

Formal Theory of Communication Topology in Concurrent ML

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1 Mathematical Artifacts

$$f(x) = x^2$$

```

1 type thread_id
2 val spawn: (unit -> unit) -> thread_id
3
4 type 'a chan
5 val channel : unit -> 'a chan
6 val recv : 'a chan -> 'a
7 val send : ('a chan * 'a) -> unit
8
9
10
11
12 signature SERV = sig
13     type serv
14     val make : unit -> serv
15     val call : serv * int -> int
16 end
17
18 structure Serv : SERV = struct
19     datatype serv = S of (int * int chan) chan
20
21     fun make () = let
22         val reqCh = channel ()
23         fun loop state = let
24             val (v, replCh) = recv reqCh in
25             send (replCh, state);
26             loop v end in
27         spawn (fn () => loop 0);
28         S reqCh end
29
30     fun call (server, v) = let
31         val S reqCh = server
32         val replCh = channel () in
33         send (reqCh, (v, replCh));
34         recv replCh end end
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11   val thenEvt: 'a event * ('a -> 'b event) -> 'b event
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4      val z = ref (4 * 73)
5      val x = 4 in
6      z := 1;
7      x * !z end
8
1
2      let
3      val f = fn x => x 1
4      val g = fn y => y + 2
5      val h = fn z => z + 3 in
6      (f g) + (f h) end
7
8
1
2      datatype 'a list = Nil | Cons 'a ('a list)
3
4      inductive sorted ::
5      ('a => 'a => bool) =>
6      'a list => bool where
7      Nil : sorted P Nil |
8      Single : sorted P (Cons x Nil) |
9      Cons :
10      P x y =>
11      sorted P (Cons y ys) =>
12      sorted P (Cons x (Cons y ys))
13
1
2      datatype nat = Z | S nat
3
4      inductive lte :: nat => nat => bool where
5      Eq : lte n n |
6      Lt : lte n1 n2 => lte n1 (S n2)
7
8      theorem "
9      sorted lte
10      (Cons (Z) (Cons (S Z)
11      (Cons (S Z) (Cons
12      (S (S (S Z))) Nil))))"
13      apply (rule Cons)
14      apply (rule Lt)
15      apply (rule Eq)
16      apply (rule Cons)
17      apply (rule Eq)
18      apply (rule Cons)
19      apply (rule Lt)
20      apply (rule Lt)
21      apply (rule Eq)
22      apply (rule Single)

```

```

22     done
23

```

```

1
2  definition True :: bool where
3      True  $\equiv ((\lambda x::\text{bool}. x) = (\lambda x. x))$ 
4
5  definition False :: bool where
6      False  $\equiv (\forall P. P)$ 
7
8

```

```

1
2  signature CHAN = sig
3      type 'a chan
4      val channel: unit -> 'a chan
5      val send: 'a chan * 'a -> unit
6      val recv: 'a chan -> 'a
7  end
8

```

```

1
2  structure ManyToManyChan : CHAN = struct
3      type message_queue = 'a option ref queue
4
5      datatype 'a chan_content =
6          Send of (condition * 'a) queue |
7          Recv of (condition * 'a option ref) queue |
8          Inac
9
10     datatype 'a chan =
11         Ch of 'a chan_content ref * mutex_lock
12
13     fun channel () = Ch (ref Inac, mutexLock ())
14
15     fun send (Ch (conRef, lock)) m =
16         acquire lock;
17         (case !conRef of
18             Recv q => let
19                 val (recvCond, mopRef) = dequeue q in
20                 mopRef := Some m;
21                 if (isEmpty q) then conRef := Inac else ();
22                 release lock; signal recvCond; () end |
23             Send q => let
24                 val sendCond = condition () in
25                 enqueue (q, (sendCond, m));
26                 release lock; wait sendCond; () end |
27             Inac => let
28                 val sendCond = condition () in
29                 conRef := Send (queue [(sendCond, m)]);

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30         release lock; wait sendCond; () end)
31
32 fun recv (Ch (conRef, lock)) =
33     acquire lock;
34     (case !conRef of
35         Send q => let
36             val (sendCond, m) = dequeue q in
37             if (isEmpty q) then
38                 conRef := Inac
39             else
40                 ();
41             release lock; signal sendCond; m end |
42         Recv q => let
43             val recvCond = condition ()
44             val mopRef = ref None in
45             enqueue (q, (recvCond, mopRef));
46             release lock; wait recvCond;
47             valOf (!mopRef) end |
48         Inac => let
49             val recvCond = condition ()
50             val mopRef = ref None in
51             conRef := Recv (queue [(recvCond, mopRef)]);
52             release lock; wait recvCond;
53             valOf (!mopRef) end)
54
55 end
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22         acquire lock;
23         (let
24             val (recvCond, mopRef) = dequeue q in
25             mopRef := Some m;
26             if (isEmpty q) then conRef := Inac else ();
27             release lock; signal (recvCond);
28             () end) |
29         Send _ => raise NeverHappens end
30
31     fun recv (Ch (conRef, lock)) =
32         acquire lock;
33         (case !conRef of
34             Inac => let
35                 val recvCond = condition ()
36                 val mopRef = ref None in
37                 conRef := Recv (queue [(recvCond, mopRef)]);
38                 release lock; wait recvCond;
39                 valOf (!mopRef) end |
40             Recv q => let
41                 val recvCond = condition ()
42                 val mopRef = ref None in
43                 enqueue (q, (recvCond, mopRef));
44                 release lock; wait recvCond;
45                 valOf (!mopRef) end |
46             Send (sendCond, m) =>
47                 conRef := Inac;
48                 release lock;
49                 signal sendCond;
50                 m end)
51
52     end
53
54
55 1     structure FanInChan : CHAN = struct
56 2
57 3     datatype 'a chan_content =
58 4         Send of (condition * 'a) queue |
59 5         Recv of condition * 'a option ref |
60 6         Inac
61 7
62 8     datatype 'a chan =
63 9         Ch of 'a chan_content ref * mutex_lock
64 10
65 11     fun channel () = Ch (ref Inac, mutexLock ())
66 12
67 13     fun send (Ch (conRef, lock)) m =
68 14         acquire lock;
69 15         case !conRef of
70 16             Recv (recvCond, mopRef) =>
71 17                 mopRef := Some m; conRef := Inac;

```

```

18         release lock; signal recvCond;
19         () |
20     Send q => let
21         val sendCond = condition () in
22         enqueue (q, (sendCond, m));
23         release lock; wait sendCond;
24         () end |
25     Inac => let
26         val sendCond = condition () in
27         conRef := Send (queue [(sendCond, m)])
28         release lock; wait sendCond; () end
29
30 fun recv (Ch (conRef, lock)) = let
31     val recvCond = condition ()
32     val mopRef = ref None in
33     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
34         Inac => (* conRef already set *)
35             wait recvCond; valOf (!mopRef) |
36         Send q =>
37             (* the current thread is the only one
38              * that updates the state from this state *)
39             acquire lock;
40             (let
41                 val (sendCond, m) = dequeue q in
42                 if (isEmpty q) then conRef := Inac else ();
43                 release lock; signal sendCond; m end) |
44         Recv _ => raise NeverHappens end end
45
46

```

```

1
2 structure OneToOneChan : CHAN = struct
3
4     datatype 'a chan_content =
5         Send of condition * 'a |
6         Recv of condition * 'a option ref |
7         Inac
8
9     datatype 'a chan = Ch of 'a chan_content ref
10
11 fun channel () = Ch (ref Inac)
12
13 fun send (Ch conRef) m = let
14     val sendCond = condition () in
15     case cas (conRef, Inac, Send (sendCond, m)) of
16         Inac =>
17             (* conRef already set to Send *)
18             wait sendCond; () |
19         Recv (recvCond, mopRef) =>
20             (* the current thread is the only one

```



```

21      /* that accesses conRef for this state */
22      mopRef := Some m; conRef := Inac;
23      signal recvCond; () |
24      Send _ => raise NeverHappens end end
25
26
27 fun recv (Ch conRef) = let
28   val recvCond = condition ();
29   val mopRef = ref None in
30   case cas (conRef, Inac, Recv (recvCond, mopRef)) of
31     Inac => (* conRef already set to Recv*)
32     wait recvCond; valOf (!mopRef) |
33     Send (sendCond, m) =>
34     (* the current thread is the only one
35      /* that accesses conRef for this state */
36      conRef := Inac; signal sendCond; m |
37     Recv _ => raise NeverHappens end end
38
39 end
40
41
42 1 structure OneShotChan : CHAN = struct
43 2
44 3 datatype 'a chan_content =
45 4   Send of condition * 'a |
46 5   Recv of condition * 'a option ref |
47 6   Inac
48 7
49 8 datatype 'a chan = Ch of 'a chan_content ref *
50 mutex_lock
51 9
52 10 fun channel () = Ch (ref Inac, lock ())
53 11
54 12 fun send (Ch (conRef, lock)) m = let
55 13   val sendCond = condition () in
56 14   case (conRef, Inac, Send (sendCond, m)) of
57 15     Inac =>
58 16     (* conRef already set to Send*)
59 17     wait sendCond; () |
60 18     Recv (recvCond, mopRef) =>
61 19     mopRef := Some m; signal recvCond;
62 20     () |
63 21     Send _ => raise NeverHappens end end
64 22
65 23
66 24 fun recv (Ch (conRef, lock)) = let
67 25   val recvCond = condition ()
68 26   val mopRef = ref None in
69 27   case (conRef, Inac, Recv (recvCond, mopRef)) of
70 28     Inac =>

```

```

29      (* conRef already set to Recv*)
30      wait recvCond; valOf (!mopRef) |
31      Send (sendCond, m) =>
32          acquire lock; signal sendCond;
33          (* never releases lock;
34          -* blocks others forever *)
35          m |
36      Recv _ =>
37          acquire lock;
38          (* never able to acquire lock;
39          -* blocked forever *)
40          raise NeverHappens end end
41
42 end
43
1  structure OneShotToOneChan : CHAN = struct
2
3      datatype 'a chan =
4          Ch of condition * condition * 'a option ref
5
6      fun channel () =
7          Ch (condition (), condition (), ref None)
8
9      fun send (Ch (sendCond, recvCond, mopRef)) m =
10          mopRef := Some m; signal recvCond;
11          wait sendCond; ()
12
13      fun recv (Ch (sendCond, recvCond, mopRef)) =
14          wait recvCond; signal sendCond;
15          valOf (!mopRef)
16
17      end
18
1
2      datatype var = Var string
3
4      datatype exp =
5          Let var boundexp exp |
6          Result var
7
8      boundexp =
9          Unit |
10         Chan |
11         Prim prim |
12         Spawn exp |
13         Sync var |
14         Fst var |
15         Snd var |

```

```

16     Case var var exp var exp |
17     App var var and
18
19   prim =
20     SendEvt var var |
21     RecvEvt var |
22     Pair var var |
23     Left var |
24     Right var |
25     Abs var var ex
26
27
1   datatype ctrl_label =
2     LNext var | LSpawn var | LCall var | LReturn var
3
4   type_synonym ctrl_path = (ctrl_label list)
5
6   datatype chan = Ch ctrl_path var
7
8   datatype val =
9     VUnit | VChan chan | VClsr prim (var  $\rightarrow$  val)
10
11   datatype ctn = Ctn var exp (var  $\rightarrow$  val)
12
13   datatype state = State exp (var  $\rightarrow$  val) (ctn list)
14
15
1
2   inductive seq_step ::
3     bind * (var  $\rightarrow$  val))  $\Rightarrow$  val  $\Rightarrow$  bool where
4     Let_Unit:
5       seq_step (Unit, env) VUnit |
6     Let_Prim:
7       seq_step (Prim p, env) (VClsr p env) |
8     Let_Fst:
9       env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
10       envp x1 = Some v  $\Rightarrow$ 
11       seq_step (FST xp, env) v |
12     Let_Snd:
13       env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
14       envp x2 = Some v  $\Rightarrow$ 
15       seq_step (SND xp, env) v
16
17   inductive seq_step_up ::
18     bind * (var  $\rightarrow$  val))  $\Rightarrow$  exp * val_env  $\Rightarrow$  bool where
19     Let_Case_Left:
20       env xs = Some (VClsr (Left x1') envl)  $\Rightarrow$ 
21       envl x1' = Some vl  $\Rightarrow$ 

```

```

22     seq_step_up
23     (Case xs xl el xr er, env)
24     (el, env(xl ↦ vl)) |
25   Let_Case_Right:
26     env xs = Some (VClsr (Right xr') envr) ⇒
27     envr xr' = Some vr ⇒
28     seq_step_up
29     (Case xs xl el xr er, env)
30     (er, env(xr ↦ vr)) |
31   Let_App:
32     env f = Some (VClsr (Abs fp xp el) envl) ⇒
33     env xa = Some va ⇒
34     seq_step_up
35     (App f xa, env)
36     (el, envl(
37       fl ↦ (VClsr (Abs fp xp el) envl),
38       xl ↦ va))
39
40
1   type_synonym trace_pool = "control_path → state"
2
3   inductive leaf ::
4     trace_pool ⇒ control_path ⇒ bool where
5     Intro:
6       trpl pi ≠ None ⇒
7       (∄ pi' . trpl pi' ≠ None ∧ strict_prefix pi pi') ⇒
8       leaf trpl pi
9
10

```