CAP. V

5.1

$$X(n) = \sum_{i=1}^{p} \phi_{p,i} X(n-i) + e(n)$$

5.3 AR(1)

$$X(n) = \frac{1}{2} X(n-i) + e(n)$$

Como $\phi_{1,1} = \frac{1}{2} < 1$

$$E\{X(n)\} = E\left\{\frac{1}{2} X(n-i) + e(n)\right\} = 0$$

Por formula 5.7:

$$V\{X(n)\} = \sigma_X^2 = \frac{\sigma_N^2}{1-\phi_{1,1}^2} = \frac{4}{3}$$

5.4 AR(1), μ_X y σ_X ya las calcule. Por formula 5.9, 5.10 y 5.12 respectivamente:

$$R_{XX}(k) = \phi_{1,1}^k \sigma_X^2 = \frac{2^{-k+2}}{3}$$
$$r_{XX}(k) = \phi_{1,1}^k = 2^{-k}$$
$$S_{XX}(f) = \frac{\sigma_N^2}{1 - 2\phi_{1,1} cos(2\pi f) + \phi_{1,1}^2} = \frac{1}{1 - \frac{8}{3} cos(2\pi f) + \frac{16}{9}}$$

5.13 AR(2), por formula 5.25 y 5.26 respectivamente:

$$\begin{cases} r_{XX}(1)\left(1-\phi_{2,2}\right) = \phi_{2,1} \\ \left[r_{XX}(2) - \phi_{2,2}\right]\left(1-\phi_{2,2}\right) = \phi_{2,1}^2 \end{cases}$$

$\phi_{2,2} = -0.2$ y $\phi_{2,1} = 0.6$

5.21 MA(1)

$$X(n) = 0.8e(n-1) + e(n)$$

Por formulas 5.40, 5.42, 5.43:

$$\mu_X = 0$$
$$\sigma_X^2 = (\theta_{1,1}^2 + 1)\sigma_N^2 = (0.8^2 + 1) = 1.64$$
$$r_{XX}(k) = \delta(k) + \frac{\theta_{1,1}}{1+\theta_{1,1}^2}\delta(k-1) = \delta(k) + \frac{20}{41}\delta(k-1)$$
$$S_{XX}(f) = \sigma_N^2\left[\theta_{1,1}^2 + 2\theta_{1,1} cos(2\pi f) + 1\right] = 1.64 + 1.6 cos(2\pi f) + 1$$
$$\phi_{i,i} = \frac{(-1)^{i-1}\theta_{1,1}^i\left(1-\theta_{1,1}^2\right)}{1-\theta_{1,1}^{2i+2}} = \frac{(-1)^{i-1} 0.8^i \; 0.36}{1-0.8^{2i+2}}$$
$$\phi_{1,1} = \frac{20}{41}$$
$$\phi_{2,2} = -\frac{400}{1281}$$

5.29

$$X(n) = (2k - n)d$$

Es un proceso homogéneo, es decir, no depende del tiempo (ejemplo: tirar una moneda y ver que sale no depende del tiempo), por lo tanto es Markov.

$$p(0) = \begin{bmatrix} \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} \rightarrow P\left[X(0) = 0\right] = 1$$

$$P(1) = \begin{bmatrix} \dots & 0 & 1/2 & 0 & 0 & 0 & \dots & 0 \\ \dots & 1/2 & 0 & 1/2 & 0 & 0 & \dots & 0 \\ \dots & 0 & 1/2 & 0 & 1/2 & 0 & \dots & 0 \\ \dots & 0 & 0 & 1/2 & 0 & 1/2 & \dots & 0 \\ \dots & 0 & 0 & 0 & 1/2 & 0 & \dots & 0 \\ \dots & 0 & 0 & 0 & 0 & 1/2 & \dots & 0 \\ \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & 0 \end{bmatrix}$$

5.31

$$X(n+1) = X(n) - 1$$

$$P[U(n) = 0] = P[U(n) = 1] = P[U(n) = 2] = \frac{1}{3}$$

$X(n)$ es la cantidad de mensajes, mientras que $U(n)$ es la taza de arribos. Condición inicial: $X(0) = 1$ (En el instante 0 hay 1 mensaje), por lo tanto

$$p(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Sabiendo que tengo 1 mensaje en $t = 1$ y $\frac{1}{3}$ de probabilidad de que llegue 0, 1 o 2 mensajes:

$$p(1) = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 0 \\ \vdots \end{bmatrix}$$

De la misma forma:

$$p(1) = \begin{bmatrix} 1/3\ 1/3 + 1/3\ 1/3 + 0 \\ 1/3\ 1/3 + 1/3\ 1/3 + 1/3\ 1/3 \\ 1/3\ 1/3 + 1/3\ 1/3 + 1/3\ 1/3 \\ 1/3\ 1/3 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Es decir, p representa la probabilidad de tener n mensajes en un t dado, con una condicion inicial fijada. Por otro lado, P representa la probabilidad de tener n mensajes en un t dado para todas las condiciones iniciales.

$$P(1) = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \dots \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \dots \\ 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & \dots \\ 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \end{bmatrix}$$

5.37 Ecuación 5.72:

$$\lim_{\epsilon\to 0} P_{k,j}(\epsilon) = \lambda_{k,j}\epsilon \quad k \neq j$$
$$= 1 + \lambda_{k,j}\epsilon \quad k = j$$

$$P_{i,j}(\tau) = \sum_k P_{i,k}(t)P_{k,j}(\tau - t) = \underbrace{\sum_{k\neq j} P_{i,k}(t)\lambda_{kj}(\tau - t) + P_{i,j}\left[1 + \lambda_{kj}(\tau - t)\right]}_{\text{Usando la propiedad de este ejercicio}}$$
$$= \sum_{k\neq j} P_{i,k}(t)\lambda_{kj}(\tau - t) + P_{i,j} + P_{i,j}\lambda_{kj}(\tau - t)$$
$$= \sum_k P_{i,k}(t)\lambda_{k,j}(\tau - t) + P_{i,j}(t)$$

$$P_{i,j}(\tau) = \sum_k P_{i,k}(t)\lambda_{k,j}(\tau - t) + P_{i,j}(t)$$
$$P_{i,j}(\tau) - P_{i,j}(t) = \sum_k P_{i,k}(t)\lambda_{k,j}(\tau - t)$$
$$\frac{P_{i,j}(\tau) - P_{i,j}(t)}{\tau - t} = \sum_k P_{i,k}(t)\lambda_{k,j}$$

5.38

$$p_1(t) = \frac{\mu}{\lambda + \mu} + e^{-(\mu+\lambda)t} \cdot \frac{\lambda p_1(0) - \mu p_2(0)}{\lambda + \mu}$$
$$p_2(t) = 1 - p_1(t)$$

Con $p_1(0) = 0$ y $p_2(0) = 1$

$$p_1(t) = -\frac{\mu}{\lambda + \mu}\left[1 + e^{-(\mu+\lambda)t}\right]$$
$$p_2(t) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu}e^{-(\mu+\lambda)t}$$

$$\lim_{t\to\infty} p_1(t) = -\frac{\mu}{\lambda + \mu} \text{ y } \lim_{t\to\infty} p_2(t) = 1 - \frac{\mu}{\lambda + \mu} = \frac{\lambda}{\lambda + \mu}$$

5.48 Se define $\lambda_a = \frac{5}{6}\frac{jobs}{min}$ y $\lambda_d = 1\frac{jobs}{min}$, con $\rho = \frac{\lambda_a}{\lambda_d}$ y el Service Time $E\{S\} = \frac{1}{\lambda_d}$
a)

$$E\{W\} = \frac{\rho}{1-\rho}E\{S\} = 5\ min$$

b) $\lambda_d = 2\frac{jobs}{min} \to E\{W\} = \frac{5}{14}$
c)$\lambda_a = \frac{5}{3}\frac{jobs}{min} \quad \lambda_d = 2\frac{jobs}{min} \to E\{W\} = 2.5$

5.54

CAP. VI

6.1

$$\mathcal{F}_{Y|H_0}(y|H_0) = 1 \quad 0 \leq 1y \leq 1$$
$$\mathcal{F}_{Y|H_1}(y|H_1) = 2y \quad 0 \leq 1y \leq 1$$

a)

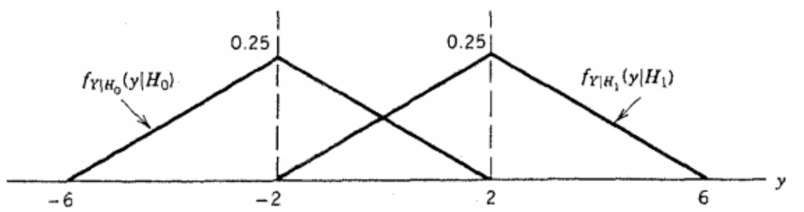
$$L(y) = \frac{2y}{1} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1/2}{1/2}$$
$$y \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2}$$

b)

$$\begin{aligned} P_e = P(H_0)P(D_1|H_0) + P(H_1)P(D_0|H_1) &= \frac{1}{2}\int_{1/2}^{1} f_{Y|H_0}(y|H_0)\ dy + \frac{1}{2}\int_{0}^{1/2} f_{Y|H_1}(y|H_1)\ dy \\ &= \frac{1}{2}\int_{1/2}^{1} 1\ dy + \frac{1}{2}\int_{0}^{1/2} 2y\ dy = \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \end{aligned}$$

Si me hubiera dado mayor a 1/2, me estaría delatando que está mal, no puedo tener una probabilidad mayor que a la del error.

6.4



$$f_{Y|H_0}(y|H_0) = \frac{1}{4}\left(1 - \left|\frac{x+2}{4}\right|\right) \quad -6 < y < 2$$
$$f_{Y|H_1}(y|H_1) = \frac{1}{4}\left(1 - \left|\frac{x-2}{4}\right|\right) \quad -2 < y < 6$$
$$\frac{f_{Y|H_1}(y|H_1)}{f_{Y|H_0}(y|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)\,(C_{10} - C_{00})}{P(H_1)\,(C_{01} - C_{11})}$$

a) Con $-2 \leq y \leq 2$:

$$\frac{1/8 + y/16}{1/8 - y/16} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1/3}{2/3}$$
$$y \underset{H_0}{\overset{H_1}{\gtrless}} -\frac{2}{3}$$

Costo medio (**formula**):

$$\overline{C} = C_{00}P(H_0)P(D_0|H_0) + C_{01}P(H_1)P(D_0|H_1) + C_{10}P(H_0)P(D_1|H_0) + C_{11}P(H_1)P(D_1|H_1)$$

$$\overline{C} = 1 \cdot \frac{1}{3} \cdot \frac{7}{9} + 3 \cdot \frac{2}{3} \cdot \frac{1}{18} + 3 \cdot \frac{1}{3} \cdot \frac{2}{9} + 1 \cdot \frac{2}{3} \cdot \frac{17}{18} = \frac{11}{9} \approx 1.2222$$

b)

$$\overline{C} = 1 \cdot \frac{1}{3} \cdot \frac{7}{8} + 3 \cdot \frac{2}{3} \cdot \frac{1}{8} + 3 \cdot \frac{1}{3} \cdot \frac{1}{8} + 1 \cdot \frac{2}{3} \cdot \frac{7}{8} = \frac{5}{4} = 1.25$$

6.7

a)

$$P_F = P(D_1|H_0)$$
$$P_M = P(D_0|H_1)$$

a) $\bar{C} = C_{00}(1-P_F) + C_{10}\cdot P_F + P(H_1)\left[(C_{11}-C_{00}) + (C_{01}-C_{11})\cdot P_M - (C_{10}-C_{00})\cdot P_F\right] =$

$$= C_{00}\cdot\left[(1-P_F) - P(H_1) + P(H_1)\cdot P_F\right] + C_{10}\left(P_F - P(H_1)\cdot P_F\right)$$

$$+ C_{01}\cdot P(H_1)\cdot P_M + C_{11}\cdot\left[P(H_1) - P(H_1)\cdot P_M\right] =$$

$$= C_{00}\cdot\left[(1-P_F) - P(H_1)(1-P_F)\right] + C_{10}\cdot P_F\cdot\left[1-P(H_1)\right] + C_{01}\cdot P(H_1)\cdot P_M +$$

$$+ C_{11}\cdot P(H_1)\cdot(1-P_M) =$$

$$= C_{00}\cdot\left[(1-P_F) - P(H_1) + P(H_1)\cdot P_F\right] + C_{10}\left(P_F - P(H_1)\cdot P_F\right)$$

$$+ C_{01}\cdot P(H_1)\cdot P_M + C_{11}\cdot\left[P(H_1) - P(H_1)\cdot P_M\right] =$$

$$= C_{00}\cdot\left[(1-P_F) - P(H_1)(1-P_F)\right] + C_{10}\cdot P_F\cdot\left[1-P(H_1)\right] + C_{01}\cdot P(H_1)\cdot P_M +$$

$$+ C_{11}\cdot P(H_1)\cdot(1-P_M) =$$

$$= C_{00}\cdot(1-P_F)\left[1-P(H_1)\right] + C_{10}\cdot P_F\cdot P(H_0) + C_{01}\cdot P(H_1)\cdot P_M + C_{11}\cdot P(H_1)\cdot(1-P_M)$$

$$= C_{00}\cdot\left[1-P(D_1|H_0)\right]\cdot P(H_0) + C_{10}\cdot P(H_0)\cdot P(D_1|H_0) + C_{01}\cdot P(H_1)\cdot P(D_0|H_1) + C_{11}\cdot P(H_1)\cdot\left[1-P(D_0|H_1)\right]$$

$$= C_{00}\cdot P(D_0|H_0)\cdot P(H_0) + C_{10}\cdot P(H_0)\cdot P(D_1|H_0) + C_{01}\cdot P(H_1)\cdot P(D_0|H_1) + C_{11}\cdot P(H_1)\cdot P(D_1|H_1)$$

b)

6.13

$$f_{Y|H_0}(y|H_0) = \frac{1}{2}e^{-y/2} \quad 0 < y$$
$$f_{Y|H_1}(y|H_1) = \frac{1}{4}e^{-y/4} \quad 0 < y$$

a)

$$\frac{f_{Y_1,Y_2|H_1}(y_1,y_2|H_1)}{f_{Y_1,Y_2|H_0}(y_1,y_2|H_0)} = \frac{\frac{1}{16}e^{-(y_1/4)-(y_2/4)}}{\frac{1}{4}e^{-(y_1/2)-(y_2/2)}} \underset{H_0}{\overset{H_1}{\gtrless}} 1$$

$$e^{(y_1/4)+(y_2/4)} \underset{H_0}{\overset{H_1}{\gtrless}} 4$$

$$y_1 + y_2 \underset{H_0}{\overset{H_1}{\gtrless}} 4\ Ln(4)$$

b)

$$\begin{aligned} P_M = P(D_0|H_1) = & \int_0^{4Ln(4)} \int_0^{4Ln(4)-y_2} f_{Y_1,Y_2|H_1}(y_1,y_2|H_1)dy_1dy_2 \\ = & \int_0^{4Ln(4)} \frac{e^{-y_2/4}}{4}\frac{4-e^{y_2/4}}{4}dy_2 = -\frac{Ln(4)-3}{4} \approx 0.4034 \end{aligned}$$

$$\begin{aligned} P_F = P(D_1|H_0) = & \int_0^{4Ln(4)} \int_{4Ln(4)-y_1}^{\infty} f_{Y_1,Y_2|H_0}(y_1,y_2|H_0)dy_2dy_1 + \int_{4Ln(4)}^{\infty}\int_0^{\infty} f_{Y_1,Y_2|H_0}(y_1,y_2|H_0)dy_2dy_1 \\ = & \int_0^{4Ln(4)} \frac{e^{-y_1/2}}{2}\frac{e^{-y_1/2}}{16}dy_1 + \int_{4Ln(4)}^{\infty} \frac{e^{-y_1/2}}{2}dy_1 = \frac{255}{8192} + \frac{1}{16} \approx 0.0936 \end{aligned}$$

CAP. VII

7.7
Pag. 387.

$$\begin{bmatrix} R_{SS}(0) & R_{SS}(1) & R_{SS}(2) \\ R_{SS}(1) & R_{SS}(0) & R_{SS}(1) \\ R_{SS}(2) & R_{SS}(1) & R_{SS}(0) \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} R_{SS}(1) \\ R_{SS}(2) \\ R_{SS}(3) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.9 \\ 0.7 & 0.9 & 1 \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.7 \\ 0.6 \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\hat{S}(n) = h_1 S(n-1) + h_2 S(n-2) + h_3 S(n-3)$$

7.14

$$\Sigma_{XX} = \begin{bmatrix} 2 & 1 & 1/2 \\ 1 & 3 & 1 \\ 1/2 & 1 & 4 \end{bmatrix}$$

Se sabe que:

$$\gamma(n,n) = 1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \gamma(2,1) & 1 & 0 \\ \gamma(3,1) & \gamma(3,2) & 1 \end{bmatrix} \cdot \begin{bmatrix} X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} V_X(1) \\ V_X(2) \\ V_X(3) \end{bmatrix}$$

Como la media es nula

$$C_{XX}(t_1,t_2) = R_{XX}(t_1,t_2) - \mu_X(t_1)\mu_X(t_2) = R_{XX}(t_1,t_2)$$

Eq. 7.70:

$$\gamma(2,1) = -\frac{R_{XX}(1,2)}{R_{XX}(1,1)} = -\frac{1}{2}$$

Las **innovaciones** son ortogonales, por lo tanto:

$$E\{V_X(1)V_X(3)\} = 0$$
$$E\{X(1)\left[\gamma(3,1)X(1) + \gamma(3,2)X(2) + X(3)\right]\} = 0$$
$$\gamma(3,1)R_{XX}(1,1) + \gamma(3,2)R_{XX}(1,2) + R_{XX}(1,3) = 0$$
$$2\gamma(3,1) + \gamma(3,2) + \frac{1}{2} = 0$$

$$E\{V_X(2)V_X(3)\} = 0$$
$$E\{\left[X(1)\gamma(2,1) + X(2)\right]\left[\gamma(3,1)X(1) + \gamma(3,2)X(2) + X(3)\right]\} = 0$$
$$-\gamma(3,1) - \frac{\gamma(3,2)}{2} - \frac{1}{4} + \gamma(3,1) + 3\gamma(3,2) + 1 = 0$$
$$\gamma(3,2) = -\frac{3}{4}\cdot\frac{2}{5} = -\frac{3}{10}$$
$$\gamma(3,1) = -\frac{1}{10}$$

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.1 & -0.3 & 1 \end{bmatrix} \Longrightarrow \mathbb{L} = \Gamma^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0.3 & 1 \end{bmatrix}$$