

Formal Theory of Communication Topology in Concurrent ML

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1 Mathematical Artifacts

$$f(x) = x^2$$

```

1  type thread_id
2  val spawn: (unit -> unit) -> thread_id
3
4  type 'a chan
5  val channel : unit -> 'a chan
6  val recv : 'a chan -> 'a
7  val send : ('a chan * 'a) -> unit
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11   val thenEvt: 'a event * ('a -> 'b event) -> 'b event
12
13
1   val server = Serv.make ()
2   val _ = spawn (fn () => Serv.call (server, 35))
3   val _ = spawn (fn () =>
4       Serv.call (server, 12);
5       Serv.call (server, 13))
6   val _ = spawn (fn () => Serv.call (server, 81))
7   val _ = spawn (fn () => Serv.call (server, 44))
8
1   structure Serv : SERV = struct
2     datatype serv = S of (int * int chan) chan
3
4     fun make () = let
5
6       val reqChn = FanIn.channel()
7
8       fun loop state = let
9         val (v, replCh) = FanIn.recv reqChn in
10        OneShot.send (replCh, state);
11        loop v end in
12
13      spawn (fn () => loop 0);
14      S reqChn end
15
16    fun call (server, v) = let
17      val S reqChn = server
18      val replChn = OneShot.channel () in
19      FanIn.send (reqCh, (v, replCh));
20      OneShot.recv replChn end
21
22    end
23
1   let
2     val w = 4
3     val x = ref 1
4     val y = ref 2
5     val z = (!x + 1) + (!y + 2) + (w - 3)
6     val w = 1 in
7     y := 0;
8     (!y + 2) - (!x + 1) * (w - 3) end
9
1   let
2     val x = 1
3     val y = 2

```

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4      val z = ref (4 * 73)
5      val x = 4 in
6      z := 1;
7      x * !z end
8
1
2      let
3          val f = fn x => x 1
4          val g = fn y => y + 2
5          val h = fn z => z + 3 in
6          (f g) + (f h) end
7
8
1
2      datatype 'a list = Nil | Cons 'a ('a list)
3
4      inductive sorted ::
5          ('a => 'a => bool) =>
6          'a list => bool where
7          Nil : sorted P Nil |
8          Single : sorted P (Cons x Nil) |
9          Cons :
10             P x y =>
11             sorted P (Cons y ys) =>
12             sorted P (Cons x (Cons y ys))
13
1
2      datatype nat = Z | S nat
3
4      inductive lte :: nat => nat => bool where
5          Eq : lte n n |
6          Lt : lte n1 n2 => lte n1 (S n2)
7
8      theorem "
9          sorted lte
10             (Cons (Z) (Cons (S Z)
11                 (Cons (S Z) (Cons
12                     (S (S (S Z))) Nil))))"
13      apply (rule Cons)
14      apply (rule Lt)
15      apply (rule Eq)
16      apply (rule Cons)
17      apply (rule Eq)
18      apply (rule Cons)
19      apply (rule Lt)
20      apply (rule Lt)
21      apply (rule Eq)
22      apply (rule Single)

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```

22     done
23

```

```

1
2  definition True :: bool where
3      True  $\equiv$  (( $\lambda x :: \text{bool}.$  x) = ( $\lambda x.$  x))
4
5  definition False :: bool where
6      False  $\equiv$  ( $\forall P.$  P)
7
8

```

```

1
2  signature CHAN = sig
3      type 'a chan
4      val channel: unit -> 'a chan
5      val send: 'a chan * 'a -> unit
6      val recv: 'a chan -> 'a
7  end
8

```

```

1
2  structure ManyToManyChan : CHAN = struct
3      type message_queue = 'a option ref queue
4
5      datatype 'a chan_content =
6          Send of (condition * 'a) queue |
7          Recv of (condition * 'a option ref) queue |
8          Inac
9
10     datatype 'a chan =
11         Chn of 'a chan_content ref * mutex_lock
12
13     fun channel () = Chn (ref Inac, mutexLock ())
14
15     fun send (Chn (conRef, lock)) m =
16         acquire lock;
17         (case !conRef of
18             Recv q => let
19                 val (recvCond, mopRef) = dequeue q in
20                 mopRef := Some m;
21                 if (isEmpty q) then conRef := Inac else ();
22                 release lock; signal recvCond; () end |
23             Send q => let
24                 val sendCond = condition () in
25                 enqueue (q, (sendCond, m));
26                 release lock; wait sendCond; () end |
27             Inac => let
28                 val sendCond = condition () in
29                 conRef := Send (queue [(sendCond, m)]);

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30         release lock; wait sendCond; () end)
31
32 fun recv (Chn (conRef, lock)) =
33     acquire lock;
34     (case !conRef of
35         Send q => let
36             val (sendCond, m) = dequeue q in
37             if (isEmpty q) then
38                 conRef := Inac
39             else
40                 ();
41             release lock; signal sendCond; m end |
42         Recv q => let
43             val recvCond = condition ()
44             val mopRef = ref None in
45             enqueue (q, (recvCond, mopRef));
46             release lock; wait recvCond;
47             valOf (!mopRef) end |
48         Inac => let
49             val recvCond = condition ()
50             val mopRef = ref None in
51             conRef := Recv (queue [(recvCond, mopRef)]);
52             release lock; wait recvCond;
53             valOf (!mopRef) end)
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55 end
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22         acquire lock;
23         (let
24             val (recvCond, mopRef) = dequeue q in
25             mopRef := Some m;
26             if (isEmpty q) then conRef := Inac else ();
27             release lock; signal (recvCond);
28             () end) |
29         Send _ => raise NeverHappens end
30
31     fun recv (Chn (conRef, lock)) =
32         acquire lock;
33         (case !conRef of
34             Inac => let
35                 val recvCond = condition ()
36                 val mopRef = ref None in
37                 conRef := Recv (queue [(recvCond, mopRef)]);
38                 release lock; wait recvCond;
39                 valOf (!mopRef) end |
40             Recv q => let
41                 val recvCond = condition ()
42                 val mopRef = ref None in
43                 enqueue (q, (recvCond, mopRef));
44                 release lock; wait recvCond;
45                 valOf (!mopRef) end |
46             Send (sendCond, m) =>
47                 conRef := Inac;
48                 release lock;
49                 signal sendCond;
50                 m end)
51
52     end
53
54
55 1     structure FanInChan : CHAN = struct
56 2
57 3     datatype 'a chan_content =
58 4         Send of (condition * 'a) queue |
59 5         Recv of condition * 'a option ref |
60 6         Inac
61 7
62 8     datatype 'a chan =
63 9         Chn of 'a chan_content ref * mutex_lock
64 10
65 11     fun channel () = Chn (ref Inac, mutexLock ())
66 12
67 13     fun send (Chn (conRef, lock)) m =
68 14         acquire lock;
69 15         case !conRef of
70 16             Recv (recvCond, mopRef) =>
71 17                 mopRef := Some m; conRef := Inac;

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```

18         release lock; signal recvCond;
19         () |
20     Send q => let
21         val sendCond = condition () in
22         enqueue (q, (sendCond, m));
23         release lock; wait sendCond;
24         () end |
25     Inac => let
26         val sendCond = condition () in
27         conRef := Send (queue [(sendCond, m)])
28         release lock; wait sendCond; () end
29
30 fun recv (Chn (conRef, lock)) = let
31     val recvCond = condition ()
32     val mopRef = ref None in
33     case cas (conRef, Inac, Recv (recvCond, mopRef)) of
34         Inac => (* conRef already set *)
35             wait recvCond; valOf (!mopRef) |
36         Send q =>
37             (* the current thread is the only one
38              * that updates the state from this state *)
39             acquire lock;
40             (let
41                 val (sendCond, m) = dequeue q in
42                 if (isEmpty q) then conRef := Inac else ();
43                 release lock; signal sendCond; m end) |
44         Recv _ => raise NeverHappens end end
45
46

```

```

1
2 structure OneToOneChan : CHAN = struct
3
4     datatype 'a chan_content =
5         Send of condition * 'a |
6         Recv of condition * 'a option ref |
7         Inac
8
9     datatype 'a chan = Chn of 'a chan_content ref
10
11 fun channel () = Chn (ref Inac)
12
13 fun send (Chn conRef) m = let
14     val sendCond = condition () in
15     case cas (conRef, Inac, Send (sendCond, m)) of
16         Inac =>
17             (* conRef already set to Send *)
18             wait sendCond; () |
19         Recv (recvCond, mopRef) =>
20             (* the current thread is the only one

```



```

21      -* that accesses conRef for this state *
22      mopRef := Some m; conRef := Inac;
23      signal recvCond; () |
24      Send _ => raise NeverHappens end end
25
26
27 fun recv (Chn conRef) = let
28   val recvCond = condition ();
29   val mopRef = ref None in
30   case cas (conRef, Inac, Recv (recvCond, mopRef)) of
31     Inac => (* conRef already set to Recv*)
32     wait recvCond; valOf (!mopRef) |
33     Send (sendCond, m) =>
34       (* the current thread is the only one
35        -* that accesses conRef for this state *)
36       conRef := Inac; signal sendCond; m |
37     Recv _ => raise NeverHappens end end
38
39 end
40
41
42 1 structure OneShotChan : CHAN = struct
43 2
44 3 datatype 'a chan_content =
45 4   Send of condition * 'a |
46 5   Recv of condition * 'a option ref |
47 6   Inac
48 7
49 8 datatype 'a chan = Chn of 'a chan_content ref *
50 mutex_lock
51 9
52 10 fun channel () = Chn (ref Inac, lock ())
53 11
54 12 fun send (Chn (conRef, lock)) m = let
55 13   val sendCond = condition () in
56 14   case (conRef, Inac, Send (sendCond, m)) of
57 15     Inac =>
58 16       (* conRef already set to Send*)
59 17       wait sendCond; () |
60 18     Recv (recvCond, mopRef) =>
61 19       mopRef := Some m; signal recvCond;
62 20       () |
63 21     Send _ => raise NeverHappens end end
64 22
65 23
66 24 fun recv (Chn (conRef, lock)) = let
67 25   val recvCond = condition ()
68 26   val mopRef = ref None in
69 27   case (conRef, Inac, Recv (recvCond, mopRef)) of
70 28     Inac =>

```

```

29         (* conRef already set to Recv*)
30         wait recvCond; valOf (!mopRef) |
31     Send (sendCond, m) =>
32         acquire lock; signal sendCond;
33         (* never releases lock;
34         -* blocks others forever *)
35         m |
36     Recv _ =>
37         acquire lock;
38         (* never able to acquire lock;
39         -* blocked forever *)
40         raise NeverHappens end end
41
42 end
43
1  structure OneShotToOneChan : CHAN = struct
2
3      datatype 'a chan =
4          Chn of condition * condition * 'a option ref
5
6      fun channel () =
7          Chn (condition (), condition (), ref None)
8
9      fun send (Chn (sendCond, recvCond, mopRef)) m =
10          mopRef := Some m; signal recvCond;
11          wait sendCond; ()
12
13      fun recv (Chn (sendCond, recvCond, mopRef)) =
14          wait recvCond; signal sendCond;
15          valOf (!mopRef)
16
17      end
18

```

2 Syntax

```

1
2      datatype var = Var string
3
4      datatype exp =
5          Let var boundexp exp |
6          Rslt var
7
8      boundexp =
9          Unt |
10         MkChn |
11         Prim prim |

```

```

12     Spwn exp |
13     Sync var |
14     Fst var |
15     Snd var |
16     Case var var exp var exp |
17     App var var and
18
19   prim =
20     SendEvt var var |
21     RecvEvt var |
22     Pair var var |
23     Lft var |
24     Rht var |
25     Abs var var ex
26
27

```

3 Dynamic Semantics

```

1   datatype ctrl_label =
2     LNxt var | LSpwn var | LCall var | LRtn var
3
4   type_synonym ctrl_path = (ctrl_label list)
5
6   datatype chan = Chn ctrl_path var
7
8   datatype val =
9     VUnt | VChn chan | VClsr prim (var  $\rightarrow$  val)
10
11   datatype ctn = Ctn var exp (var  $\rightarrow$  val)
12
13   datatype state = Stt exp (var  $\rightarrow$  val) (ctn list)
14
15
16
17   inductive seq_step ::
18     bind * (var  $\rightarrow$  val))  $\Rightarrow$  val  $\Rightarrow$  bool where
19     LetUnt:
20       seq_step (Unt, env) VUnt |
21     LetPrim:
22       seq_step (Prim p, env) (VClsr p env) |
23     LetFst:
24       env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
25       envp x1 = Some v  $\Rightarrow$ 
26       seq_step (Fst xp, env) v |
27     LetSnd:
28       env xp = Some (VClsr (Pair x1 x2) envp)  $\Rightarrow$ 
29       envp x2 = Some v  $\Rightarrow$ 

```

```

15     seq_step (Snd xp, env) v
16
17 inductive seq_step_up ::
18   bind * (var  $\rightarrow$  val)  $\Rightarrow$  exp * val_env  $\Rightarrow$  bool where
19   LetCaseLft:
20     env xs = Some (VClsr (Lft xl') envl)  $\Rightarrow$ 
21     envl xl' = Some vl  $\Rightarrow$ 
22     seq_step_up
23       (Case xs xl el xr er, env)
24       (el, env(xl  $\mapsto$  vl)) |
25   LetCaseRht:
26     env xs = Some (VClsr (Rht xr') envr)  $\Rightarrow$ 
27     envr xr' = Some vr  $\Rightarrow$ 
28     seq_step_up
29       (Case xs xl el xr er, env)
30       (er, env(xr  $\mapsto$  vr)) |
31   LetApp:
32     env f = Some (VClsr (Abs fp xp el) envl)  $\Rightarrow$ 
33     env xa = Some va  $\Rightarrow$ 
34     seq_step_up
35       (App f xa, env)
36       (el, envl(
37         fp  $\mapsto$  (VClsr (Abs fp xp el) envl),
38         xp  $\mapsto$  va))
39
40
41 type_synonym cmmn_set = (ctrl_path * chan * ctrl_path)
42 set
43
44 type_synonym trace_pool = ctrl_path  $\rightarrow$  state
45
46 inductive leaf ::
47   trace_pool  $\Rightarrow$  ctrl_path  $\Rightarrow$  bool where
48   intro:
49     trpl pi  $\neq$  None  $\Rightarrow$ 
50     ( $\nexists$  pi' . trpl pi'  $\neq$  None  $\wedge$  strict_prefix pi pi')  $\Rightarrow$ 
51     leaf trpl pi
52
53 inductive concur_step ::
54   trace_pool * cmmn_set  $\Rightarrow$ 
55   trace_pool * cmmn_set  $\Rightarrow$ 
56   bool where
57   Seq_Sttep_Down:
58     leaf trpl pi  $\Rightarrow$ 
59     trpl pi = Some
60       (Stt (Rslt x) env
61         ((Ctn xk ek envk) # k))  $\Rightarrow$ 
62     env x = Some v  $\Rightarrow$ 
63     concur_step
64       (trpl, ys)

```

```

64         (trpl(pi @ [LRtn xk] ↦
65           (Stt ek (envk(xk ↦ v)) k)), ys) |
66 Seq_Step:
67   leaf trpl pi ⇒
68   trpl pi = Some
69     (Stt (Let x b e) env k) ⇒
70   seq_step (b, env) v ⇒
71   concur_step
72     (trpl, ys)
73     (trpl(pi @ [LNxt x] ↦
74       (Stt e (env(x ↦ v)) k)), ys) |
75 Seq_Step_Up:
76   leaf trpl pi ⇒
77   trpl pi = Some
78     (Stt (Let x b e) env k) ⇒
79   seq_step_up (b, env) (e', env') ⇒
80   concur_step
81     (trpl, ys)
82     (trpl(pi @ [LCall x] ↦
83       (Stt e' env'
84         ((Ctn x e env) # k))), ys) |
85 LetMkCh:
86   leaf trpl pi ⇒
87   trpl pi = Some (Stt (Let x MkChn e) env k) ⇒
88   concur_step
89     (trpl, ys)
90     (trpl(pi @ [LNxt x] ↦
91       (Stt e (env(x ↦ (VChn (Chn pi x)))) k)), ys) |
92 LetSpwn:
93   leaf trpl pi ⇒
94   trpl pi = Some
95     (Stt (Let x (Spwn ec) e) env k) ⇒
96   concur_step
97     (trpl, ys)
98     (trpl(
99       pi @ [LNxt x] ↦
100         (St e (env(x ↦ VUnt)) k),
101       pi @ [LSpwn x] ↦
102         (St ec env []), ys) |
103 LetSync:
104   leaf trpl pis ⇒
105   trpl pis = Some
106     (Stt (Let xs (Sync xse) es) envs ks) ⇒
107   envs xse = Some
108     (VClsr (SendEvt xsc xm) envse) ⇒
109   leaf trpl pir ⇒
110   trpl pir = Some
111     (Stt (Let xr (Sync xre) er) envr kr) ⇒
112   envr xre = Some
113     (VClsr (RecvEvt xrc) envre) ⇒

```

```

114     envse xsc = Some (VChn c) ==>
115     envre xrc = Some (VChn c) ==>
116     envse xm = Some vm ==>
117     concur_step
118       (trpl, ys)
119       (trpl(
120         pis @ [LNxt xs] ↦
121         (Stt es (envs(xs ↦ VUnt)) ks),
122         pir @ [LNxt xr] ↦
123         (Stt er (envr(xr ↦ vm)) kr)),
124       ys ∪ {(pis, c, pir)})
125
126
1
2   inductive star :: ('a ⇒ 'a ⇒ bool) ⇒ 'a ⇒ 'a ⇒ bool
3   for r where
4   refl: star r x x |
5   step: r x y ==> star r y z ==> star r x z

```

4 Dynamic Communication

```

1   inductive is_send_path ::
2   trace_pool ⇒ chan ⇒
3   control_path ⇒ bool where
4   intro:
5     trpl piy = Some
6     (Stt (Let xy (Sync xe) en) env k) ==>
7     env xe = Some
8     (VClSr (SendEvt xsc xm) enve) ==>
9     enve xsc = Some (VChn c) ==>
10    is_send_path trpl c piy
11
12  inductive is_recv_path ::
13  trace_pool ⇒ chan ⇒
14  control_path ⇒ bool where
15  intro:
16    trpl piy = Some
17    (Stt (Let xy (Sync xe) en) env k) ==>
18    env xe = Some
19    (VClSr (RecvEvt xrc) enve) ==>
20    enve xrc = Some (VChn c) ==>
21    is_recv_path trpl c piy
22
23  inductive every_two ::
24  ('a ⇒ bool) ⇒
25  ('a ⇒ 'a ⇒ bool) ⇒
26  bool where
27  intro: (∀ pi1 pi2 .

```

```

28         p x1 →
29         p x2 →
30         r x1 x2) ⇒
31     every_two p r
32
33 inductive ordered :: 'a list ⇒ 'a list ⇒ bool where
34     left: prefix pi1 pi2 ⇒ ordered pi1 pi2 |
35     right: prefix pi2 pi1 ⇒ ordered pi1 pi2
36
37 inductive one_shot :: trace_pool ⇒ chan ⇒ bool where
38     intro:
39         every_two
40         (is_send_path trpl c) op= ⇒
41         one_shot trpl c
42
43 inductive fan_out :: trace_pool ⇒ chan ⇒ bool where
44     intro:
45         every_two
46         (is_send_path trpl c) ordered ⇒
47         fan_out trpl c
48
49 inductive fan_in :: trace_pool ⇒ chan ⇒ bool where
50     intro:
51         every_two
52         (is_recv_path trpl c) ordered ⇒
53         fan_in trpl c
54
55 inductive one_to_one :: trace_pool ⇒ chan ⇒ bool where
56     intro:
57         fan_out trpl c ⇒
58         fan_in trpl c ⇒
59         one_to_one trpl c
60
61

```

5 Static Semantics

```

1
2 datatype abstract_value =
3     AChn var |
4     AUnt |
5     APrim prim
6
7 type_synonym abstract_env = var ⇒ abstract_value set
8
9 fun rslt_var :: exp ⇒ var where
10     rslt_var (Rslt x) = x |
11     rslt_var (Let _ _ e) = (rslt_var e)
12

```

```

13
14 inductive may_be_eval ::
15   abstract_env * abstract_env ⇒
16   exp ⇒ bool where
17   Result:
18     may_be_eval (V, C) (RESULT x) |
19   Let_Unt:
20     {AUnt} ⊆ V x ⇒
21     may_be_eval (V, C) e ⇒
22     may_be_eval (V, C) (Let x Unt e) |
23   Let_Chn:
24     {AChn x} ⊆ V x ⇒
25     may_be_eval (V, C) e ⇒
26     may_be_eval (V, C) (Let x (MkChn) e) |
27   Let_SendEvt :
28     {APrim (SendEvt xc xm)} ⊆ V x ⇒
29     may_be_eval (V, C) e ⇒
30     may_be_eval (V, C) (Let x (Prim (SendEvt xc xm)) e)
31 |
32   Let_RecvEvt :
33     {APrim (RecvEvt xc)} ⊆ V x ⇒
34     may_be_eval (V, C) e ⇒
35     may_be_eval (V, C) (Let x (Prim (RecvEvt xc)) e) |
36   Let_Pair :
37     {APrim (Pair x1 x2)} ⊆ V x ⇒
38     may_be_eval (V, C) e ⇒
39     may_be_eval (V, C) (Let x (Pair x1 x2) e) |
40   Let_Left :
41     {APrim (Left xp)} ⊆ V x ⇒
42     may_be_eval (V, C) e ⇒
43     may_be_eval (V, C) (Let x (Left xp) e) |
44   Let_Right:
45     {APrim (Right xp)} ⊆ V x ⇒
46     may_be_eval (V, C) e ⇒
47     may_be_eval (V, C) (Let x (Right xp) e) |
48   Let_Abs :
49     {APrim (Abs f' x' e')} ⊆ V f' ⇒
50     may_be_eval (V, C) e' ⇒
51     {APrim (Abs f' x' e')} ⊆ V x ⇒
52     may_be_eval (V, C) e ⇒
53     may_be_eval (V, C) (Let x (Abs f' x' e') e) |
54   Let_Spawn:
55     {AUnt} ⊆ V x ⇒
56     may_be_eval (V, C) ec ⇒
57     may_be_eval (V, C) e ⇒
58     may_be_eval (V, C) (Let x (Spwn ec) e) |
59   Let_Sync :
60     ∀ xsc xm xc .
61     (APrim (SendEvt xsc xm)) ∈ V xe →
62     AChn xc ∈ V xsc →

```



```

62      {AUnt}  $\subseteq V\ x \wedge V\ x_m \subseteq C\ x_c \implies$ 
63       $\forall\ xrc\ xc\ .$ 
64      (APrim (RecvEvt xrc))  $\in V\ xe \implies$ 
65      AChn xc  $\in V\ xrc \implies$ 
66      C xc  $\subseteq V\ x \implies$ 
67      may_be_eval (V, C) e  $\implies$ 
68      may_be_eval (V, C) (Let x (Syync xe) e) |
69  Let_Fst:
70       $\forall\ x1\ x2.$ 
71      (APrim (Pair x1 x2))  $\in V\ xp \implies$ 
72      V x1  $\subseteq V\ x \implies$ 
73      may_be_eval (V, C) e  $\implies$ 
74      may_be_eval (V, C) (Let x (Fst xp) e) |
75  Let_Snd:
76       $\forall\ x1\ x2\ .$ 
77      (APrim (Pair x1 x2))  $\in V\ xp \implies$ 
78      V x2  $\subseteq V\ x \implies$ 
79      may_be_eval (V, C) e  $\implies$ 
80      may_be_eval (V, C) (Let x (Snd xp) e) |
81  Let_Case:
82       $\forall\ x1'.$ 
83      (APrim (Left x1'))  $\in V\ xs \implies$ 
84      V x1'  $\subseteq V\ x1 \wedge V\ (rslt\_var\ el) \subseteq V\ x \wedge$ 
85      may_be_eval (V, C) el  $\implies$ 
86       $\forall\ xr'.$ 
87      (APrim (Right xr'))  $\in V\ xs \implies$ 
88      V xr'  $\subseteq V\ xr \wedge V\ (rslt\_var\ er) \subseteq V\ x \wedge$ 
89      may_be_eval (V, C) er  $\implies$ 
90      may_be_eval (V, C) e  $\implies$ 
91      may_be_eval (V, C) (Let x (Case xs x1 el xr er) e) |
92  Let_App:
93       $\forall\ f'\ x'\ e'.$ 
94      (APrim (Abs f' x' e'))  $\in V\ f \implies$ 
95      V xa  $\subseteq V\ x' \wedge$ 
96      V (rslt_var e')  $\subseteq V\ x \implies$ 
97      may_be_eval (V, C) e  $\implies$ 
98      may_be_eval (V, C) (Let x (App f xa) e)
99
100
101  fun abstract :: val  $\Rightarrow$  abstract_value where
102      abstract VUnt = AUnt |
103      abstract VChn (Chn pi x) = AChn x |
104      abstract VClsr p env = APrim p
105
106
107
108  inductive
109      may_be_eval_val ::
110      abstract_env * abstract_env  $\Rightarrow$  val  $\Rightarrow$  bool and
111      may_be_eval_env ::

```

```

112     abstract_env * abstract_env  $\Rightarrow$  val_env  $\Rightarrow$  bool where
113 Unt:
114     may_be_eval_val (V, C) VUnt |
115 Chan:
116     may_be_eval_val (V, C) VChn c |
117 SendEvt:
118     may_be_eval_env (V, C) env  $\Rightarrow$ 
119     may_be_eval_val (V, C) (VClsr (SendEvt _ _) env) |
120 RecvEvt:
121     may_be_eval_env (V, C) env  $\Rightarrow$ 
122     may_be_eval_val (V, C) (VClsr (RecvEvt _ _) env) |
123 Left:
124     may_be_eval_env (V, C) env  $\Rightarrow$ 
125     may_be_eval_val (V, C) (VClsr (Left _ _) env) |
126 Right:
127     may_be_eval_env (V, C) env  $\Rightarrow$ 
128     may_be_eval_val (V, C) (VClsr (Right _ _) env) |
129 Abs:
130     {(APrim (Abs f x e))}  $\subseteq$  V f  $\Rightarrow$ 
131     may_be_eval (V, C) e  $\Rightarrow$ 
132     may_be_eval_env (V, C) env  $\Rightarrow$ 
133     may_be_eval_val (V, C) (VClsr (Abs f x e) env) |
134 Pair:
135     may_be_eval_env (V, C) env  $\Rightarrow$ 
136     may_be_eval_val (V, C) (VClsr (Pair _ _) env) |
137 intro:
138      $\forall x v .$ 
139     env x = Some v  $\rightarrow$ 
140     {abstract v}  $\subseteq$  V x  $\wedge$  may_be_eval_val (V, C) v  $\Rightarrow$ 
141     may_be_eval_env (V, C) env
142
143
144 inductive may_be_eval_stack ::
145     abstract_env * abstract_env  $\Rightarrow$ 
146     abstract_value set  $\Rightarrow$  cont list  $\Rightarrow$  bool where
147 Empty:
148     may_be_eval_stack (V, C) valset [] |
149 Nonempty:
150     valset  $\subseteq$  V x  $\Rightarrow$ 
151     may_be_eval (V, C) e  $\Rightarrow$ 
152     may_be_eval_env (V, C) env  $\Rightarrow$ 
153     may_be_eval_stack (V, C) (V (rslt_var e)) k  $\Rightarrow$ 
154     may_be_eval_stack (V, C) valset ((Ctn x e env) # k)
155
156
157 inductive may_be_eval_state ::
158     abstract_env * abstract_env  $\Rightarrow$ 
159     state  $\Rightarrow$  bool where
160 intro:
161     may_be_eval (V, C) e  $\Rightarrow$ 

```

```

162     may_be_eval_env (V, C) env  $\impl$ 
163     may_be_eval_stack (V, C) (V (rslt_var e)) k  $\impl$ 
164     may_be_eval_state (V, C) (Stt e env k)
165
166 inductive may_be_eval_pool ::
167     abstract_env * abstract_env  $\Rightarrow$ 
168     trace_pool  $\Rightarrow$  bool where
169 intro:
170      $\forall$  pi st .
171     trpl pi = Some st  $\longrightarrow$ 
172     may_be_eval_state (V, C) st  $\impl$ 
173     may_be_eval_pool (V, C) trpl
174
175

```