Climate Risk Hedging

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Introduction

Chapter 1

Climate Risks

Before going straight on climate risks, we need to agree on the definition of *risk* in Finance. Suppose that a stock return follows an AR(1) process:

$$R_t = \phi R_{t-1} + \varepsilon_t \tag{1.1}$$

with ε_t is a *white noise* process, like a coin flip: completely unpredictable. We generaly consider that it is mean zero $E_t(\varepsilon_{t+1}) = E(\varepsilon_{t+1}) = 0$ and constant variance $\sigma_t(\varepsilon_{t+1}) = \sigma(\varepsilon_{t+1}) = \sigma^2_\varepsilon$. Such white noise is also unpredictable from its past: the *autocorrelation* $\text{Corr}(\varepsilon_t, \varepsilon_{t+1}) = 0$. Below is an example of such white noise process.

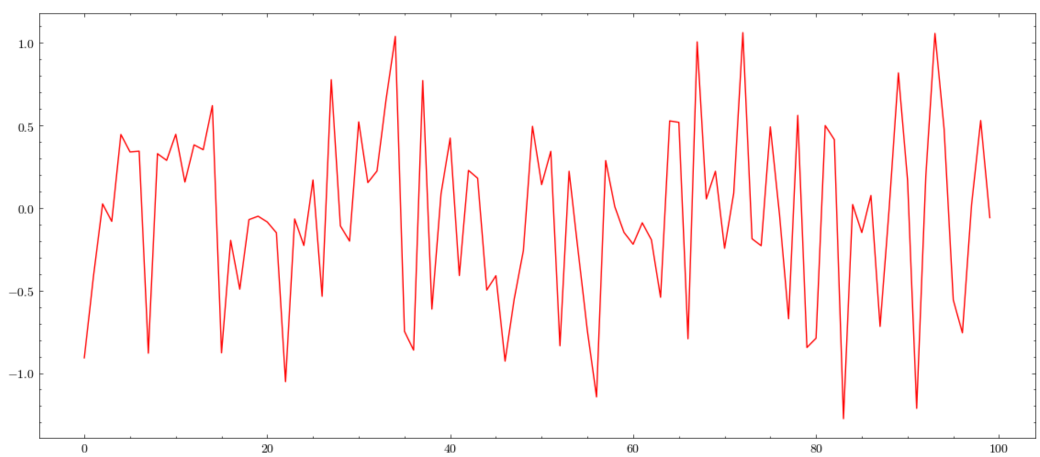

Figure 1.1: ε_t with $\sigma_\varepsilon = 0.5$

The conditional expectation of the stock return is:

$$\begin{aligned} E_t(R_{t+1}) &= \phi R_t + E_t(\varepsilon_{t+1}) \\ &= \phi R_t \end{aligned} \tag{1.2}$$

because $E_t(\varepsilon_{t+1}) = 0$.

The actual or *realized* return will vary around this expectation. The unexpected return is:

$$\begin{aligned} R_{t+1} - E_t(R_{t+1}) &= \phi R_t + \varepsilon_{t+1} - \phi R_t \\ &= \varepsilon_{t+1} \end{aligned} \tag{1.3}$$

and conditional variance of the stock return is:

$$\begin{aligned} \sigma_t^2(R_{t+1}) &= \sigma_t^2(\varepsilon_{t+1}) \\ &= \sigma_\varepsilon^2 \end{aligned} \tag{1.4}$$

due to the definition of the white noise process with constant variance, and the fact that we already know R_t at time t, so $\sigma_t^2(R_t) = 0$.

Risk is the unexpected in Finance. This is the fact that returns are not what we expected. Either higher or lower. So, what matters is this ε_{t+1} in our stock return process and its variance σ_ε^2.

Figure 1.2 shows the stock return process with increasing uncertainty σ_ε^2. The more uncertain the process, the more the realized returns will vary around the expected returns.

With this in mind, we can now move to the question of climate risks.

1.1 Climate Risks and Asset Prices

With a simple one-period model, we can show how climate change-related issues can add uncertainty to asset prices (thus, risks). We will consider two channels through which climate change adds uncertainty: the cash-flows channel and the discount rate channel.

1.1.1 Cash-Flows Channel

Let's define a payoff X_{t+1} as the cash-flows provided to the investors per dollar invested in the stock at time t:

$$X_{t+1} = \frac{CF_{t+1}}{P_t} \tag{1.5}$$

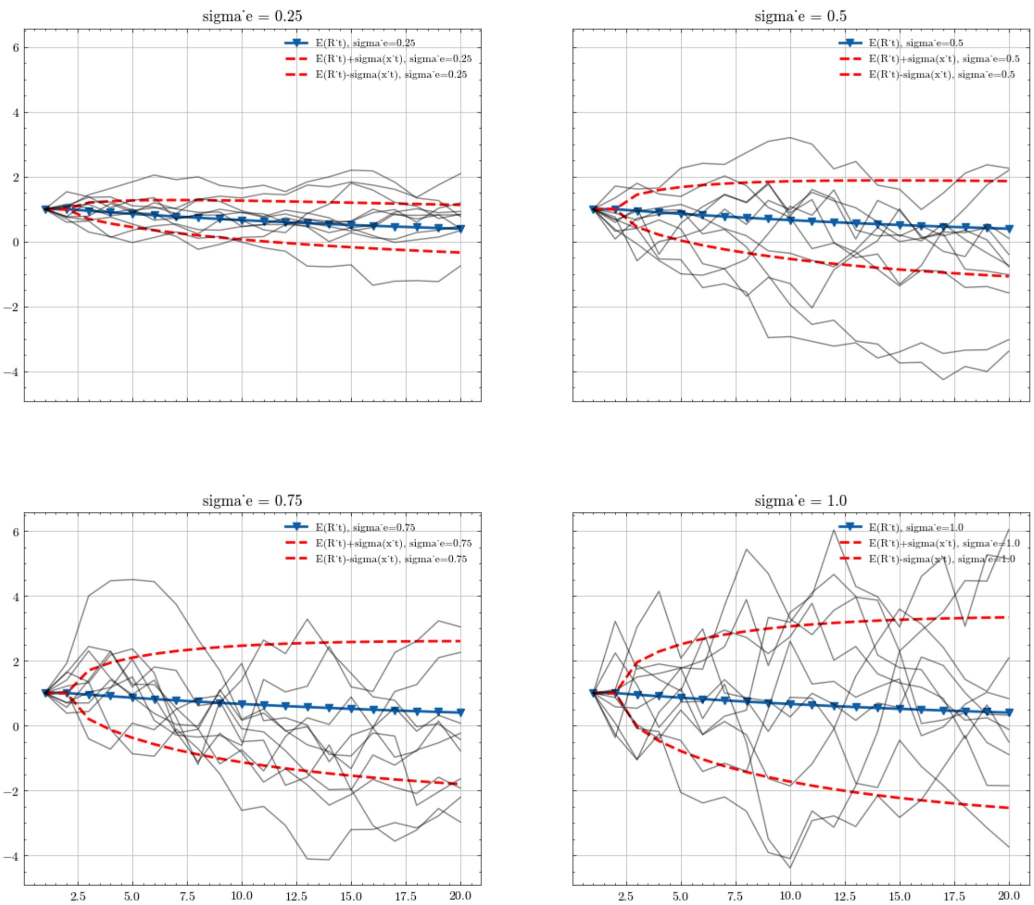

Figure 1.2: Simulation with increasing uncertainty (σ_ε^2) in an AR(1) process.

Analysts form forecasts about the future cash-flows of the firm based on information available at time t. The expectations of the payoff are a function of climate-related conditions CC_{t+1}. CC_{t+1} could represents any climate-related issues that may affect the firm's cash-flows. But to make the things more concrete, we can think of CC_{t+1} as a carbon tax. The expected payoff is then:

$$E_t(X_{t+1}) = \beta_{cc} E_t(CC_{t+1}) \tag{1.6}$$

with β_{cc} the sensitivities of the payoff to the carbon tax. Of course, the payoff may also depend on other factors like macroeconomic conditions. But for the sake of simplicity, we will focus on the carbon tax example. In order to forecast the future payoff, analysts needs to forecast future carbon tax $E_t(CC_{t+1})$.

Suppose that the carbon tax CC_{t+1} is an AR(1) process:

$$CC_{t+1} = \phi CC_t + \tilde{CC}_{t+1} \tag{1.7}$$

with $\tilde{CC}_{t+1}$ a white noise process representing unexpected changes in the carbon tax. Again, $E_t(\tilde{CC}_{t+1}) = 0$ and $\sigma^2_t(\tilde{CC}_{t+1}) = \sigma^2_{\tilde{CC}}$. The expected value of the carbon tax at time $t+1$ is:

$$E_t(CC_{t+1}) = \phi CC_t \tag{1.8}$$

The analysts may for example think that the carbon tax is supposed to increase by 5%. For the sake of the illustration, we simulate the carbon tax process with $\phi = 1.05$ and $\sigma^2_{\tilde{CC}} = 0.25$ with 10 periods in figure 1.3 shows the carbon tax process.

Due to the possibility of unanticipated changes to the carbon tax, the realized payoff may also differ from the expected one:

$$X_{t+1} - E_t(X_{t+1}) = \beta_{CC} \tilde{CC}_{t+1} + \varepsilon_{t+1} \tag{1.9}$$

Therefore, we can say that carbon tax uncertainty adds uncertainty to the payoff, because:

- The highest the uncertainty in the carbon tax process ($\sigma^2_{\tilde{CC}}$), the more the realized payoff will vary around the expected payoff (see figure 1.4).
- The higher the sensitivity of the payoff to the carbon tax (β_{CC}), the more the realized payoff will vary around the expected payoff (see figure 1.5).

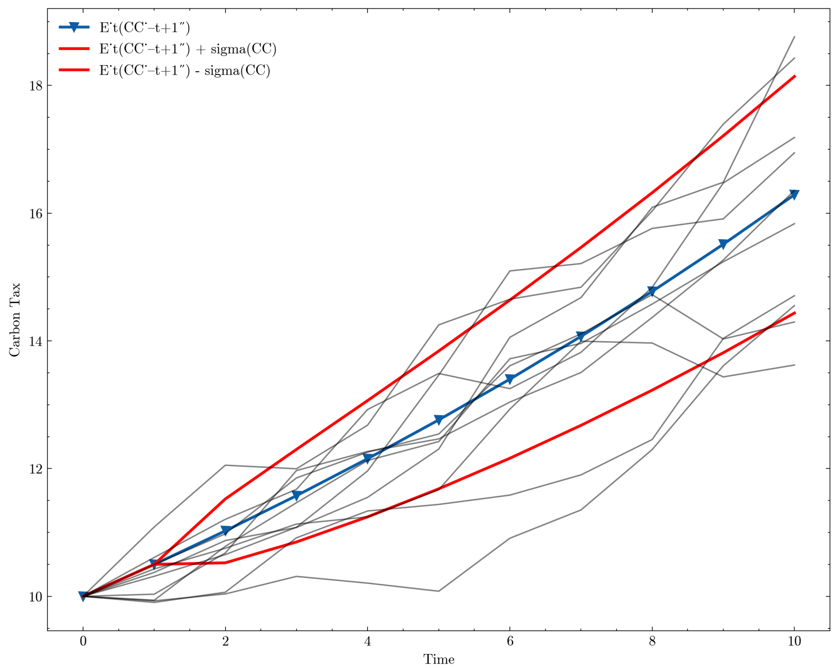

Figure 1.3: Simulation of the carbon tax process.

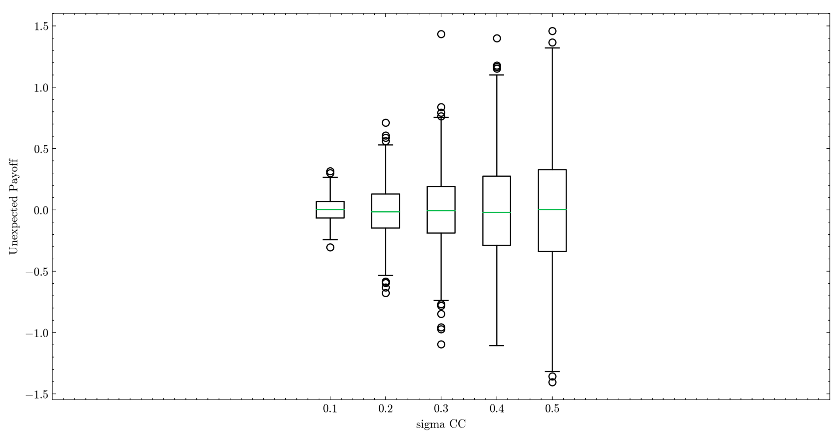

Figure 1.4: Simulation of the payoff process with different values for $\sigma^2_{\tilde{C}C}$.

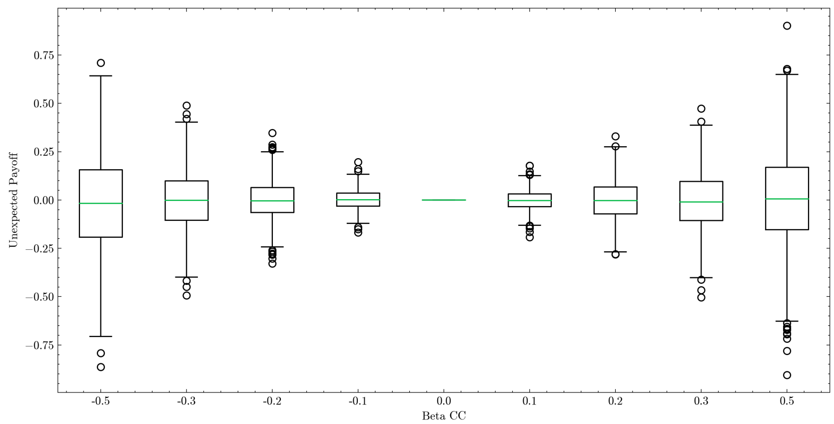

Figure 1.5: Simulation of the payoff process with different values for β_{CC}.

1.1.2 Discount Rate Channel

Now, how the stock is priced? The price of the stock is the discounted value of the cash-flows per dollar invested in the stock. The cash-flows are discounted according to a required rate of return for the firm:

$$P_{t+1} = \frac{X_{t+1}}{\rho_{t+1}} \tag{1.10}$$

If we assume that $\beta_m = 0$, we have a time-varying discount rate ρ_{t+1}:

$$\rho_{t+1} = 1 - \frac{\beta_{CC}}{\gamma} d_{t+1} \tag{1.11}$$

with d_{t+1} the perception of climate risks by the average investor at time $t+1$, and γ the constant risk aversion of the average investor.

The price in P_{t+1}, once the shock are realized at time $t+1$, will be:

$$P_{t+1} = \frac{X_{t+1}}{1 - \frac{\beta_{CC}}{\gamma} d_{t+1}} \tag{1.12}$$

Which can be approximated as:

$$P_{t+1} \approx X_{t+1} + \frac{\beta_{CC}}{\gamma} d_{t+1} \tag{1.13}$$

It's expected value at time t is:

$$E_t(P_{t+1}) = E_t(X_{t+1}) + \frac{\beta_{CC}}{\gamma} E_t(d_{t+1}) \tag{1.14}$$

And the unexpected return is:

$$R_{t+1} - E_t(R_{t+1}) = P_{t+1} - E_t(P_{t+1}) \tag{1.15}$$

because X_{t+1} is the payoff per dollar invested in the stock at time t, which corresponds to the definition of a return in a one-period model $R_{t+1} = \frac{CF_{t+1}}{P_t}$.

We now can substitute P_{t+1} and $E_t(P_{t+1})$ in the equation above:

$$R_{t+1} - E_t(R_{t+1}) = X_{t+1} + \frac{\beta_{CC}}{\gamma} d_{t+1} - E_t(X_{t+1}) - \frac{\beta_{CC}}{\gamma} E_t(d_{t+1}) \tag{1.16}$$

We recognize the unexpected payoff $X_{t+1} - E_t(X_{t+1})$:

$$R_{t+1} - E_t(R_{t+1}) = X_{t+1} - E_t(X_{t+1}) + \frac{\beta_{CC}}{\gamma} d_{t+1} - \frac{\beta_{CC}}{\gamma} E_t(d_{t+1}) \tag{1.17}$$

We can rewrite the unexpected payoff as a function of the shocks:

$$R_{t+1} - E_t(R_{t+1}) = \beta_m \tilde{M}_{t+1} + \beta_{CC} \tilde{CC}_{t+1} + \varepsilon_{t+1} + \frac{\beta_{CC}}{\gamma} d_{t+1} - \frac{\beta_{CC}}{\gamma} E_t(d_{t+1}) \tag{1.18}$$

Let's drop the macroeconomic component as we assumed that $\beta_m = 0$:

$$R_{t+1} - E_t(R_{t+1}) = \beta_{CC} \tilde{CC}_{t+1} + \varepsilon_{t+1} + \frac{\beta_{CC}}{\gamma} d_{t+1} - \frac{\beta_{CC}}{\gamma} E_t(d_{t+1}) \tag{1.19}$$

We can factorize the second part, with the unexpected change in the average perception of climate risks d:

$$R_{t+1} - E_t(R_{t+1}) = \beta_{CC} \tilde{CC}_{t+1} + \varepsilon_{t+1} + \frac{\beta_{CC}}{\gamma} (d_{t+1} - E_t(d_{t+1})) \tag{1.20}$$

We group together the terms related to the climate change:

$$R_{t+1} - E_t(R_{t+1}) = \beta_{CC}\tilde{CC}_{t+1} + \varepsilon_{t+1} + \beta_{CC}\left(\frac{1}{\gamma}(d_{t+1} - E_t(d_{t+1}))\right) \quad (1.21)$$

$$R_{t+1} - E_t(R_{t+1}) = \beta_{CC}\left(\tilde{CC}_{t+1} + \frac{1}{\gamma}(d_{t+1} - E_t(d_{t+1}))\right) + \varepsilon_{t+1} \quad (1.22)$$

What does it says? In our simple model, we have related the unexpected returns to the unexpected changes in climate risks. It can either comes from the realization of an unexpected climate event $\tilde{CC}_{t+1}$, or from the unexpected change in the average perception of climate risks $d_{t+1} - E_t(d_{t+1})$.

Now what drives the changes in asset prices? Ultimately, we are interested in the changes in asset prices, source of potential *returns*. The changes in expectations about future cash flows or discount rate. So, we are not so much interested in the level of expectations of climate risks, but in the changes in beliefs about climate risks.

Investors form expectations about climate risks in an horizon h with information available (those are conditional expectations). Each period, new information arrives and investors update their beliefs about climate risks:

$$\Delta E_t(CC_{t+h}) = E_t(CC_{t+h}) - E_{t-1}(CC_{t+h}) \quad (1.23)$$

with $\Delta E_t(CC_{t+h})$ the *innovation* or *news* in climate risk. On the other hand, we have the *unexpected* returns $\tilde{R}_t$:

$$\tilde{R}_t = R_t - E_{t-1}(R_t) \quad (1.24)$$

with R_t the *realized* returns and $E_{t-1}(R_t)$ the expected returns based on information available at time $t-1$. Because investors reprice assets based on the arrival of new information, we can expect that the innovation in climate risk is reflected in the unexpected returns:

$$\tilde{R}_t = \beta\Delta E_t(CC_{t+h}) + \varepsilon_t \quad (1.25)$$

with β non-null as long as we expect that changes in climate risks affect investors expectations about future cash flows or discount rate.

1.2 Measuring Climate Risks

We have seen that what matters for asset prices are expectations about climate risks. What matters about returns are the changes in expectations about climate risks.

For more traditional macroeconomic factors, creating a time series that capture expectations of these factors is not so difficult. You may use data from the central bank, the government, *etc.* They publish leading indicators, surveys, *etc.* The task is more challenging for climate risks.

A common approach in the literature (see Engle et al. (2020) [**?**]) is to use newspapers coverage of climate events as a proxy for the average investor's beliefs about climate risks. As they noted, when there are events that plausibly contains information about changes in climate risk, this will likely leads to newspaper coverage of these events. Newspapers may even be the direct source that investors use to update their beliefs about climate risk. Following the initial work from Engle et al. (2020) [**?**], researchers have developed a number of climate news series series capturing a variety of different climate risks. Each measure is signed such that a large number corresponds to "bad news".

1.3 Tag Index

1.4 Similarity Index

1.5 Conclusion

In what follow we propose a method to construct hedge portfolios with tradable assets that mimic the behavior of climate risks.

Chapter 2

Climate Risks Exposure

2.1 Mimicking Approach

Ross (1976) [?] introduced the concept of *arbitrage pricing theory* (APT). In this model, the expected return of an asset is a linear function of a set of risk factors. Famous examples of risk factors are the *Fama-French factors* (see Fama and French (1993) [?]). Those factors are the excess return of the market, the excess return of small cap stocks over big cap stocks and the excess return of high book-to-market stocks over low book-to-market stocks:

$$E(R_i) = \beta_m R_m + \beta_{smb} R_{smb} + \beta_{hml} R_{hml} \tag{2.1}$$

with $E(R_i)$ the expected return of asset i, R_m the excess return of the market, R_{smb} the excess return of small cap stocks over big cap stocks, R_{hml} the excess return of high book-to-market stocks over low book-to-market stocks, β_m the market beta of asset i, β_{smb} the size beta of asset i and β_{hml} the value beta of asset i. Those factors are tradable, as they are directly traded in financial markets (you can buy the market, small cap stocks and high book-to-market stocks and short sell the opposite side of the trade).

Macroeconomic factors are examples of *non-tradable factors* (think about inflation, industrial growth, *etc*). Economic conditions have pervasive effects on asset returns (see Flannery and Protopapadakis (2002) [?]). A standard way to tackle the problem of non-tradable factors is to use factor mimicking portfolios (FMPs), such as in Jurczenko and Teiletche (2022) [?]. That is, to construct a portfolio of tradable assets that mimics the behavior of non-tradable factors.

Climate risks are non-tradable factors, as they are not directly traded in financial markets (see Jurczenko and Teiletche (2023) [?]). We can use the same approach of FMPs to construct a portfolio of tradable assets that mimics the behavior of climate risks.

$$\Delta E_t(CC_{t+h}) = w^T \tilde{R}_t + \varepsilon_t \tag{2.2}$$

FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD ML macro FMPs vs underlying macro factors

2.2 Narrative Approach

Green Factor from Pastor

FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD ML macro FMPs vs underlying macro factors

2.3 Scenario-Based Approach

2.4 Risk Premia

Problem with short time series to infer risk premia

2.5 Conclusion

Chapter 3

Climate Risks Hedging

An investor might be seeking to hedge the climate risks to improve the risk-return profile of a portfolio.

3.1 Hedging a Fund with Climate Risk Hedging Portfolio

A practical way to would be to determine a combination of an existing portfolio p with, climate FMPs that minimizes the variance of the combined portfolio returns.

More precisely, let's assume that the investors determines a vector "tilt" ω that represents the weights of the FMPs in the combined portfolio.

The vector ω would be determined by:

$$\min_{\omega} \quad T^{-1} \sum_{t=1}^{T} (R_t^p - \omega^T H_t)^2 \tag{3.1}$$

3.2 Backtesting a Climate Risk Hedging Strategy

Figure 3 – Macro Risk Contributions

Figure 4 – Endowment portfolio and its macro-hedged version: Quarterly returns and Maximum Drawdowns