

# Climate Risk Hedging

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# Introduction



# Chapter 1

## ESG Preferences and Climate Risk Premia

### 1.1 Capital Asset Pricing Model (CAPM)

### 1.2 ESG Preferences

#### 1.2.1 Expected Utility and Optimal Portfolio

##### Setting the Investor's Expected Utility

Let's assume a single period model, from  $t = 0$  to  $t = 1$ . We have  $N$  stocks.

We have a  $N \times 1$  vector of returns  $\tilde{r}_1$  at period 1, assumed to be normally distributed:

$$\tilde{r}_1 = \mu + \tilde{\epsilon}_1 \tag{1.1}$$

with  $\mu$  the equilibrium expected excess returns and  $\tilde{\epsilon}_1$  the random component of the returns  $\tilde{\epsilon}_1 \sim N(0, \Sigma)$ .

The investor  $i$  has an exponential CARA utility function, with  $\tilde{W}_{1,i}$  the wealth at period 1, and  $X_i$  the  $N \times 1$  vector of portfolio weights.

PLACEHOLDER

Figure 1.1: Efficient Frontier

$$V(\tilde{W}_{1,i}, X_i) = -\exp(-A_i \tilde{W}_{1,i} - b_i^T X_i) \quad (1.2)$$

with  $A_i$  agent's absolute risk aversion,  $b_i$  an  $N \times 1$  vector of nonpecuniary benefits.

$$b_i = d_i g \quad (1.3)$$

with  $g$  an  $N \times 1$  vector and  $d_i \geq 0$  a scalar measuring the agent's taste for the nonpecuniary benefits.

The expectation of agent  $i$ 's in period 0 are:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = E_0(-\exp(-A_i \tilde{W}_{1,i} - b_i^T X_i)) \quad (1.4)$$

We can replace  $\tilde{W}_{1,i}$  by the relation  $\tilde{W}_{1,i} = W_{0,i}(1 + r_f + X_i^T \tilde{r}_1)$  and define  $a_i := A_i W_{0,i}$ . The idea is to make out from the expectation the terms that we know about (in period 0), and reexpress the terms within the expectation as a function of the portfolio weights  $X_i$ . The last two steps use the fact that  $\tilde{r}_1$  is normally distributed with mean  $\mu$  and variance  $\Sigma$ .

$$\begin{aligned} E_0(V(\tilde{W}_{1,i}, X_i)) &= E_0(-\exp(-A_i W_{0,i}(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)) \\ &= E_0(-\exp(-a_i(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)) \\ &= E_0(-\exp(-a_i(1 + r_f) - a_i X_i^T \tilde{r}_1 - b_i^T X_i)) \\ &= -\exp(-a_i(1 + r_f)) E_0(-\exp(-a_i X_i^T \tilde{r}_1 - b_i^T X_i)) \\ &= -\exp(-a_i(1 + r_f)) E_0(-\exp(-a_i X_i^T (\tilde{r}_1 + \frac{b_i}{a_i}))) \quad (1.5) \\ &= -\exp(-a_i(1 + r_f)) \exp(-a_i X_i^T (E_0(\tilde{r}_1) + \frac{b_i}{a_i}) + \frac{1}{2} a_i^2 X_i^T \text{Var}(\tilde{r}_1) X_i) \\ &= -\exp(-a_i(1 + r_f)) \exp(-a_i X_i^T (\mu + \frac{b_i}{a_i}) + \frac{1}{2} a_i^2 X_i^T \Sigma X_i) \end{aligned}$$

### Solving for the Investor's Optimal Portfolio

The investors choose their optimal portfolios at time 0. The optimal portfolio  $X_i$  is the one that maximizes the expected utility. To find it, we differentiate the expected utility with respect to  $X_i$  and set it to zero, to obtain the first-order condition.

We are going to do it step by step:



1. Combine the Exponential Terms:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp\left(-a_i(1 + r_f) - a_i X_i^T\left(\mu + \frac{b_i}{a_i}\right) + \frac{1}{2}a_i^2 X_i^T \Sigma X_i\right) \quad (1.6)$$

and let  $f(X_i)$  be the exponent:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp f(X_i) \quad (1.7)$$

2. Differentiate  $f(X_i)$  with respect to  $X_i$ . We have the chain rule:

$$\frac{\partial h}{\partial X_i} = \frac{\partial h}{\partial f} \frac{\partial f}{\partial X_i} \quad (1.8)$$

If  $h = -\exp(f)$ , then  $\frac{\partial h}{\partial f} = -\exp(f)$ . Therefore we have:

$$\frac{\partial h}{\partial X_i} = -\exp(f) \frac{\partial f}{\partial X_i} \quad (1.9)$$

To tackle the derivative of  $f(X_i)$ , we use two rules. First  $\frac{\partial x^T b}{\partial x} = b$  and  $\frac{\partial x^T A x}{\partial x} = 2Ax$  if  $A$  is symmetric. We have:

$$\frac{\partial f}{\partial X_i} = -a_i\left(\mu + \frac{b_i}{a_i}\right) + a_i^2 \Sigma X_i \quad (1.10)$$

Combining:

$$\frac{\partial h}{\partial X_i} = -\exp(f)\left(-a_i\left(\mu + \frac{b_i}{a_i}\right) + a_i^2 \Sigma X_i\right) \quad (1.11)$$

3. Set the derivative to zero:

$$\begin{aligned} -\exp(f)\left(-a_i\left(\mu + \frac{b_i}{a_i}\right) + a_i^2 \Sigma X_i\right) &= 0 \\ -a_i\left(\mu + \frac{b_i}{a_i}\right) + a_i^2 \Sigma X_i &= 0 \end{aligned} \quad (1.12)$$

where the exponential term is always positive, so we can drop it.

## PLACEHOLDER

Figure 1.2: Efficient Frontier with ESG Preferences

4. Rearrange and solve for  $X_i$ :

$$\begin{aligned}
 a_i^2 \Sigma X_i &= a_i \left( \mu + \frac{b_i}{a_i} \right) \\
 a_i \Sigma X_i &= \mu + \frac{b_i}{a_i} \\
 \Sigma X_i &= \frac{1}{a_i} \left( \mu + \frac{b_i}{a_i} \right) \\
 X_i &= \frac{1}{a_i} \Sigma^{-1} \left( \mu + \frac{b_i}{a_i} \right)
 \end{aligned} \tag{1.13}$$

For the sake of simplicity, we assume that  $a_i = a$  for all investors. We now have:

$$\begin{aligned}
 X_i &= \frac{1}{a} \Sigma^{-1} \left( \mu + \frac{b_i}{a} \right) \\
 &= \frac{1}{a} \Sigma^{-1} \left( \mu + \frac{d_i}{a} g \right)
 \end{aligned} \tag{1.14}$$

Therefore, the optimal portfolio differs across investors due to the ESG characteristics  $g$  of the stocks and the investors' taste for nonpecuniary benefits  $d_i$ .

## 1.2.2 Heterogeneous Investors and Expected Returns

### Heterogeneous Market

The  $n$ th element of investor  $i$ 's portfolio weight vector  $X_i$  is:

$$X_{i,n} = \frac{W_{0,i,n}}{W_{0,i}} \tag{1.15}$$

with  $W_{0,i,n}$  the wealth invested in stock  $n$  by investor  $i$  at time 0.

The total wealth invested in stock  $n$  at time 0 is:

$$W_{0,n} := \int_i W_{0,i,n} di \tag{1.16}$$

The  $n$ th element of the market-weight vector  $w_m$  is:

$$w_{m,n} = \frac{W_{0,n}}{W_0} \quad (1.17)$$

We can now express  $W_{0,n}$  in terms of individual investors' wealths by using the definition of  $W_{0,n}$ :

$$w_{m,n} = \frac{1}{W_0} \int_i W_{0,i,n} di \quad (1.18)$$

We now that  $W_{0,i,n} = W_{0,i} X_{i,n}$ , so we can rewrite the equation:

$$w_{m,n} = \frac{1}{W_0} \int_i W_{0,i} X_{i,n} di \quad (1.19)$$

Defining  $\omega_i = \frac{W_{0,i}}{W_0}$ , we have:

$$\begin{aligned} w_{m,n} &= \int_i \frac{W_{0,i}}{W_0} X_{i,n} di \\ &= \int_i \omega_i X_{i,n} di \end{aligned} \quad (1.20)$$

We can now plug in  $X_i$  to obtain  $w_m$  the vector of market weights:

$$\begin{aligned} w_m &= \int_i \omega_i X_i di \\ &= \int_i \omega_i \frac{1}{a} \Sigma^{-1} (\mu + \frac{d_i}{a} g)_n di \\ &= \frac{1}{a} \Sigma^{-1} \mu \left( \int_i \omega_i di \right) + \frac{1}{a^2} \Sigma^{-1} g \left( \int_i \omega_i d_i di \right) \end{aligned} \quad (1.21)$$

We have  $\int_i \omega_i di = 1$  and we define  $\bar{d} := \int_i d_i di \geq 0$ , the wealth-weighted mean of ESG tastes  $d_i$  across agents. Therefore:

$$w_m = \frac{1}{a} \Sigma^{-1} \mu + \frac{1}{a^2} \Sigma^{-1} g \bar{d} \quad (1.22)$$

## Expected Returns

Starting from the the vector of market weights  $w_m$ , we now can solve for  $\mu$  the vector of expected returns. We have:

$$\begin{aligned}
w_m &= \frac{1}{a}\Sigma^{-1}\mu + \frac{1}{a^2}\Sigma^{-1}g\bar{d} \\
aw_m &= \Sigma^{-1}\mu + \frac{1}{a}\Sigma^{-1}g\bar{d} \\
aw_m - \frac{1}{a}\Sigma^{-1}g\bar{d} &= \Sigma^{-1}\mu \\
\Sigma(aw_m - \frac{1}{a}\Sigma^{-1}g\bar{d}) &= \mu \\
\mu &= a\Sigma w_m - \frac{1}{a}\Sigma\Sigma^{-1}g\bar{d} \\
\mu &= a\Sigma w_m - \frac{1}{a}g\bar{d}
\end{aligned} \tag{1.23}$$

Multiplying by  $w_m$ , we find the market equity premium  $\mu_m = w_m^T\mu$ :

$$\begin{aligned}
\mu_m &= aw_m^T\Sigma w_m - \frac{\bar{d}}{a}w_m^Tg \\
&= a\sigma_m^2 - \frac{\bar{d}}{a}w_m^Tg
\end{aligned} \tag{1.24}$$

where  $\sigma_m^2 = w_m^T\Sigma w_m$  is the market return variance.

## Expected Excess Returns

### 1.2.3 ESG Portfolio

#### Portfolio Tilts

#### Factor Pricing with the ESG Portfolio

## 1.3 Climate Risk

## Chapter 2

# Sources of ESG Factor and Climate Risk

### 2.1 Market Risk

### 2.2 ESG Factor Risk

### 2.3 Climate Risk

