

# Formal Theory of Communication Topology in Concurrent ML

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## 1 Mathematical Artifacts

```

1 type thread_id
2 val spawn : (unit -> unit) -> thread_id
3
4 type 'a chan
5 val channel : unit -> 'a chan
6 val recv : 'a chan -> 'a
7 val send : ('a chan * 'a) -> unit
8
9
10
11 signature SERV = sig
12   type serv
13   val make : unit -> serv
14   val call : serv * int -> int
15 end
16
17 structure Serv : SERV = struct
18   datatype serv = S of (int * int chan) chan
19
20   fun make () = let
21     val reqChn = channel ()
22     fun loop state = let
23       val (v, replCh) = recv reqChn in
24       send (replCh, state);
25       loop v end in
26     spawn (fn () => loop 0);
27     S reqChn end
28
29   fun call (server, v) = let
30     val S reqChn = server
31     val replChn = channel () in
32     send (reqChn, (v, replCh));
33     recv replChn end end
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1  val server = Serv.make ()
2  val _ = spawn (fn () => Serv.call (server, 35))
3  val _ = spawn (fn () =>
4    Serv.call (server, 12);
5    Serv.call (server, 13))
6  val _ = spawn (fn () => Serv.call (server, 81))
7  val _ = spawn (fn () => Serv.call (server, 44))
8

1  structure Serv : SERV = struct
2    datatype serv = S of (int * int chan) chan
3
4    fun make () = let
5
6      val reqChn = FanIn.channel()
7
8      fun loop state = let
9        val (v, replCh) = FanIn.recv reqChn in
10       OneShot.send (replCh, state);
11       loop v end in
12
13     spawn (fn () => loop 0);
14     S reqChn end
15
16   fun call (server, v) = let
17     val S reqChn = server
18     val replChn = OneShot.channel () in
19     FanIn.send (reqCh, (v, replCh));
20     OneShot.recv replChn end
21
22   end
23

1  let
2    val w = 4
3    val x = ref 1
4    val y = ref 2
5    val z = (!x + 1) + (!y + 2) + (w - 3)
6    val w = 1 in
7    y := 0;
8    (!y + 2) - (!x + 1) * (w - 3) end
9

1  let
2    val x = 1
3    val y = 2
4    val z = ref (4 * 73)

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5     val x = 4 in
6     z := 1;
7     x * !z end
8
1
2   let
3     val f = fn x => x 1
4     val g = fn y => y + 2
5     val h = fn z => z + 3 in
6     (f g) + (f h) end
7
1
2   datatype 'a list = Nil | Cons 'a ('a list)
3
4   inductive
5     sorted ::
6     ('a => 'a => bool) =>
7     'a list => bool where
8     Nil : sorted P Nil |
9     Single : sorted P (Cons x Nil) |
10    Cons :
11      P x y =>
12      sorted P (Cons y ys) =>
13      sorted P (Cons x (Cons y ys))
14
1   datatype nat = Z | S nat
2
3   inductive
4     lte ::
5     nat => nat => bool where
6     Eq : lte n n |
7     Lt : lte n1 n2 => lte n1 (S n2)
8
9   theorem "
10     sorted lte
11     (Cons (Z) (Cons (S Z)
12       (Cons (S Z) (Cons
13         (S (S (S Z))) Nil))))"
14   apply (rule Cons)
15   apply (rule Lt)
16   apply (rule Eq)
17   apply (rule Cons)
18   apply (rule Eq)
19   apply (rule Cons)
20   apply (rule Lt)
21   apply (rule Lt)
22   apply (rule Eq)
23   apply (rule Single)

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24     done
25
1  definition True :: bool where
2      True  $\equiv ((\lambda x :: \text{bool}. x) = (\lambda x. x))$ 
3
4  definition False :: bool where
5      False  $\equiv (\forall P. P)$ 
6
7
1  signature CHAN = sig
2      type 'a chan
3      val channel : unit -> 'a chan
4      val send : 'a chan * 'a -> unit
5      val recv : 'a chan -> 'a
6      end
7
1
2  structure ManyToManyChan : CHAN = struct
3      type message_queue = 'a option ref queue
4
5      datatype 'a chan_content =
6          Send of (condition * 'a) queue |
7          Recv of (condition * 'a option ref) queue |
8          Inac
9
10     datatype 'a chan =
11         Chn of 'a chan_content ref * mutex_lock
12
13     fun channel () = Chn (ref Inac, mutexLock ())
14
15     fun send (Chn (conRef, lock)) m =
16         acquire lock;
17         (case !conRef of
18             Recv q => let
19                 val (recvCond, mopRef) = dequeue q in
20                 mopRef := Some m;
21                 if (isEmpty q) then conRef := Inac else ();
22                 release lock; signal recvCond; () end |
23             Send q => let
24                 val sendCond = condition () in
25                 enqueue (q, (sendCond, m));
26                 release lock; wait sendCond; () end |
27             Inac => let
28                 val sendCond = condition () in
29                 conRef := Send (queue [(sendCond, m)]);
30                 release lock; wait sendCond; () end)
31
32     fun recv (Chn (conRef, lock)) =

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33     acquire lock;
34     (case !conRef of
35     Send q => let
36         val (sendCond, m) = dequeue q in
37         if (isEmpty q) then
38             conRef := Inac
39         else
40             ();
41         release lock; signal sendCond; m end |
42     Recv q => let
43         val recvCond = condition ()
44         val mopRef = ref None in
45         enqueue (q, (recvCond, mopRef));
46         release lock; wait recvCond;
47         valOf (!mopRef) end |
48     Inac => let
49         val recvCond = condition ()
50         val mopRef = ref None in
51         conRef := Recv (queue [(recvCond, mopRef)]);
52         release lock; wait recvCond;
53         valOf (!mopRef) end)
54
55 end
56
57
1
2     structure FanOutChan : CHAN = struct
3
4     datatype 'a chan_content =
5         Send of condition * 'a |
6         Recv of (condition * 'a option ref) queue |
7         Inac
8
9     datatype 'a chan =
10         Chn of 'a chan_content ref * mutex_lock
11
12     fun channel () = Chn (ref Inac, mutexLock ())
13
14     fun send (Chn (conRef, lock)) m = let
15         val sendCond = condition () in
16         case cas (conRef, Inac, Send (sendCond, m)) of
17             Inac => (* conRef already set *)
18                 wait sendCond; () |
19             Recv q =>
20                 (* the current thread is
21                  * the only one that updates from this state *)
22                 acquire lock;
23                 (let
24                     val (recvCond, mopRef) = dequeue q in

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25         mopRef := Some m;
26         if (isEmpty q) then conRef := Inac else ();
27         release lock; signal (recvCond);
28         () end) |
29     Send _ => raise NeverHappens end
30
31 fun recv (Chn (conRef, lock)) =
32     acquire lock;
33     (case !conRef of
34         Inac => let
35             val recvCond = condition ()
36             val mopRef = ref None in
37             conRef := Recv (queue [(recvCond, mopRef)]);
38             release lock; wait recvCond;
39             valOf (!mopRef) end |
40         Recv q => let
41             val recvCond = condition ()
42             val mopRef = ref None in
43             enqueue (q, (recvCond, mopRef));
44             release lock; wait recvCond;
45             valOf (!mopRef) end |
46         Send (sendCond, m) =>
47             conRef := Inac;
48             release lock;
49             signal sendCond;
50             m end)
51
52 end
53
54
55 1 structure FanInChan : CHAN = struct
56 2
57 3 datatype 'a chan_content =
58 4     Send of (condition * 'a) queue |
59 5     Recv of condition * 'a option ref |
60 6     Inac
61 7
62 8 datatype 'a chan =
63 9     Chn of 'a chan_content ref * mutex_lock
64 10
65 11 fun channel () = Chn (ref Inac, mutexLock ())
66 12
67 13 fun send (Chn (conRef, lock)) m =
68 14     acquire lock;
69 15     case !conRef of
70 16     Recv (recvCond, mopRef) =>
71 17         mopRef := Some m; conRef := Inac;
72 18         release lock; signal recvCond;
73 19         () |
74 20     Send q => let

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```

21     val sendCond = condition () in
22     enqueue (q, (sendCond, m));
23     release lock; wait sendCond;
24     () end |
25   Inac => let
26     val sendCond = condition () in
27     conRef := Send (queue [(sendCond, m)])
28     release lock; wait sendCond; () end
29
30   fun recv (Chn (conRef, lock)) = let
31     val recvCond = condition ()
32     val mopRef = ref None in
33     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
34       Inac => (* conRef already set *)
35         wait recvCond; valOf (!mopRef) |
36       Send q =>
37         (* the current thread is the only one
38          * that updates the state from this state *)
39         acquire lock;
40         (let
41           val (sendCond, m) = dequeue q in
42           if (isEmpty q) then conRef := Inac else ();
43           release lock; signal sendCond; m end) |
44       Recv _ => raise NeverHappens end end
45
46

```

```

1
2 structure OneToOneChan : CHAN = struct
3
4   datatype 'a chan_content =
5     Send of condition * 'a |
6     Recv of condition * 'a option ref |
7     Inac
8
9   datatype 'a chan = Chn of 'a chan_content ref
10
11   fun channel () = Chn (ref Inac)
12
13   fun send (Chn conRef) m = let
14     val sendCond = condition () in
15     case cas (conRef, Inac, Send (sendCond, m)) of
16       Inac =>
17         (* conRef already set to Send *)
18         wait sendCond; () |
19       Recv (recvCond, mopRef) =>
20         (* the current thread is the only one
21          * that accesses conRef for this state *)
22         mopRef := Some m; conRef := Inac;
23         signal recvCond; () |

```



```

24     Send _ => raise NeverHappens end end
25
26
27 fun recv (Chn conRef) = let
28     val recvCond = condition ();
29     val mopRef = ref None in
30     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
31         Inac => (* conRef already set to Recv*)
32             wait recvCond; valOf (!mopRef) |
33         Send (sendCond, m) =>
34             (* the current thread is the only one
35              * that accesses conRef for this state *)
36             conRef := Inac; signal sendCond; m |
37         Recv _ => raise NeverHappens end end
38
39 end
40
1  structure OneShotChan : CHAN = struct
2
3  datatype 'a chan_content =
4      Send of condition * 'a |
5      Recv of condition * 'a option ref |
6      Inac
7
8  datatype 'a chan = Chn of 'a chan_content ref * mutex_lock
9
10 fun channel () = Chn (ref Inac, lock ())
11
12 fun send (Chn (conRef, lock)) m = let
13     val sendCond = condition () in
14     case (conRef, Inac, Send (sendCond, m)) of
15         Inac =>
16             (* conRef already set to Send*)
17             wait sendCond; () |
18         Recv (recvCond, mopRef) =>
19             mopRef := Some m; signal recvCond;
20             () |
21         Send _ => raise NeverHappens end end
22
23
24 fun recv (Chn (conRef, lock)) = let
25     val recvCond = condition ()
26     val mopRef = ref None in
27     case (conRef, Inac, Recv (recvCond, mopRef)) of
28         Inac =>
29             (* conRef already set to Recv*)
30             wait recvCond; valOf (!mopRef) |
31         Send (sendCond, m) =>
32             acquire lock; signal sendCond;

```

```

33         (* never relases lock;
34         -* blocks others forever *)
35         m |
36     Recv _ =>
37         acquire lock;
38         (* never able to acquire lock;
39         -* blocked forever *)
40         raise NeverHappens end end
41
42 end
43
1 structure OneShotToOneChan : CHAN = struct
2
3     datatype 'a chan =
4         Chn of condition * condition * 'a option ref
5
6     fun channel () =
7         Chn (condition (), condition (), ref None)
8
9     fun send (Chn (sendCond, recvCond, mopRef)) m =
10         mopRef := Some m; signal recvCond;
11         wait sendCond; ()
12
13     fun recv (Chn (sendCond, recvCond, mopRef)) =
14         wait recvCond; signal sendCond;
15         valOf (!mopRef)
16
17 end
18

```

## 2 Syntax

```

1
2     datatype var = Var string
3
4     datatype exp =
5         Let var boundexp exp |
6         Rslt var
7
8     boundexp =
9         Unt |
10        MkChn |
11        Prim prim |
12        Spwn exp |
13        Sync var |
14        Fst var |
15        Snd var |
16        Case var var exp var exp |

```

```

17   App var var and
18
19   prim =
20     SendEvt var var |
21     RecvEvt var |
22     Pair var var |
23     Lft var |
24     Rht var |
25     Abs var var ex

```

### 3 Dynamic Semantics

```

1   datatype ctrl_label =
2     LNxt var | LSpwn var | LCall var | LRtn var
3
4   type_synonym ctrl_path = (ctrl_label list)
5
6   datatype chan = Chn ctrl_path var
7
8   datatype val =
9     VUnt | VChn chan | VClsr prim (var  $\rightarrow$  val)
10
11  datatype ctn = Ctn var exp (var  $\rightarrow$  val)
12
13  datatype state = Stt exp (var  $\rightarrow$  val) (ctn list)
14
15

```

```

1
2  inductive
3    seq_step ::
4      bind * (var  $\rightarrow$  val))  $\Rightarrow$  val  $\Rightarrow$  bool where
5    LetUnt :
6      seq_step (Unt, env) VUnt |
7    LetPrim :
8      seq_step (Prim p, env) (VClsr p env) |
9    LetFst :
10     env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
11     envp x1 = Some v  $\Rightarrow$ 
12     seq_step (Fst xp, env) v |
13    LetSnd :
14     env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
15     envp x2 = Some v  $\Rightarrow$ 
16     seq_step (Snd xp, env) v
17
18
19

```

```

1

```

```

2
3 inductive
4   seq_step_up ::
5     bind * (var  $\rightarrow$  val)  $\Rightarrow$ 
6     exp * val_env  $\Rightarrow$  bool where
7   LetCaseLft :
8     env xs = Some (VClsr (Lft xl') envl)  $\Rightarrow$ 
9     envl xl' = Some vl  $\Rightarrow$ 
10    seq_step_up
11      (Case xs xl el xr er, env)
12      (el, env(xl  $\mapsto$  vl)) |
13   LetCaseRht :
14     env xs = Some (VClsr (Rht xr') envr)  $\Rightarrow$ 
15     envr xr' = Some vr  $\Rightarrow$ 
16    seq_step_up
17      (Case xs xl el xr er, env)
18      (er, env(xr  $\mapsto$  vr)) |
19   LetApp :
20     env f = Some (VClsr (Abs fp xp el) envl)  $\Rightarrow$ 
21     env xa = Some va  $\Rightarrow$ 
22    seq_step_up
23      (App f xa, env)
24      (el, envl(
25        fp  $\mapsto$  (VClsr (Abs fp xp el) envl),
26        xp  $\mapsto$  va))
27
28
1
2
3 type_synonym cmmn_set = (ctrl_path * chan * ctrl_path) set
4
5 type_synonym trace_pool = ctrl_path  $\rightarrow$  state
6
7 inductive
8   leaf ::
9     trace_pool  $\Rightarrow$  ctrl_path  $\Rightarrow$  bool where
10  intro :
11    trpl pi  $\neq$  None  $\Rightarrow$ 
12    ( $\nexists$  pi' . trpl pi'  $\neq$  None  $\wedge$  strict_prefix pi pi')  $\Rightarrow$ 
13    leaf trpl pi
14
15
1
2 inductive
3   concur_step ::
4     trace_pool * cmmn_set  $\Rightarrow$ 
5     trace_pool * cmmn_set  $\Rightarrow$ 
6     bool where

```

```

7   Seq_Sttstep_Down :
8     leaf trpl pi ==>
9     trpl pi = Some
10      (Stt (Rslt x) env
11       ((Ctn xk ek envk) # k)) ==>
12     env x = Some v ==>
13     concur_step
14       (trpl, ys)
15       (trpl(pi @ [LRtn xk] ↦
16        (Stt ek (envk(xk ↦ v)) k))), ys) |
17   Seq_Step :
18     leaf trpl pi ==>
19     trpl pi = Some
20      (Stt (Let x b e) env k) ==>
21     seq_step (b, env) v ==>
22     concur_step
23       (trpl, ys)
24       (trpl(pi @ [LNxt x] ↦
25        (Stt e (env(x ↦ v)) k), ys) |
26   Seq_Step_Up :
27     leaf trpl pi ==>
28     trpl pi = Some
29      (Stt (Let x b e) env k) ==>
30     seq_step_up (b, env) (e', env') ==>
31     concur_step
32       (trpl, ys)
33       (trpl(pi @ [LCall x] ↦
34        (Stt e' env'
35         ((Ctn x e env) # k))), ys) |
36   LetMkCh :
37     leaf trpl pi ==>
38     trpl pi = Some (Stt (Let x MkChn e) env k) ==>
39     concur_step
40       (trpl, ys)
41       (trpl(pi @ [LNxt x] ↦
42        (Stt e (env(x ↦ (VChn (Chn pi x)))) k))), ys) |
43   LetSpwn :
44     leaf trpl pi ==>
45     trpl pi = Some
46      (Stt (Let x (Spwn ec) e) env k) ==>
47     concur_step
48       (trpl, ys)
49       (trpl(
50        pi @ [LNxt x] ↦
51        (St e (env(x ↦ VUnt)) k),
52        pi @ [LSpwn x] ↦
53        (St ec env []), ys) |
54   LetSync :
55     leaf trpl pis ==>
56     trpl pis = Some

```

```

57      (Stt (Let xs (Sync xse) es) envs ks)  $\Rightarrow$ 
58      envs xse = Some
59      (VClSr (SendEvt xsc xm) envse)  $\Rightarrow$ 
60      leaf trpl pir  $\Rightarrow$ 
61      trpl pir = Some
62      (Stt (Let xr (Sync xre) er) envr kr)  $\Rightarrow$ 
63      envr xre = Some
64      (VClSr (RecvEvt xrc) envre)  $\Rightarrow$ 
65      envse xsc = Some (VChn c)  $\Rightarrow$ 
66      envre xrc = Some (VChn c)  $\Rightarrow$ 
67      envse xm = Some vm  $\Rightarrow$ 
68      concur_step
69      (trpl, ys)
70      (trpl(
71        pis @ [LNxt xs]  $\mapsto$ 
72        (Stt es (envs(xs  $\mapsto$  VUnt)) ks),
73        pir @ [LNxt xr]  $\mapsto$ 
74        (Stt er (envr(xr  $\mapsto$  vm)) kr)),
75        ys  $\cup$  {(pis, c, pir)})
76
77
1  inductive
2  star ::
3    ('a  $\Rightarrow$  'a  $\Rightarrow$  bool)  $\Rightarrow$ 
4    'a  $\Rightarrow$  'a  $\Rightarrow$  bool for r where
5  refl : star r x x |
6  step : r x y  $\Rightarrow$  star r y z  $\Rightarrow$  star r x z
7

```

## 4 Dynamic Communication

```

1  inductive
2  is_send_path ::
3    trace_pool  $\Rightarrow$  chan  $\Rightarrow$ 
4    control_path  $\Rightarrow$  bool where
5  intro :
6    trpl piy = Some
7    (Stt (Let xy (Sync xe) en) env k)  $\Rightarrow$ 
8    env xe = Some
9    (VClSr (SendEvt xsc xm) enve)  $\Rightarrow$ 
10   enve xsc = Some (VChn c)  $\Rightarrow$ 
11   is_send_path trpl c piy
12
13 inductive
14 is_recv_path ::
15   trace_pool  $\Rightarrow$  chan  $\Rightarrow$ 
16   control_path  $\Rightarrow$  bool where
17   intro :

```

```

18   trpl piy = Some
19   (Stt (Let xy (Sync xe) en) env k) ==>
20   env xe = Some
21   (VClsr (RecvEvt xrc) enve) ==>
22   enve xrc = Some (VChn c) ==>
23   is_recv_path trpl c piy
24
25

```

```

1
2   inductive
3     every_two ::
4       ('a => bool) =>
5       ('a => 'a => bool) => bool where
6   intro : (∀ pi1 pi2 .
7     p x1 ->
8     p x2 ->
9     r x1 x2) ==>
10    every_two p r
11
12   inductive
13     ordered ::
14       'a list => 'a list => bool where
15   left : prefix pi1 pi2 ==> ordered pi1 pi2 |
16   right : prefix pi2 pi1 ==> ordered pi1 pi2
17
18

```

```

1
2   inductive one_shot :: trace_pool => chan => bool where
3   intro :
4     every_two
5     (is_send_path trpl c) op= ==>
6     one_shot trpl c
7
8   inductive fan_out :: trace_pool => chan => bool where
9   intro :
10     every_two
11     (is_send_path trpl c) ordered ==>
12     fan_out trpl c
13
14   inductive fan_in :: trace_pool => chan => bool where
15   intro :
16     every_two
17     (is_recv_path trpl c) ordered ==>
18     fan_in trpl c
19
20   inductive one_to_one :: trace_pool => chan => bool where
21   intro :
22     fan_out trpl c ==>

```

```

23     fan_in trpl c  $\Rightarrow$ 
24     one_to_one trpl c
25
26

```

## 5 Static Semantics

```

1
2  datatype abstract_value =
3    AChn var |
4    AUnt |
5    APrim prim
6
7  type_synonym abstract_env = var  $\Rightarrow$  abstract_value set
8
9  fun rslt_var :: exp  $\Rightarrow$  var where
10    rslt_var (Rslt x) = x |
11    rslt_var (Let _ _ e) = (rslt_var e)
12
13

```

```

1
2
3  inductive
4    static_eval_exp ::
5      abstract_env * abstract_env  $\Rightarrow$ 
6      exp  $\Rightarrow$  bool where
7    Rslt :
8      static_eval_exp (V, C) (Rslt x) |
9    let_unt :
10      {AUnt}  $\subseteq$  V x  $\Rightarrow$ 
11      static_eval_exp (V, C) e  $\Rightarrow$ 
12      static_eval_exp (V, C) (Let x Unt e) |
13    let_chan :
14      {AChn x}  $\subseteq$  V x  $\Rightarrow$ 
15      static_eval_exp (V, C) e  $\Rightarrow$ 
16      static_eval_exp (V, C) (Let x (MkChn) e) |
17    let_send_evt :
18      {APrim (SendEvt xc xm)}  $\subseteq$  V x  $\Rightarrow$ 
19      static_eval_exp (V, C) e  $\Rightarrow$ 
20      static_eval_exp (V, C)
21        (Let x (Prim (SendEvt xc xm)) e) |
22    let_recv_evt :
23      {APrim (RecvEvt xc)}  $\subseteq$  V x  $\Rightarrow$ 
24      static_eval_exp (V, C) e  $\Rightarrow$ 
25      static_eval_exp (V, C)
26        (Let x (Prim (RecvEvt xc)) e) |
27    let_pair :
28      {APrim (Pair x1 x2)}  $\subseteq$  V x  $\Rightarrow$ 

```



```

29     static_eval_exp (V, C) e  $\impl$ 
30     static_eval_exp (V, C) (Let x (Pair x1 x2) e) |
31 let_left :
32     {APrim (Left xp)}  $\subseteq$  V x  $\impl$ 
33     static_eval_exp (V, C) e  $\impl$ 
34     static_eval_exp (V, C) (Let x (Left xp) e) |
35 let_right :
36     {APrim (Right xp)}  $\subseteq$  V x  $\impl$ 
37     static_eval_exp (V, C) e  $\impl$ 
38     static_eval_exp (V, C) (Let x (Right xp) e) |
39 let_abs :
40     {APrim (Abs f' x' e')}  $\subseteq$  V f'  $\impl$ 
41     static_eval_exp (V, C) e'  $\impl$ 
42     {APrim (Abs f' x' e')}  $\subseteq$  V x  $\impl$ 
43     static_eval_exp (V, C) e  $\impl$ 
44     static_eval_exp (V, C) (Let x (Abs f' x' e') e) |
45 let_spawn :
46     {AUnt}  $\subseteq$  V x  $\impl$ 
47     static_eval_exp (V, C) ec  $\impl$ 
48     static_eval_exp (V, C) e  $\impl$ 
49     static_eval_exp (V, C) (Let x (Spwn ec) e) |
50 let_sync :
51      $\forall$  xsc xm xc .
52     (APrim (SendEvt xsc xm))  $\in$  V xe  $\longrightarrow$ 
53     AChn xc  $\in$  V xsc  $\longrightarrow$ 
54     {AUnt}  $\subseteq$  V x  $\wedge$  V xm  $\subseteq$  C xc  $\impl$ 
55      $\forall$  xrc xc .
56     (APrim (RecvEvt xrc))  $\in$  V xe  $\longrightarrow$ 
57     AChn xc  $\in$  V xrc  $\longrightarrow$ 
58     C xc  $\subseteq$  V x  $\impl$ 
59     static_eval_exp (V, C) e  $\impl$ 
60     static_eval_exp (V, C) (Let x (Syync xe) e) |
61 let_fst :
62      $\forall$  x1 x2.
63     (APrim (Pair x1 x2))  $\in$  V xp  $\longrightarrow$ 
64     V x1  $\subseteq$  V x  $\impl$ 
65     static_eval_exp (V, C) e  $\impl$ 
66     static_eval_exp (V, C) (Let x (Fst xp) e) |
67 let_snd :
68      $\forall$  x1 x2 .
69     (APrim (Pair x1 x2)  $\in$  V xp  $\longrightarrow$ 
70     V x2  $\subseteq$  V x  $\impl$ 
71     static_eval_exp (V, C) e  $\impl$ 
72     static_eval_exp (V, C) (Let x (Snd xp) e) |
73 let_case :
74      $\forall$  x1' .
75     (APrim (Left x1'))  $\in$  V xs  $\longrightarrow$ 
76     V x1'  $\subseteq$  V x1  $\wedge$  V (rslt_var el)  $\subseteq$  V x  $\wedge$ 
77     static_eval_exp (V, C) el  $\impl$ 
78      $\forall$  xr' .

```

```

79      (APrim (Right xr')) ∈ V xs →
80      V xr' ⊆ V xr ∧ V (rslt_var er) ⊆ V x ∧
81      static_eval_exp (V, C) er ⇒
82      static_eval_exp (V, C) e ⇒
83      static_eval_exp (V, C) (Let x (Case xs xl el xr er) e)
84  |
85  let_app :
86  ∀ f' x' e' .
87    (APrim (Abs f' x' e') ∈ V f →
88    V xa ⊆ V x' ∧
89    V (rslt_var e') ⊆ V x ⇒
90    static_eval_exp (V, C) e ⇒
91    static_eval_exp (V, C) (Let x (App f xa) e)
92
1   inductive static_reachable :: exp ⇒ exp ⇒ bool where
2     Refl :
3       static_reachable e e |
4     let_Spawn_Child
5       static_reachable ec e ⇒
6       static_reachable (Let x (Spwn ec) en) e |
7     let_Case_Left :
8       static_reachable el e ⇒
9       static_reachable (Let x (case xs xl el xr er) en) e |
10    let_Case_Right :
11      static_reachable er e ⇒
12      static_reachable (Let x (case xs xl el xr er) en) e |
13    let_Abs_Body : "
14      static_reachable eb e ⇒
15      static_reachable (Let x (Abs f xp eb) en) e |
16    Let :
17      static_reachable en e ⇒
18      static_reachable (Let x b en) e
19
20
21
22
23 fun abstract :: val ⇒ abstract_value where
24   abstract VUnt = AUnt |
25   abstract VChn (Chn pi x) = AChn x |
26   abstract VClsr p env = APrim p
27
28
29
30
31 locale semantics_sound =
32   assumes
33     exp_always_not_static_bound_sound :
34       env' x = Some v ⇒
35       (V, C) static_eval e ⇒

```

```

6      ([[  $\mapsto$  (Stt e ( $\lambda$  _ . None) [])], H) star concur_step (
  trpl', H')  $\impl$ 
7      trpl' pi = Some (Stt e' env' k')  $\impl$ 
8      {|v|}  $\subseteq$  V x and
9
10     exp_always_exp_not_static_reachable_sound:
11     ([[  $\mapsto$  (Stt e0 ( $\lambda$  _ . None) [])], { }) star concur_step
  (trpl', H')  $\impl$ 
12     trpl' pi' = Some (Stt e' env' k')  $\impl$ 
13     static_reachable e0 e'
14

```

```

1
2 inductive
3   static_eval_val ::
4     abstract_env * abstract_env  $\Rightarrow$  val  $\Rightarrow$  bool and
5   static_eval_env ::
6     abstract_env * abstract_env  $\Rightarrow$  val_env  $\Rightarrow$  bool where
7   Unt :
8     static_eval_val (V, C) VUnt |
9   Chan :
10    static_eval_val (V, C) VChn c |
11   SendEvt :
12    static_eval_env (V, C) env  $\impl$ 
13    static_eval_val (V, C) (VClsr (SendEvt _ _) env) |
14   RecvEvt :
15    static_eval_env (V, C) env  $\impl$ 
16    static_eval_val (V, C) (VClsr (RecvEvt _) env) |
17   Left :
18    static_eval_env (V, C) env  $\impl$ 
19    static_eval_val (V, C) (VClsr (Left _) env) |
20   Right :
21    static_eval_env (V, C) env  $\impl$ 
22    static_eval_val (V, C) (VClsr (Right _) env) |
23   Abs :
24    {(APrim (Abs f x e))}  $\subseteq$  V f  $\impl$ 
25    static_eval_exp (V, C) e  $\impl$ 
26    static_eval_env (V, C) env  $\impl$ 
27    static_eval_val (V, C) (VClsr (Abs f x e) env) |
28   Pair :
29    static_eval_env (V, C) env  $\impl$ 
30    static_eval_val (V, C) (VClsr (Pair _ _) env) |
31   intro :
32      $\forall$  x v .
33     env x = Some v  $\impl$ 
34     {abstract v}  $\subseteq$  V x  $\wedge$  static_eval_val (V, C) v  $\impl$ 
35     static_eval_env (V, C) env
36
37

```

```

1
2 inductive static_eval_stack ::
3   abstract_env * abstract_env  $\Rightarrow$ 
4   abstract_value set  $\Rightarrow$  cont list  $\Rightarrow$  bool where
5   Empty :
6     static_eval_stack (V, C) valset [] |
7   Nonempty :
8     valset  $\subseteq$  V x  $\Rightarrow$ 
9     static_eval_exp (V, C) e  $\Rightarrow$ 
10    static_eval_env (V, C) env  $\Rightarrow$ 
11    static_eval_stack (V, C) (V (rslt_var e)) k  $\Rightarrow$ 
12    static_eval_stack (V, C) valset ((Ctn x e env) # k)
13
14
15 inductive static_eval_state ::
16   abstract_env * abstract_env  $\Rightarrow$ 
17   state  $\Rightarrow$  bool where
18   intro :
19     static_eval_exp (V, C) e  $\Rightarrow$ 
20     static_eval_env (V, C) env  $\Rightarrow$ 
21     static_eval_stack (V, C) (V (rslt_var e)) k  $\Rightarrow$ 
22     static_eval_state (V, C) (Stt e env k)
23
24 inductive static_eval_pool ::
25   abstract_env * abstract_env  $\Rightarrow$ 
26   trace_pool  $\Rightarrow$  bool where
27   intro :
28      $\forall$  pi st .
29     trpl pi = Some st  $\rightarrow$ 
30     static_eval_state (V, C) st  $\Rightarrow$ 
31     static_eval_pool (V, C) trpl
32
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```

```

1
2 theorem static_eval_preserved_under_concur_step :
3   static_eval_pool (V, C) trpl  $\Rightarrow$ 
4   concur_step (trpl, ys) (trpl', ys')  $\Rightarrow$ 
5   static_eval_pool (V, C) trpl'
6 proof sketch
7 qed
8
9 theorem static_eval_preserved_under_concur_step_star :
10  static_eval_pool (V, C) trpl  $\Rightarrow$ 
11  star concur_step (trpl, ys) (trpl', ys')  $\Rightarrow$ 
12  static_eval_pool (V, C) trpl'
13 proof sketch
14 qed
15

```

```

1
2 theorem trace_pool_snapshot_not_static_bound_sound :
3   env x = Some v  $\implies$ 
4   trpl pi = Some (Stt e env k)  $\implies$ 
5   static_eval_pool (V, C) trpl  $\implies$ 
6   {abstract v}  $\subseteq$  V x
7 proof sketch
8 qed
9

1
2 theorem trace_pool_always_not_static_bound_sound :
3   env' x = Some v  $\implies$ 
4   static_eval_pool (V, C) trpl  $\implies$ 
5   star_concur_step (trpl, ys) (trpl', ys')  $\implies$ 
6   trpl' pi = Some (Stt e' env' k')  $\implies$ 
7   {abstract v}  $\subseteq$  V x
8 proof sketch
9 qed
10
11

1
2 inductive
3   static_reachable_left :: exp  $\Rightarrow$  exp  $\Rightarrow$  bool where
4   Refl :
5     static_reachable_left e0 e0 |
6   let_Spawn_Child :
7     static_reachable_left e0 (Let x (Spwn ec) en)  $\implies$ 
8     static_reachable_left e0 ec |
9   let_Case_Left :
10    static_reachable_left e0 (Let x (case xs xl el xr er)
11    en)  $\implies$ 
12    static_reachable_left e0 el |
13  let_Case_Right :
14    static_reachable_left e0 (Let x (case xs xl el xr er)
15    en)  $\implies$ 
16    static_reachable_left e0 er |
17  let_Abs_Body :
18    static_reachable_left e0 (Let x (Abs f xp eb) en)  $\implies$ 
19    static_reachable_left e0 eb |
20  Let :
21    static_reachable_left e0 (Let x b en)  $\implies$ 
22    static_reachable_left e0 en
23
24 inductive
25   static_reachable_over_prim :: exp  $\Rightarrow$  prim  $\Rightarrow$  bool where
26   SendEvt :
27     static_reachable_over_prim e0 (SendEvt xC xM) |
28   RecvEvt :

```

```

27     static_reachable_over_prim e0 (RecvEvt xC) |
28   Pair :
29     static_reachable_over_prim e0 (Pair x1 x2) |
30   Left :
31     static_reachable_over_prim e0 (Left x) |
32   Right :
33     static_reachable_over_prim e0 (Right x) |
34   Abs :
35     static_reachable_left e0 eb  $\implies$ 
36     static_reachable_over_prim e0 (Abs fp xp eb)
37
38   inductive
39     static_reachable_over_env :: exp  $\Rightarrow$  env  $\Rightarrow$  bool and
40     static_reachable_over_val :: exp  $\Rightarrow$  val  $\Rightarrow$  bool where
41   VUnt :
42     static_reachable_over_val e0 VUnt |
43   VChn :
44     static_reachable_over_val e0 (VChn c) |
45   VClsr : "
46     static_reachable_over_prim e0 p  $\implies$ 
47     static_reachable_over_env e0 env'  $\implies$ 
48     static_reachable_over_val e0 (VClsr p env') |
49   intro : "
50      $\forall$  x v .
51     env x = Some v  $\longrightarrow$ 
52     static_reachable_over_val e0 v  $\implies$ 
53     static_reachable_over_env e0 env
54
55   inductive
56     static_reachable_over_stack ::
57     exp  $\Rightarrow$  cont list  $\Rightarrow$  bool where
58   Empty :
59     static_reachable_over_stack e0 [] |
60   Nonempty :
61     static_reachable_left e0 ek  $\implies$ 
62     static_reachable_over_env e0 envk  $\implies$ 
63     static_reachable_over_stack e0 k  $\implies$ 
64     static_reachable_over_stack e0 ((Ctn xk ek envk) # k)
65
66   inductive
67     static_reachable_over_state ::
68     exp  $\Rightarrow$  state  $\Rightarrow$  bool where
69   intro :
70     static_reachable_left e0 e  $\implies$ 
71     static_reachable_over_env e0 env  $\implies$ 
72     static_reachable_over_stack e0 k  $\implies$ 
73     static_reachable_over_state e0 (Ctn e env k)
74
75

```

```

1 lemma static_reachable_trans :
2   static_reachable ez ey  $\implies$ 
3   static_reachable ey ex  $\implies$ 
4   static_reachable ez ex
5 proof sketch
6 qed
7
8
9 lemma static_reachable_over_state_preserved :
10  concur_step (trpl, ys) (trpl', ys')  $\implies$ 
11   $\forall$  pi st.
12    trpl pi = Some st  $\longrightarrow$ 
13    static_reachable_over_state e0 st  $\implies$ 
14    trpl' pi' = Some st'  $\implies$ 
15    static_reachable_over_state e0 st'
16 proof sketch
17 qed
18
19
20 lemma state_always_exp_not_static_reachable_sound :
21  star concur_step (trpl0, ys0) (trpl', ys')  $\implies$ 
22  trpl0 = [[]  $\mapsto$  (Stt e0 ( $\lambda$  _ . None) [])]  $\implies$ 
23  trpl' pi' = Some st'  $\implies$ 
24  static_reachable_over_state e0 st'
25 proof sketch
26 qed
27
28
29 interpretation semantics_sound
30 proof sketch
31 qed
32

```

## 6 Static Communication

```

1 locale communication_sound =
2   fixes
3     static_one_shot :: abstract_env  $\Rightarrow$  exp  $\Rightarrow$  var  $\Rightarrow$  bool and
4     static_fan_out :: abstract_env  $\Rightarrow$  exp  $\Rightarrow$  var  $\Rightarrow$  bool and
5     static_fan_in :: abstract_env  $\Rightarrow$  exp  $\Rightarrow$  var  $\Rightarrow$  bool and
6     static_one_to_one :: abstract_env  $\Rightarrow$  exp  $\Rightarrow$  var  $\Rightarrow$  bool
7
8   assumes
9     static_one_shot_sound:
10      static_one_shot V e xC  $\implies$ 
11      static_eval (V, C) e  $\implies$ 
12      star concur_step ([[]  $\mapsto$  (Stt e ( $\lambda$  _ . None) [])], {x})
13      (trpl', H')  $\implies$ 

```

```

13     one_shot trpl' (Ch pi xC) and
14
15   static_fan_out_sound:
16     static_fan_out V e xC  $\Rightarrow$ 
17     (V, C) static_eval e  $\Rightarrow$ 
18     star concur_step ([[  $\mapsto$  (Stt e ( $\lambda$  _ . None) []) ], {}])
19   (trpl', H')  $\Rightarrow$ 
20     fan_out trpl' (Ch pi xC) and
21
22   static_fan_in_sound:
23     static_fan_in V e xC  $\Rightarrow$ 
24     (V, C) static_eval e  $\Rightarrow$ 
25     star concur_step ([[  $\mapsto$  (Stt e ( $\lambda$  _ . None) []) ], {}])
26   (trpl', H')  $\Rightarrow$ 
27     fan_in trpl' (Ch pi xC) and
28
29   static_one_to_one_sound: "
30     static_one_to_one V e xC  $\Rightarrow$ 
31     (V, C) static_eval e  $\Rightarrow$ 
32     star concur_step ([[  $\mapsto$  (Stt e ( $\lambda$  _ . None) []) ], {}])
33   (trpl', H')  $\Rightarrow$ 
34     one_to_one trpl' (Ch pi xC)"
35
36
37 datatype node_label = NLet var | NRslt var
38
39 fun top_node_label :: exp  $\Rightarrow$  node_label where
40   top_node_label (Let x b e) = NLet x |
41   top_node_label (Rslt y) = NRslt y
42
43 type_synonym node_set = node_label set
44
45 type_synonym node_map = node_label  $\Rightarrow$  var set
46
47 inductive
48   static_static_send_node_label ::
49     abstract_env  $\Rightarrow$  exp  $\Rightarrow$ 
50     var  $\Rightarrow$  node_label  $\Rightarrow$  bool where
51   intro:
52     {AChn xC}  $\subseteq$  V xSC  $\Rightarrow$ 
53     {APrim (SendEvt xSC xM)}  $\subseteq$  V xE  $\Rightarrow$ 
54     static_reachable e (Let x (Sync xE) e')  $\Rightarrow$ 
55     static_static_send_node_label V e xC (NLet x)
56
57 inductive
58   static_static_recv_node_label ::
59     abstract_env  $\Rightarrow$  exp  $\Rightarrow$ 
60     var  $\Rightarrow$  node_label  $\Rightarrow$  bool where
61   intro:
62     {AChn xC}  $\subseteq$  V xRC  $\Rightarrow$ 

```



```

27      {APrim (RecvEvt xRC)}  $\subseteq$  V xE  $\implies$ 
28      static_reachable e (Let x (Sync xE) e')  $\implies$ 
29      static_static_recv_node_label V e xC (NLet x)
30
31

```

## 7 Static Communication part A

```

1
2  datatype edge_label = ENxt | ESpwn | ECall | ERtn
3
4  type_synonym flow_label = node_label * edge_label *
    node_label
5
6  type_synonym flow_set = flow_label set
7
8  type_synonym step_label = node_label * edge_label
9
10 type_synonym abstract_path = step_label list
11
12
13 inductive
14   static_traversable ::
15     abstract_env  $\Rightarrow$ 
16     (node_label * edge_label * node_label) set  $\Rightarrow$ 
17     exp  $\Rightarrow$  bool  where
18   Rslt:
19     static_traversable V F (Rslt x) |
20   let_Unit:
21     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
22     static_traversable V F e  $\implies$ 
23     static_traversable V F (Let x Unt e) |
24   let_Chan:
25     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
26     static_traversable V F e  $\implies$ 
27     static_traversable V F (Let x MkChn e) |
28   let_SendEvt:
29     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
30     static_traversable V F e  $\implies$ 
31     static_traversable V F (Let x = (SendEvt xc xm) e) |
32   let_RecvEvt:
33     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
34     static_traversable V F e  $\implies$ 
35     static_traversable V F (Let x = (RecvEvt xc) in e) |
36   let_Pair:
37     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
38     static_traversable V F e  $\implies$ 
39     static_traversable V F (Let x (Pair x1 x2) e) |
40   let_Left:

```

```

29     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
30     static_traversable V F e  $\implies$ 
31     static_traversable V F (Let x (Lft xp) e) |
32 let_Right:
33     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
34     static_traversable V F e  $\implies$ 
35     static_traversable V F (Let x (Rht xp) e)" |
36 let_Abs:
37     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
38     static_traversable V F eb  $\implies$ 
39     static_traversable V F e  $\implies$ 
40     static_traversable V F (Let x (Abs f xp eb) e) |
41 let_Spawn:
42     {
43       (NLet x, ENxt, top_node_label e),
44       (NLet x, ESawn, top_node_label ec)}
45      $\subseteq$  F  $\implies$ 
46     static_traversable V F ec  $\implies$ 
47     static_traversable V F e  $\implies$ 
48     static_traversable V F (Let x (Spwn ec) e) |
49 let_Sync:
50     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
51     static_traversable V F e  $\implies$ 
52     static_traversable V F (Let x (Sync xSE) e) |
53 let_Fst:
54     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
55     static_traversable V F e  $\implies$ 
56     static_traversable V F (Let x (Fst xp) e) |
57 let_Snd:
58     {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
59     static_traversable V F e  $\implies$ 
60     static_traversable V F (Let x (Snd xp) e) |
61 let_Case:
62     {
63       (NLet x, ECall, top_node_label el),
64       (NLet x, ECall, top_node_label er),
65       (NRslt (rslt_var el), ERtn, top_node_label e),
66       (NRslt (rslt_var er), ERtn, top_node_label e)}
67      $\subseteq$  F  $\implies$ 
68     static_traversable V F el  $\implies$ 
69     static_traversable V F er  $\implies$ 
70     static_traversable V F e  $\implies$ 
71     static_traversable V F (Let x (Case xs xl el xr er) e)
72 |
73 let_App:
74      $\forall$  fp xp eb .
75     APrim (Abs fp xp eb)  $\in$  V f  $\longrightarrow$ 
76     {
77       (NLet x, ECall, top_node_label eb),
78       (NRslt (rslt_var eb), ERtn, top_node_label e)}

```

```

78       $\subseteq F \implies$ 
79      static_traversable V F e  $\implies$ 
80      static_traversable V F (Let x (App f xa) e)
81
82
1   inductive static_traceable ::
2     abstract_env  $\Rightarrow$ 
3     flow_set  $\Rightarrow$  node_label  $\Rightarrow$ 
4     (node_label  $\Rightarrow$  bool)  $\Rightarrow$ 
5     abstract_path  $\Rightarrow$  bool where
6   Empty:
7     isEnd start  $\implies$ 
8     static_traceable V F start isEnd [] |
9   Edge:
10    isEnd end  $\implies$ 
11    {(start, edge, end)}  $\subseteq F \implies$ 
12    static_traceable V F start isEnd [(start, edge)] |
13   Step:
14    static_traceable V F middle isEnd ((middle, edge') #
15    path)  $\implies$ 
16    {(start, edge, middle)}  $\subseteq F \implies$ 
17    static_traceable V F start isEnd ((start, edge) # (
18    middle, edge') # path)
19
20   inductive static_inclusive ::
21     abstract_path  $\Rightarrow$  abstract_path  $\Rightarrow$  bool where
22   Prefix1:
23     prefix pi1 pi2  $\implies$ 
24     static_inclusive pi1 pi2 |
25   Prefix2:
26     prefix pi2 pi1  $\implies$ 
27     static_inclusive pi1 pi2 |
28   Spawn1:
29     static_inclusive (pi @ (NLet x, ESpwn) # pi1) (pi @ (
30     NLet x, ENxt) # pi2) |
31   Spawn2:
32     static_inclusive (pi @ (NLet x, ENxt) # pi1) (pi @ (NLet
33     x, ESpwn) # pi2)
34
35   inductive
36     singular ::
37       abstract_path  $\Rightarrow$ 
38       abstract_path  $\Rightarrow$  bool where
39   equal:
40     pi1 = pi2  $\implies$ 
41     singular pi1 pi2 |
42   exclusive:

```

```

9      /(static_inclusive pi1 pi2) ==>
10     singular pi1 pi2
11
12   inductive
13     noncompetitive ::
14       abstract_path => abstract_path => bool where
15     ordered:
16       ordered pi1 pi2 ==>
17       noncompetitive pi1 pi2 |
18     exclusive:
19       /(not_inclusive pi1 pi2) ==>
20       noncompetitive pi1 pi2
21
22   1   inductive
23     2   static_one_shot ::
24       3   abstract_env => exp =>
25       4   var => bool where
26     5   Sync:
27       6   every_two
28       7   (static_traceable V F (top_node_label e)
29       8   (static_static_send_node_label V e xC))
30       9   singular ==>
31     10   static_traversable V F e ==>
32     11   static_one_shot V e xC
33
34   13  inductive
35     14  static_one_to_one ::
36       15  abstract_env => exp =>
37       16  var => bool where
38     17  Sync:
39       18  every_two
40       19  (static_traceable V F
41       20  (top_node_label e) (
42  static_static_send_node_label V e xC))
43       21  noncompetitive ==>
44       22  every_two
45       23  (static_traceable V F
46       24  (top_node_label e) (
47  static_static_recv_node_label V e xC))
48       25  noncompetitive ==>
49       26  static_traversable V F e ==>
50       27  static_one_to_one V e xC
51
52   29  inductive
53     30  static_fan_out ::
54       31  abstract_env => exp =>
55       32  var => bool where
56     33  Sync:
57       34  every_two

```

```

35         (static_traceable V F
36         (top_node_label e) (static_static_send_node_label
V e xC))
37         noncompetitive  $\Rightarrow$ 
38         static_traversable V F e  $\Rightarrow$ 
39         static_fan_out V e xC
40
41     inductive
42     static_fan_in ::
43     abstract_env  $\Rightarrow$  exp  $\Rightarrow$ 
44     var  $\Rightarrow$  bool where
45     Sync:
46     every_two (static_traceable V F (top_node_label e) (
static_static_recv_node_label V e xC)) noncompetitive  $\Rightarrow$ 
47     static_traversable V F e  $\Rightarrow$ 
48     static_fan_in V e xC
49
50
1 locale communication_sound_A =
2   Static_Communication.communication_sound static_one_shot
   static_fan_out static_fan_in static_one_to_one
3
1 inductive paths_correspond ::
2   control_path  $\Rightarrow$  abstract_path  $\Rightarrow$  bool where
3   Empty:
4     paths_correspond [] [] |
5   Next:
6     paths_correspond pi path  $\Rightarrow$ 
7     paths_correspond (pi @ [LNext x]) (path @ [(NLet x,
ENxt)]) |
8   Spawn:
9     paths_correspond pi path  $\Rightarrow$ 
10    paths_correspond (pi @ [LSpawn x]) (path @ [(NLet x,
ESpwn)]) |
11  Call:
12    paths_correspond pi path  $\Rightarrow$ 
13    paths_correspond (pi @ [LCall x]) (path @ [(NLet x,
ECall)]) |
14  Rtn:
15    paths_correspond pi path  $\Rightarrow$ 
16    paths_correspond (pi @ [LRtn x]) (path @ [(NRslt x,
ERtn)])
17
1 lemma not_static_inclusive_sound: "
2   star concur_step
3   ([[]  $\mapsto$  (Stt e ( $\lambda$  _ . None) [])], { })
4   (trpl', ys')  $\Rightarrow$ 

```

```

5   trpl' pi1 ≠ None ⇒
6   trpl' pi2 ≠ None ⇒
7   paths_correspond pi1 path1 ⇒
8   paths_correspond pi2 path2 ⇒
9   static_inclusive path1 path2"
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```

inductive
   
 static\_traversable\_env ::
   
 abstract\_env ⇒ flow\_set ⇒ env ⇒ bool and
   
 static\_traversable\_val ::
   
 abstract\_env ⇒ flow\_set ⇒ val ⇒ bool where
   
 Intro:
   
 ∀ x v .
   
 env x = Some v →
   
 {rslt\_var v} ⊆ V x ∧ static\_traversable\_val V F v
   
 ⇒
   
 static\_traversable\_env V F env |
   
 Unit:
   
 static\_traversable\_val V F VUnit |
   
 Chan:
   
 static\_traversable\_val V F (VChn c) |
   
 SendEvt:
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (SendEvt \_ \_) env) |
   
 RecvEvt:
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (RecvEvt \_) env) |
   
 Left:
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (Left \_) env) |
   
 Right:
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (Right \_) env) |
   
 Abs:
   
 static\_traversable V F e ⇒
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (Abs f x e) env) |
   
 Pair:
   
 static\_traversable\_env V F env ⇒
   
 static\_traversable\_val V F (VClsr (Pair \_ \_) env)

inductive static\_traversable\_stack ::
   
 abstract\_env ⇒ flow\_set ⇒ cont list ⇒ bool where
   
 Empty:
   
 static\_traversable\_stack V F [] |

```

42   Nonempty:
43     static_traversable V F e ==>
44     static_traversable_env V F env ==>
45     static_traversable_stack V F k ==>
46     static_traversable_stack V F ((Ctn x e env) # k))
47
48   inductive
49     static_traversable_pool ::
50     abstract_env => flow_set =>
51     trace_pool => bool   where
52   Intro:
53     ∀ pi e env k .
54     E pi = Some (Stt e env k) →
55     static_traversable V F e ∧
56     static_traversable_env V F env ∧
57     static_traversable_stack V F k ==>
58     static_traversable_pool V F E
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```

1
2 lemma not_static_traceable_sound: "
3   trpl' pi = Some (Stt (Let x b en) env k)  $\implies$ 
4   star concur_step
5     ([[[]  $\mapsto$  (Stt e ( $\lambda$  _ . None) [])], {}))
6     (trpl', ys')  $\implies$ 
7   static_eval (V, C) e  $\implies$ 
8   static_traversable V F e  $\implies$ 
9   isEnd (NLet x)  $\implies$ 
10   $\exists$  path .
11    paths_correspond pi path  $\wedge$ 
12    static_traceable V F (top_node_label e) isEnd path
13  proof sketch
14  qed
15

```

```

1
2
3 interpretation communication_sound_A
4   proof -
5
6

```

## 8 Static Communication part B

```

1 datatype edge_label = ENxt | ESpwn | ESend var | ECall |
2   ERtn var
3 type_synonym flow_label = (node_label * edge_label *
4   node_label)
5 type_synonym flow_set = flow_label set
6
7 type_synonym step_label = (node_label * edge_label)
8
9 type_synonym abstract_path = step_label list
10
11
12 inductive static_traversable :: abstract_env  $\Rightarrow$  flow_set
13    $\Rightarrow$  (var  $\Rightarrow$  node_label  $\Rightarrow$  bool)  $\Rightarrow$  exp  $\Rightarrow$  bool where
14   result:
15     static_traversable V F static_recv_site (Rslt x) |
16   let_Unit:
17     {(NLet x , ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
18     static_traversable V F static_recv_site e  $\implies$ 
19     static_traversable V F static_recv_site (Let x Unt e
20   ) |
21   let_Chan:

```



```

10      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
11      static_traversable V F static_recv_site e  $\implies$ 
12      static_traversable V F static_recv_site (Let x MkChn
    e) |
13  let_SendEvt:
14      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
15      static_traversable V F static_recv_site e  $\implies$ 
16      static_traversable V F static_recv_site (Let x (Prim (
    SendEvt xc xm)) e) |
17  let_RecvEvt:
18      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
19      static_traversable V F static_recv_site e  $\implies$ 
20      static_traversable V F static_recv_site (Let x (Prim (
    RecvEvt xc)) e) |
21  let_Pair:
22      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
23      static_traversable V F static_recv_site e  $\implies$ 
24      static_traversable V F static_recv_site (Let x (Prim (
    Pair x1 x2)) e) |
25  let_Left:
26      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
27      static_traversable V F static_recv_site e  $\implies$ 
28      static_traversable V F static_recv_site (Let x (Prim (
    Lft xp)) e) |
29  let_Right:
30      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
31      static_traversable V F static_recv_site e  $\implies$ 
32      static_traversable V F static_recv_site (Let x (Prim (
    Rght xp)) e) |
33  let_Abs:
34      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
35      static_traversable V F static_recv_site eb  $\implies$ 
36      static_traversable V F static_recv_site e  $\implies$ 
37      static_traversable V F static_recv_site (Let x (Prim (
    Abs f xp eb)) e) |
38  let_Spawn:
39      {(NLet x, ENxt, top_node_label e),
40       (NLet x, ESpawn, top_node_label ec)}  $\subseteq$  F  $\implies$ 
41      static_traversable V F static_recv_site ec  $\implies$ 
42      static_traversable V F static_recv_site e  $\implies$ 
43      static_traversable V F static_recv_site (Let x (Spwn ec)
    e) |
44  let_Sync:
45      {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
46      ( $\forall$  xSC xM xC y .
47        $\{^{\sim}\text{SendEvt xSC xM}\} \subseteq V \text{ xSE} \longrightarrow$ 
48        $\{^{\sim}\text{Chan xC}\} \subseteq V \text{ xSC} \longrightarrow$ 
49       static_recv_site xC (NLet y)  $\longrightarrow$ 
50       {(NLet x, ESend xSE, NLet y)}  $\subseteq$  F)  $\implies$ 
51      static_traversable V F static_recv_site e  $\implies$ 

```

```

52   static_traversable V F static_recv_site (Let x (Sync xSE
    ) e) |
53 let_Fst:
54   {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
55   static_traversable V F static_recv_site e  $\implies$ 
56   static_traversable V F static_recv_site (Let x (Fst xp
    ) e) |
57 let_Snd:
58   {(NLet x, ENxt, top_node_label e)}  $\subseteq$  F  $\implies$ 
59   static_traversable V F static_recv_site e  $\implies$ 
60   static_traversable V F static_recv_site (Let x (Snd xp
    ) e) |
61 let_Case:
62   {
63     (NLet x, ECall, top_node_label el),
64     (NLet x, ECall, top_node_label er),
65     (NRslt (rslt_var el), ERtn x, top_node_label e),
66     (NRslt (rslt_var er), ERtn x, top_node_label e)}  $\subseteq$  F
     $\implies$ 
67   static_traversable V F static_recv_site el  $\implies$ 
68   static_traversable V F static_recv_site er  $\implies$ 
69   static_traversable V F static_recv_site e  $\implies$ 
70   static_traversable V F static_recv_site (Let x (Case
    xs xl el xr er) e) |
71 let_App:
72   ( $\forall$  f' xp eb .  $\hat{\text{Abs}}$  f' xp eb  $\in$  V f  $\longrightarrow$ 
73     {(NLet x, ECall, top_node_label eb),
74      (NRslt (rslt_var eb), ERtn x, top_node_label e)})
     $\subseteq$  F)  $\implies$ 
75   static_traversable V F static_recv_site e  $\implies$ 
76   static_traversable V F static_recv_site (Let x (App f xa
    ) e)
77
78
79 1 inductive
80 2   static_built_on_chan :: "abstract_env  $\Rightarrow$  node_map  $\Rightarrow$  var
     $\Rightarrow$  var  $\Rightarrow$  bool"
81 3 where
82 4   Chan:
83 5     AChn xc  $\in$  V x  $\implies$ 
84 6     static_built_on_chan V Ln xc x |
85 7   SendEvt:
86 8     APrim (SendEvt xsc xm)  $\in$  V x  $\implies$ 
87 9     static_built_on_chan V Ln xc xsc  $\vee$ 
    static_built_on_chan V Ln xc xm  $\implies$ 
88 10    static_built_on_chan V Ln xc x |
89 11  RecvEvt:
90 12    APrim (RecvEvt xrc)  $\in$  V x  $\implies$ 
91 13    static_built_on_chan V Ln xc xrc  $\implies$ 

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14   static_built_on_chan V Ln xc x |
15 Pair:
16   APrim (Pair x1 x2) ∈ V x ⇒
17   static_built_on_chan V Ln xc x1 ∨ static_built_on_chan
    V Ln xc x2 ⇒
18   static_built_on_chan V Ln xc x |
19 Left:
20   APrim (Left xa) ∈ V x ⇒
21   static_built_on_chan V Ln xc xa ⇒
22   static_built_on_chan V Ln xc x |
23 Right:
24   APrim (Right xa) ∈ V x ⇒
25   static_built_on_chan V Ln xc xa ⇒
26   static_built_on_chan V Ln xc x |
27 Abs:
28   APrim (Abs f xp eb) ∈ V x ⇒
29   /Set.is_empty (Ln (nodeLabel eb) - {xp}) ⇒
30   static_built_on_chan V Ln xc x
31

1 fun chan_set ::
2   abstract_env ⇒ node_map ⇒ var ⇒ var ⇒ var set" where
3   chan_set V Ln xc x = (if (static_built_on_chan V Ln xc x
    ) then {x} else {})
4

1 inductive static_live_chan ::
2   abstract_env ⇒ node_map ⇒ node_map ⇒ var ⇒ exp ⇒ bool
    where
3   Result:
4     chan_set V Ln xc y = Ln (NRslt y) ⇒
5     static_live_chan V Ln Lx xc (Rslt y) |
6 Let_Unit:
7   static_live_chan V Ln Lx xc e ⇒
8   Ln (top_node_label e) = Lx (NLet x) ⇒
9   Lx (NLet x) = Ln (NLet x) ⇒
10  static_live_chan V Ln Lx xc (Let x Unt e) |
11 Let_Chan:
12  static_live_chan V Ln Lx xc e ⇒
13  Ln (top_node_label e) = Lx (NLet x) ⇒
14  (Lx (NLet x) - {x}) = Ln (NLet x) ⇒
15  static_live_chan V Ln Lx xc (Let x MkChn e) |
16 Let_SendEvt:
17  static_live_chan V Ln Lx xc e ⇒
18  Ln (top_node_label e) = Lx (NLet x) ⇒
19  (Lx (NLet x) - {x}) ∪ chan_set V Ln xc xsc ∪
20  chan_set V Ln xc xm = Ln (NLet x) ⇒
21  static_live_chan V Ln Lx xc (Let x (Prim (SendEvt xsc xm
    )) e) |
22 Let_RecvEvt:

```

```

23   static_live_chan V Ln Lx xc e  $\implies$ 
24   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
25   (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xrc = Ln (NLet x)
     $\implies$ 
26   static_live_chan V Ln Lx xc (Let x (Prim (RecvEvt xrc))
    e) |
27 Let_Pair:
28   static_live_chan V Ln Lx xc e  $\implies$ 
29   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
30   (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc x1  $\cup$ 
31   chan_set V Ln xc x2 = Ln (NLet x)  $\implies$ 
32   static_live_chan V Ln Lx xc (Let x (Prim (Pair x1 x2))
    e) |
33 Let_Left:
34   static_live_chan V Ln Lx xc e  $\implies$ 
35   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
36   (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xa = Ln (NLet x
    )  $\implies$ 
37   static_live_chan V Ln Lx xc (Let x (Prim (Lft xa)) e)
    |
38 Let_Right:
39   static_live_chan V Ln Lx xc e  $\implies$ 
40   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
41   (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xa = Ln (NLet x
    )  $\implies$ 
42   static_live_chan V Ln Lx xc (Let x (Prim (Rght xa)) e)
    |
43 Let_Abs:
44   static_live_chan V Ln Lx xc e  $\implies$ 
45   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
46   static_live_chan V Ln Lx xc eb  $\implies$ 
47   (Lx (NLet x) - {x})  $\cup$ 
48   (Ln (top_node_label eb) - {xp}) = Ln (NLet x)  $\implies$ 
49   static_live_chan V Ln Lx xc (Let x (Prim (Abs f xp eb)
    ) e) |
50 Let_Spawn:
51   static_live_chan V Ln Lx xc e  $\implies$ 
52   static_live_chan V Ln Lx xc ec  $\implies$ 
53   Ln (top_node_label e)  $\cup$  Ln (top_node_label ec) = Lx (
    NLet x)  $\implies$ 
54   (Lx (NLet x) - {x}) = Ln (NLet x)  $\implies$ 
55   static_live_chan V Ln Lx xc (Let x (Spwn ec) e) |
56 Let_Sync:
57   static_live_chan V Ln Lx xc e  $\implies$ 
58   Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
59   (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xe = Ln (NLet x
    )  $\implies$ 
60   static_live_chan V Ln Lx xc (Let x (Sync xe) e) |
61 Let_Fst:
62   static_live_chan V Ln Lx xc e  $\implies$ 

```

```

63     Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
64     (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xa = Ln (NLet x
    )  $\implies$ 
65     static_live_chan V Ln Lx xc (Let x (Fst xa) e) |
66 Let_Snd:
67     static_live_chan V Ln Lx xc e  $\implies$ 
68     Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
69     (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xa = Ln (NLet x
    )  $\implies$ 
70     static_live_chan V Ln Lx xc (Let x (Snd xa) e) |
71 Let_Case:
72     static_live_chan V Ln Lx xc e  $\implies$ 
73     Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
74     static_live_chan V Ln Lx xc el  $\implies$ 
75     static_live_chan V Ln Lx xc er  $\implies$ 
76     (Lx (NLet x) - {x})  $\cup$  chan_set V Ln xc xs  $\cup$ 
77     (Ln (top_node_label el) - {xl})  $\cup$ 
78     (Ln (top_node_label er) - {xr}) = Ln (NLet x)  $\implies$ 
79     static_live_chan V Ln Lx xc (Let x (Case xs xl el xr
    er) e) |
80 Let_App:
81     static_live_chan V Ln Lx xc e  $\implies$ 
82     Ln (top_node_label e) = Lx (NLet x)  $\implies$ 
83     (Lx (NLet x) - {x})  $\cup$ 
84     chan_set V Ln xc f  $\cup$ 
85     chan_set V Ln xc xa = Ln (NLet x)  $\implies$ 
86     static_live_chan V Ln Lx xc (Let x (App f xa) e)
87
88
89

```

```

1
2 inductive static_traceable ::
3   flow_set  $\Rightarrow$  node_label
4    $\Rightarrow$  abstract_path  $\Rightarrow$  bool" where
5   Empty:
6     static_traceable F end [] |
7   Edge:
8     (start, edge, end)  $\in$  F  $\implies$ 
9     static_traceable F end [(start, edge)] |
10  Step:
11    static_traceable F end ((middle, edge') # post)  $\implies$ 
12    (start, edge, middle)  $\in$  F  $\implies$ 
13    path = [(start, edge), (middle, edge')] @ post  $\implies$ 
14    static_traceable F end path
15
16

```

1

```

2   inductive static_live_traversable :: "flow_set  $\Rightarrow$  node_map
    $\Rightarrow$  node_map  $\Rightarrow$  flow_label  $\Rightarrow$  bool" where
3   Next: "
4     (l, ENxt, l')  $\in$  F  $\Rightarrow$ 
5     /Set.is_empty (Lx l)  $\Rightarrow$ 
6     /Set.is_empty (Ln l')  $\Rightarrow$ 
7     static_live_traversable F Ln Lx (l, ENxt, l')
8   " |
9   Spawn: "
10    (l, ESpwn, l')  $\in$  F  $\Rightarrow$ 
11    /Set.is_empty (Lx l)  $\Rightarrow$ 
12    /Set.is_empty (Ln l')  $\Rightarrow$ 
13    static_live_traversable F Ln Lx (l, ESpwn, l')
14  " |
15  Call_Live_Outer: "
16    (l, ECall, l')  $\in$  F  $\Rightarrow$ 
17    /Set.is_empty (Lx l)  $\Rightarrow$ 
18    static_live_traversable F Ln Lx (l, ECall, l')
19  " |
20  Call_Live_Inner: "
21    (l, ECall, l')  $\in$  F  $\Rightarrow$ 
22    /Set.is_empty (Ln l')  $\Rightarrow$ 
23    static_live_traversable F Ln Lx (l, ECall, l')
24  " |
25  Return: "
26    (l, ERtn x, l')  $\in$  F  $\Rightarrow$ 
27    /Set.is_empty (Ln l')  $\Rightarrow$ 
28    static_live_traversable F Ln Lx (l, ERtn x, l')
29  " |
30  Send: "
31    ((NLet xSend), ESend xE, (NLet xRecv))  $\in$  F  $\Rightarrow$ 
32    {xE}  $\subseteq$  (Ln (NLet xSend))  $\Rightarrow$ 
33    static_live_traversable F Ln Lx ((NLet xSend), ESend xE,
34    (NLet xRecv))
35  "
36
1
2
3   inductive static_live_traceable :: "abstract_env  $\Rightarrow$  flow_set
    $\Rightarrow$  node_map  $\Rightarrow$  node_map  $\Rightarrow$  node_label  $\Rightarrow$  (node_label  $\Rightarrow$ 
   bool)  $\Rightarrow$  abstract_path  $\Rightarrow$  bool" where
4   Empty:
5     isEnd start  $\Rightarrow$ 
6     static_live_traceable V F Ln Lx start isEnd [] |
7   Edge:
8     isEnd end  $\Rightarrow$ 
9     static_live_traversable F Ln Lx (start, edge, end)  $\Rightarrow$ 
10    static_live_traceable V F Ln Lx start isEnd [(start,

```

```

    edge)] |
11 Step:
12   static_live_traceable V F Ln Lx middle isEnd ((middle,
    edge') # path) ==>
13   static_live_traversable F Ln Lx (start, edge, middle) ==>
14   static_live_traceable V F Ln Lx start isEnd ((start,
    edge) # (middle, edge') # path) |
15 Pre_Return:
16   static_live_traceable V F Ln Lx (NRslt y) isEnd ((NRslt
    y, ERtn x) # post) ==>
17   static_traceable F (NRslt y) pre ==>
18   /static_balanced (pre @ [(NRslt y, ERtn x)]) ==>
19   /Set.is_empty (Lx (NLet x)) ==>
20   path = pre @ (NRslt y, ERtn x) # post ==>
21   static_live_traceable V F Ln Lx start isEnd path
22
23
1
2
3 inductive static_inclusive ::
4   abstract_path => abstract_path => bool where
5 Prefix1:
6   prefix pi1 pi2 ==>
7   pi1 static_inclusive pi2 |
8 Prefix2:
9   prefix pi2 pi1 ==>
10  pi1 static_inclusive pi2 |
11 Spawn1:
12  static_inclusive (pi @ (NLet x, ESpwn) # pi1) (pi @ (
    NLet x, ENxt) # pi2) |
13 Spawn2:
14  static_inclusive (pi @ (NLet x, ENxt) # pi1
    static_inclusive) (pi @ (NLet x, ESpwn) # pi2) |
15 Send1:
16  static_inclusive (pi @ (NLet x, ESend xE) # pi1) (pi @ (
    NLet x, ENxt) # pi2) |
17 Send2:
18  static_inclusive (pi @ (NLet x, ENxt) # pi1) (pi @ (NLet
    x, ESend xE) # pi2)
19
20
1
2
3
4 inductive singular ::
5   abstract_path => abstract_path => bool where
6   equal:
7     pi1 = pi2 ==>

```

```

8     singular pi1 pi2 |
9     exclusive:
10    /(pi1 static_inclusive pi2) ==>
11    singular pi1 pi2
12
13    inductive noncompetitive ::
14    abstract_path => abstract_path => bool" where
15    ordered:
16    ordered pi1 pi2 ==>
17    noncompetitive pi1 pi2 |
18    exclusive:
19    /(pi1 static_inclusive pi2) ==>
20    noncompetitive pi1 pi2
21
22 inductive static_one_shot :: abstract_env => exp => var =>
    bool where
23   Sync:
24   every_two (static_live_traceable V F Ln Lx (NLet xC) (
    static_send_node_label V e xC)) singular ==>
25   static_live_chan V Ln Lx xC e ==>
26   static_traversable V F (static_recv_node_label V e) e ==>
27   static_one_shot V e xC
28
29 inductive static_one_to_one :: abstract_env => exp => var =>
    bool where
30   Sync:
31   every_two (static_live_traceable V F Ln Lx (NLet xC) (
    static_send_node_label V e xC)) noncompetitive ==>
32   every_two (static_live_traceable V F Ln Lx (NLet xC) (
    static_recv_node_label V e xC)) noncompetitive ==>
33   static_live_chan V Ln Lx xC e ==>
34   static_traversable V F (static_recv_node_label V e) e ==>
35   static_one_to_one V e xC
36
37 inductive static_fan_out :: abstract_env => exp => var =>
    bool where
38   Sync:
39   every_two (static_live_traceable V F Ln Lx (NLet xC) (
    static_send_node_label V e xC)) noncompetitive ==>
40   static_live_chan V Ln Lx xC e ==>
41   static_traversable V F (static_recv_node_label V e) e ==>
42   static_fan_out V e xC
43
44 inductive static_fan_in :: abstract_env => exp => var => bool
    where
45   Sync:
46   every_two (static_live_traceable V F Ln Lx (NLet xC) (
    static_recv_node_label V e xC)) noncompetitive ==>
47   static_live_chan V Ln Lx xC e ==>
48   static_traversable V F (static_recv_node_label V e) e ==>

```



```

49     static_fan_in V e xC
50
51
1  locale communication_sound_B =
2    Static_Communication.communication_sound static_one_shot
    static_fan_out static_fan_in static_one_to_one
3
1
2
1
1 interpretation communication_sound_B
2 proof sketch
3 qed
4
1
1
1
1
2
1
2
3 let lp = fun lp x =>
4   let z1 = case x of
5     L y => let z2 = lp y in z2 |
6     R () => let z3 = () in z3
7   in ()
8   in
9
10 let mksr = fun _ x =>
11   let ch1 = mkChan () in
12   let z4 = (lp (L (L (R ()))) in
13   let srv = fun srv x =>
14     let p = sync (recv_evt ch1) in
15     let v1 = fst p in
16     let ch2 = snd p in
17     let z5 = sync (send_evt ch2 x) in
18     let z6 = srv v1 in ()
19   in
20   let z7 = spawn (
21     let z8 = srv (R ()) in ()) in

```

```

22     ch1 in
23
24   let rqst = fun _ pair =>
25     let ch3 = fst pair in
26     let v2 = snd pair in
27     let ch4 = chan () in
28     let z9 = sync (send_evt ch3 (v2, ch4)) in
29     let v3 = sync (recv_evt ch4) in
30     v3 in
31
32   let srvr = mksr () in
33   let z10 = spawn (
34     let z11 = rqst (srvr, R ()) in ())
35     in
36   let z12 = rqst (srvr, L (R ())) in
37   ()
38
39
1
1

```