

题目 15

$$\begin{aligned}
 f(x) &= 2 \cos x (\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}) - \sqrt{3} \frac{1 - \cos 2x}{2} + \frac{1}{2} \sin 2x \\
 &= \sin x \cos x + \sqrt{3} \cos^2 x - \sqrt{3} (\frac{1}{2} - \frac{1}{2} \cos 2x) + \frac{1}{2} \sin 2x \\
 &= \sin 2x + \sqrt{3} \frac{\cos 2x + 1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cos 2x \\
 &= \sin 2x + \sqrt{3} \cos 2x \\
 &= 2 \sin (2x + \frac{\pi}{3})
 \end{aligned}$$

所以函数周期为 π , 振幅为 2, 初相为 $\frac{\pi}{3}$, 单调递增区间 $-\frac{\pi}{2} + 2k\pi \leq 2x + \frac{\pi}{3} \leq \frac{\pi}{2} + 2k\pi \Rightarrow x \in (-\frac{5\pi}{12} + k\pi, \frac{\pi}{12} + k\pi), k \in Z$

题目 17

(1) 最大温差: $30^\circ C - 10^\circ C = 20^\circ C$

(2)

$$\begin{cases} A + b = 30 \\ -A + b = 10 \end{cases} \Rightarrow \begin{cases} A = 10 \\ b = 20 \end{cases}$$

周期 $T = 2 \times (14 - 6) = 16 \Rightarrow \omega = \frac{2\pi}{16} = \frac{\pi}{8}$ 将 $(16, 30)$ 代入函数得 $10 \sin(\frac{\pi}{8} \times 16 + \phi) + 20 = 30 \Rightarrow \phi = -\frac{5\pi}{4} + 2k\pi \Rightarrow \phi = \frac{3\pi}{4}$

题 18

周期 $T = 2 \times (\frac{11\pi}{12} - \frac{5\pi}{12}) = \pi \Rightarrow \omega = \frac{2\pi}{T} = 2, A = 3$, 将 $(\frac{5\pi}{12}, 3)$ 代入 $y = 3 \sin(2x + \phi)$ 有

$$2 \times \frac{5\pi}{12} + \phi = \frac{\pi}{2} + 2k\pi \Rightarrow \phi = -\frac{\pi}{3} + 3k\pi, (k \in Z)$$

所以函数表达式为

$$y = 3 \sin(2x - \frac{\pi}{3})$$

题目 19 (1)

$$\begin{aligned}
 f(x) &= \sqrt{3} \cos^2 \omega x + \frac{1}{2} \sin 2\omega x + a \\
 &= \sqrt{3} \frac{\cos 2\omega x + 1}{2} + \frac{1}{2} \sin 2\omega x + a \\
 &= \frac{\sqrt{3}}{2} \cos 2\omega x + \frac{1}{2} \sin 2\omega x + \frac{\sqrt{3}}{2} + a \\
 &= \sin(2\omega x + \frac{\pi}{3}) + \frac{\sqrt{3}}{2} + a
 \end{aligned}$$

$$2\omega \times \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2} \implies \omega = \frac{1}{2}$$

(2)

$$f(x) = \sin\left(x + \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} + a, x \in \left[-\frac{\pi}{3}, \frac{5\pi}{6}\right] \implies x + \frac{\pi}{3} \in \left[0, \frac{7\pi}{6}\right]$$

最小值

$$\sin \frac{7\pi}{6} + \frac{\sqrt{3}}{2} + a = -\frac{1}{2} + \frac{\sqrt{3}}{2} + a = \sqrt{3}$$

所以

$$a = \frac{\sqrt{3} + 1}{2}$$

21 题 (1)

$$\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) \in [-\sqrt{2}, \sqrt{2}] \implies a \in [-\sqrt{2}, \sqrt{2}] \quad \text{有解}$$

(2) 当 $x \in [0, \pi]$ 时

$$x + \frac{\pi}{4} \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \quad \text{有两解} \implies x_1 \in \left[0, \frac{\pi}{4}\right), x_2 \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right] \implies a \in [1, \sqrt{2}), \text{此时两根关于 } x = \frac{\pi}{4} \text{ 对称}$$

即两根之和为 $\frac{\pi}{2}$