CS6046: Assignment 1

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1 Question 1

The loss function for perceptron can be written as

$$l(w,(x,y)) = 1_{[yw^Tx> \leq 0]} \tag{1}$$

So we convexify the problem by finding a surrogate convex loss function and we choose hinge loss because its a convex function and upper bounds the perceptron's 0-1 loss function.

$$f_t(w) = max\left\{0, 1 - y_t w^T x_t\right\} \tag{2}$$

Note that when perceptron makes a mistake, hinge loss function is sent by the adversary and when the perceptron is correct, then the function $f_t(w_t) = 0$

In this setting, we perform OGD (online gradient descent) and the below equation is the update rule for OGD, where $z_t \in \partial f_t(w_t)$

$$w_{t+1} = w_t - \eta z_t \tag{3}$$

In this case, when $y_t w^T x_t > 0$ since $f_t = 0$, $z_t = 0$. Otherwise, note that

$$f_t(w_t) = 1 - y_t w^T x_t \implies z_t = -y_t x_t$$

So the update equation becomes,

$$w_{t+1} = \begin{cases} w_t & if\ y_t w_t^T x_t > 0 \\ w_t + \eta x_t y_t & otherwise \end{cases} \tag{4}$$

Lets denote the set $\mathcal{M}$ to be the set of timesteps where the perceptron algorithm made a mistake. We know that at all those rounds we have the update happening.

Using the OGD regret bound equation,

$$\sum_{t=0}^{T} f_t(w_t) - \sum_{t=0}^{T} f_t(u) \leq \frac{1}{2\eta}||u||_2^2 + \sum_{t=0}^{T} \frac{\eta}{2}||z_t||_2^2$$

Using the bound on the surrogate loss function, $\sum_{t=0}^{T} f_t(w_t) \geq |\mathcal{M}|$, and $R = max_t||x_t||$, we get,

$$|\mathcal{M}| - \sum_{t=1}^{T} f_t(u) \leq \frac{1}{2\eta}||u||_2^2 + \frac{\eta}{2}|\mathcal{M}|R^2$$

Setting $\eta = \frac{||u||}{R\sqrt{|\mathcal{M}|}}$ (obtained by differentiating and setting the RHS to 0), we obtain

$$|\mathcal{M}| - R||u||\sqrt{|\mathcal{M}|} - \sum_{t=1}^{T} f_t(u) \leq 0 \tag{5}$$

Now invoke the result from quadratic equations that, the inequality $x - b\sqrt{x} - c \leq 0$ implies that $x \leq c + b^2 + b\sqrt{c}$, we obtain a bound on the mistake set cardinality.

$$|\mathcal{M}| \leq \sum_t f_t(u) + R||u||\sqrt{\sum_t f_t(u)} + R^2||u||^2 \tag{6}$$

Now taking the margin condition into consideration ($y_t u^T x_t \geq 1$ for all t) then,

$$|\mathcal{M}| = y^2$$

From 5 we get,

$$y^2 - R||u||y - \sum_{t=1}^{T} f_t(u) \leq 0$$

Now analysing the roots of the polynomial,

$$\sqrt{|\mathcal{M}|} = \left(R||u|| \pm \sqrt{R^2||u||^2 + 4\sum_{t=1}^{T} f_t(u)} \right) /2$$

Taking the positive root ($y \geq 0$) and using the margin condition,

$$|\mathcal{M}| \leq R^2||u||^2 \tag{7}$$

We can take $||u|| = 1/\gamma$ for our calculation purposes, then the bound becomes R^2/γ^2. Technically, the mistake set $|\mathcal{M}|$ doesnt depend on η until $\eta > 0$. So we can set it to any value, but for the sake of getting an upperbound, we set it to the value as we did previously.

2 Question 2

The adversary is not realizable. This is because, upon checking with each individual experts, the adversary doesn't seem to match any of the experts in the dataset. So it can be decided that its non realizable. From plots, we can observe that RWM algorithm has near linear mistakes. Had the adversary been realizable we would have got better mistake bound in RWM ($O(\sqrt{T})$). But weighted majority seems to perform well when compared to all algorithms and it might be inferred that we have a combination of experts who deliver correct output (adversary is non realizable in this case as well).

3 Question 3

Here we investigate the effect of η on the regret curves and plot for different η. Note that we initialize η with the theoretical value for the experiments increase it by 100x and decrease it by 10x and plot the regret curves. η values chosen are the values which give minimum regret from the graphs shown below.

Regret for the perturbed leader compares similar to that of L2 and entropic regularizer. It is a kind of regularizer which addes random noise to the noise vector obtained. Since this is a randomized algorithm, upon solving the regret bound for the algorithm I expect $O(\sqrt{Td})$ (need to take expectation).

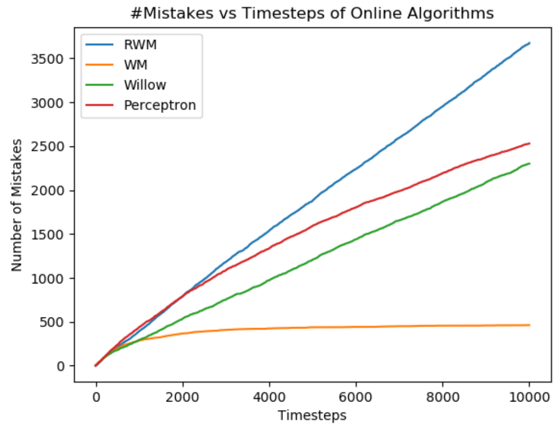

Figure 1: Mistake plots for the given list algorithms. RWM: Randomized weighted majority, WM: Weighted Majority. WM performs the best out of all the available algorithms.

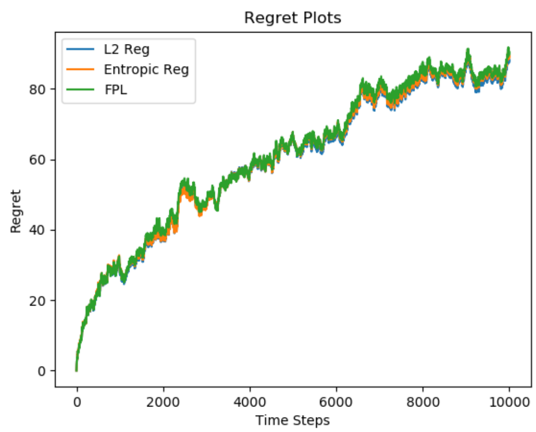

Figure 2: The regret graphs as a function of timestep. The final algorithm is averaged over 100 trails of running the algorithm. We see all the three algorithms perform the same

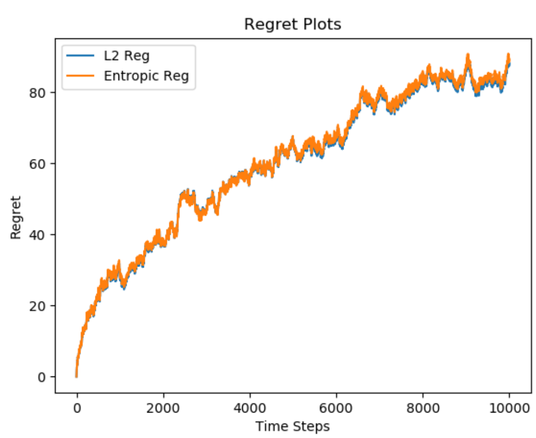

Figure 3: The regret graphs as a function of timestep for algorithms with L2 and Entropic regularizer

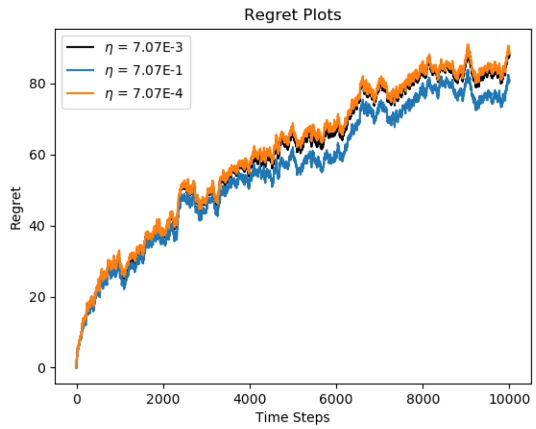

Figure 4: The regret graphs as a function of timestep for L2 regularizer, with different values of η (as indicated in the graph)

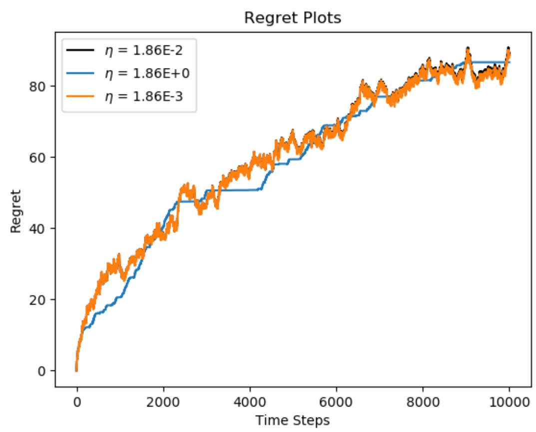

Figure 5: The regret graphs as a function of timestep for Entropic regularizer, with different values of η (as indicated in the graph)

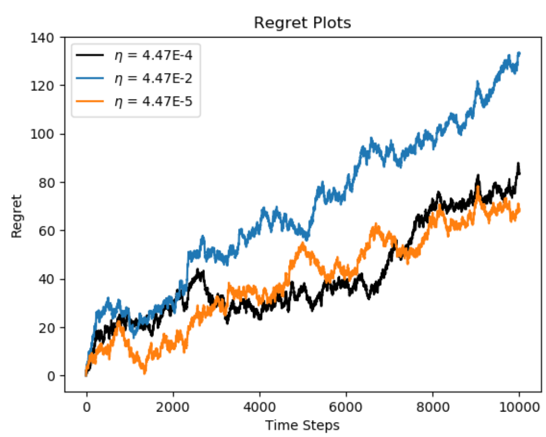

Figure 6: The regret graphs as a function of timestep for Perturbed leader regularizer, with different values of η (as indicated in the graph)