

题目 15

$$\begin{aligned}f(x) &= 2 \cos x \left( \sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} \right) - \sqrt{3} \frac{1 - \cos 2x}{2} + \frac{1}{2} \sin 2x \\&= \sin x \cos x + \sqrt{3} \cos^2 x - \sqrt{3} \left( \frac{1}{2} - \frac{1}{2} \cos 2x \right) + \frac{1}{2} \sin 2x \\&= \sin 2x + \sqrt{3} \frac{\cos 2x + 1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cos 2x \\&= \sin 2x + \sqrt{3} \cos 2x \\&= 2 \sin \left( 2x + \frac{\pi}{3} \right)\end{aligned}$$

所以函数周期为  $\pi$ , 振幅为 2, 初相为  $\frac{\pi}{3}$ , 单调递增区间

$$\begin{aligned}-\frac{\pi}{2} + 2k\pi &\leq 2x + \frac{\pi}{3} \leq \frac{\pi}{2} + 2k\pi \\ \implies x &\in \left( -\frac{5\pi}{12} + k\pi, \frac{\pi}{12} + k\pi \right), k \in \mathbb{Z}\end{aligned}$$

题目 17

(1) 最大温差:  $30^\circ\text{C} - 10^\circ\text{C} = 20^\circ\text{C}$

(2)

$$\begin{cases} A + b = 30 \\ -A + b = 10 \end{cases} \implies \begin{cases} A = 10 \\ b = 20 \end{cases}$$

周期

$$T = 2 \times (14 - 6) = 16 \implies \omega = \frac{2\pi}{16} = \frac{\pi}{8}$$

将 (16, 30) 代入函数得

$$\begin{aligned}10 \sin \left( \frac{\pi}{8} \times 16 + \phi \right) + 20 &= 30 \\ \implies \phi &= -\frac{5\pi}{4} + 2k\pi \implies \phi = \frac{3\pi}{4}\end{aligned}$$

题 18

周期

$$T = 2 \times \left( \frac{11\pi}{12} - \frac{5\pi}{12} \right) = \pi \implies \omega = \frac{2\pi}{T} = 2$$

$A = 3$ , 将

$$\left( \frac{5\pi}{12}, 3 \right)$$

代入

$$y = 3 \sin(2x + \phi)$$

有

$$2 \times \frac{5\pi}{12} + \phi = \frac{\pi}{2} + 2k\pi \implies \phi = -\frac{\pi}{3} + 3k\pi, (k \in \mathbb{Z})$$

所以函数表达式为

$$y = 3 \sin(2x - \frac{\pi}{3})$$

题目 19 (1)

$$\begin{aligned} f(x) &= \sqrt{3} \cos^2 \omega x + \frac{1}{2} \sin 2\omega x + a \\ &= \sqrt{3} \frac{\cos 2\omega x + 1}{2} + \frac{1}{2} \sin 2\omega x + a \\ &= \frac{\sqrt{3}}{2} \cos 2\omega x + \frac{1}{2} \sin 2\omega x + \frac{\sqrt{3}}{2} + a \\ &= \sin(2\omega x + \frac{\pi}{3}) + \frac{\sqrt{3}}{2} + a \\ 2\omega \times \frac{\pi}{6} + \frac{\pi}{3} &= \frac{\pi}{2} \implies \omega = \frac{1}{2} \end{aligned}$$

(2)

$$f(x) = \sin(x + \frac{\pi}{3}) + \frac{\sqrt{3}}{2} + a, x \in [-\frac{\pi}{3}, \frac{5\pi}{6}] \implies x + \frac{\pi}{3} \in [0, \frac{7\pi}{6}]$$

最小值

$$\sin \frac{7\pi}{6} + \frac{\sqrt{3}}{2} + a = -\frac{1}{2} + \frac{\sqrt{3}}{2} + a = \sqrt{3}$$

所以

$$a = \frac{\sqrt{3} + 1}{2}$$

21 题 (1)

$$\sin x + \cos x = \sqrt{2} \sin(x + \frac{\pi}{4}) \in [-\sqrt{2}, \sqrt{2}] \implies a \in [-\sqrt{2}, \sqrt{2}] \quad \text{有解}$$

(2) 当  $x \in [0, \pi]$  时

$$x + \frac{\pi}{4} \in [\frac{\pi}{4}, \frac{5\pi}{4}] \quad \text{有两解} \implies x_1 \in [0, \frac{\pi}{4}), x_2 \in (\frac{\pi}{4}, \frac{\pi}{2}] \implies a \in [1, \sqrt{2}), \text{此时两根关于 } x = \frac{\pi}{4} \text{ 对称}$$

即两根之和为  $\frac{\pi}{2}$