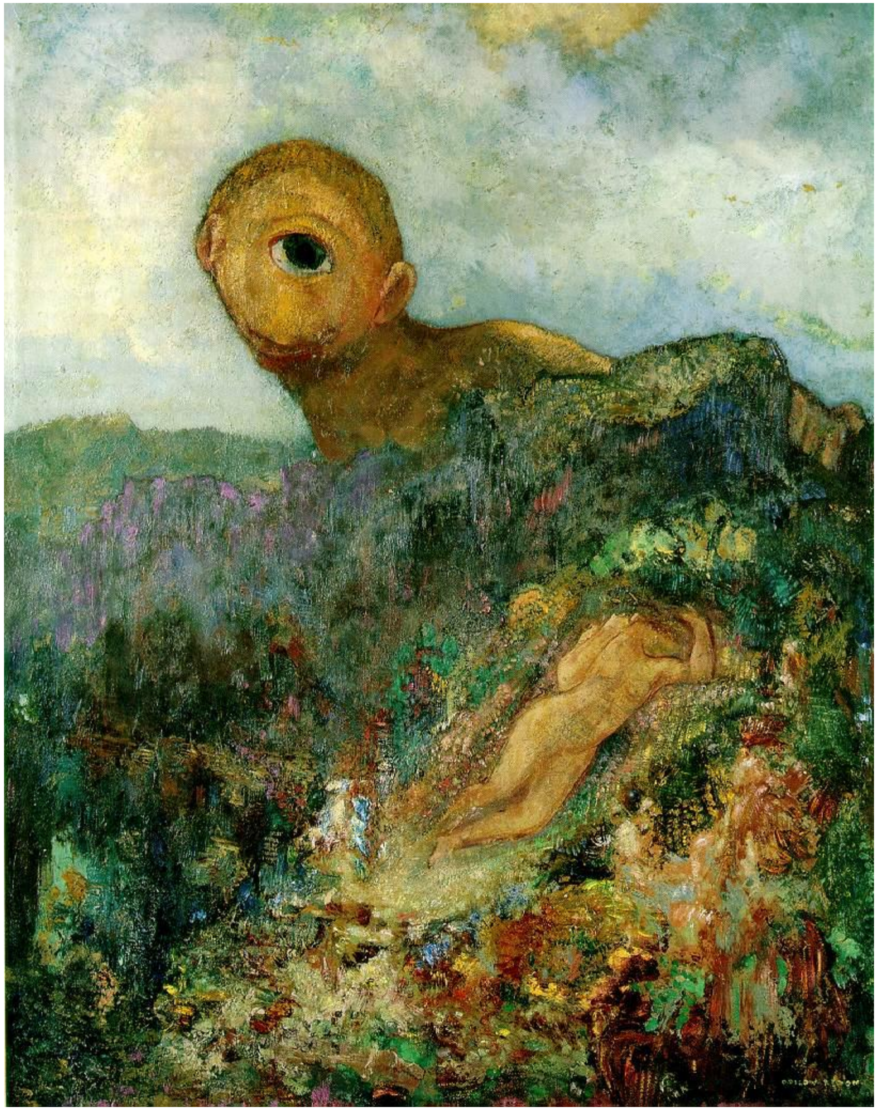
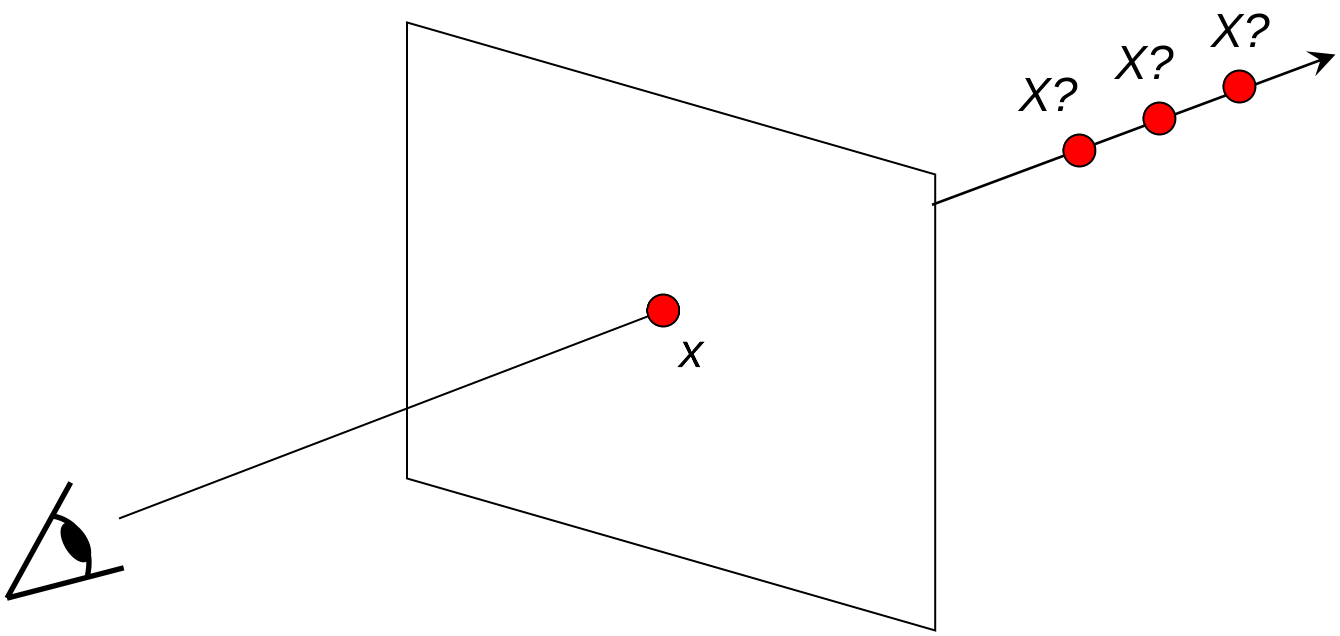
Single-view geometry



Odilon Redon, Cyclops, 1914

Our goal: Recovery of 3D structure

- Recovery of structure from one image is inherently ambiguous



Our goal: Recovery of 3D structure

- Recovery of structure from one image is inherently ambiguous

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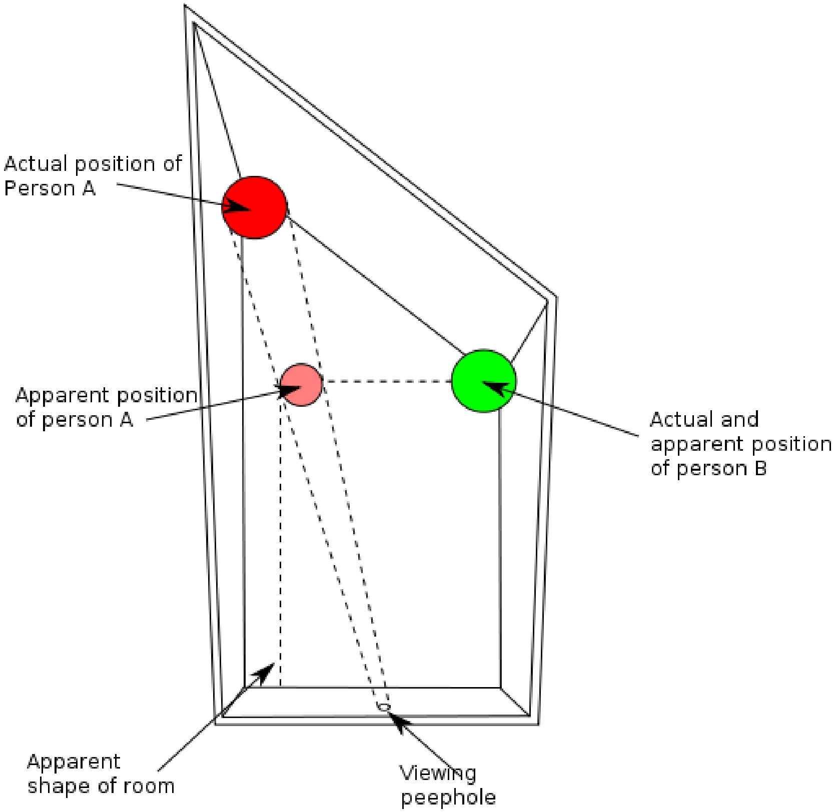
- Recovery of structure from one image is inherently ambiguous

Ames Room

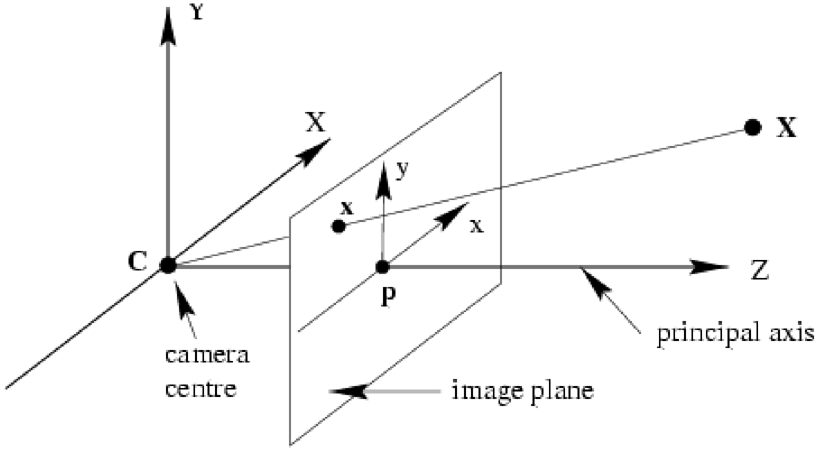


http://en.wikipedia.org/wiki/Ames_room

Our goal: Recovery of 3D structure

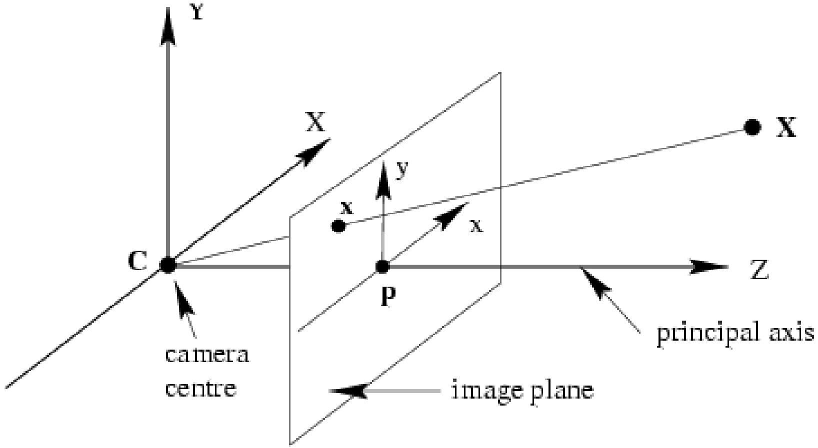
- We will need *multi-view geometry*

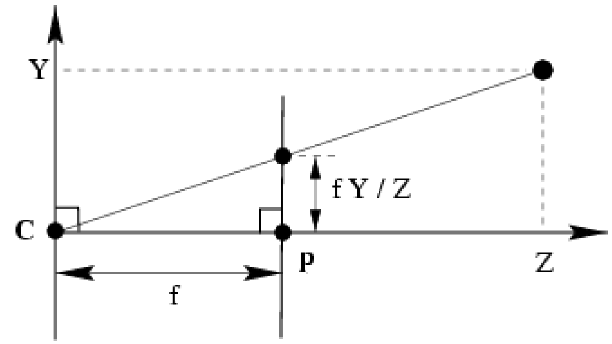
Recall: Pinhole camera model



- **Principal axis:** line from the camera center perpendicular to the image plane
- **Normalized (camera) coordinate system:** camera center is at the origin and the principal axis is the z-axis

Recall: Pinhole camera model



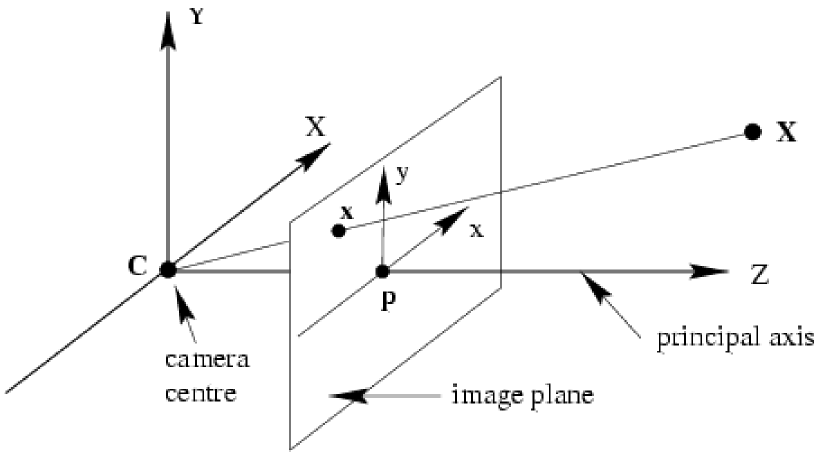


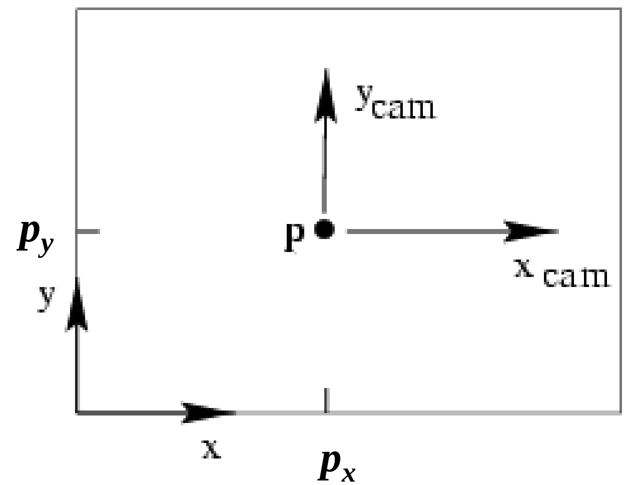
$$(X, Y, Z) \mapsto (f\,X / Z, f\,Y / Z)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f\,X \\ f\,Y \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{PX}$$

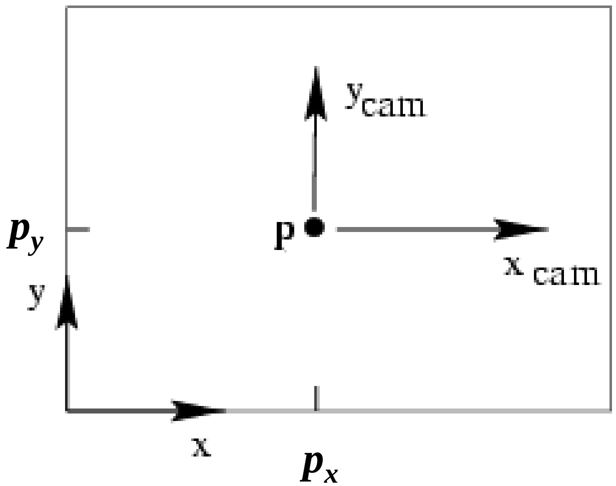
Principal point





- **Principal point (p):** point where principal axis intersects the image plane (origin of normalized coordinate system)
- Normalized coordinate system: origin is at the principal point
- Image coordinate system: origin is in the corner
- How to go from normalized coordinate system to image coordinate system?

Principal point offset

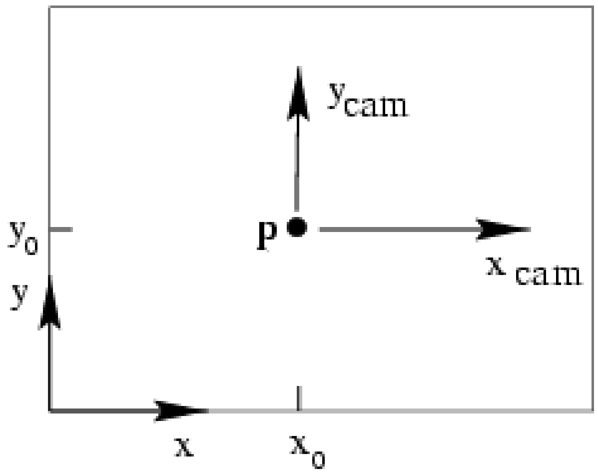


principal point: (p_x, p_y)

$$(X, Y, Z) \mapsto (f X / Z + p_x, f Y / Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f X + Z p_x \\ f Y + Z p_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal point offset

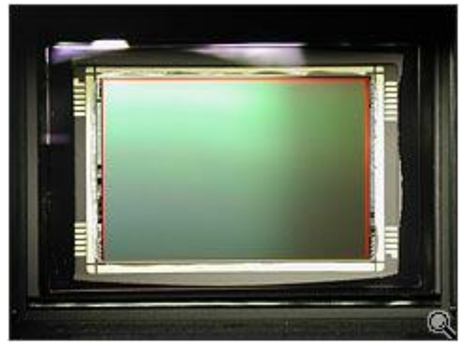


principal point: (p_x, p_y)

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$ calibration matrix $$\mathrm{P} = \mathrm{K}[\mathrm{I} \mid 0]$$

Pixel coordinates

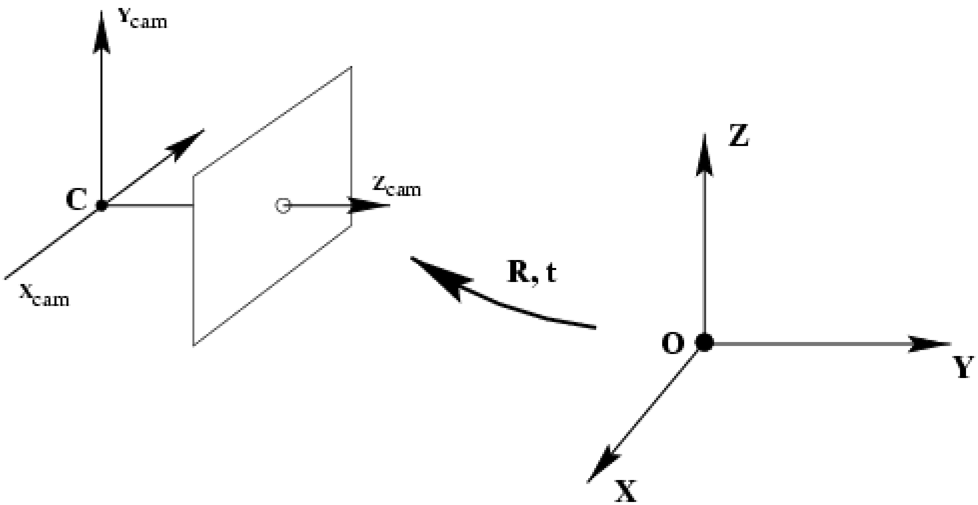


Pixel size: $\dfrac{1}{m_x} \times \dfrac{1}{m_y}$

m_x pixels per meter in horizontal direction,
m_y pixels per meter in vertical direction

$$K = \underbrace{\begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix}}_{\text{pixels/m}} \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\text{m}} = \underbrace{\begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}}_{\text{pixels}}$$

Camera rotation and translation



- In general, the camera coordinate frame will be related to the world coordinate frame by a rotation and a translation

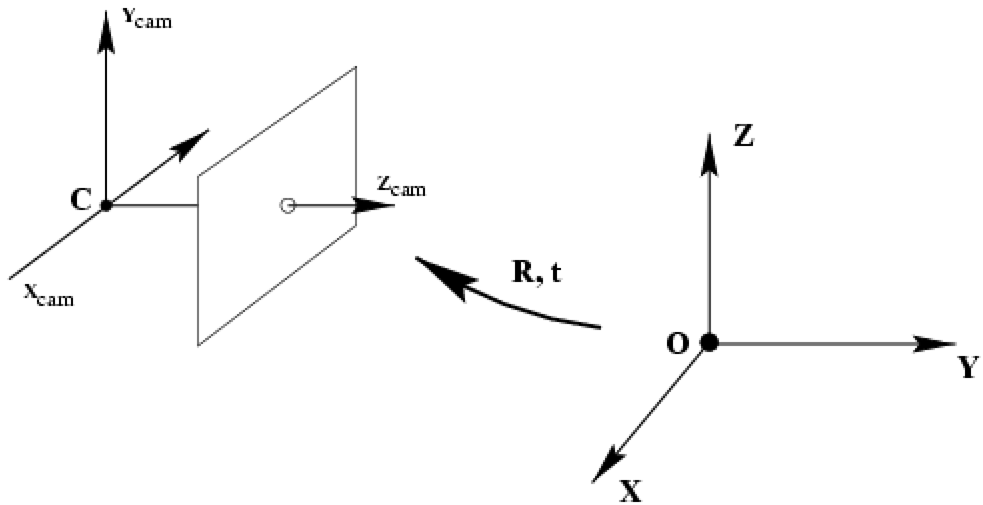
$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

coords. of point in camera frame

coords. of a point in world frame (nonhomogeneous)

coords. of camera center in world frame

Camera rotation and translation



In non-homogeneous coordinates:

$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

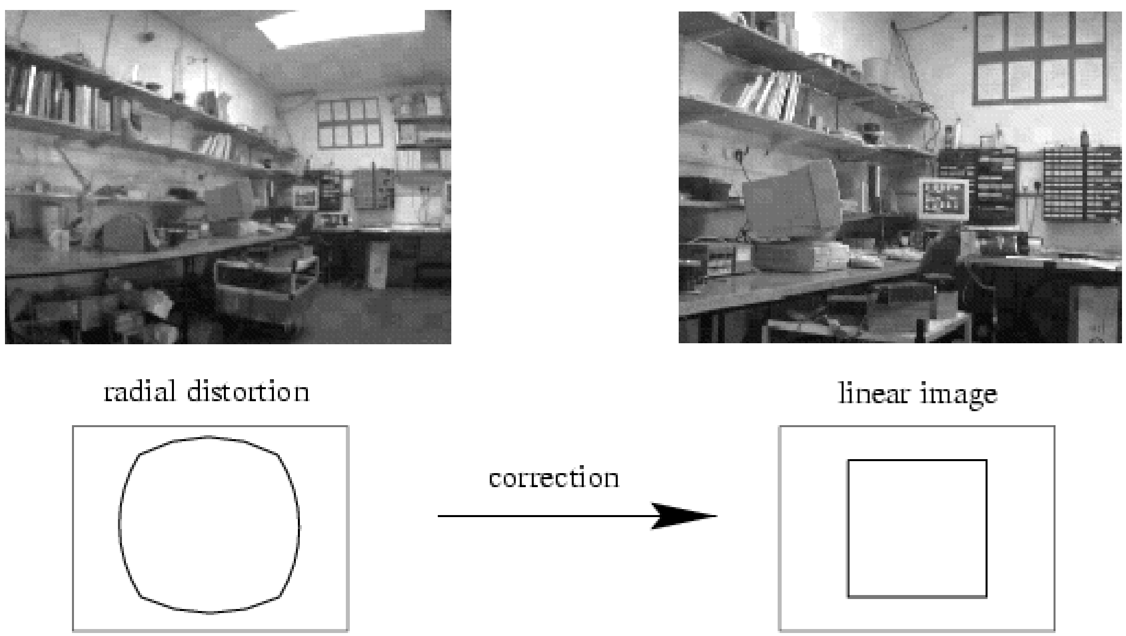
$$x = K[I \mid 0] X_{cam} = K\left[R \mid -R\tilde{C}\right] X \qquad P = K[R \mid t], \qquad t = -R\tilde{C}$$

Note: C is the null space of the camera projection matrix (PC=0)

Camera parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$



Camera parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*
- Extrinsic parameters
  - Rotation and translation relative to world coordinate system

Camera calibration

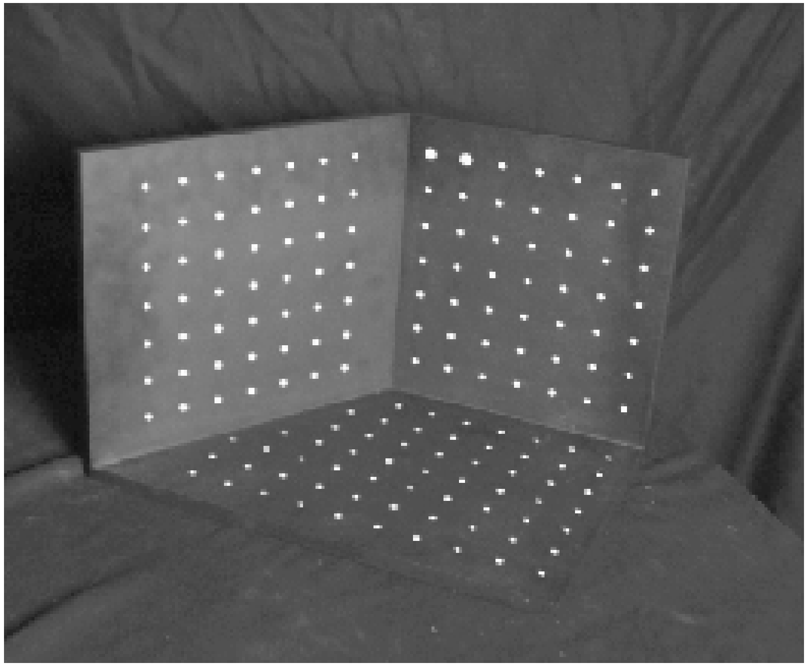
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

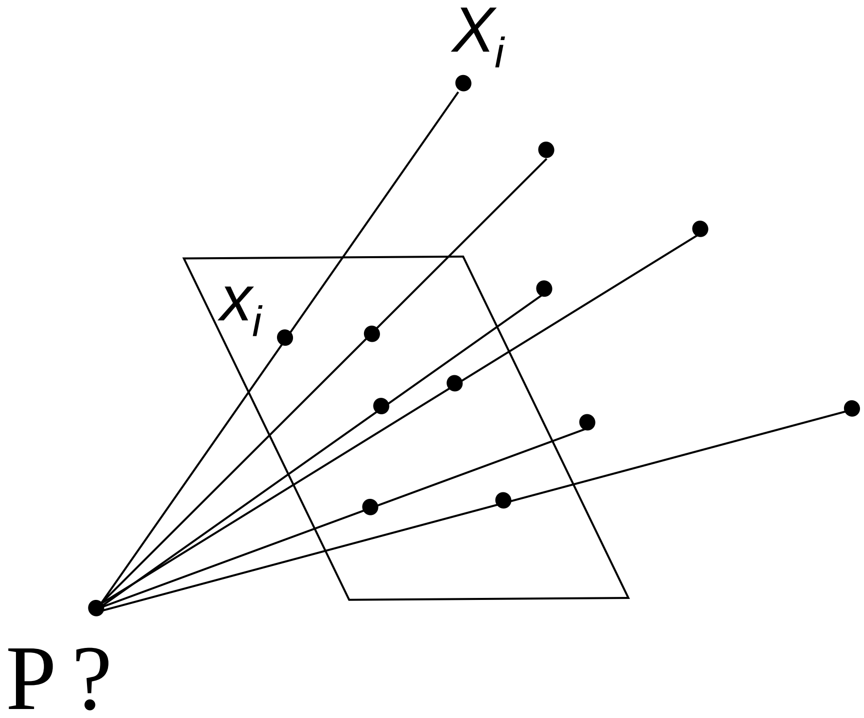
$$\begin{bmatrix}\lambda x \\ \lambda y \\ \lambda\end{bmatrix} = \begin{bmatrix}* & * & * & * \\ * & * & * & * \\ * & * & * & *\end{bmatrix}\begin{bmatrix}X \\ Y \\ Z \\ 1\end{bmatrix}$$

Source: D. Hoiem

Camera calibration

- Given n points with known 3D coordinates X_i and known image projections x_i, estimate the camera parameters





Camera calibration: Linear method

$$\lambda \mathrm{x}_i = \mathrm{P}\mathrm{X}_i \qquad \mathrm{x}_i \times \mathrm{P}\mathrm{X}_i = 0 \qquad \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathrm{P}_1^T \mathrm{X}_i \\ \mathrm{P}_2^T \mathrm{X}_i \\ \mathrm{P}_3^T \mathrm{X}_i \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & -\mathrm{X}_i^T & y_i \mathrm{X}_i^T \\ \mathrm{X}_i^T & 0 & -x_i \mathrm{X}_i^T \\ -y_i \mathrm{X}_i^T & x_i \mathrm{X}_i^T & 0 \end{bmatrix} \begin{pmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{pmatrix} = 0$$

Two linearly independent equations

Camera calibration: Linear method

$$\begin{bmatrix} 0^T & \mathrm{X}_1^T & -y_1\mathrm{X}_1^T \\ \mathrm{X}_1^T & 0^T & -x_1\mathrm{X}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathrm{X}_n^T & -y_n\mathrm{X}_n^T \\ \mathrm{X}_n^T & 0^T & -x_n\mathrm{X}_n^T \end{bmatrix} \begin{pmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{pmatrix} = 0 \qquad \mathrm{Ap} = 0$$

- P has 11 degrees of freedom (12 parameters, but scale is arbitrary)
- One 2D/3D correspondence gives us two linearly independent equations
- Homogeneous least squares
- 6 correspondences needed for a minimal solution

Camera calibration: Linear method

$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \cdots & \cdots & \cdots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \qquad \mathrm{Ap} = 0$$

- Note: for coplanar points that satisfy $\Pi^T X=0$, we will get degenerate solutions $(\Pi,0,0)$, $(0,\Pi,0)$, or $(0,0,\Pi)$

Camera calibration: Linear method

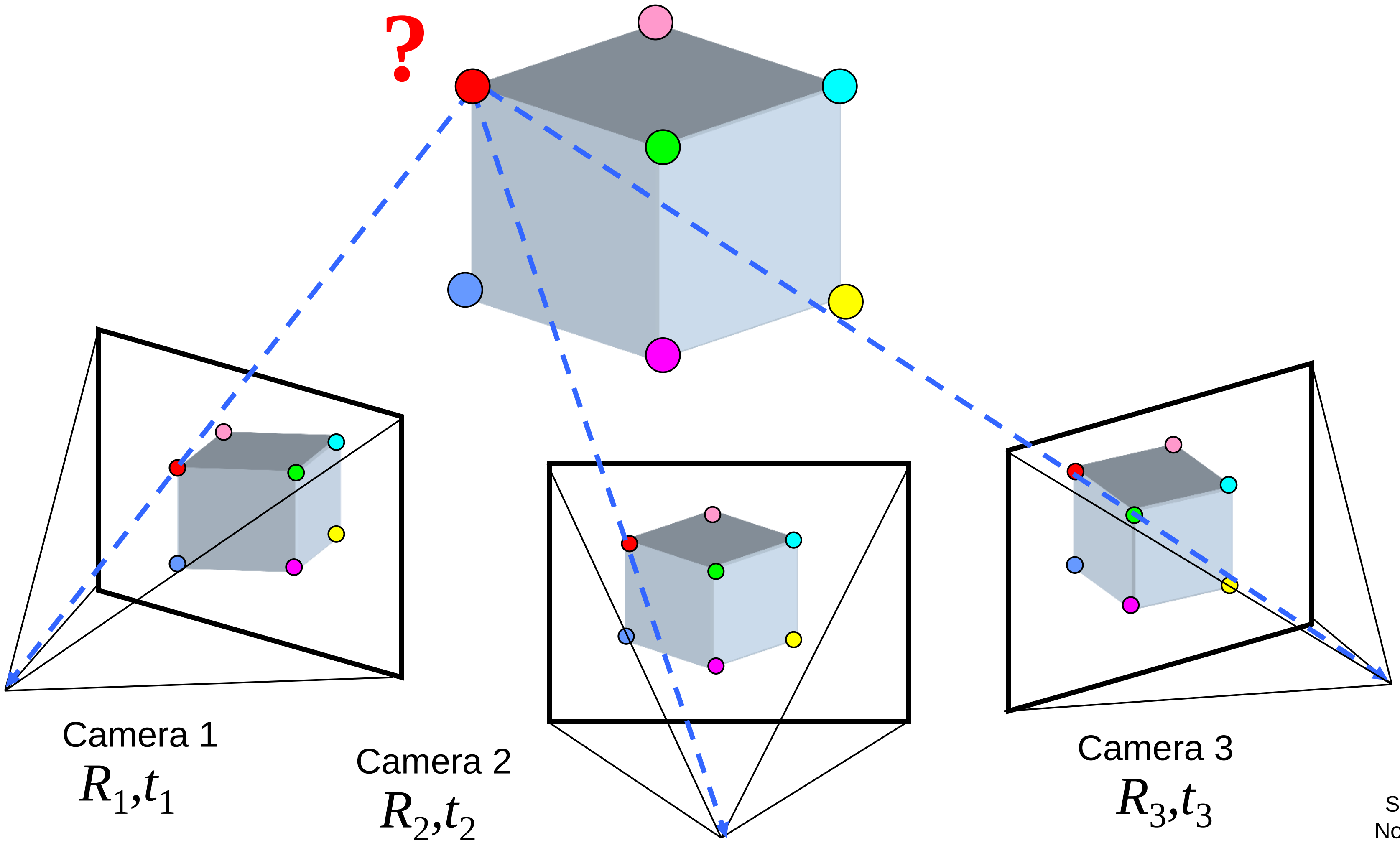
- Advantages: easy to formulate and solve
- Disadvantages
  - Doesn’t directly tell you camera parameters
  - Doesn’t model radial distortion
  - Can’t impose constraints, such as known focal length and orthogonality

- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimize error using Newton’s method or other non-linear optimization

Source: D. Hoiem

Multi-view geometry problems

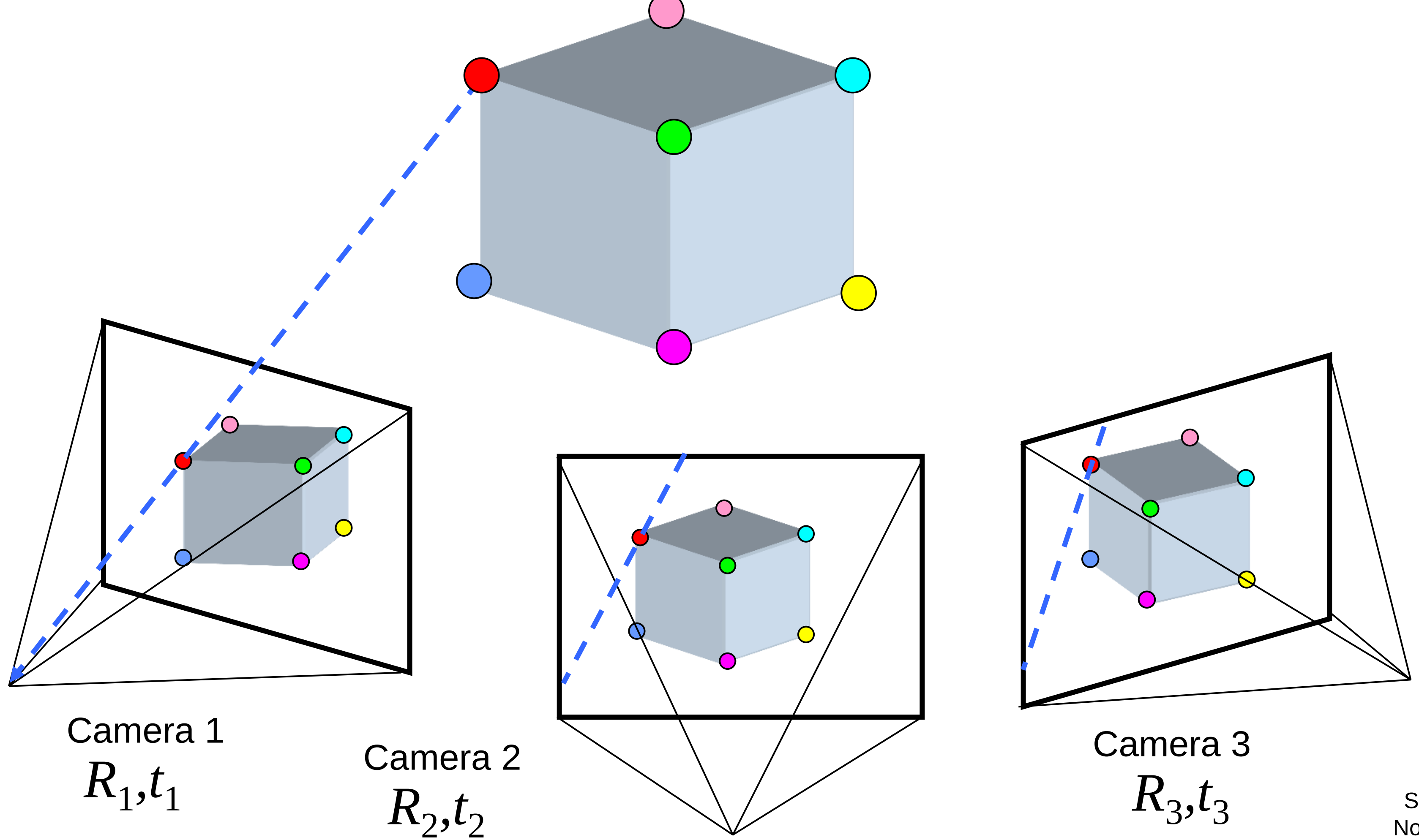
- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



Slide credit: Noah Snavely

Multi-view geometry problems

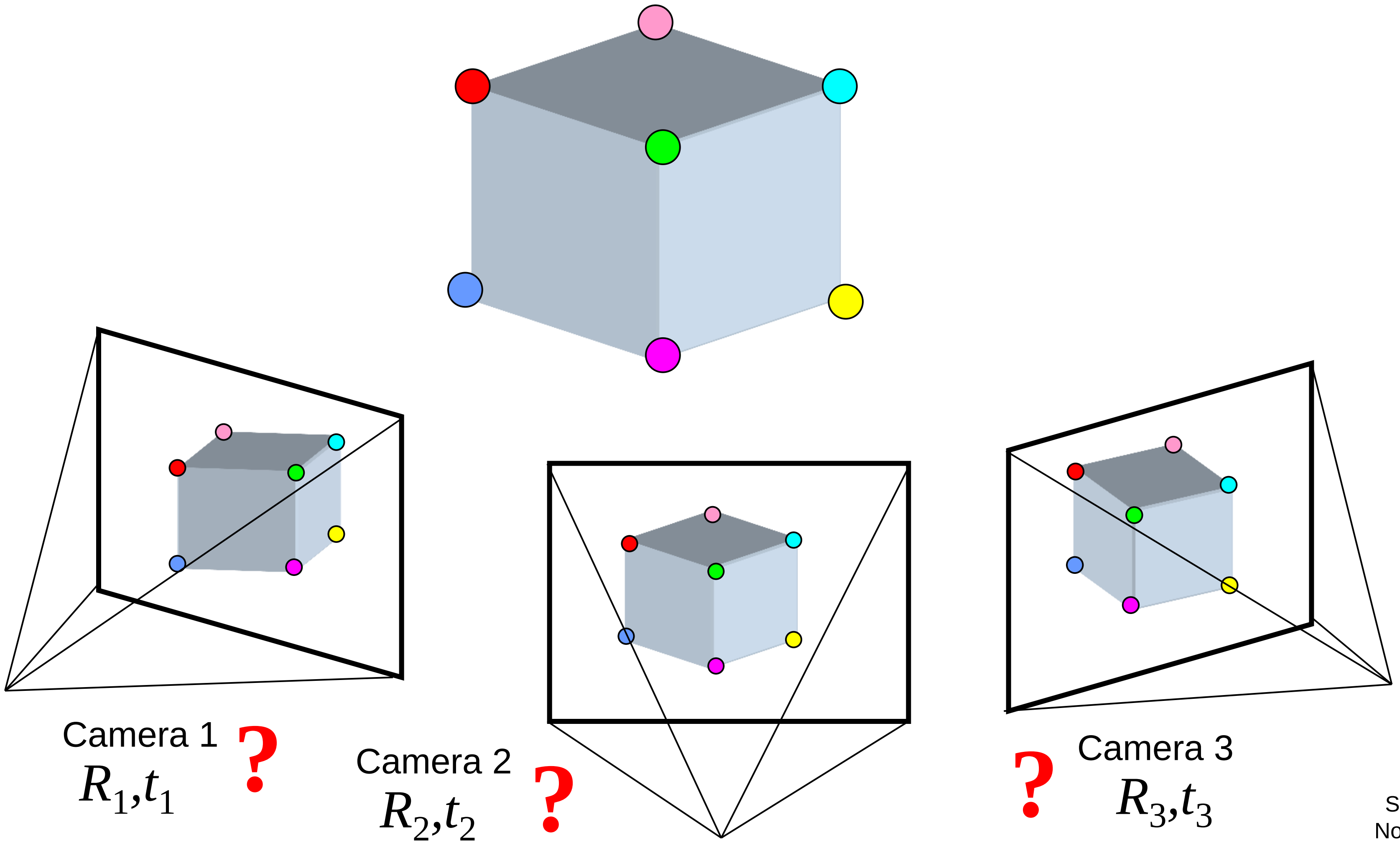
- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?



Slide credit: Noah Snavely

Multi-view geometry problems

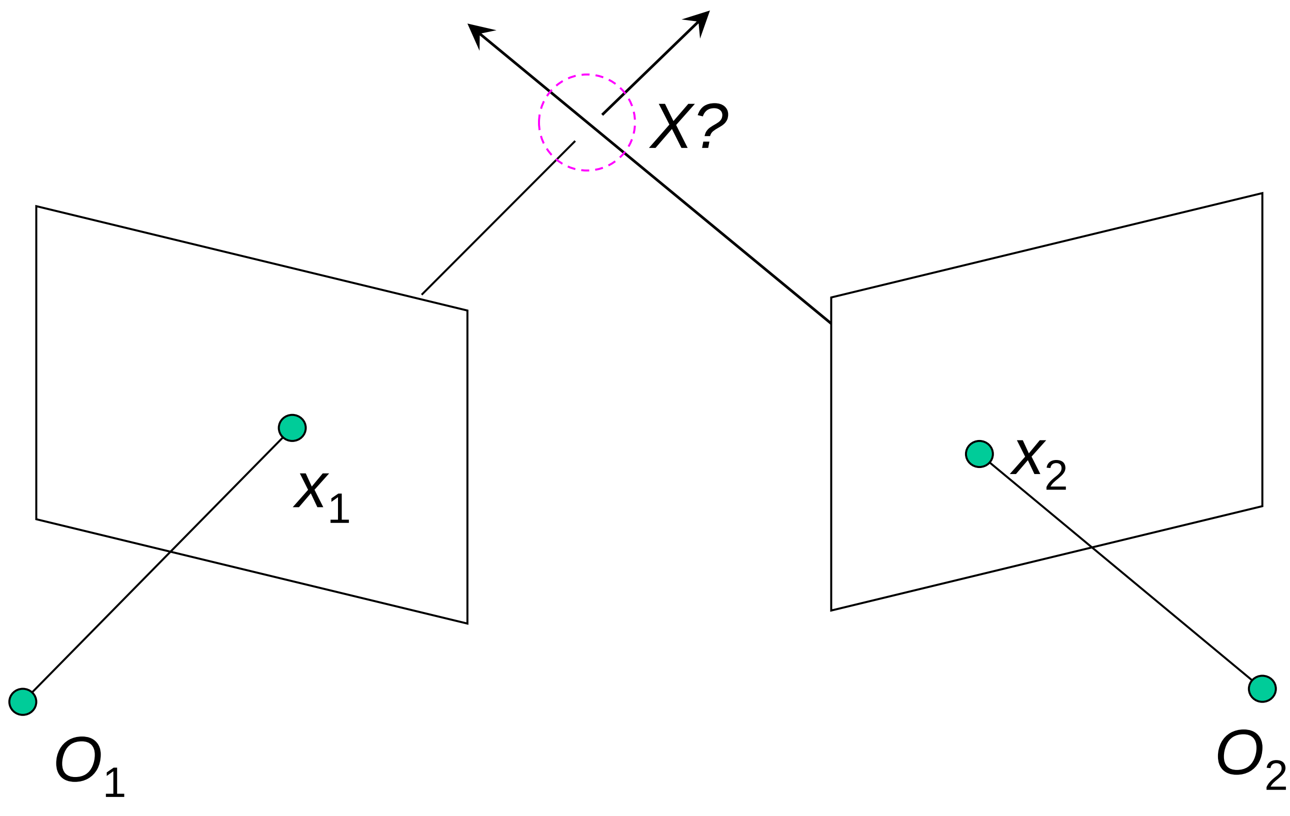
- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters



Slide credit: Noah Snavely

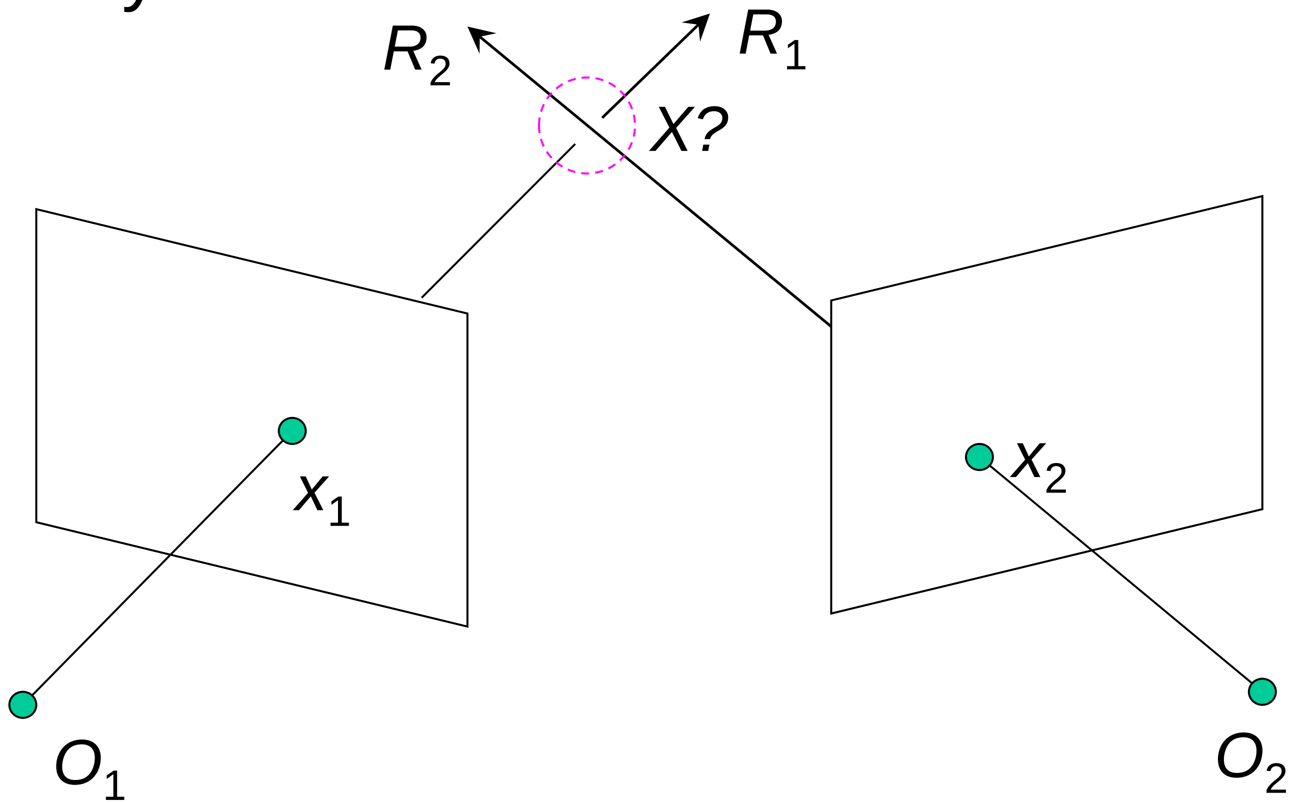
Triangulation

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



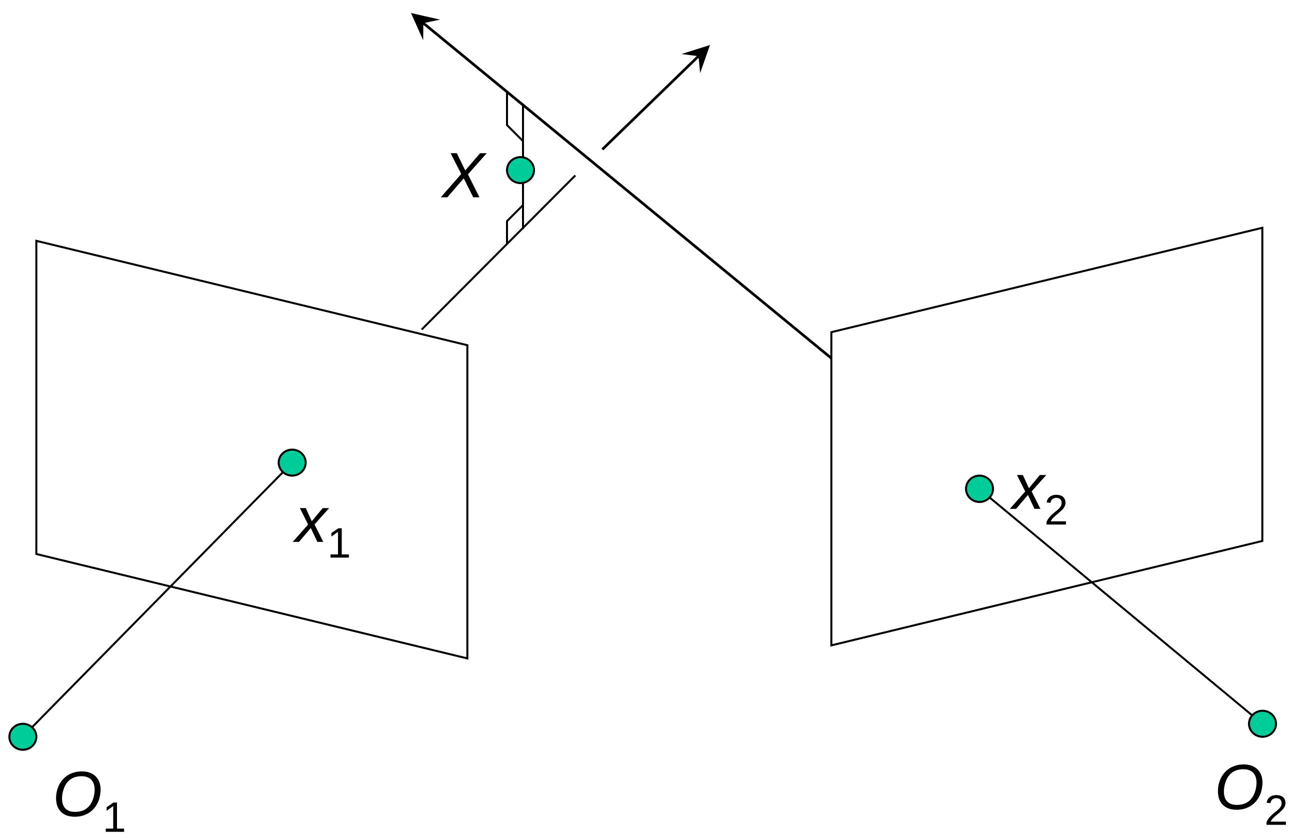
Triangulation

- We want to intersect the two visual rays corresponding to x_1 and x_2, but because of noise and numerical errors, they don't meet exactly



Triangulation: Geometric approach

- Find shortest segment connecting the two viewing rays and let X be the midpoint of that segment



Triangulation: Linear approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}]\mathrm{P}_1 \mathrm{X} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}]\mathrm{P}_2 \mathrm{X} = 0$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Triangulation: Linear approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}]\mathrm{P}_1 \mathrm{X} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}]\mathrm{P}_2 \mathrm{X} = 0$$

Two independent equations each in terms of three unknown entries of X

Triangulation: Nonlinear approach

Find X that minimizes

$$d^2(x_1, P_1 X) + d^2(x_2, P_2 X)$$