Climate Transition Risk and Asset Pricing

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Chapter 1

States of the World and Asset Pricing

Assets give a *payoff* x_{t+1}. In our focus on stocks, $x_{t+1} = p_{t+1} + d_{t+1}$, where p_{t+1} is the price of the stock at time $t+1$ and d_{t+1} is the dividend paid at time $t+1$. x_{t+1} is a random variable, like a coin-flip - we don't know at t what it will be at $t+1$. But we can assign probabilities to the possible outcomes of x_{t+1}. We can think of the *randomness* of x_{t+1} as being due to the randomness of the *state of the world* at $t+1$. x_{t+1} takes on different values in different *states of the world*. We have:

$$E(x_{t+1}) = \sum_s \pi(s)x(s) \tag{1.1}$$

where $E(x_{t+1})$ is the expected value of x_{t+1}, $\pi(s)$ is the probability of state s, and $x(s)$ is the value of x_{t+1} in state s.

The question we are trying to answer here is: what is the price or value p_t of the payoff x_{t+1} at time t?

1.1 Utility and Asset Pricing

We want to find the value of the payoff x_{t+1} at time t to an investor. We describe the investor's preferences using an *utility function*:

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}) \tag{1.2}$$

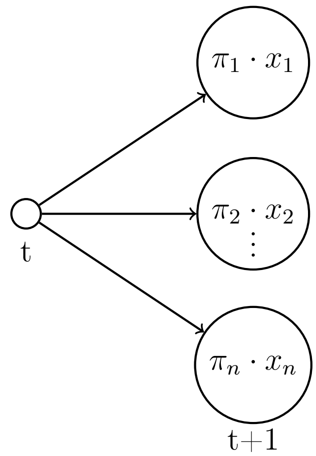

Figure 1.1: States of the world at time $t + 1$

where c_t is consumption at time t, c_{t+1} is consumption at time $t + 1$, $u(c_t)$ is the utility of consumption at time t. β is the *discount factor* that measures how much the investor values consumption at time $t + 1$ relative to consumption at time t.

The point of the utility function is to capture the investor's aversion to *risk* and *delay*, and discounting prices accordingly. Asset pricing depends on what are people willing to pay for. It depends on how impatient and risk averse they are. An example is the logarithmic utility function. In that case, if $u(c) = \log(c)$, then $u'(c) = 1/c$.

$u(c)$ is the level of utility from consumption. It can be described as the level of happiness of the investor. $u'(c)$ is the marginal utility of consumption. It can be roughly described as hunger of the investor. If $u'(c) > 0$ when $u(c)$ rises, it means that people always want more consumption. If $u'(c) < 0$ when $u(c)$ rises, then $u(c)$ is concave, and hunger decrease as you eat more. β is a number, typically 0.95. People prefer money now than later, they dislike delay. β captures their impatience. c_{t+1} is random. You don't know at time t how things will turn out, what c_{t+1} will be.

Example X

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A more useful utility function is the *power utility function*:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{1.3}$$

In that case, $u'(c) = c^{-\gamma}$, and γ is the *risk aversion* parameter. If $\gamma = 1$, then $u(c) = \log(c)$ and $u'(c) = 1/c$.

This functional form lets you have more or less curved function, that is more or less risk aversion.

Example X

power utility function and marginal utility function

1.2 Risk Valuation

To apply what we have learned so far to risk valuation, we can start from the definition of the covariance:

$$\text{Cov}(m, x) = E(mx) - E(m)E(x) \tag{1.4}$$

Thus,

$$p = E(mx) = \text{Cov}(m, x) + E(m)E(x) \tag{1.5}$$

$$p = \frac{1}{R^f}E(x) + \text{Cov}(m, x) \tag{1.6}$$

with the approximation

$$m_{t+1} \approx 1 - \delta - \gamma \Delta c_{t+1} \tag{1.7}$$

we have

$$p_t^i \approx \frac{1}{R^f}E(x_{t+1}^i) + \text{Cov}(x_{t+1}^i, \Delta c_{t+1}) \tag{1.8}$$

That is, the price is lower if the asset do well when consumption is low, and vice versa. Prices are higher for assets that provide insurance against consumption risks. Prices are low for assets that, if you buy them, make your consumption more risky.

In risk valuation, the covariance term really matters. The same m and the same x can have different prices depending on the covariance between m and x.

Example 1

Suppose there are two states u and d tomorrow, with probability 1/2 each. In state u, consumption is high, and in state d, consumption is low.

$$p_t = E(m, x) = \frac{1}{2} m_u x_u + \frac{1}{2} m_d x_d \tag{1.9}$$

Suppose x pays off well in good times if $x_u = 2$ and $x_d = 1$. Suppose also that $m_u = 0.5$ and $m_d = 1$. Then,

$$p_t = \frac{1}{2} \times 0.5 \times 2 + \frac{1}{2} \times 1 \times 1 = 1 \tag{1.10}$$

Now suppose we keep the same volatility but x pays off well in bad times and badly in good times, with $x_u = 1$ and $x_d = 2$.
Then,

$$p_t = \frac{1}{2} \times 0.5 \times 1 + \frac{1}{2} \times 1 \times 2 = 1.25 \tag{1.11}$$

The payoff is worth more in the second case because it pays off more in bad times, that is when m is high (hungry) rather than m is low (full). m acts like a price: it says that payoffs delivered in the bad state of nature d worth more than payoffs delivered in the good state of nature u.

In the previous part, we had the discount factor of the form:

$$p_t^i = \frac{E(x_{t+1}^i)}{ER^i} \tag{1.12}$$

where ER^i is the expected return on the asset. Our new version is:

$$p_t^i = E(m_{t+1} x_{t+1}^i) \tag{1.13}$$

where m_{t+1} is the stochastic discount factor. It is stochastic in the sense that it is unknown at time t, and therefore is inside the expectation. It is the same for all assets. Different covariance of m_{t+1} with x^i_{t+1} gives different risk adjustments for different assets.

1.3 Risk and Beta

In this section, we will see that $p = E(mx)$ implies:

$$E(R^{ei}) = -R^f \text{Cov}(R^{ei}, m) = \beta_{R^{ei},m}\lambda_m \tag{1.14}$$

where $\beta_{R^{ei},m}$ is the beta of the asset i with respect to m.

1.3.1 Expected Excess Returns and Covariance

We start with the fact that if *excess* returns or zero-cost portfolios have price 0, then, when excess returns $R^e = R^i - R^f$ is the payoff, $p = E(mx)$ implies:

$$0 = E_t(m_{t+1}R^e_{t+1}) \tag{1.15}$$

Using again the definition of the covariance and looking to obtain the betas:

$$\text{Cov}(m, x) = E(mx) - E(m)E(x) \tag{1.16}$$

$$E(mx) = \text{Cov}(m, x) + E(m)E(x) \tag{1.17}$$

Then

$$0 = E(mR^e) = E(m)E(R^e) + \text{Cov}(m, R^e) \tag{1.18}$$

$$E(m)E(R^e) = -\text{Cov}(m, R^e) \tag{1.19}$$

$$E(R^e) = -R^f \text{Cov}(m, R^e) \tag{1.20}$$

1.3.2 Betas Formulation

We can reformulate our previous result in terms of betas:

$$E(R^e) = \frac{\text{Cov}(m, R^e)}{var(m)}[-R^f var(m)] \tag{1.21}$$

$$E(R^e) = \beta_{R^e,m}\lambda_m \tag{1.22}$$

We can now link it directly to consumption. If $m_{t+1} = a - bf_{t+1}$ then:

$$E(R^e) = -R^f \text{Cov}(R^e, m) = R^f \times b \times cov(R^e, f) \tag{1.23}$$

$$E(R^e) = \frac{\text{Cov}(R^e, f)}{var(f)}[R^f \times b \times var(f)] = \beta_{R^e,f}\lambda_f \tag{1.24}$$

So, using $m_{t+1} \approx 1 - \delta - \gamma\Delta c_{t+1}$, we have:

$$E(R^e) \approx -\text{Cov}(1 - \delta - \gamma\Delta c_{t+1}, R^e) \approx \gamma\text{Cov}(\Delta c_{t+1}, R^e) \tag{1.25}$$

where we also have assumed that $R^f \approx 1$. We finally have:

$$E(R^e) \approx \beta_{R^e,\Delta c} \times \lambda_c \tag{1.26}$$

1.3.3 Interpreting Excess Return and Beta Relationship

In regression terms, it means that we can run time series regressions to find betas:

$$R^i_{t+1} = \alpha_i + \beta_{i,\Delta c}\Delta c_{t+1} + \epsilon^i_{t+1} \tag{1.27}$$

The average returns should indeed be linearly related to betas:

$$E(R^i) = R^f + \beta_{i,\Delta c}\lambda_c \tag{1.28}$$

where β is the right hand variable (the usual x) and λ is the slope (usually the β).

The superscript i emphasizes that this is about why average returns of one asset are higher than of another (cross section). This is not about the fluctuation in ex-post return or predicting returns. $E(R^i)$ is the reward, β_i

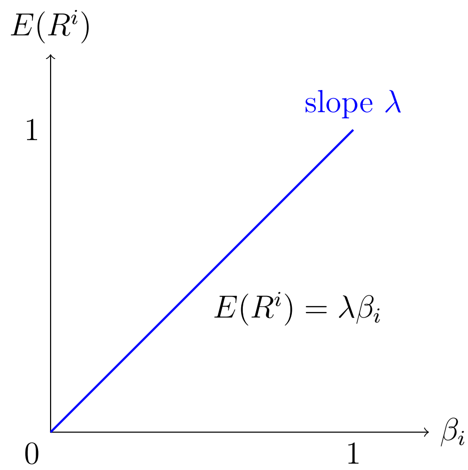

Figure 1.2: Expected Return and Beta Relationship

the quantity of risk that varies across assets i, and λ is the price of risk (it is common to all assets).

An asset must offer high $E(R^i)$ (reward) to compensate the investors for high β_i (risk). Assets that covary negatively with m, hence positively with consumption growth, must pay a higher average return.

Example X

Given a certain amount of volatility (price must move at some point), price (risk-discount) depends on when good/bad performance occurs. Average returns are high if beta on m or Δc is large. Stocks must pay high returns if they tend to go down in bad times. Price is depressed if a payoff is low in bad times, when the investor is hungry (high m, low Δc), therefore we have high $E(R^i)$. Price is high if a payoff is high in bad times, when the investor is full (low m, high Δc). Therefore we have low $E(R^i)$. Higher γ (risk aversion) implies larger price effects.

The variance $\sigma(R^e)$ of an individual asset does not matter, only its covariance with m (*e.g.* consumption growth) matters. Recall that we have the following:

$$R^{ei} = \beta_{i,m} m + \epsilon^i \tag{1.29}$$

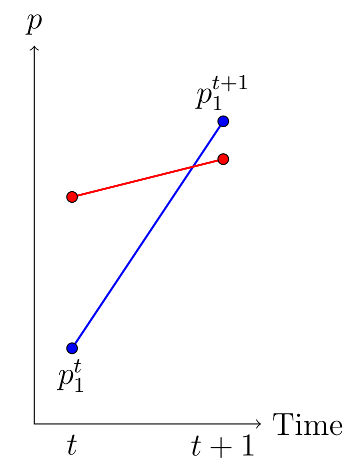

Figure 1.3: High $E(R^i)$ = Low p^i

Therefore the variance is:

$$var(R^{ei}) = \beta_{i,m}^2 \sigma_m^2 + \sigma_{\epsilon^i}^2 \tag{1.30}$$

and the second component of variance has no effect on mean returns.

1.4 CAPM and Multifactor models

We can relate what we have seen so far to the CAPM and Fama-French model. This underlines the economic rationale for $E(hml)$, $E(smb)$, and so forth.

To do so, we start from:

$$E(R^{ei}) = R^f \mathrm{Cov}(R^{ei}, m) = \beta_{R^{ei},m} \lambda_m \tag{1.31}$$

We find reasons to say:

$$m = a - b \times f \tag{1.32}$$

to get

$$E(R^{ei}) = \beta_{R^{ei},f} \lambda_f \tag{1.33}$$

with the full algebra to be:

$$E(R^{ei}) = R^f \mathrm{Cov}(R^{ei}, f \times b) = R^f \mathrm{Cov}(R^{ei}, f) \times b = \frac{\mathrm{Cov}(R^{ei}, f)}{var(f)} \times [R^f \times b \times var(f)] \tag{1.34}$$

The intuition is to say that if low f indicates bad times, when people are hungry, then assets which pay off badly in times of low f must have low prices and deliver high expected returns.

1.5 Conclusion

What we have seen here is the beginning of all asset pricing models. In the empirical version, we'll see that we use risk factors instead of Δc.

Chapter 2

Climate Transition Risk and Asset Pricing

Uncertainty about states of world with climate risks is a key driver of asset prices. Uncertainty about the states of the world with climate risks can be thought with the uncertainty about the climate scenarios.

2.1 Climate Transition Risk Dynamic

The evolution of aggregate consumption growth Δc_{t+1} is described as:

$$\Delta c_{t+1} = \mu + x_t + J_{t+1} \tag{2.1}$$

where μ is the unconditional average, x_t represents the time-varying expected consumption growth and J_{t+1} is the climate transition shock to the economy. It can be referred to the abrupt introduction of a carbon tax or a sudden shift in climate policy.

Example X

Sample Paths: Trend Growth and Above-Trend Growth

The expected consumption growth is:

$$x_{t+1} = \mu_x + \rho x_t + \phi J_{t+1} \tag{2.2}$$

with a persistence parameter ρ and innovations also driven by the shock J_{t+1}. The parameter ϕ captures the way the climate shock J_{t+1} affects future path of consumption growth.

Example X

Sample Paths: Above-Trend Growth,With andWithout a Climate Transition Shock

In any period, the transition shock J_{t+1} can take the value $-\varepsilon \in (0, 1)$ with probability λ_t or 0 with probability $(1 - \lambda_t)$. The probability λ_t is itself time-varying, with the dynamics:

$$\lambda_{t+1} = \mu_\lambda + \alpha\lambda_t + \eta x_t + \xi J_{t+1} \tag{2.3}$$

in addition to an autoregressive term, it depends on lagged measures of economic activity (x_t) as well as on the current shock J_{t+1}.

Example X

Time-varying transition risk

We think about climate transition risk in terms of a relatively low-probability catastrophic event that could dramatically impact the economy.

The parameter λ_t captures the conditional probability of such an abrupt shift occurring and ε captures the magnitude of the shock.

The occurence of the abrupt low-carbon shift also affects the future path of the economy, since it directly affects the expected consumption growth x_t. Specifically, when $\phi > 0$, the transition shock reduces not only consumption immediately but also future expected consumption growth.

Example X

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When instead, $\phi < 0$, there is a partial mean reversion after a transition shock. This case has an especially interesting interpretation when modelling climate transition: it captures the ability of the economy to shift towards a low-carbon economy.

Example X

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Through the parameter ξ, a transition realization also affects future climate transition risk λ_t.

Example X

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Finally, the model allows for feedback effects between climate change and the economy. Climate risks affect the economy (when the climate disaster occurs) and the economy affects climate risks through the effects of x_t on λ_{t+1} (modulated through the parameter η): when economic activity is high, climate risk increases. But when the climate shock materializes, consumption is low.

Example X

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2.2 Climate Transition Risk Valuation

Once the physical process of climate change is modeled, one need to choose an utility function for the representative investor. It implies a stochastic discount factor (SDF) that prices assets in the economy, at equilibrium.

When the uncertainty emanates directly from the climate process itself, climate damage tends to be unexpectedly high in times when consumption is low because climate disaster realization are a primary driver of reduced consumption.

2.3 Climate Transition Risk Beta and Excess Returns

Assets that are positively exposed to climate risk - that is, assets with low payoffs when climate damages are high - thus tend to require positive risk premia. On the other hand, assets that are negatively exposed to climate risk - assets that payoffs primarily when climate are realized - tend to require negative risk premia since these assets provide an insurance against bad (high marginal utility) states of the world.

DERIVATION OF BETAS

Example X

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2.4 Conclusion

We've focused on climate transition risk, but same approach can be applied to physical risk.

Chapter 3

Empirical Methods for Asset Pricing

Chapter 4

The Narrative Approach

Chapter 5

Mimicking Portfolio

Chapter 6

Quantity-Based Approach