Climate Risk Hedging

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May 20, 2024

Contents

Chapter 1

Introduction

We consider alternative approaches for climate risk hedging. All approaches share the same goal: to be long stocks that do well in periods with unexpectedly bad news about climate risks, and short stocks that do badly in those scenarios.

Chapter 2

Modelling Returns

2.1 Returns as Random Variables

We model stock returns as *random variables*. A random variable can take one of many values, with an associated probability. For example, the gross return on a stock can be one of four values as shown in Table 2.1.

R	π
1.1	0.6
1.2	0.1
0.7	0.25
0.0	0.05

Table 2.1: Example of a gross return distribution.

Each value is a possible *realization* of the random variable. You can experiment with this in Python using the following code:

```
import numpy as np

# Define the possible returns and their probabilities
returns = np.array([1.1, 1.2, 0.7, 0.0])
probabilities = np.array([0.6, 0.1, 0.25, 0.05])

# Generate a random return
print(np.random.choice(returns, size=1, p=probabilities))
```

Of course, stock returns can take on many more values than just four, but this is a simple example.

The *distribution* of the random variable is a listing of the values it can take, along with their associated probabilities. For example, the distribution of the random variable in Table 2.1 is:

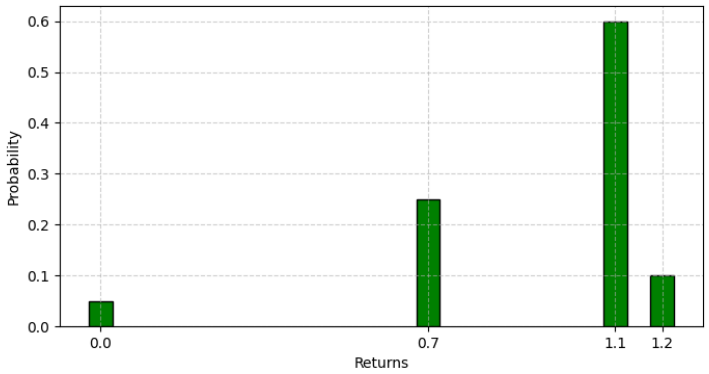

Figure 2.1: Example of a random variable distribution.

Another way to think of a random variable is as a *function* that maps "states of the world" to real numbers. For example, the random variable in Table 2.1 could be:

R	States of the world	π
1.1	New product works, competitor burns down	0.6
1.2	New product works, competitor ok.	0.1
0.7	Only old products work	0.25
0.0	Factory burns down, no insurance.	0.05

Table 2.2: Random variable as a function mapping states of the world to real numbers.

The probability really describes the external events that define the states of the world. Usually, we can't name those events, so we just use the probabilities that the stock return takes on various values.

In the end, all random variables have a discrete number of values, as in our example. Stock prices are only listed to 1/8 dollar, all payments are rounded to the nearest cent, etc. However, we often think of *continuous* random variables, that can be any real number. Corresponding to the discrete probabilities above, we now have a continuous probability *density*, usually

denoted $f(R)$. The density tells you the probability per unit of R. For example, $f(R_0)\Delta R$ tells you the probability that the return is between R_0 and $R_0 + \Delta R$.

A common assumption is that returns (or log returns) are normally distributed. This means that the density is given by the formula:

$$f(R) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(R-\mu)^2}{2\sigma^2}\right) \tag{2.1}$$

The graph of this function looks like this:

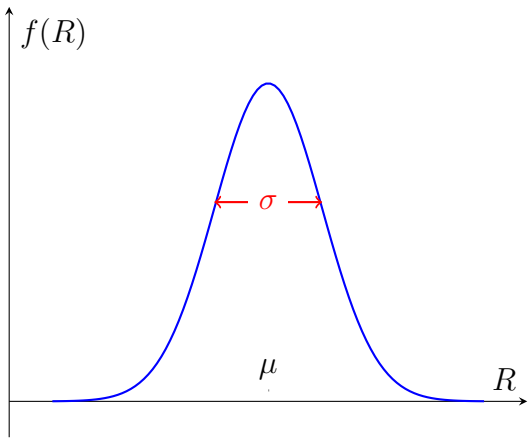

Figure 2.2: The probability density function of normally distributed returns

About 30% (31.73% to be exact) of the probability of a normal distribution is more than one standard deviation away from the mean. About 5% of the probability is more than two standard deviations away from the mean (in fact 4.55%, the 5% probability lines is at 1.96 standard deviation). That means that there is only one chance in 20 that the return will be more than two standard deviations away from the mean in a normally distributed world. In reality, stock returns have "fat tails", meaning that they are more likely to take on extreme values than a normal distribution would suggest.

You can experiment modelling returns with a normal distribution in Python:

```
import numpy as np

# Parameters for the normal distribution
mu = 1  # Mean
```

```
sigma = 0.05  # Standard deviation

print(np.random.normal(mu, sigma, 1))
```

2.2 Expected Value and Variance of Returns

Rather than plot the entire distribution, we usually summarize the behavior of a random variable with two numbers: the *mean* and the *variance*.

We denote the values that R can takes on a R_i, with associated probabilities π_i. The mean of R is given by:

$$E(R) = \sum_i R_i \pi_i \tag{2.2}$$

The mean is a measure of *central tendency*. It tells you where R is on average. A high mean stock return is obviously better than a low mean stock return.

The *variance* of R is given by:

$$\sigma^2(R) = E[(R - E(R))^2] = \sum_i \pi_i (R_i - E(R))^2 \tag{2.3}$$

Since squares are always positive, variance tells you how much R is far away from its mean. It measures the spread of the distribution. High variance is not a good thing. It will be our measure of *risk*.

Using our previous example, the Python code is:

```
import numpy as np

# Define the values that R can take (R_i) and their
                                associated probabilities (pi_i
                                )
values = np.array([1.1, 1.2, 0.7, 0.0])  # Example values R_i
probabilities = np.array([0.6, 0.1, 0.25, 0.05])  #
                                Corresponding probabilities
                                pi_i

# Calculate the expected value (mean) of R
expected_value = np.sum(values * probabilities)
print("Expected Value E(R):", expected_value)

# Calculate the variance of R
```

```
variance = np.sum((values - expected_value)**2 *
                                  probabilities)
print("Variance of R:", variance)
```

The *covariance* is:

$$\text{Cov}(R^a, R^b) = E[(R^a - E(R^a))(R^b - E(R^b))] = \sum_i \pi_i [(R_i^a - E(R^a))(R_i^b - E(R^b))] \tag{2.4}$$

It measures the tendency of two returns to move together. It's positive if they tend to move in the same direction, negative if one tends to go up when the other goes down. It's zero if they are independent.

The size of the covariance depends on the unit of measurement. For example, if we measure one return in cents, the covariance goes up by a factor of 100, even though the relationship between the two returns is the same. The *correlation coefficient* resolves this problem:

$$\text{Corr}(R^a, R^b) = \frac{\text{Cov}(R^a, R^b)}{\sigma(R^a)\sigma(R^b)} \tag{2.5}$$

The correlation coefficient is always between -1 and 1.

Keeping the same example as before but adding a second return, the Python code is:

```
import numpy as np

# Define the values and probabilities for two random
                                  variables R^a and R^b
values_a = np.array([1.1, 1.2, 0.7, 0.0])  # Example values R
                                  ^a_i
values_b = np.array([1.0, 1.3, 0.4, 0.1])  # Example values R
                                  ^b_i
probabilities = np.array([0.6, 0.1, 0.25, 0.05])  #
                                  Corresponding probabilities
                                  pi_i

# Calculate the expected values (means) of R^a and R^b
expected_value_a = np.sum(values_a * probabilities)
expected_value_b = np.sum(values_b * probabilities)

# Calculate the covariance between R^a and R^b
```

```
covariance = np.sum(probabilities * (values_a -
                                     expected_value_a) * (values_b
                                     - expected_value_b))
print("Covariance between R^a and R^b:", covariance)

# Calculate the variances of R^a and R^b
variance_a = np.sum((values_a - expected_value_a)**2 *
                                     probabilities)
variance_b = np.sum((values_b - expected_value_b)**2 *
                                     probabilities)

# Calculate the standard deviations of R^a and R^b
std_dev_a = np.sqrt(variance_a)
std_dev_b = np.sqrt(variance_b)

# Calculate the correlation coefficient
correlation_coefficient = covariance / (std_dev_a * std_dev_b
                                     )
print("Correlation Coefficient between R^a and R^b:",
                                     correlation_coefficient)
```

2.3 Returns Algebra

We will have to do a lot of manipulation of random variables. For example, we will want to know what is the mean and standard deviation of a *portfolio* of two returns. To derive any of these rules, we have to keep in mind the definitions in the previous section.

First interesting fact: constants come out of expectations and expectations of sums are equals to sums of expectations. For example, if c and d are real numbers:

$$E(cR^a) = cE(R^a) \tag{2.6}$$

$$E(R^a + R^b) = E(R^a) + E(R^b) \tag{2.7}$$

$$E(cR^a + dR^b) = cE(R^a) + dE(R^b) \tag{2.8}$$

Let's check this with Python:

```python
import numpy as np

# Define the values for random variable R^a and its
                                associated probabilities
values_a = np.array([1.1, 1.2, 0.7, 0.0])  # Example values R
                                ^a_i
probabilities = np.array([0.6, 0.1, 0.25, 0.05])  #
                                Corresponding probabilities
                                pi_i

# Calculate the expected value of R^a
expected_value_a = np.sum(values_a * probabilities)
print("Expected Value E(R^a):", expected_value_a)

# Define the constant c
c = 0.5

# Calculate the expected value of cR^a using the definition
expected_value_cRa = np.sum(c * values_a * probabilities)
print("Expected Value E(cR^a):", expected_value_cRa)

# Calculate c times the expected value of R^a
c_times_expected_value_a = c * expected_value_a
print("c * E(R^a):", c_times_expected_value_a)

# Check if E(cR^a) is numerically equal to cE(R^a)
if np.isclose(expected_value_cRa, c_times_expected_value_a):
    print("Numerically verified that E(cR^a) = cE(R^a)")
else:
    print("Numerical discrepancy found")
```

Variance of sums works like taking a square:

$$\sigma^2(R^a + R^b) = \sigma^2(R^a) + \sigma^2(R^b) + 2\text{Cov}(R^a, R^b) \tag{2.9}$$

With constants c and d:

$$\sigma^2(cR^a + dR^b) = c^2\sigma^2(R^a) + d^2\sigma^2(R^b) + 2cd\text{Cov}(R^a, R^b) \tag{2.10}$$

The covariance works linearly with constants:

$$\text{Cov}(cR^a, dR^b) = cd\text{Cov}(R^a, R^b) \tag{2.11}$$

In Python:

```
import numpy as np

# Define the values and probabilities for two random
                                variables R^a and R^b
values_a = np.array([1.1, 1.2, 0.7, 0.0])  # Example values R
                                ^a_i
values_b = np.array([1.0, 1.3, 0.4, 0.1])  # Example values R
                                ^b_i
probabilities = np.array([0.6, 0.1, 0.25, 0.05])  #
                                Corresponding probabilities
                                pi_i

# Calculate the expected values (means) of R^a and R^b
expected_value_a = np.sum(values_a * probabilities)
expected_value_b = np.sum(values_b * probabilities)

# Calculate the covariance between R^a and R^b
covariance_ab = np.sum(probabilities * (values_a -
                                expected_value_a) * (values_b
                                - expected_value_b))
print("Covariance Cov(R^a, R^b):", covariance_ab)

# Constants c and d
c = 0.5  # Scaling factor for R^a
d = 0.8  # Scaling factor for R^b

# Calculate the covariance between cR^a and dR^b
covariance_cRa_dRb = np.sum(probabilities * (c * (values_a -
                                expected_value_a)) * (d * (
                                values_b - expected_value_b)))
print("Covariance Cov(cR^a, dR^b):", covariance_cRa_dRb)

# Calculate cd * Cov(R^a, R^b)
cd_covariance_ab = c * d * covariance_ab
print("cd * Cov(R^a, R^b):", cd_covariance_ab)

# Check if Cov(cR^a, dR^b) is numerically equal to cd * Cov(R
                                ^a, R^b)
if np.isclose(covariance_cRa_dRb, cd_covariance_ab):
    print("Numerically verified that Cov(cR^a, dR^b) = cd *
                                    Cov(R^a, R^b)")
else:
    print("Numerical discrepancy found")
```

Normal distributions have an extra property. Linear combinations of nor-

mally distributed random variables are again normally distributed. Precisely, if R^a and R^b are normally distributed and:

$$R^p = cR^a + dR^b \tag{2.12}$$

then R^p is normally distributed.

A variable R is *lognormally* distributed if $r := \ln(R)$ is normally distributed. This is a nice model for stock returns since you can never lose more than all your money (*i.e.* you can never see $R < 0$). A lognormal distribution captures this property. A normal distribution always includes events in which $R < 0$, which is not possible for stock returns.

2.4 Sample Returns

So far, we have worked as if we knew the probabilities of each return. In reality, we don't. We have to *estimate* them from *sample returns*. Similarly, if we don't know the mean, variance, regression coefficients, etc., we have to estimate them as well. That's what *statistics* is about.

The *average* or *sample mean* is:

$$\bar{R} = \frac{1}{T}\sum_{i=1}^{N} R_t \tag{2.13}$$

where $\{R_0, R_1, \ldots, R_T\}$ is a *sample* of data on a stock returns. We need to keep the *sample mean* and the true, or *population mean* separate in our heads. For example, the true probabilities that a coin will lands heads is 0.5, so the mean of a bet on a coin toss (1$ if heads, -1$ if tails) is $0.5\times 1 + 0.5\times -1 = 0$. A *sample* of coin tosses might be $\{H, T, H, H, T\}$, In that *sample*, the frequency of heads is $3/5 = 0.6$ and the frequency of tails is $2/5 = 0.4$. So the *sample mean* of a coin toss bet is $0.6 \times 1 + 0.4 \times -1 = 0.2$.

As the sample gets bigger, the *sample mean* will gets closer to the *population mean*. That property of the sample mean is called *consistency*. It makes it a good estimator of the population mean. But the sample and population mean are not the same thing for any finite sample.

Also, sample means approach population means only if you are repeatedly doing the same thing, like tossing a fair coin. This may not be true for stock returns. If there are days when expected returns are high and days when they

are low, then the average return will not necessarily recover the expected return.

The *sample variance* is:

$$s^2 = \hat{\sigma}^2 = \frac{1}{T-1}\sum_{i=1}^{T}(R_t - \bar{R})^2 \tag{2.14}$$

The sample mean and sample variance vary from sample to sample. Let's flip a coin, with $Heads = +1$ and $Tails = -1$. If we get $\{H, T, H, H, T\}$, the sample mean is 0.2, but if we get $\{H, H, H, H, H\}$, the sample mean is 1. The true population mean, of course, is 0. Thus, the sample mean, the standard deviation, and other statistics are also *random variables*, as they vary from sample to sample. They are random variables that depends on the whole sample, not just what happened one day, but they are random variables nonetheless. The population mean and variance are just numbers.

We can then ask, "how much does the sample mean vary from sample to sample?" If a mutual fund manager tells you that his fund has a mean return of 10% while the market has a mean return of 8%, you want to know if it was just luck or if it means that his true, population mean, which you are likely to earn in the next 5 years, is 2% higher than the market. In other words, was the *realization* of the random variable called "my estimate of manager A's mean return" near the mean of the true or population mean of the random variable called "manager A's return"?

Figuring out the variation of the sample mean is a good use of statistics. The sample mean is:

$$\bar{R} = \frac{1}{T}\sum_{i=1}^{T} R_t \tag{2.15}$$

Therefore,

$$E(\bar{R}) = \frac{1}{T}\sum_{i=1}^{T} E(R_t) = E(R) \tag{2.16}$$

assuming all the R_t are drawn from the same distribution. This verifies that the sample mean is *unbiased*. On average, across many samples, the sample mean will reveal the true mean. The variance of the sample mean is:

$$\sigma^2(\bar{R}) = \sigma^2(\frac{1}{T}\sum_{i=1}^{T} R_t) = \frac{1}{T^2}\sum_{i=1}^{T}\sigma^2(R_t) + \text{covariance terms} \tag{2.17}$$

If we assume that all the covariances are zero (R_t are independent), then the variance of the sample mean is:

$$\sigma^2(\bar{R}) = \frac{1}{T}\sigma^2(R) \tag{2.18}$$

and the standard deviation:

$$\sigma(\bar{R}) = \frac{1}{\sqrt{T}}\sigma(R) \tag{2.19}$$

For stock returns, $\text{Cov}(R_t, R_{t+1}) = 0$ is a good assumption. It's a great assumption for coin tosses: seeing heads this time makes not more likely to see heads next time.

We don't know σ. We have to *estimate* the sampling variation of the sample mean by using your estimate of σ, namely the sample standard deviation. If we use hats to denote estimate:

$$\hat{\sigma}(\bar{R}) = \frac{1}{\sqrt{T}}\hat{\sigma}(R) \tag{2.20}$$

The classic use of this formula is to give a standard error or measure of uncertainty of the sample mean, and to test whether the sample mean is equal to some value, usually zero.

The test is based on a *condifence interval*. Assuming normal distribution, the confidence interval for the mean is the sample mean plus or minus 2 (in fact 1.96) standard errors. The meaning of this interval is that if the true mean was outside the interval, there would be less than 5% chance of seeing a sample mean as high or low as the one you got.

There is an easier method. We can just calculate the probability that the sample mean comes out at its actual value (or higher) given the null hypothesis, *i.e.* calculate the area under the distribution of the sample mean past the sample mean we have, given an assumption (hypothesis) about the true mean. This is the *p-value*.

Usually, tests are run using the *t-distribution*. When you take account of sampling variation in $\hat{\sigma}$, you can show that the ratio:

$$\sqrt{T}\frac{\bar{R} - E(R)}{\hat{\sigma}} \tag{2.21}$$

is not a normal distribution with mean zero and variance one, but a *t-distribution* with t degrees of freedom.

2.5 Conclusion

2.6 Exercises

2.6.1

Compute the expected return $E(R)$ for the $R = [1.1, 1.2, 0.7, 0.0]$ with probabilities $\pi = [0.6, 0.1, 0.25, 0.05]$ with Python.

2.6.2

Derive $E(cR^a) = cE(R^a)$ using the definition of expectations $E(R) = \sum_i R_i \pi_i$.

2.6.3

Derive the variance of sums formula:

$$\sigma^2(R^a + R^b) = \sigma^2(R^a) + \sigma^2(R^b) + 2\text{Cov}(R^a, R^b) \tag{2.22}$$

using the definition of variance $\sigma^2(R) = E[(R - E(R))^2]$ and the definition of covariance $\text{Cov}(R^a, R^b) = E[(R^a - E(R^a))(R^b - E(R^b))]$.

2.6.4

Express the expected return $E(R^p)$ and variance $\sigma^2(R^p)$ of the portfolio R^p with returns R^a and R^b and weights c and d.

2.7 Solutions

2.7.1

```
import numpy as np

# Define the returns and their corresponding probabilities
returns = np.array([1.1, 1.2, 0.7, 0.0])  # Values of R
probabilities = np.array([0.6, 0.1, 0.25, 0.05])  #
                                Corresponding probabilities pi

# Calculate the expected return E(R)
expected_return = np.sum(returns * probabilities)
print("Expected Return E(R):", expected_return)
```

2.7.2

$$E(cR^a) = \sum_i \pi_i c R_i^a = c \sum_i \pi_i R_i^a = cE(R^a) \tag{2.23}$$

2.7.3

$$\begin{aligned}
\sigma^2(R^a + R^b) &= E[(R^a + R^b - E(R^a + R^b))^2] && (2.24)\\
&= E[(R^a + R^b - E(R^a) - E(R^b))^2] && (2.25)\\
&= E[(R^a - E(R^a) + R^b - E(R^b))^2] && (2.26)\\
&= E[(R^a - E(R^a))^2 + (R^b - E(R^b))^2 + 2(R^a - E(R^a))(R^b - E(R^b))] && (2.27)\\
&= \sigma^2(R^a) + \sigma^2(R^b) + 2\text{Cov}(R^a, R^b) && (2.28)
\end{aligned}$$

2.7.4

Its mean is:

$$E(R^p) = cE(R^a) + dE(R^b) \tag{2.29}$$

and its variance is:

$$\sigma^2(R^p) = c^2\sigma^2(R^a) + d^2\sigma^2(R^b) + 2cd\text{Cov}(R^a, R^b) \tag{2.30}$$

Chapter 3

Regression

We will run regression, for example of a return on the market return:

$$R_t = \alpha + \beta R_{m,t} + \epsilon_t \tag{3.1}$$

where R_t is the return on the asset, $R_{m,t}$ is the return on the market portfolio, α is the intercept, β is the slope coefficient and ϵ_t is the regression residual.

We may sometimes run multiple regressions of returns on the return of several portfolios, for example:

$$R_t = \alpha + \beta R_{m,t} + \gamma R_{p,t} + \epsilon_t \tag{3.2}$$

where R_p is the return on the portfolio of interest.

The generic form is:

$$y_t = \alpha + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \ldots + \beta_n x_{n,t} + \epsilon_t \tag{3.3}$$

3.1 β estimation

Starting with:

$$y_t = \alpha + \beta x_t + \epsilon_t \tag{3.4}$$

With the usual assumption that errors are uncorrelated, we have the right hand variables $E(\epsilon_t x_t) = 0$ and $E(\epsilon_t) = 0$.

Multiplying both sides by $x_t - E(x_t)$ and taking expectations:

$$\beta = \frac{Cov(y, x)}{Var(x)} \tag{3.5}$$

3.2 Matrix Algebra

3.3 Matrix Form

3.4 OLS vs. GLS

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Chapter 5

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Predictability and Returns

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Chapter 7

Tracking Portfolio for News

Chapter 8

Climate Hedge Targets

One challenge with designing portfolios that hedge climate risks is that there is no unique way of choosing the hedge target. Climate change is a complex phenomenon and presents a variety of risks, including physical risks such as rising sea levels and transition risks such as the dangers to certain business models from regulations to curb emissions. Different risks may be relevant for different investors, and these risks are imperfectly correlated. In addition, climate change is a long-run threat, and we would thus ideally build portfolios that hedge the long-run realizations of climate risk, something difficult to produce in practice. To overcome these challenges, Engle *et al.* (2020) [**?**] argue that the objective of hedging long-run realizations of a given climate risk can be achieved by constructing a sequence of short-lived hedges against *news* (one-period innovation in expectations) about future realizations of the risk. Following the initial work of Engle *et al.* (2020), researchers have developped a variety of climate news series, capturing a variety of climate risks.

8.1 Climate News Series

Describe some climate new series.

8.2 Climate News Innovation

Building on the work of Engle *et al.* (2020), we use the $AR(1)$ innovations of each climate news series as the hedge targets. For a given climate news series

c, we denote these $AR(1)$ innovation in month t as $CC_{c,t}$.

8.2.1 Climate News Shock

8.3 Portfolio Exposure to Climate News Innovations

8.3.1 Multifactor Regression

8.3.2 Climate News Innovations as a Risk Factor

Chapter 9

Climate Risk Mimicking Portfolio

The mimicking portfolio approach combines a pre-determined set of assets into a portfolio that is maximally correlated with a given climate change shock, using historical data. To obtain the mimicking portfolios, we estimate the following regression model:

$$CC_t = wR_t + \epsilon_t \tag{9.1}$$

where CC_t denotes the (mean zero) climate hedge target in month t, w is a vector of N portfolio weights, R_t is the $N \times 1$ vector of demeaned excess returns and ϵ_t is the regression residual. The portfolio weights are estimated each month using a rolling window of T months of historical data.