

# Formal Theory of Communication Topology in Concurrent ML

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## 1 Mathematical Artifacts

```

1  type thread_id
2  val spawn : (unit -> unit) -> thread_id
3
4  type 'a chan
5  val channel : unit -> 'a chan
6  val recv : 'a chan -> 'a
7  val send : ('a chan * 'a) -> unit
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1  val server = Serv.make ()
2  val _ = spawn (fn () => Serv.call (server, 35))
3  val _ = spawn (fn () =>
4      Serv.call (server, 12);
5      Serv.call (server, 13))
6  val _ = spawn (fn () => Serv.call (server, 81))
7  val _ = spawn (fn () => Serv.call (server, 44))
8

1  structure Serv : SERV = struct
2      datatype serv = S of (int * int chan) chan
3
4      fun make () = let
5
6          val reqChn = FanIn.channel()
7
8          fun loop state = let
9              val (v, replCh) = FanIn.recv reqChn in
10                 OneShot.send (replCh, state);
11                 loop v end in
12
13             spawn (fn () => loop 0);
14             S reqChn end
15
16     fun call (server, v) = let
17         val S reqChn = server
18         val replChn = OneShot.channel () in
19             FanIn.send (reqCh, (v, replCh));
20             OneShot.recv replChn end
21
22     end
23

1  let
2      val w = 4
3      val x = ref 1
4      val y = ref 2
5      val z = (!x + 1) + (!y + 2) + (w - 3)
6      val w = 1 in
7          y := 0;
8          (!y + 2) - (!x + 1) * (w - 3) end
9

1  let
2      val x = 1
3      val y = 2
4      val z = ref (4 * 73)

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5     val x = 4 in
6     z := 1;
7     x * !z end
8
1
2   let
3     val f = fn x => x 1
4     val g = fn y => y + 2
5     val h = fn z => z + 3 in
6     (f g) + (f h) end
7
1
2   datatype 'a list = Nil | Cons 'a ('a list)
3
4   inductive
5     sorted ::
6       ('a => 'a => bool) =>
7       'a list => bool where
8     Nil : sorted P Nil |
9     Single : sorted P (Cons x Nil) |
10    Cons :
11      P x y =>
12      sorted P (Cons y ys) =>
13      sorted P (Cons x (Cons y ys))
14
1   datatype nat = Z | S nat
2
3   inductive
4     lte ::
5       nat => nat => bool where
6     Eq : lte n n |
7     Lt : lte n1 n2 => lte n1 (S n2)
8
9   theorem "
10     sorted lte
11     (Cons (Z) (Cons (S Z)
12       (Cons (S Z) (Cons
13         (S (S (S Z))) Nil))))"
14   apply (rule Cons)
15   apply (rule Lt)
16   apply (rule Eq)
17   apply (rule Cons)
18   apply (rule Eq)
19   apply (rule Cons)
20   apply (rule Lt)
21   apply (rule Lt)
22   apply (rule Eq)
23   apply (rule Single)

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24     done
25
1   definition True :: bool where
2     True  $\equiv ((\lambda x :: \text{bool}. x) = (\lambda x. x))$ 
3
4   definition False :: bool where
5     False  $\equiv (\forall P. P)$ 
6
7
1   signature CHAN = sig
2     type 'a chan
3     val channel : unit -> 'a chan
4     val send : 'a chan * 'a -> unit
5     val recv : 'a chan -> 'a
6     end
7
1
2   structure ManyToManyChan : CHAN = struct
3     type message_queue = 'a option ref queue
4
5     datatype 'a chan_content =
6       Send of (condition * 'a) queue |
7       Recv of (condition * 'a option ref) queue |
8       Inac
9
10    datatype 'a chan =
11      Chn of 'a chan_content ref * mutex_lock
12
13    fun channel () = Chn (ref Inac, mutexLock ())
14
15    fun send (Chn (conRef, lock)) m =
16      acquire lock;
17      (case !conRef of
18        Recv q => let
19          val (recvCond, mopRef) = dequeue q in
20            mopRef := Some m;
21            if (isEmpty q) then conRef := Inac else ();
22            release lock; signal recvCond; () end |
23        Send q => let
24          val sendCond = condition () in
25            enqueue (q, (sendCond, m));
26            release lock; wait sendCond; () end |
27        Inac => let
28          val sendCond = condition () in
29            conRef := Send (queue [(sendCond, m)]);
30            release lock; wait sendCond; () end)
31
32    fun recv (Chn (conRef, lock)) =

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33     acquire lock;
34     (case !conRef of
35     Send q => let
36         val (sendCond, m) = dequeue q in
37         if (isEmpty q) then
38             conRef := Inac
39         else
40             ();
41         release lock; signal sendCond; m end |
42     Recv q => let
43         val recvCond = condition ()
44         val mopRef = ref None in
45         enqueue (q, (recvCond, mopRef));
46         release lock; wait recvCond;
47         valOf (!mopRef) end |
48     Inac => let
49         val recvCond = condition ()
50         val mopRef = ref None in
51         conRef := Recv (queue [(recvCond, mopRef)]);
52         release lock; wait recvCond;
53         valOf (!mopRef) end)
54
55 end
56
57
1
2     structure FanOutChan : CHAN = struct
3
4     datatype 'a chan_content =
5         Send of condition * 'a |
6         Recv of (condition * 'a option ref) queue |
7         Inac
8
9     datatype 'a chan =
10         Chn of 'a chan_content ref * mutex_lock
11
12     fun channel () = Chn (ref Inac, mutexLock ())
13
14     fun send (Chn (conRef, lock)) m = let
15         val sendCond = condition () in
16         case cas (conRef, Inac, Send (sendCond, m)) of
17             Inac => (* conRef already set *)
18                 wait sendCond; () |
19             Recv q =>
20                 (* the current thread is
21                  * the only one that updates from this state *)
22                 acquire lock;
23                 (let
24                     val (recvCond, mopRef) = dequeue q in

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```

25         mopRef := Some m;
26         if (isEmpty q) then conRef := Inac else ();
27         release lock; signal (recvCond);
28         () end) |
29     Send _ => raise NeverHappens end
30
31 fun recv (Chn (conRef, lock)) =
32     acquire lock;
33     (case !conRef of
34         Inac => let
35             val recvCond = condition ()
36             val mopRef = ref None in
37             conRef := Recv (queue [(recvCond, mopRef)]);
38             release lock; wait recvCond;
39             valOf (!mopRef) end |
40         Recv q => let
41             val recvCond = condition ()
42             val mopRef = ref None in
43             enqueue (q, (recvCond, mopRef));
44             release lock; wait recvCond;
45             valOf (!mopRef) end |
46         Send (sendCond, m) =>
47             conRef := Inac;
48             release lock;
49             signal sendCond;
50             m end)
51
52 end
53
54
55 1 structure FanInChan : CHAN = struct
56 2
57 3 datatype 'a chan_content =
58 4     Send of (condition * 'a) queue |
59 5     Recv of condition * 'a option ref |
60 6     Inac
61 7
62 8 datatype 'a chan =
63 9     Chn of 'a chan_content ref * mutex_lock
64 10
65 11 fun channel () = Chn (ref Inac, mutexLock ())
66 12
67 13 fun send (Chn (conRef, lock)) m =
68 14     acquire lock;
69 15     case !conRef of
70 16     Recv (recvCond, mopRef) =>
71 17         mopRef := Some m; conRef := Inac;
72 18         release lock; signal recvCond;
73 19         () |
74 20     Send q => let

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21     val sendCond = condition () in
22     enqueue (q, (sendCond, m));
23     release lock; wait sendCond;
24     () end |
25   Inac => let
26     val sendCond = condition () in
27     conRef := Send (queue [(sendCond, m)])
28     release lock; wait sendCond; () end
29
30   fun recv (Chn (conRef, lock)) = let
31     val recvCond = condition ()
32     val mopRef = ref None in
33     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
34       Inac => (* conRef already set *)
35         wait recvCond; valOf (!mopRef) |
36       Send q =>
37         (* the current thread is the only one
38          * that updates the state from this state *)
39         acquire lock;
40         (let
41           val (sendCond, m) = dequeue q in
42           if (isEmpty q) then conRef := Inac else ();
43           release lock; signal sendCond; m end) |
44       Recv _ => raise NeverHappens end end
45
46

```

```

1
2 structure OneToOneChan : CHAN = struct
3
4   datatype 'a chan_content =
5     Send of condition * 'a |
6     Recv of condition * 'a option ref |
7     Inac
8
9   datatype 'a chan = Chn of 'a chan_content ref
10
11   fun channel () = Chn (ref Inac)
12
13   fun send (Chn conRef) m = let
14     val sendCond = condition () in
15     case cas (conRef, Inac, Send (sendCond, m)) of
16       Inac =>
17         (* conRef already set to Send *)
18         wait sendCond; () |
19       Recv (recvCond, mopRef) =>
20         (* the current thread is the only one
21          * that accesses conRef for this state *)
22         mopRef := Some m; conRef := Inac;
23         signal recvCond; () |

```



```

24     Send _ => raise NeverHappens end end
25
26
27 fun recv (Chn conRef) = let
28     val recvCond = condition ();
29     val mopRef = ref None in
30     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
31         Inac => (* conRef already set to Recv*)
32             wait recvCond; valOf (!mopRef) |
33         Send (sendCond, m) =>
34             (* the current thread is the only one
35              * that accesses conRef for this state *)
36             conRef := Inac; signal sendCond; m |
37         Recv _ => raise NeverHappens end end
38
39 end
40
1  structure OneShotChan : CHAN = struct
2
3  datatype 'a chan_content =
4      Send of condition * 'a |
5      Recv of condition * 'a option ref |
6      Inac
7
8  datatype 'a chan = Chn of 'a chan_content ref * mutex_lock
9
10 fun channel () = Chn (ref Inac, lock ())
11
12 fun send (Chn (conRef, lock)) m = let
13     val sendCond = condition () in
14     case (conRef, Inac, Send (sendCond, m)) of
15         Inac =>
16             (* conRef already set to Send*)
17             wait sendCond; () |
18         Recv (recvCond, mopRef) =>
19             mopRef := Some m; signal recvCond;
20             () |
21         Send _ => raise NeverHappens end end
22
23
24 fun recv (Chn (conRef, lock)) = let
25     val recvCond = condition ()
26     val mopRef = ref None in
27     case (conRef, Inac, Recv (recvCond, mopRef)) of
28         Inac =>
29             (* conRef already set to Recv*)
30             wait recvCond; valOf (!mopRef) |
31         Send (sendCond, m) =>
32             acquire lock; signal sendCond;

```

```

33         (* never relases lock;
34         -* blocks others forever *)
35         m |
36     Recv _ =>
37         acquire lock;
38         (* never able to acquire lock;
39         -* blocked forever *)
40         raise NeverHappens end end
41
42 end
43
1 structure OneShotToOneChan : CHAN = struct
2
3     datatype 'a chan =
4         Chn of condition * condition * 'a option ref
5
6     fun channel () =
7         Chn (condition (), condition (), ref None)
8
9     fun send (Chn (sendCond, recvCond, mopRef)) m =
10         mopRef := Some m; signal recvCond;
11         wait sendCond; ()
12
13     fun recv (Chn (sendCond, recvCond, mopRef)) =
14         wait recvCond; signal sendCond;
15         valOf (!mopRef)
16
17 end
18

```

## 2 Syntax

```

1
2     datatype var = Var string
3
4     datatype exp =
5         Let var boundexp exp |
6         Rslt var
7
8     boundexp =
9         Unt |
10        MkChn |
11        Prim prim |
12        Spwn exp |
13        Sync var |
14        Fst var |
15        Snd var |
16        Case var var exp var exp |

```

```

17   App var var and
18
19   prim =
20     SendEvt var var |
21     RecvEvt var |
22     Pair var var |
23     Lft var |
24     Rht var |
25     Abs var var ex

```

### 3 Dynamic Semantics

```

1   datatype ctrl_label =
2     LNxt var | LSpwn var | LCall var | LRtn var
3
4   type_synonym ctrl_path = (ctrl_label list)
5
6   datatype chan = Chn ctrl_path var
7
8   datatype val =
9     VUnt | VChn chan | VClsr prim (var  $\rightarrow$  val)
10
11  datatype ctn = Ctn var exp (var  $\rightarrow$  val)
12
13  datatype state = Stt exp (var  $\rightarrow$  val) (ctn list)
14
15

```

```

1
2  inductive
3    seq_step ::
4      bind * (var  $\rightarrow$  val))  $\Rightarrow$  val  $\Rightarrow$  bool where
5    LetUnt :
6      seq_step (Unt, env) VUnt |
7    LetPrim :
8      seq_step (Prim p, env) (VClsr p env) |
9    LetFst :
10     env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
11     envp x1 = Some v  $\Rightarrow$ 
12     seq_step (Fst xp, env) v |
13    LetSnd :
14     env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
15     envp x2 = Some v  $\Rightarrow$ 
16     seq_step (Snd xp, env) v
17
18
19

```

1

```

2
3 inductive
4   seq_step_up ::
5     bind * (var  $\rightarrow$  val)  $\Rightarrow$ 
6     exp * val_env  $\Rightarrow$  bool where
7   LetCaseLft :
8     env xs = Some (VClsr (Lft xl') envl)  $\Rightarrow$ 
9     envl xl' = Some vl  $\Rightarrow$ 
10    seq_step_up
11      (Case xs xl el xr er, env)
12      (el, env(xl  $\mapsto$  vl)) |
13   LetCaseRht :
14     env xs = Some (VClsr (Rht xr') envr)  $\Rightarrow$ 
15     envr xr' = Some vr  $\Rightarrow$ 
16    seq_step_up
17      (Case xs xl el xr er, env)
18      (er, env(xr  $\mapsto$  vr)) |
19   LetApp :
20     env f = Some (VClsr (Abs fp xp el) envl)  $\Rightarrow$ 
21     env xa = Some va  $\Rightarrow$ 
22    seq_step_up
23      (App f xa, env)
24      (el, envl(
25        fp  $\mapsto$  (VClsr (Abs fp xp el) envl),
26        xp  $\mapsto$  va))
27
28
1
2
3 type_synonym cmmn_set = (ctrl_path * chan * ctrl_path) set
4
5 type_synonym trace_pool = ctrl_path  $\rightarrow$  state
6
7 inductive
8   leaf ::
9     trace_pool  $\Rightarrow$  ctrl_path  $\Rightarrow$  bool where
10  intro :
11    trpl pi  $\neq$  None  $\Rightarrow$ 
12    ( $\nexists$  pi' . trpl pi'  $\neq$  None  $\wedge$  strict_prefix pi pi')  $\Rightarrow$ 
13    leaf trpl pi
14
15
1
2 inductive
3   concur_step ::
4     trace_pool * cmmn_set  $\Rightarrow$ 
5     trace_pool * cmmn_set  $\Rightarrow$ 
6     bool where

```

```

7   Seq_Sttstep_Down :
8     leaf trpl pi ==>
9     trpl pi = Some
10      (Stt (Rslt x) env
11        ((Ctn xk ek envk) # k)) ==>
12      env x = Some v ==>
13      concur_step
14        (trpl, ys)
15        (trpl(pi @ [LRtn xk] ↦
16          (Stt ek (envk(xk ↦ v)) k))), ys) |
17   Seq_Step :
18     leaf trpl pi ==>
19     trpl pi = Some
20      (Stt (Let x b e) env k) ==>
21      seq_step (b, env) v ==>
22      concur_step
23        (trpl, ys)
24        (trpl(pi @ [LNxt x] ↦
25          (Stt e (env(x ↦ v)) k), ys) |
26   Seq_Step_Up :
27     leaf trpl pi ==>
28     trpl pi = Some
29      (Stt (Let x b e) env k) ==>
30      seq_step_up (b, env) (e', env') ==>
31      concur_step
32        (trpl, ys)
33        (trpl(pi @ [LCall x] ↦
34          (Stt e' env'
35            ((Ctn x e env) # k))), ys) |
36   LetMkCh :
37     leaf trpl pi ==>
38     trpl pi = Some (Stt (Let x MkChn e) env k) ==>
39     concur_step
40       (trpl, ys)
41       (trpl(pi @ [LNxt x] ↦
42         (Stt e (env(x ↦ (VChn (Chn pi x)))) k))), ys) |
43   LetSpwn :
44     leaf trpl pi ==>
45     trpl pi = Some
46      (Stt (Let x (Spwn ec) e) env k) ==>
47      concur_step
48        (trpl, ys)
49        (trpl(
50          pi @ [LNxt x] ↦
51            (St e (env(x ↦ VUnt)) k),
52          pi @ [LSpwn x] ↦
53            (St ec env []), ys) |
54   LetSync :
55     leaf trpl pis ==>
56     trpl pis = Some

```

```

57      (Stt (Let xs (Sync xse) es) envs ks)  $\Rightarrow$ 
58      envs xse = Some
59      (VClSr (SendEvt xsc xm) envse)  $\Rightarrow$ 
60      leaf trpl pir  $\Rightarrow$ 
61      trpl pir = Some
62      (Stt (Let xr (Sync xre) er) envr kr)  $\Rightarrow$ 
63      envr xre = Some
64      (VClSr (RecvEvt xrc) envre)  $\Rightarrow$ 
65      envse xsc = Some (VChn c)  $\Rightarrow$ 
66      envre xrc = Some (VChn c)  $\Rightarrow$ 
67      envse xm = Some vm  $\Rightarrow$ 
68      concur_step
69      (trpl, ys)
70      (trpl(
71        pis @ [LNxt xs]  $\mapsto$ 
72        (Stt es (envs(xs  $\mapsto$  VUnt)) ks),
73        pir @ [LNxt xr]  $\mapsto$ 
74        (Stt er (envr(xr  $\mapsto$  vm)) kr)),
75        ys  $\cup$  {(pis, c, pir)})
76
77
1  inductive
2  star ::
3    ('a  $\Rightarrow$  'a  $\Rightarrow$  bool)  $\Rightarrow$ 
4    'a  $\Rightarrow$  'a  $\Rightarrow$  bool for r where
5  refl : star r x x |
6  step : r x y  $\Rightarrow$  star r y z  $\Rightarrow$  star r x z
7

```

## 4 Dynamic Communication

```

1  inductive
2  is_send_path ::
3    trace_pool  $\Rightarrow$  chan  $\Rightarrow$ 
4    control_path  $\Rightarrow$  bool where
5  intro :
6    trpl piy = Some
7    (Stt (Let xy (Sync xe) en) env k)  $\Rightarrow$ 
8    env xe = Some
9    (VClSr (SendEvt xsc xm) enve)  $\Rightarrow$ 
10   enve xsc = Some (VChn c)  $\Rightarrow$ 
11   is_send_path trpl c piy
12
13 inductive
14 is_recv_path ::
15   trace_pool  $\Rightarrow$  chan  $\Rightarrow$ 
16   control_path  $\Rightarrow$  bool where
17   intro :

```

```

18   trpl piy = Some
19   (Stt (Let xy (Sync xe) en) env k) ==>
20   env xe = Some
21   (VClsr (RecvEvt xrc) enve) ==>
22   enve xrc = Some (VChn c) ==>
23   is_recv_path trpl c piy
24
25

```

```

1
2   inductive
3     every_two ::
4       ('a => bool) =>
5       ('a => 'a => bool) => bool where
6     intro : (∀ pi1 pi2 .
7       p x1 ->
8       p x2 ->
9       r x1 x2) ==>
10    every_two p r
11
12   inductive
13     ordered ::
14       'a list => 'a list => bool where
15     left : prefix pi1 pi2 ==> ordered pi1 pi2 |
16     right : prefix pi2 pi1 ==> ordered pi1 pi2
17
18

```

```

1
2   inductive one_shot :: trace_pool => chan => bool where
3     intro :
4       every_two
5       (is_send_path trpl c) op= ==>
6       one_shot trpl c
7
8   inductive fan_out :: trace_pool => chan => bool where
9     intro :
10      every_two
11      (is_send_path trpl c) ordered ==>
12      fan_out trpl c
13
14   inductive fan_in :: trace_pool => chan => bool where
15     intro :
16      every_two
17      (is_recv_path trpl c) ordered ==>
18      fan_in trpl c
19
20   inductive one_to_one :: trace_pool => chan => bool where
21     intro :
22      fan_out trpl c ==>

```

```

23     fan_in trpl c  $\Rightarrow$ 
24     one_to_one trpl c
25
26

```

## 5 Static Semantics

```

1
2  datatype abstract_value =
3    AChn var |
4    AUnt |
5    APrim prim
6
7  type_synonym abstract_env = var  $\Rightarrow$  abstract_value set
8
9  fun rslt_var :: exp  $\Rightarrow$  var where
10    rslt_var (Rslt x) = x |
11    rslt_var (Let _ _ e) = (rslt_var e)
12
13

```

```

1
2
3  inductive
4    may_be_eval_exp ::
5      abstract_env * abstract_env  $\Rightarrow$ 
6      exp  $\Rightarrow$  bool where
7    Result :
8      may_be_eval_exp (V, C) (RESULT x) |
9    let_unt :
10      {AUnt}  $\subseteq$  V x  $\Rightarrow$ 
11      may_be_eval_exp (V, C) e  $\Rightarrow$ 
12      may_be_eval_exp (V, C) (Let x Unt e) |
13    let_chan :
14      {AChn x}  $\subseteq$  V x  $\Rightarrow$ 
15      may_be_eval_exp (V, C) e  $\Rightarrow$ 
16      may_be_eval_exp (V, C) (Let x (MkChn) e) |
17    let_send_evt :
18      {APrim (SendEvt xc xm)}  $\subseteq$  V x  $\Rightarrow$ 
19      may_be_eval_exp (V, C) e  $\Rightarrow$ 
20      may_be_eval_exp (V, C)
21        (Let x (Prim (SendEvt xc xm)) e) |
22    let_recv_evt :
23      {APrim (RecvEvt xc)}  $\subseteq$  V x  $\Rightarrow$ 
24      may_be_eval_exp (V, C) e  $\Rightarrow$ 
25      may_be_eval_exp (V, C)
26        (Let x (Prim (RecvEvt xc)) e) |
27    let_pair :
28      {APrim (Pair x1 x2)}  $\subseteq$  V x  $\Rightarrow$ 

```



```

29     may_be_eval_exp (V, C) e  $\implies$ 
30     may_be_eval_exp (V, C) (Let x (Pair x1 x2) e) |
31 let_left :
32     {APrim (Left xp)}  $\subseteq$  V x  $\implies$ 
33     may_be_eval_exp (V, C) e  $\implies$ 
34     may_be_eval_exp (V, C) (Let x (Left xp) e) |
35 let_right :
36     {APrim (Right xp)}  $\subseteq$  V x  $\implies$ 
37     may_be_eval_exp (V, C) e  $\implies$ 
38     may_be_eval_exp (V, C) (Let x (Right xp) e) |
39 let_abs :
40     {APrim (Abs f' x' e')}  $\subseteq$  V f'  $\implies$ 
41     may_be_eval_exp (V, C) e'  $\implies$ 
42     {APrim (Abs f' x' e')}  $\subseteq$  V x  $\implies$ 
43     may_be_eval_exp (V, C) e  $\implies$ 
44     may_be_eval_exp (V, C) (Let x (Abs f' x' e') e) |
45 let_spawn :
46     {AUnt}  $\subseteq$  V x  $\implies$ 
47     may_be_eval_exp (V, C) ec  $\implies$ 
48     may_be_eval_exp (V, C) e  $\implies$ 
49     may_be_eval_exp (V, C) (Let x (Spwn ec) e) |
50 let_sync :
51      $\forall$  xsc xm xc .
52     (APrim (SendEvt xsc xm))  $\in$  V xe  $\longrightarrow$ 
53     AChn xc  $\in$  V xsc  $\longrightarrow$ 
54     {AUnt}  $\subseteq$  V x  $\wedge$  V xm  $\subseteq$  C xc  $\implies$ 
55      $\forall$  xrc xc .
56     (APrim (RecvEvt xrc))  $\in$  V xe  $\longrightarrow$ 
57     AChn xc  $\in$  V xrc  $\longrightarrow$ 
58     C xc  $\subseteq$  V x  $\implies$ 
59     may_be_eval_exp (V, C) e  $\implies$ 
60     may_be_eval_exp (V, C) (Let x (Syync xe) e) |
61 let_fst :
62      $\forall$  x1 x2.
63     (APrim (Pair x1 x2))  $\in$  V xp  $\longrightarrow$ 
64     V x1  $\subseteq$  V x  $\implies$ 
65     may_be_eval_exp (V, C) e  $\implies$ 
66     may_be_eval_exp (V, C) (Let x (Fst xp) e) |
67 let_snd :
68      $\forall$  x1 x2 .
69     (APrim (Pair x1 x2)  $\in$  V xp  $\longrightarrow$ 
70     V x2  $\subseteq$  V x  $\implies$ 
71     may_be_eval_exp (V, C) e  $\implies$ 
72     may_be_eval_exp (V, C) (Let x (Snd xp) e) |
73 let_case :
74      $\forall$  x1' .
75     (APrim (Left x1'))  $\in$  V xs  $\longrightarrow$ 
76     V x1'  $\subseteq$  V x1  $\wedge$  V (rslt_var el)  $\subseteq$  V x  $\wedge$ 
77     may_be_eval_exp (V, C) el  $\implies$ 
78      $\forall$  xr' .

```

```

79      (APrim (Right xr')) ∈ V xs →
80      V xr' ⊆ V xr ∧ V (rslt_var er) ⊆ V x ∧
81      may_be_eval_exp (V, C) er ⇒
82      may_be_eval_exp (V, C) e ⇒
83      may_be_eval_exp (V, C) (Let x (Case xs xl el xr er) e)
84  |
85  let_app :
86  ∀ f' x' e' .
87    (APrim (Abs f' x' e') ∈ V f →
88    V xa ⊆ V x' ∧
89    V (rslt_var e') ⊆ V x ⇒
90    may_be_eval_exp (V, C) e ⇒
91    may_be_eval_exp (V, C) (Let x (App f xa) e)
92
93
94  1
95  2
96  3 fun abstract :: val ⇒ abstract_value where
97  4   abstract VUnt = AUnt |
98  5   abstract VChn (Chn pi x) = AChn x |
99  6   abstract VClsr p env = APrim p
100  7
101  8
102
103  1
104  2 inductive
105  3   may_be_eval_val ::
106  4     abstract_env * abstract_env ⇒ val ⇒ bool and
107  5   may_be_eval_env ::
108  6     abstract_env * abstract_env ⇒ val_env ⇒ bool where
109  7   Unt :
110  8     may_be_eval_val (V, C) VUnt |
111  9   Chan :
112 10     may_be_eval_val (V, C) VChn c |
113 11   SendEvt :
114 12     may_be_eval_env (V, C) env ⇒
115 13     may_be_eval_val (V, C) (VClsr (SendEvt _ _) env) |
116 14   RecvEvt :
117 15     may_be_eval_env (V, C) env ⇒
118 16     may_be_eval_val (V, C) (VClsr (RecvEvt _) env) |
119 17   Left :
120 18     may_be_eval_env (V, C) env ⇒
121 19     may_be_eval_val (V, C) (VClsr (Left _) env) |
122 20   Right :
123 21     may_be_eval_env (V, C) env ⇒
124 22     may_be_eval_val (V, C) (VClsr (Right _) env) |
125 23   Abs :
126 24     {(APrim (Abs f x e))} ⊆ V f ⇒
127 25     may_be_eval_exp (V, C) e ⇒

```

```

26     may_be_eval_env (V, C) env  $\impl$ 
27     may_be_eval_val (V, C) (VClsr (Abs f x e) env) |
28   Pair :
29     may_be_eval_env (V, C) env  $\impl$ 
30     may_be_eval_val (V, C) (VClsr (Pair _ _) env) |
31   intro :
32      $\forall x v .$ 
33     env x = Some v  $\longrightarrow$ 
34     {abstract v}  $\subseteq V \times \wedge$  may_be_eval_val (V, C) v  $\impl$ 
35     may_be_eval_env (V, C) env
36
37
1
2   inductive may_be_eval_stack ::
3     abstract_env * abstract_env  $\Rightarrow$ 
4     abstract_value set  $\Rightarrow$  cont list  $\Rightarrow$  bool where
5   Empty :
6     may_be_eval_stack (V, C) valset [] |
7   Nonempty :
8     valset  $\subseteq V \times \impl$ 
9     may_be_eval_exp (V, C) e  $\impl$ 
10    may_be_eval_env (V, C) env  $\impl$ 
11    may_be_eval_stack (V, C) (V (rslt_var e)) k  $\impl$ 
12    may_be_eval_stack (V, C) valset ((Ctn x e env) # k)
13
14
15   inductive may_be_eval_state ::
16     abstract_env * abstract_env  $\Rightarrow$ 
17     state  $\Rightarrow$  bool where
18   intro :
19     may_be_eval_exp (V, C) e  $\impl$ 
20     may_be_eval_env (V, C) env  $\impl$ 
21     may_be_eval_stack (V, C) (V (rslt_var e)) k  $\impl$ 
22     may_be_eval_state (V, C) (Stt e env k)
23
24   inductive may_be_eval_pool ::
25     abstract_env * abstract_env  $\Rightarrow$ 
26     trace_pool  $\Rightarrow$  bool where
27   intro :
28      $\forall pi st .$ 
29     trpl pi = Some st  $\longrightarrow$ 
30     may_be_eval_state (V, C) st  $\impl$ 
31     may_be_eval_pool (V, C) trpl
32
33
1
2   theorem may_be_eval_preserved_under_concur_step : "
3     may_be_eval_pool (V, C) trpl  $\impl$ 

```

```

4   concur_step (trpl, ys) (trpl', ys')  $\Rightarrow$ 
5   may_be_stati_eval_pool (V, C) trpl'"
6 proof outline
7 qed
8
9 theorem may_be_eval_preserved_under_concur_step_star : "
10   may_be_eval_pool (V, C) trpl  $\Rightarrow$ 
11   star concur_step (trpl, ys) (trpl', ys')  $\Rightarrow$ 
12   may_be_concur_step (V, C) trpl'"
13 proof outline
14 qed
15

1
2 theorem trace_pool_snapshot_value_not_bound_sound : "
3   env x = Some v  $\Rightarrow$ 
4   trpl pi = Some (Stt e env k)  $\Rightarrow$ 
5   may_be_eval_pool (V, C) trpl  $\Rightarrow$ 
6   {abstract v}  $\subseteq$  V x "
7 proof outline
8 qed
9

1
2 theorem trace_pool_always_value_not_bound_sound : "
3   env' x = Some v  $\Rightarrow$ 
4   may_be_eval_pool (V, C) trpl  $\Rightarrow$ 
5   star concur_step (trpl, ys) (trpl', ys')  $\Rightarrow$ 
6   trpl' pi = Some (Stt e' env' k')  $\Rightarrow$ 
7   {abstract v}  $\subseteq$  V x"
8 proof outline
9 qed
10
11 theorem exp_always_value_not_bound_sound : "
12   env' x = Some v  $\Rightarrow$ 
13   may_be_eval_exp (V, C) e  $\Rightarrow$ 
14   star concur_step
15     ([[]  $\mapsto$  (Stt e ( $\lambda$  _ . None) [])], ys)
16     (trpl', ys')  $\Rightarrow$ 
17   trpl' pi = Some (Stt e' env' k')  $\Rightarrow$ 
18   {abstract v}  $\subseteq$  V x"
19 proof outline
20 qed
21
22

1 inductive is_super_exp :: exp  $\Rightarrow$  exp  $\Rightarrow$  bool where
2   Refl :
3     is_super_exp e e |
4     let_Spawn_Child

```

```

5      is_super_exp ec e ==>
6      is_super_exp (Let x (Spwn ec) en) e |
7  let_Case_Left :
8      is_super_exp el e ==>
9      is_super_exp (Let x (case xs xl el xr er) en) e |
10 let_Case_Right :
11     is_super_exp er e ==>
12     is_super_exp (Let x (case xs xl el xr er) en) e |
13 let_Abs_Body : "
14     is_super_exp eb e ==>
15     is_super_exp (Let x (Abs f xp eb) en) e |
16 Let :
17     is_super_exp en e ==>
18     is_super_exp (Let x b en) e
19
20 inductive
21   is_super_exp_left :: exp => exp => bool where
22   Refl :
23     is_super_exp_left e0 e0 |
24   let_Spawn_Child :
25     is_super_exp_left e0 (Let x (Spwn ec) en) ==>
26     is_super_exp_left e0 ec |
27   let_Case_Left :
28     is_super_exp_left e0 (Let x (case xs xl el xr er) en)
29 ==>
30     is_super_exp_left e0 el |
31   let_Case_Right :
32     is_super_exp_left e0 (Let x (case xs xl el xr er) en)
33 ==>
34     is_super_exp_left e0 er |
35   let_Abs_Body :
36     is_super_exp_left e0 (Let x (Abs f xp eb) en) ==>
37     is_super_exp_left e0 eb |
38   Let :
39     is_super_exp_left e0 (Let x b en) ==>
40     is_super_exp_left e0 en
41
42 inductive
43   is_super_exp_over_prim :: exp => prim => bool where
44   SendEvt :
45     is_super_exp_over_prim e0 (SendEvt xC xM) |
46   RecvEvt :
47     is_super_exp_over_prim e0 (RecvEvt xC) |
48   Pair :
49     is_super_exp_over_prim e0 (Pair x1 x2) |
50   Left :
51     is_super_exp_over_prim e0 (Left x) |
52   Right :
53     is_super_exp_over_prim e0 (Right x) |
54   Abs :

```

```

53     is_super_exp_left e0 eb  $\implies$ 
54     is_super_exp_over_prim e0 (Abs fp xp eb)
55
56 inductive
57   is_super_exp_over_env :: exp  $\Rightarrow$  env  $\Rightarrow$  bool and
58   is_super_exp_over_val :: exp  $\Rightarrow$  val  $\Rightarrow$  bool where
59   VUnt :
60     is_super_exp_over_val e0 VUnt |
61   VChn :
62     is_super_exp_over_val e0 (VChn c) |
63   VClsr : "
64     is_super_exp_over_prim e0 p  $\implies$ 
65     is_super_exp_over_env e0 env'  $\implies$ 
66     is_super_exp_over_val e0 (VClsr p env') |
67   intro : "
68      $\forall$  x v .
69     env x = Some v  $\longrightarrow$ 
70     is_super_exp_over_val e0 v  $\implies$ 
71     is_super_exp_over_env e0 env
72
73 inductive
74   is_super_exp_over_stack ::
75     exp  $\Rightarrow$  cont list  $\Rightarrow$  bool where
76   Empty :
77     is_super_exp_over_stack e0 [] |
78   Nonempty :
79     is_super_exp_left e0 ek  $\implies$ 
80     is_super_exp_over_env e0 envk  $\implies$ 
81     is_super_exp_over_stack e0 k  $\implies$ 
82     is_super_exp_over_stack e0 ((Ctn xk ek envk) # k)
83
84 inductive
85   is_super_exp_over_state ::
86     exp  $\Rightarrow$  state  $\Rightarrow$  bool where
87   intro :
88     is_super_exp_left e0 e  $\implies$ 
89     is_super_exp_over_env e0 env  $\implies$ 
90     is_super_exp_over_stack e0 k  $\implies$ 
91     is_super_exp_over_state e0 (Ctn e env k)
92
93
94
95 lemma is_super_exp_trans : "
96   is_super_exp ez ey  $\implies$ 
97   is_super_exp ey ex  $\implies$ 
98   is_super_exp ez ex"
99 proof outline
100 qed
101
102
103

```

```

9 lemma is_super_exp_over_state_preserved : "
10   concur_step (trpl, ys) (trpl', ys')  $\implies$ 
11    $\forall$  pi st.
12     trpl pi = Some st  $\implies$ 
13     is_super_exp_over_state e0 st  $\implies$ 
14     trpl' pi' = Some st'  $\implies$ 
15     is_super_exp_over_state e0 st'"
16 proof outline
17 qed
18
19
1 lemma state_always_exp_not_reachable_sound : "
2   star concur_step (trpl0, ys0) (trpl', ys')  $\implies$ 
3   trpl0 = [[]  $\mapsto$  (Stt e0 ( $\lambda$  _ . None) [])]  $\implies$ 
4   trpl' pi' = Some st'  $\implies$ 
5   is_super_exp_over_state e0 st' "
6 proof outline
7 qed
8
9
10 lemma exp_always_exp_not_reachable_sound : "
11   star concur_step
12     ([[]  $\mapsto$  (Stt e0 ( $\lambda$  _ . None) [])], { })
13     (trpl', H')  $\implies$ 
14     trpl' pi' = Some (Stt e' env' k')  $\implies$ 
15     is_super_exp e0 e' "
16 proof outline
17 qed
18
19

```

## 6 Static Communication

```

1 datatype node_label = NLet var | NRslt var
2
3 fun top_node_label :: exp  $\Rightarrow$  node_label where
4   top_node_label (Let x b e) = NLet x |
5   top_node_label (Rslt y) = NRslt y
6
7 type_synonym node_set = node_label set
8
9 type_synonym node_map = node_label  $\Rightarrow$  var set
10
11 inductive
12   may_be_static_send_node_label ::
13     abstract_env  $\Rightarrow$  exp  $\Rightarrow$ 
14     var  $\Rightarrow$  node_label  $\Rightarrow$  bool where
15   intro:
16     {AChn xC}  $\subseteq$  V xSC  $\implies$ 

```

```

17      {APrim (SendEvt xSC xM)}  $\subseteq$  V xE  $\implies$ 
18      is_super_exp e (Let x (Sync xE) e')  $\implies$ 
19      may_be_static_send_node_label V e xC (NLet x)
20
21  inductive
22    may_be_static_recv_node_label ::
23      abstract_env  $\Rightarrow$  exp  $\Rightarrow$ 
24      var  $\Rightarrow$  node_label  $\Rightarrow$  bool where
25    intro:
26      {AChn xC}  $\subseteq$  V xRC  $\implies$ 
27      {APrim (RecvEvt xRC)}  $\subseteq$  V xE  $\implies$ 
28      is_super_exp e (Let x (Sync xE) e')  $\implies$ 
29      may_be_static_recv_node_label V e xC (NLet x)
30
31

```