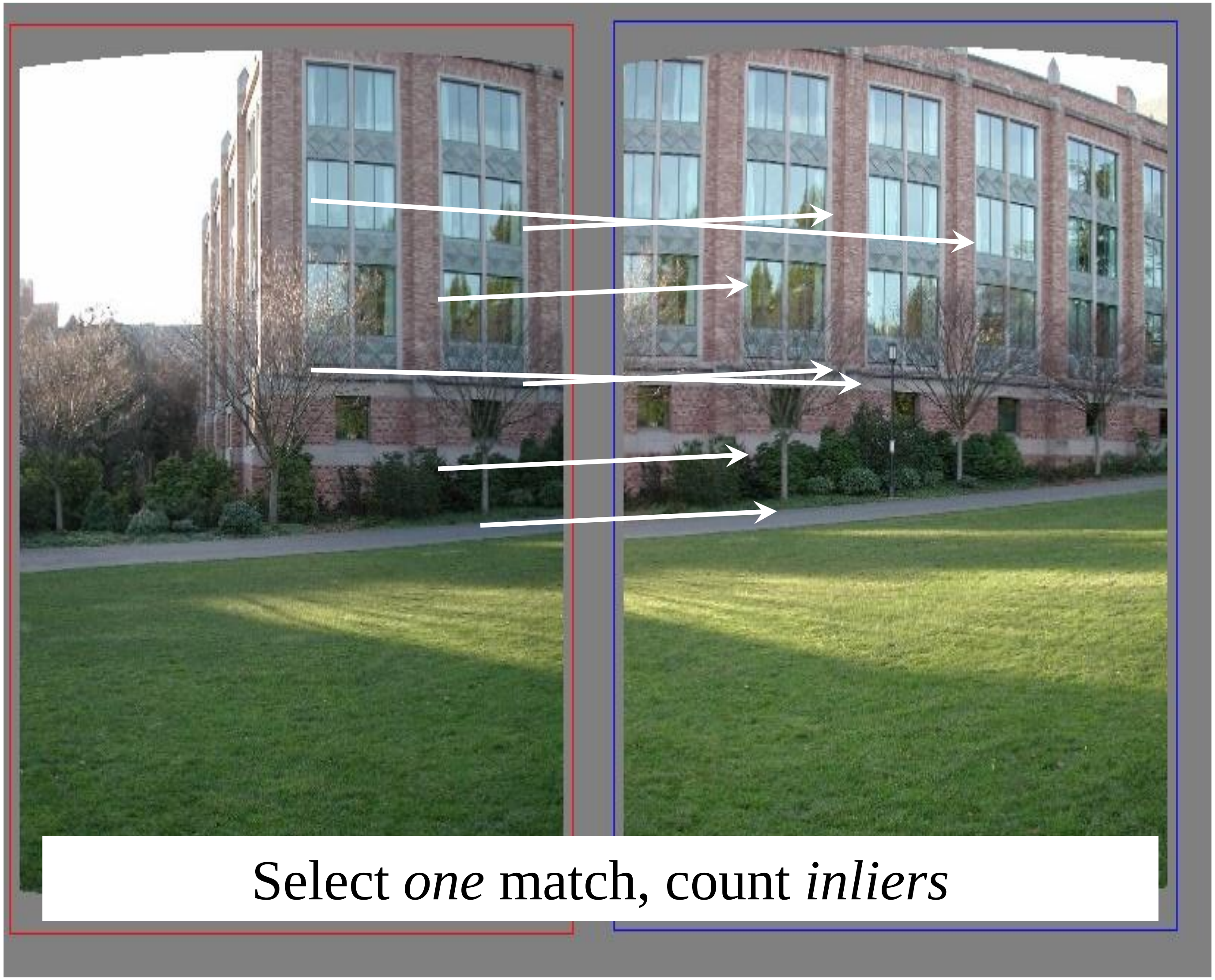
RANSAC example: Translation



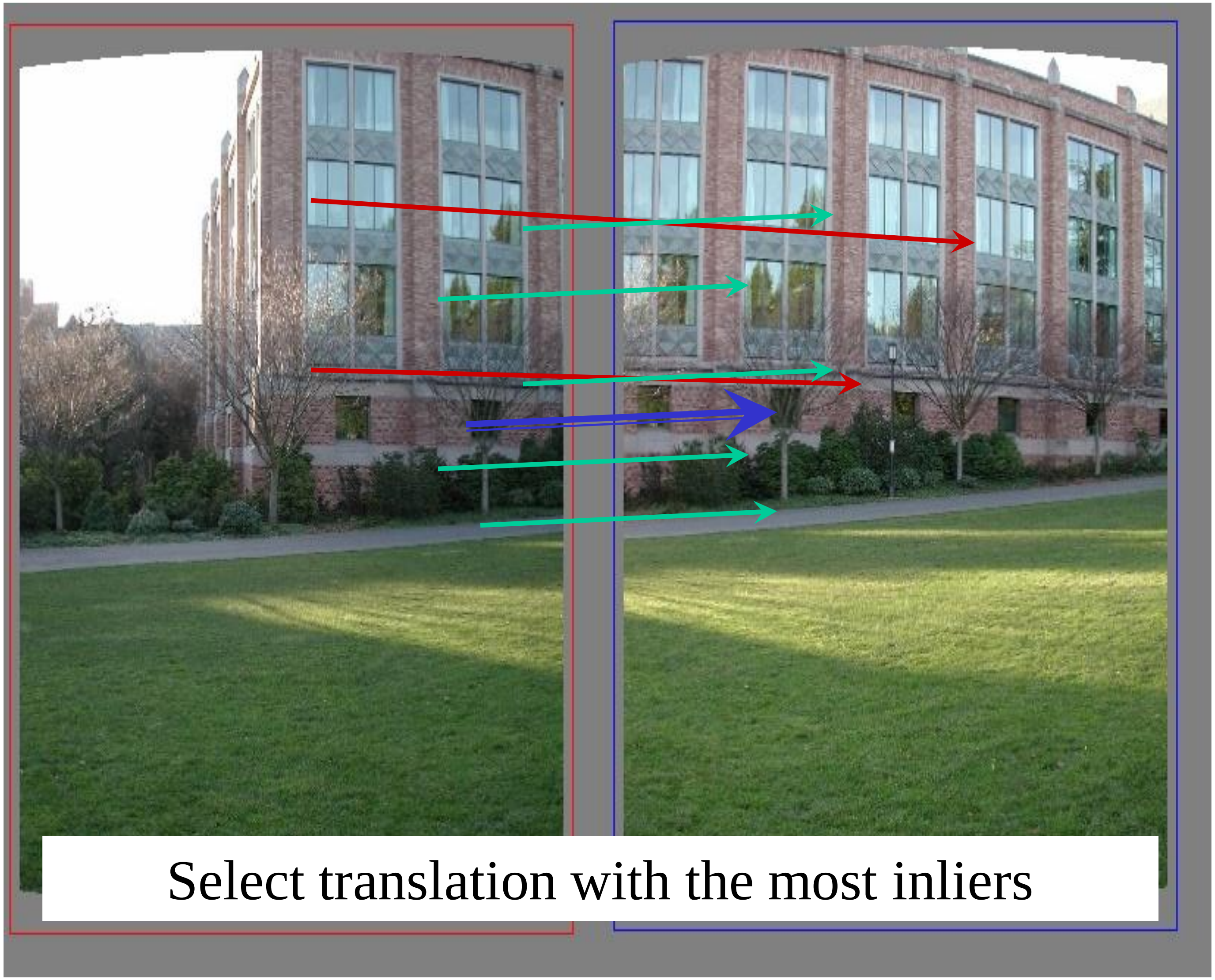
RANSAC example: Translation



RANSAC example: Translation

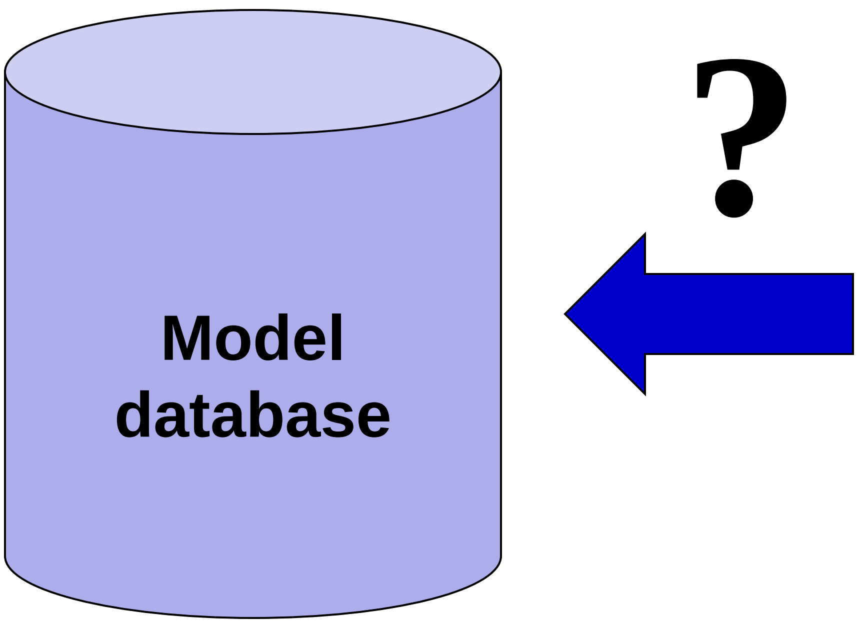


RANSAC example: Translation



Scalability: Alignment to large databases

- What if we need to align a test image with thousands or millions of images in a model database?
  - Efficient putative match generation
    - Approximate descriptor similarity search, inverted indices



Test image

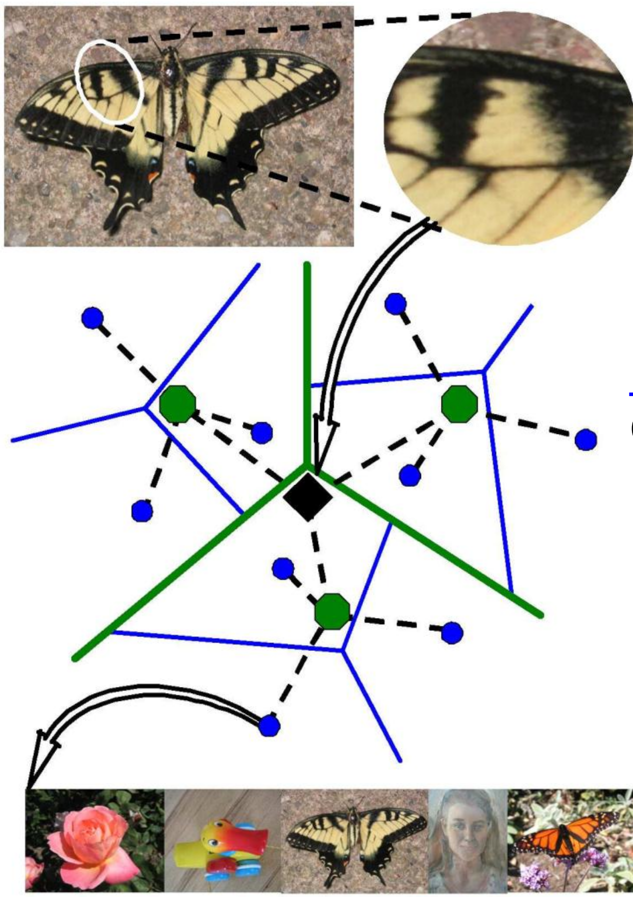
Scalability: Alignment to large databases

- What if we need to align a test image with thousands or millions of images in a model database?
  - Efficient putative match generation
    - Fast nearest neighbor search, inverted indexes

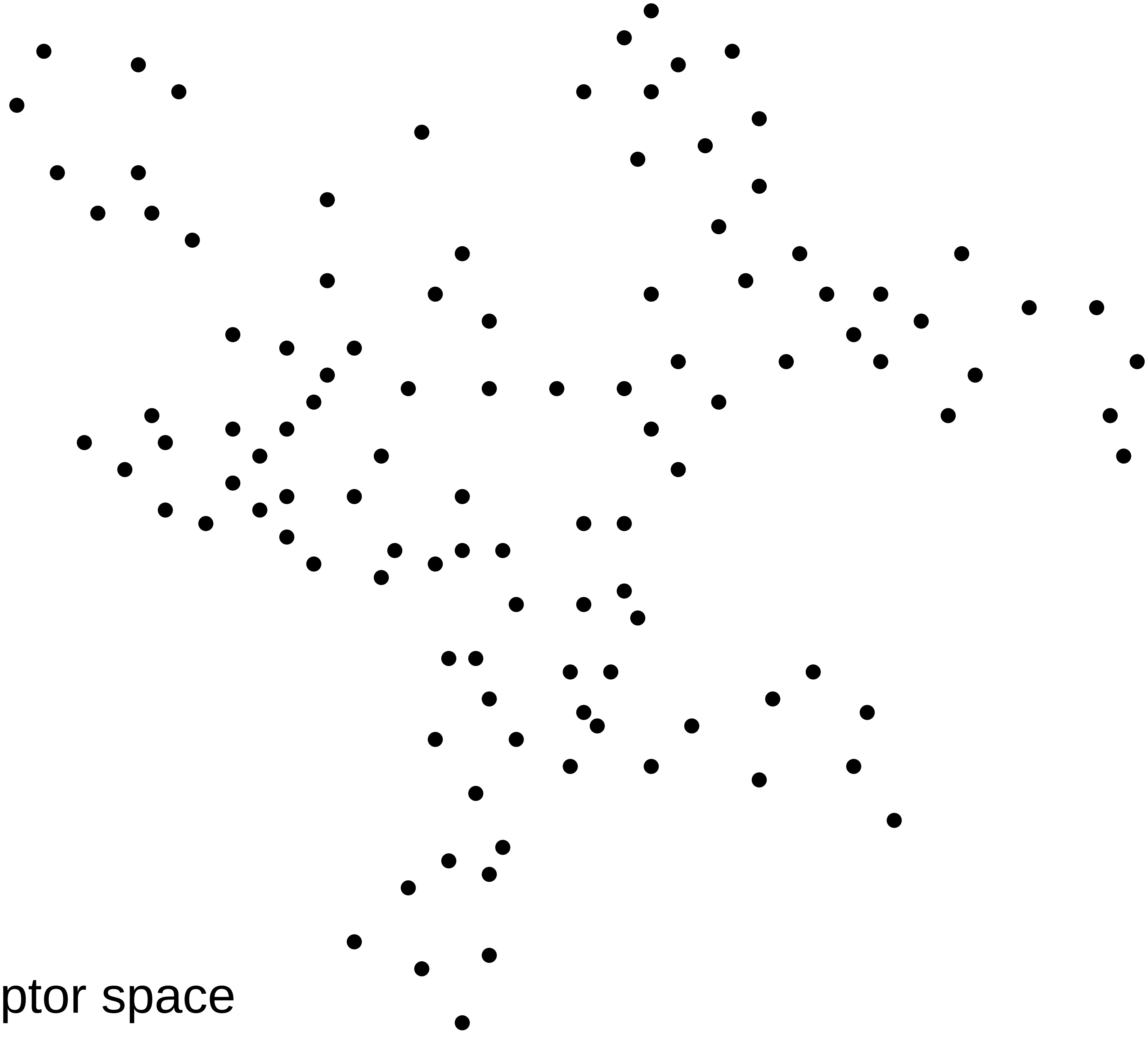
Test image

Vocabulary tree with inverted index

Database

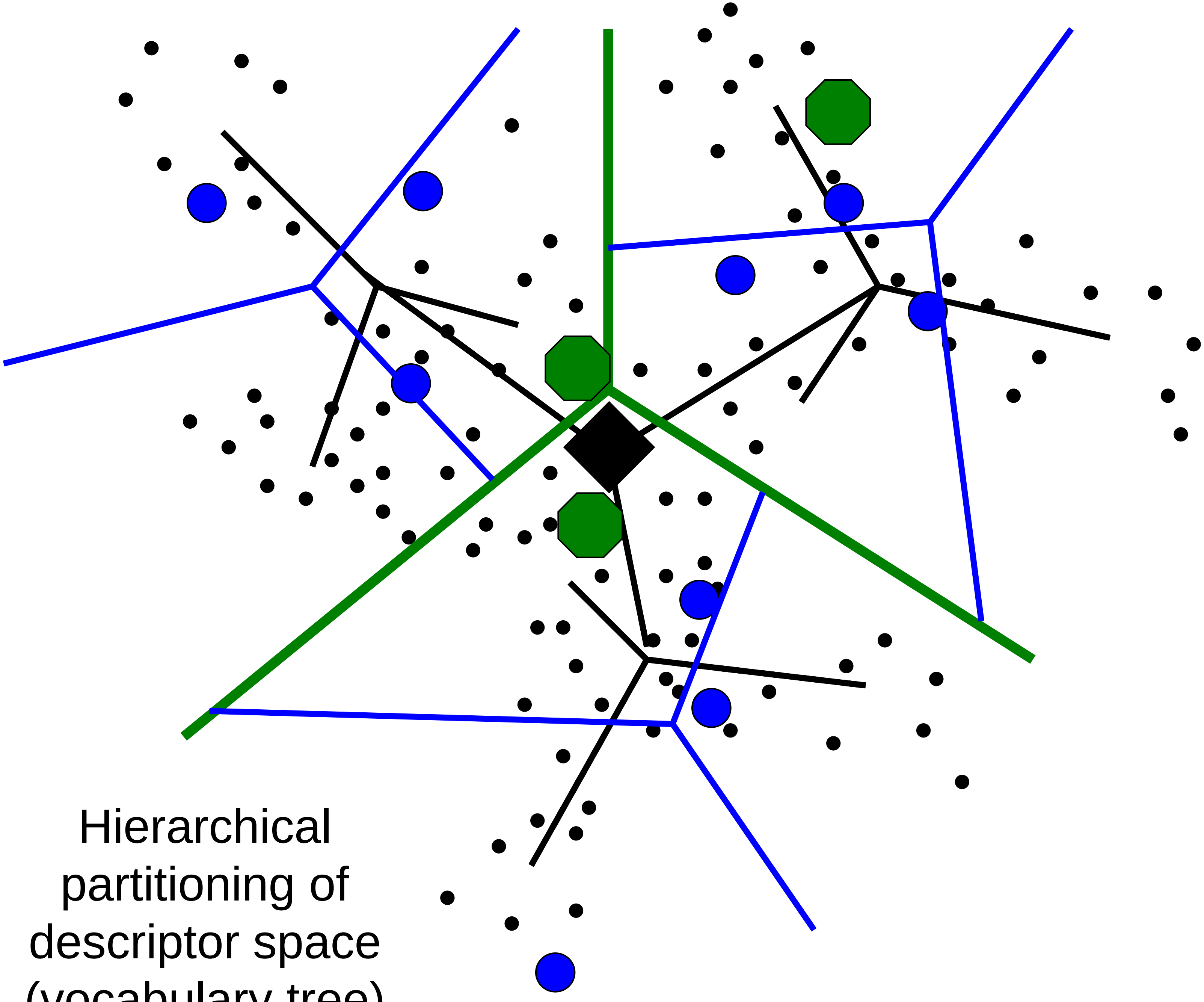


D. Nistér and H. Stewénius, [Scalable Recognition with a Vocabulary Tree](), CVPR 2006



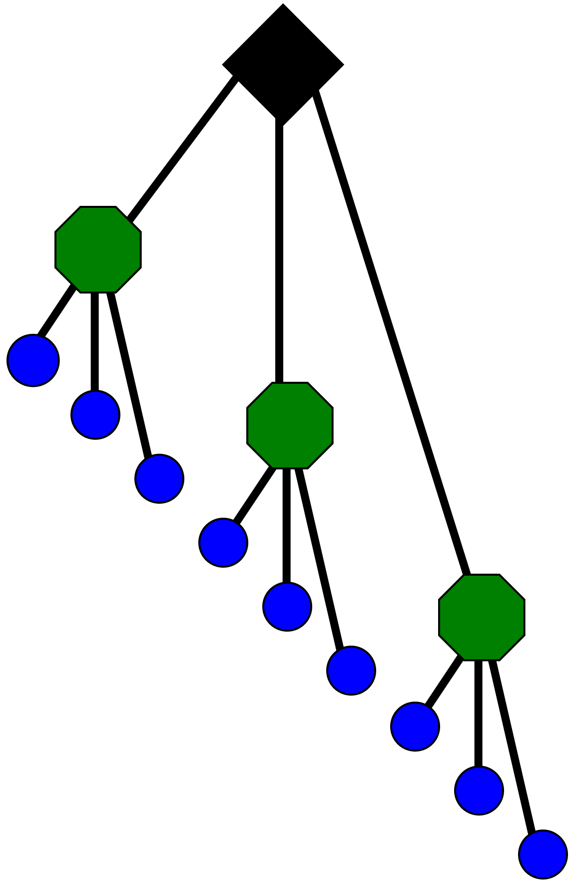
Descriptor space

Slide credit: D. Nister



Hierarchical partitioning of descriptor space (vocabulary tree)

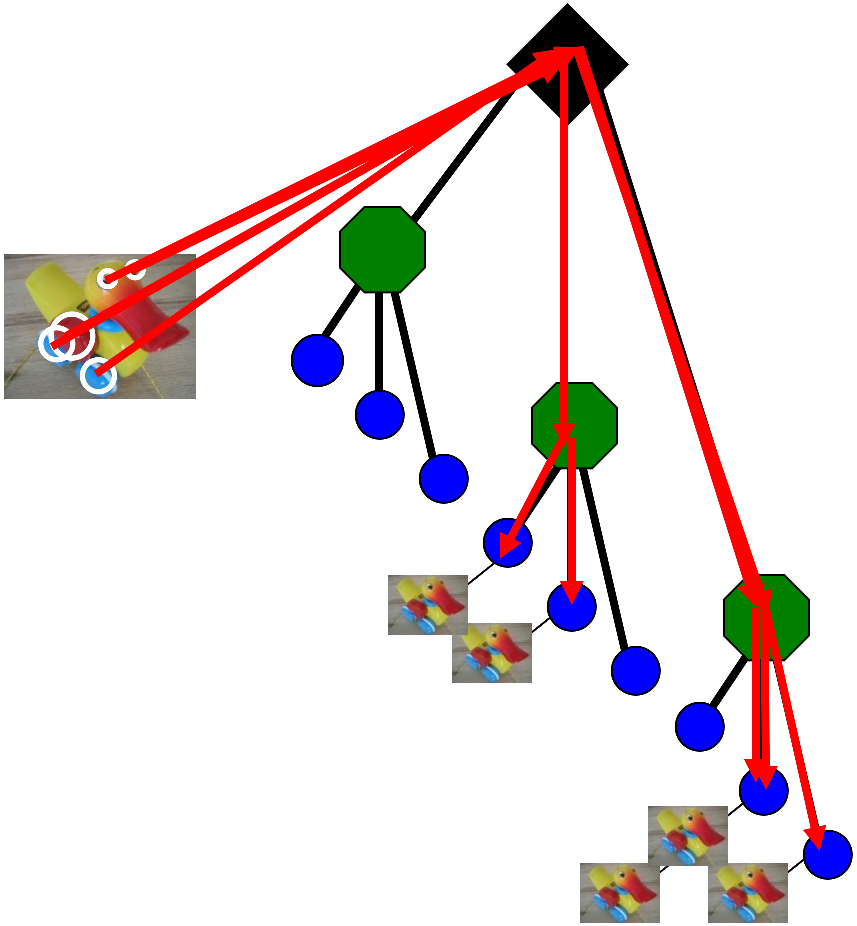
Slide credit: D. Nister



Vocabulary tree/inverted index

Slide credit: D. Nister

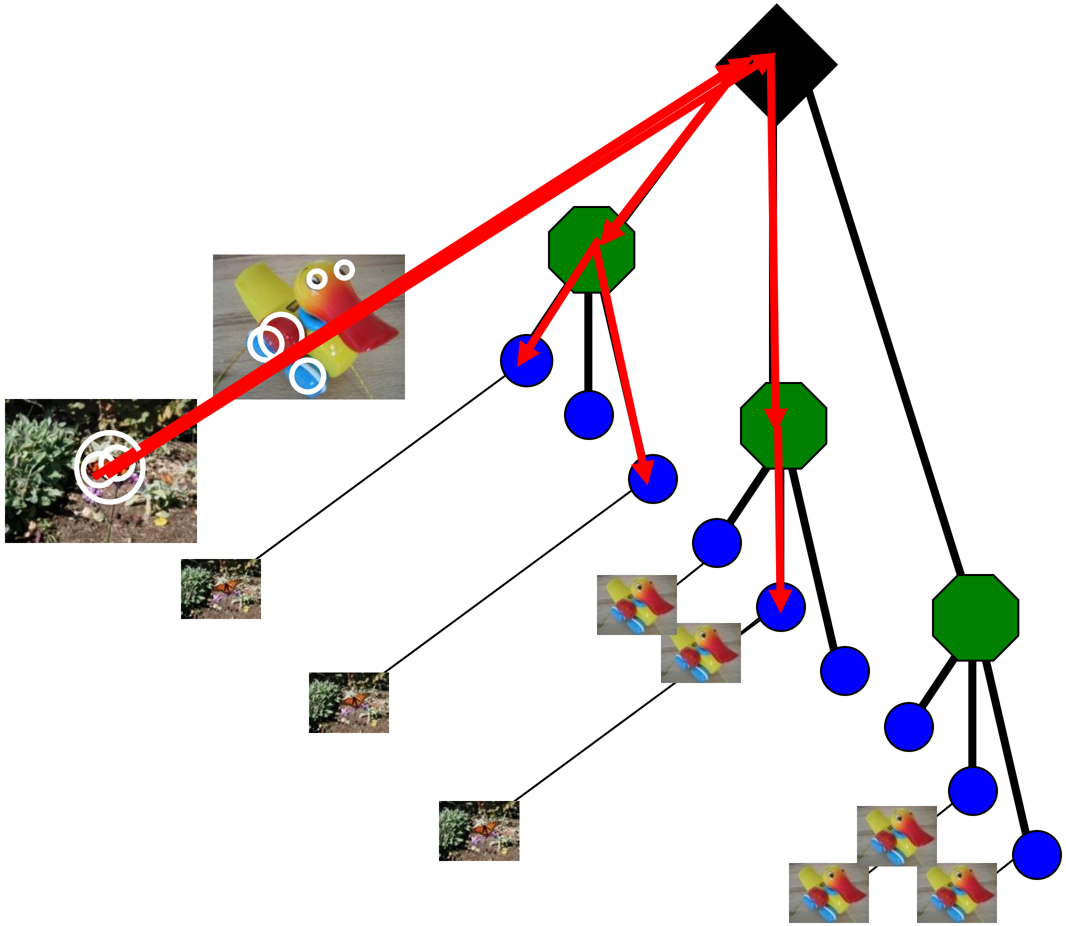
Model images



Populating the vocabulary tree/inverted index

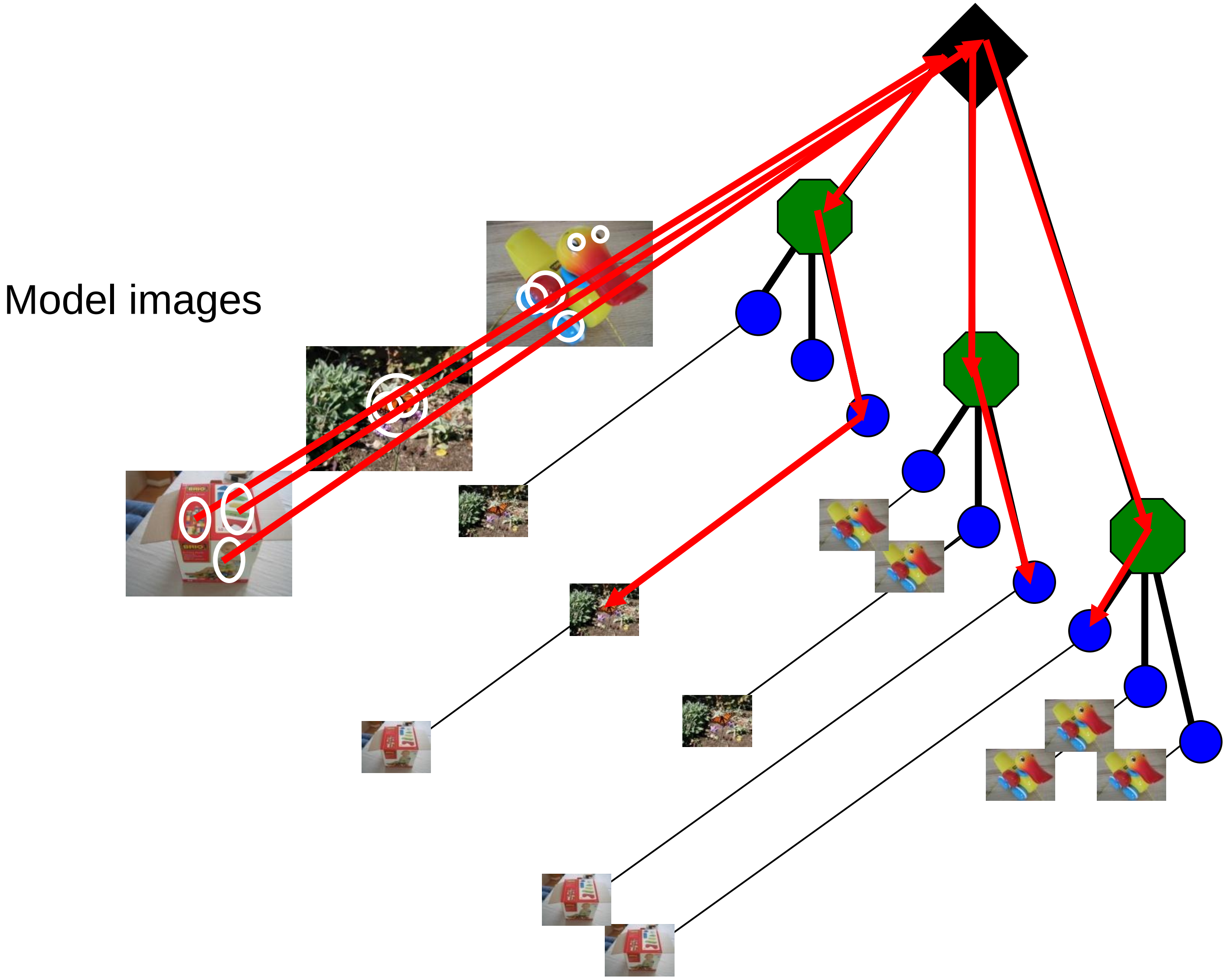
Slide credit: D. Nister

Model images



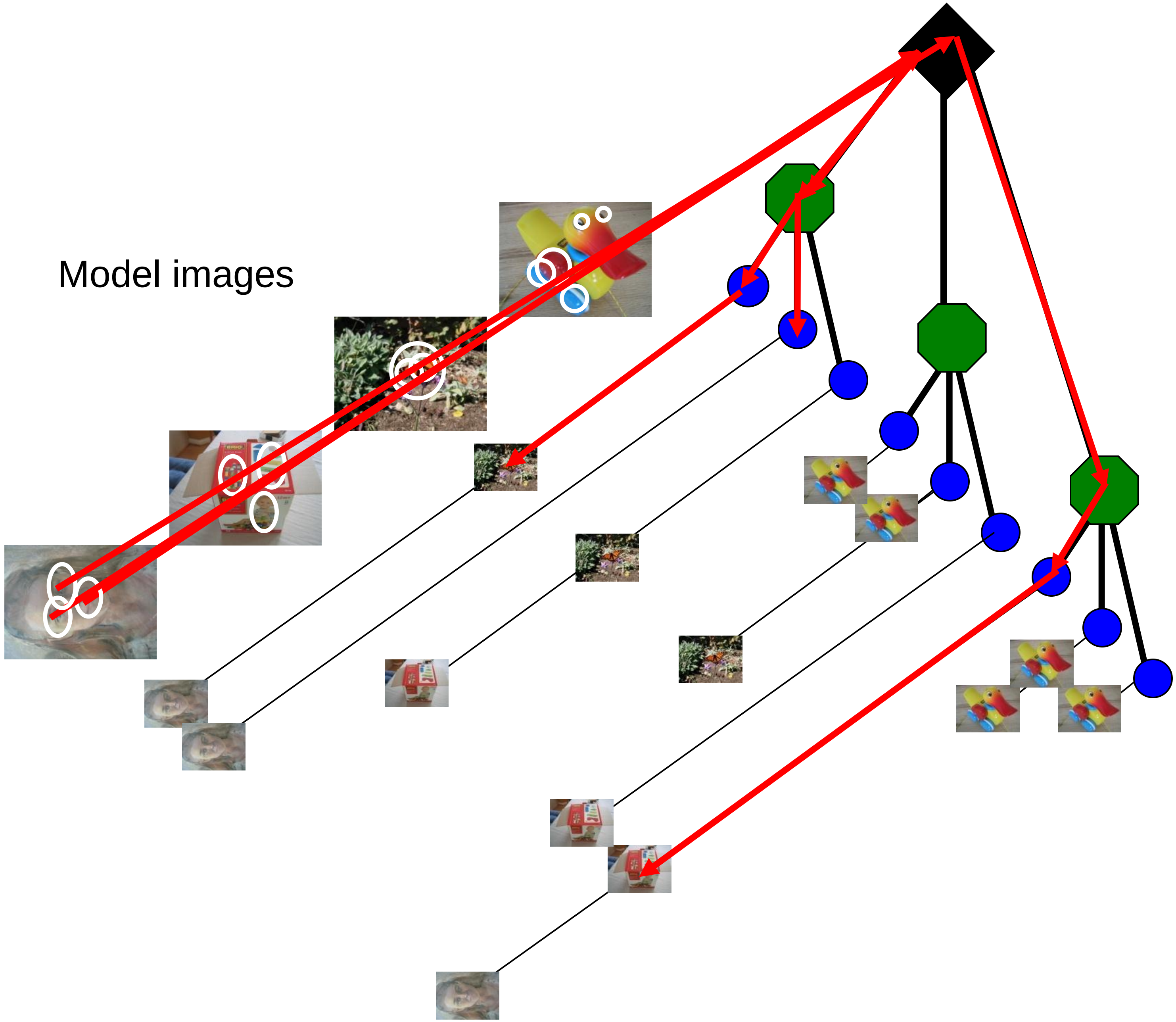
Populating the vocabulary tree/inverted index

Slide credit: D. Nister



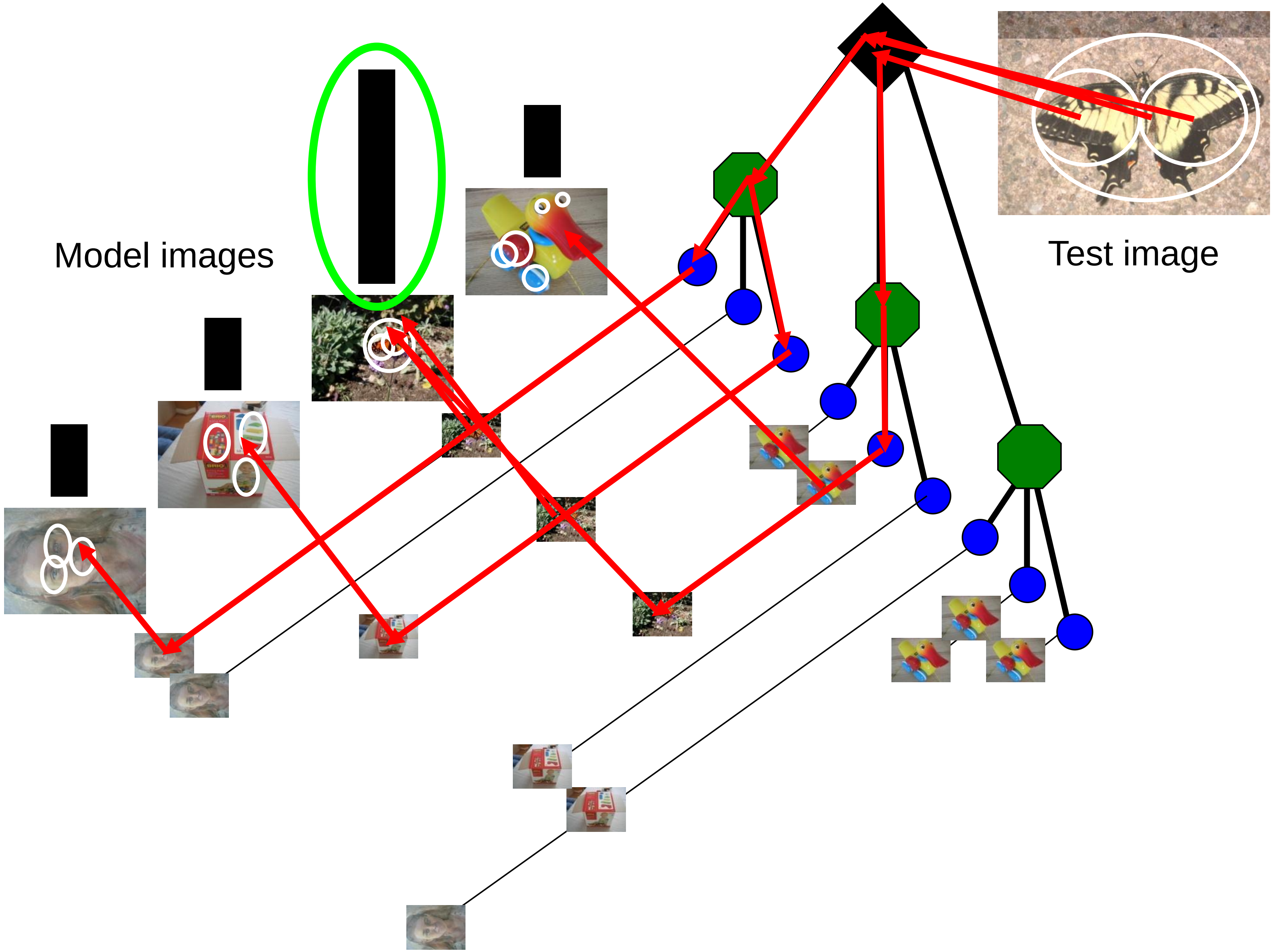
Populating the vocabulary tree/inverted index

Slide credit: D. Nister



Populating the vocabulary tree/inverted index

Slide credit: D. Nister

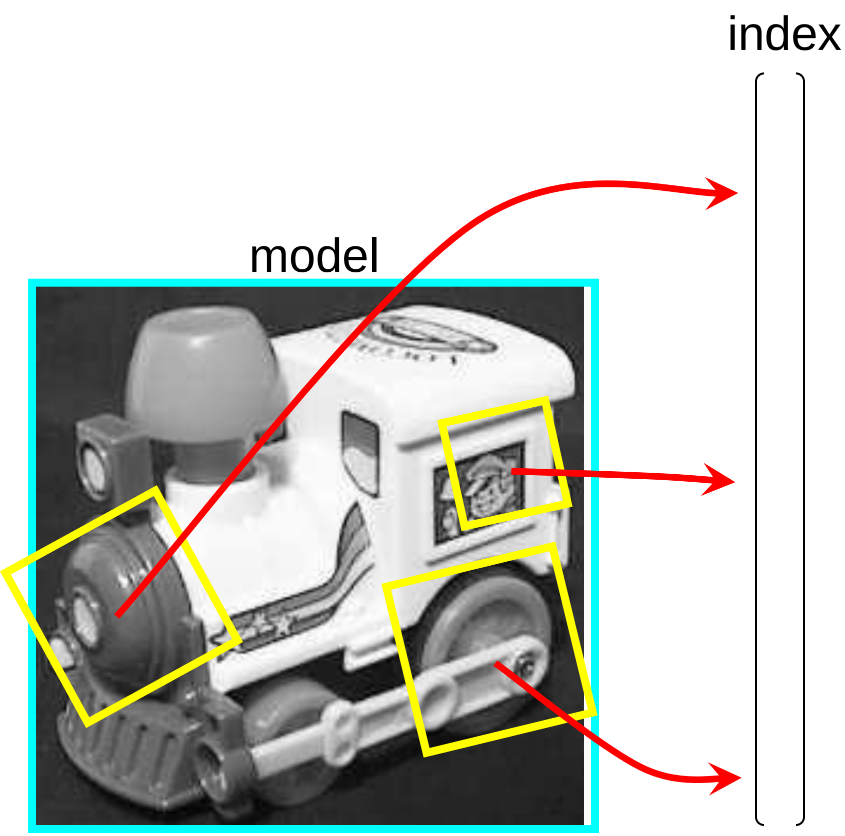


Looking up a test image

Slide credit: D. Nister

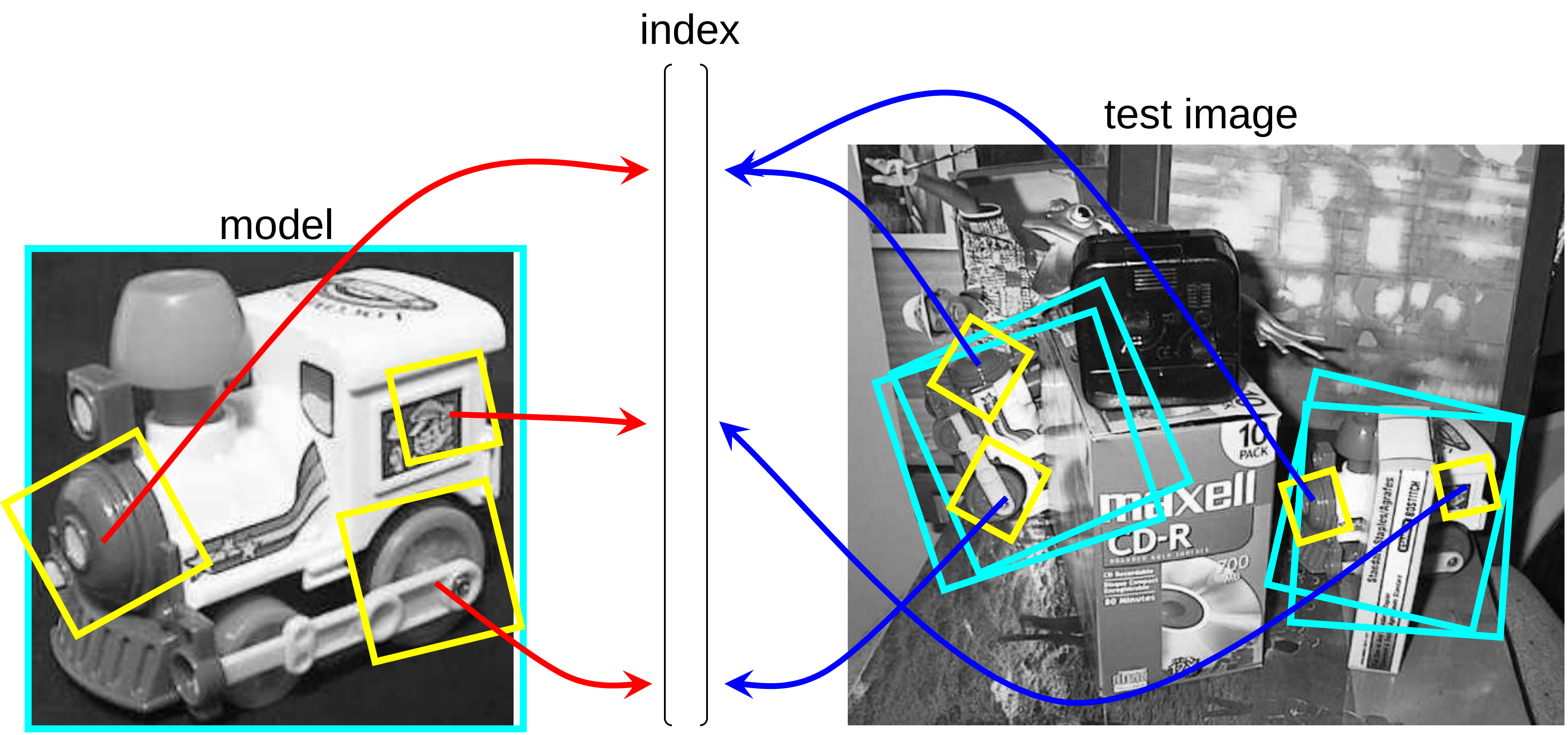
Voting for geometric transformations

- **Modeling phase:** For each model feature, record 2D location, scale, and orientation of model (relative to normalized feature coordinate frame)



Voting for geometric transformations

- **Test phase:** Each match between a test and model feature votes in a 4D Hough space (location, scale, orientation) with coarse bins
- Hypotheses receiving some minimal amount of votes can be subjected to more detailed geometric verification



Indexing with geometric invariants

- When we don't have feature descriptors, we can take n-tuples of neighboring features and compute invariant features from their geometric configurations

- Application: searching the sky: http://www.astrometry.net/