Climate Risks

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Contents

Introduction

Chapter 1

From Climate Change Uncertainty to Climate Change Ambiguity

1.1 Population Modelling

The scientific discipline of demography produces detailed population projections based assumptions about the future trend in fertilty, mortality and migration.

1.1.1 Key Dimensions of Population Modelling

Two important reasons to go beyond population size. Human population is not homogenous. And this heterogeneity is important for future growth of the population. Population growth is a direct function of the age and sex structure of the population.

1.1.2 Educational Fertility Differential

Total Fertility Rate (TFR)

Education-Specific TFR (ESTFR)

Age-Specific Fertility Rate (ASFR)

Education-Specific Age-Specific Fertility Rate (ESASFR)

Parametric model: specify ASFR relative to a reference distribution. The reference or standard ASFR can be described by a Gompertz function. This is an s-shaped function similar to a logistic function, but it is asymmetric, with faster growth at the beginning and slower growth at the end.

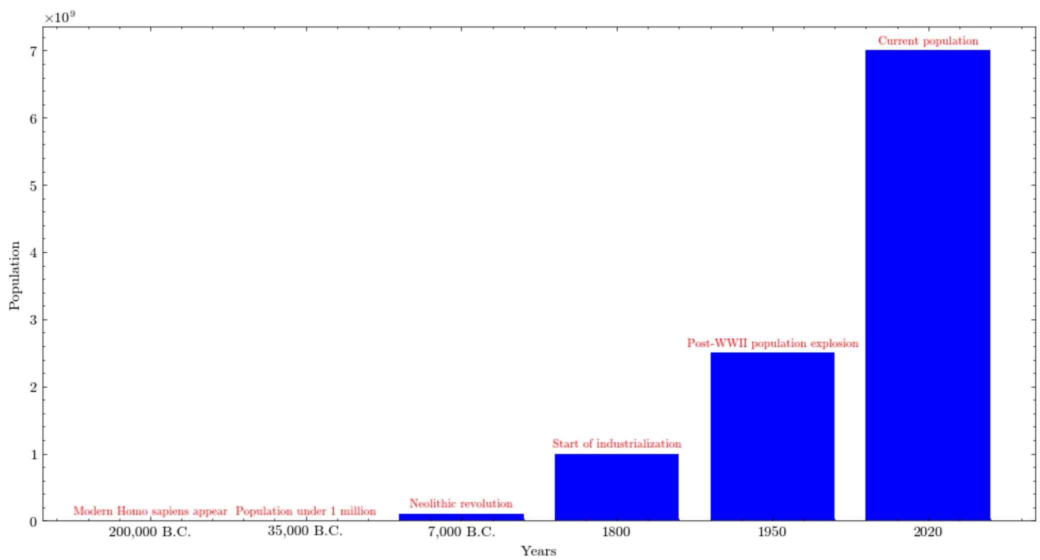

Figure 1.1: History of Human Population Growth

Age refer to the transformed age. The Gompit is defined as:

$$Y(x) = -log(-log(x)) \tag{1.1}$$

With $F(x)$ the cumulative distribution of the ASFR at age x:

$$F(x) = \int_0^x \frac{ASFR(y)}{TFR} dy \tag{1.2}$$

1.1.3 Educational Mortality Differential

1.1.4 Education Scenarios

Chapter 2

Climate Risk

2.1 Expected Utility and Optimal Portfolio

As with ESG preferences only, we start by setting up the utility function of an investor who cares about climate risk, and then derive the optimal portfolio

2.1.1 Investor's Expected Utility

Let $\tilde{C}_1$ denote climate at time 1, which is unknown at time 0. The investor utility function is now:

$$V(\tilde{W}_{1,i}, X_i, \tilde{C}_1) = -\exp\left(-A_i\tilde{W}_{1,i} - b_i^T X_i - c_i\tilde{C}_1\right) \tag{2.1}$$

where c_i is the investor's climate risk sensitivity.

Taking the expectation of the utility function from period 0, we get:

$$E_0(V(\tilde{W}_{1,i}, X_i, \tilde{C}_1)) = E_0(-\exp\left(-A_iW_{0,i} - b_i^T X_i - c_i\tilde{C}_1\right)) \tag{2.2}$$

Again, we can replace $\tilde{W}_{1,i}$ with the relation $\tilde{W}_{1,i} = W_{0,i}(1 + r_f + X_i^T\tilde{r}_1)$ and define $a_i := A_iW_{0,i}$. We still want to make out from the expectation the terms that we know about in period 0, and reexpress the terms with the expectation as a function of the portfolio weights X_i.

$$\begin{aligned}
E_0(V(\tilde{W}_{1,i}, X_i, \tilde{C}_1)) &= E_0(-\exp(-A_i W_{0,i} - b_i^T X_i - c_i \tilde{C}_1)) \\
&= E_0(-\exp(-a_i(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i - c_i \tilde{C}_1)) \\
&= -\exp(-a_i(1 + r_f))E_0(-\exp(-a_i X_i^T \tilde{r}_1 - b_i^T X_i - c_i \tilde{C}_1)) \\
&= -\exp(-a_i(1 + r_f))E_0(-\exp(-a_i X_i^T (\tilde{r}_1 + \frac{b_i}{a_i}) - c_i \tilde{C}_1)) \\
&= -\exp(-a_i(1 + r_f)) - \exp(a_i X_i^T (E_0(\tilde{r}_1) + \frac{b_i}{a_i})+ \\
&\frac{1}{2}a_i^2 X_i^T \mathrm{Var}(\tilde{\epsilon}_1) X_i + a_i c_i X_i^T \mathrm{Cov}(\tilde{\epsilon}_1, \tilde{C}_1) + \frac{1}{2}c_i^2 \mathrm{Var}(\tilde{C}_1)) \\
&= -\exp(-a_i(1 + r_f)) - \exp(-a_i X_i^T (\mu + \frac{b_i}{a_i})+ \\
&\frac{1}{2}a_i^2 X_i^T \Sigma X_i + a_i c_i X_i^T \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} + \frac{1}{2}c_i^2 \sigma^2_{\tilde{C}_1})
\end{aligned} \tag{2.3}$$

where $\sigma_{\tilde{\epsilon}_1, \tilde{C}_1} = \mathrm{Cov}(\tilde{\epsilon}_1, \tilde{C}_1)$.

2.1.2 Optimal Portfolio

Again, the investor i seeks to maximize its expected utility, by choosing the optimal portfolio weights X_i at time 0. We need to find the first order conditions for the optimization problem.

We are going to follow the same steps as in the previous chapter.

1. We combine the exponential terms:

$$\begin{aligned}
E_0(V(\tilde{W}_1, X_i, \tilde{C}_1)) = -\exp(-a_i(1 + r_f) - a_i X_i^T (\mu + \frac{b_i}{a_i})+ \\
\frac{1}{2}a_i^2 X_i^T \Sigma X_i + a_i c_i X_i^T \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} + \frac{1}{2}c_i^2 \sigma^2_{\tilde{C}_1})
\end{aligned} \tag{2.4}$$

   and let $f(X_i)$ denotes the exponent:

$$E_0(V(\tilde{W}_1, X_i, \tilde{C}_1)) = -\exp(f(X_i)) \tag{2.5}$$

2. To differentiate $f(X_i)$ with respect to X_i, we use the chain rule $\frac{\partial h}{\partial X_i} = \frac{\partial h}{\partial f}\frac{\partial f}{\partial X_i}$. If $h = -\exp(f)$, then $\frac{\partial h}{\partial f} = -\exp(f)$. Thus:

$$\frac{\partial h}{\partial X_i} = -\exp(f)\frac{\partial f}{\partial X_i} \tag{2.6}$$

3. We can again use the rules that $\frac{\partial x^T b}{\partial x} = b$ and $\frac{\partial x^T A x}{\partial x} = 2Ax$:
$$\frac{\partial f}{\partial X_i} = -a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i + a_i c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} \tag{2.7}$$
Combining:
$$\frac{\partial h}{\partial X_i} = -\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i + a_i c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) \tag{2.8}$$
4. We set the derivative to zero:
$$\begin{aligned} -\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i + a_i c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) = 0 \\ -a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i + a_i c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} = 0 \end{aligned} \tag{2.9}$$
because the exponential term is always positive.
5. We solve for X_i:
$$\begin{aligned} a_i^2 \Sigma X_i &= a_i(\mu + \frac{b_i}{a_i} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) \\ a_i \Sigma X_i &= \mu + \frac{b_i}{a_i} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} \\ \Sigma X_i &= \frac{1}{a_i}(\mu + \frac{b_i}{a_i} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) \\ X_i &= \frac{1}{a_i}\Sigma^{-1}(\mu + \frac{b_i}{a_i} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) \end{aligned} \tag{2.10}$$

Again, we will assume that $a_i = a$ for all investors:
$$X_i = \frac{1}{a}\Sigma^{-1}(\mu + \frac{b_i}{a} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) \tag{2.11}$$

2.2 Heterogeneous Climate Risk Expectations, Market Portfolio and Expected Returns

2.2.1 Heterogeneous Climate Risk Expectations and Market Portfolio

We follow the same process than in the previous chapter, now including differences in expectations c_i about climate risk $\tilde{C}_1$.

The nth elements of investor i's portfolio weight vector X_i is still:

$$X_{i,n} = \frac{W_{0,i,n}}{W_{0,i}} \tag{2.12}$$

The total wealth invested in stock n at time 0:

$$W_{0,i,n} := \int_i W_{0,i,n} di \tag{2.13}$$

The nth element of the market portfolio weight vector w_m is:

$$w_{m,n} = \frac{W_{0,m,n}}{W_{0,m}} \tag{2.14}$$

We reexpress $W_{0,n}$ in terms of individual investors' wealth by using the definition of $W_{0,n}$:

$$w_{m,n} = \frac{1}{W_0} \int_i W_{0,i,n} di \tag{2.15}$$

with $W_{0,i,n} = W_{0,i} X_{i,n}$, we can rewrite the equation:

$$\begin{aligned} w_{m,n} &= \frac{1}{W_0} \int_i W_{0,i} X_{i,n} di \\ &= \int_i \frac{W_{0,i}}{W_0} X_{i,n} di \\ &= \int_i \omega_i X_{i,n} di \end{aligned} \tag{2.16}$$

We now plug the optimal portfolio weights X_i we have found in the previous section into the equation above to obtain the market weights w_m:

$$\begin{aligned} w_{m,n} = \int_i \omega_i \frac{1}{a} \Sigma^{-1} (\mu + \frac{b_i}{a} - c_i \sigma_{\tilde{\epsilon}_1, \tilde{C}_1}) di \\ = \frac{1}{a} \Sigma^{-1} \mu (\int_i \omega_i di) + \frac{1}{a^2} \Sigma^{-1} g (\int_i \omega_i d_i di) - \frac{1}{a} \Sigma^{-1} \sigma_{\tilde{\epsilon}_1, \tilde{C}_1} (\int_i \omega_i c_i di) \end{aligned} \tag{2.17}$$

We have $\int_i \omega_i di = 1$ and $\int_i \omega_i c_i di := \bar{c} \geq 0$, the wealth-weighted average expectation about climate risk across investors. The market portfolio weights are:

$$\begin{aligned} w_m &= \frac{1}{a}\Sigma^{-1}(\mu + \frac{g}{a}\bar{d} - \bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}) \\ &= \frac{1}{a}\Sigma^{-1}\mu + \frac{g}{a^2}\Sigma^{-1}\bar{d} - \frac{1}{a}\Sigma^{-1}\bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \end{aligned} \tag{2.18}$$

2.2.2 Market Portfolio Expected Returns

Starting from the vector of marekt weights w_m, we now can solver for μ the vector of expected returns:

$$\begin{aligned} w_m &= \frac{1}{a}\Sigma^{-1}\mu + \frac{g}{a^2}\Sigma^{-1}\bar{d} - \frac{1}{a}\Sigma^{-1}\bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\ aw_m &= \Sigma^{-1}\mu + \frac{g}{a}\Sigma^{-1}\bar{d} - \Sigma^{-1}\bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\ aw_m - \frac{g}{a}\Sigma^{-1}\bar{d} + \Sigma^{-1}\bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} &= \Sigma^{-1}\mu \\ \Sigma(aw_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}) &= \mu \\ \mu = a\Sigma w_m - \frac{g}{a}\Sigma\Sigma^{-1}\bar{d} &+ \bar{c}\Sigma\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\ \mu = a\Sigma w_m - \frac{g}{a}\bar{d} &+ \bar{c}\Sigma\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \end{aligned} \tag{2.19}$$

Multiplying by w_m, we find the market equity premium ($\mu_m = w_m^T\mu$):

$$\begin{aligned} \mu_m &= aw_m^T\Sigma w_m - \frac{g}{a}w_m^T\bar{d} + \bar{c}w_m^T\Sigma\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\ &= aw_m^T\Sigma w_m + \bar{c}w_m^T\Sigma\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\ &= a\sigma_m^2 + \bar{c}\sigma_{mC} \end{aligned} \tag{2.20}$$

where we still maintain the assumption of an ESG-neutral market portfolio ($w_m^Tg = 0$), and we have the market portfolio variance $\sigma_m^2 = w_m^T\Sigma w_m$ and $w_m^T\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} = \mathrm{Cov}(\tilde{\epsilon}_m, \tilde{C}_1) = \sigma_{mC}$.

2.2.3 Expected Returns with Climate Risk

We use the last equation in the previous section to solve for a:

$$\begin{aligned} \mu_m &= a\sigma_m^2 + \bar{c}\sigma_{mC} \\ a &= \frac{\mu_m - \bar{c}\sigma_{mC}}{\sigma_m^2} \end{aligned} \tag{2.21}$$

Then, the expected excess returns can be reexpressed as:

$$\begin{aligned} \mu &= a\Sigma w_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{mC} \\ &= \frac{\mu_m - \bar{c}\sigma_{mC}}{\sigma_m^2}\Sigma w_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{mC} \end{aligned} \tag{2.22}$$

We know that $\frac{1}{\sigma_m^2}\Sigma w_m = \beta_m$, the market beta:

$$\begin{aligned} \mu &= \frac{\mu_m - \bar{c}\sigma_{mC}}{\sigma_m^2}\Sigma w_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{mC} \\ &= (\mu_m - \bar{c}\sigma_{mC})\beta_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{mC} \\ &= \mu_m\beta_m - \bar{c}\sigma_{mC}\beta_m - \frac{g}{a}\bar{d} + \bar{c}\sigma_{mC} \\ &= \mu_m\beta_m - \frac{g}{a}\bar{d} + \bar{c}(\sigma_{mC} - \beta_m\sigma_{mC}) \end{aligned} \tag{2.23}$$

We know that $\beta_m = (\frac{1}{\sigma_m^2}\sigma_{\tilde{\epsilon}_1,m})$:

$$\begin{aligned} \mu &= \mu_m\beta_m - \frac{g}{a}\bar{d} + \bar{c}(\sigma_{mC} - \beta_m\sigma_{mC}) \\ &= \mu_m\beta_m - \frac{g}{a}\bar{d} + \bar{c}(\sigma_{mC} - \frac{1}{\sigma_m^2}\sigma_{\tilde{\epsilon}_1,m}\sigma_{m,C}) \end{aligned} \tag{2.24}$$

In the multivariate regression of $\tilde{\epsilon}_1$ on $\tilde{\epsilon}_m$ and $\tilde{C}_1$, the slope coefficients are given by:

$$\begin{aligned} &\begin{bmatrix}\sigma_{\tilde{\epsilon}_1,m} & \sigma_{\tilde{\epsilon}_1,C}\end{bmatrix}\begin{bmatrix}\sigma_m^2 & \sigma_{m,C} \\ \sigma_{m,C} & \sigma_C^2\end{bmatrix}^{-1} \\ &= \frac{1}{\sigma_m^2\sigma_C^2 - \sigma_{mC}^2}\begin{bmatrix}\sigma_C^2\sigma_{\tilde{\epsilon}_1,m} - \sigma_{mC}\sigma_{\tilde{\epsilon}_1,C} & \sigma_m^2\sigma_{\tilde{\epsilon}_1,C} - \sigma_{mC}\sigma_{\tilde{\epsilon}_1,m}\end{bmatrix} \end{aligned} \tag{2.25}$$

So the second column (the coefficient of $\tilde{C}_1$) is:

$$\psi = \frac{1}{\sigma_m^2\sigma_C^2 - \sigma_{mC}^2}(\sigma_m^2\sigma_{\tilde{\epsilon}_1,C} - \sigma_{mC}\sigma_{\tilde{\epsilon}_1,m}) \tag{2.26}$$

We can use ψ to rewrite the expected returns:

$$\begin{aligned}\mu &= \mu_m\beta_m - \frac{g}{a}\bar{d} + \bar{c}(\sigma_{mC} - \frac{1}{\sigma_m^2}\sigma_{\tilde{\epsilon}_1,m}\sigma_{m,C}) \\ &= \mu_m\beta_m - \frac{g}{a}\bar{d} + \bar{c}\frac{\sigma_m^2\sigma_C^2 - \sigma_{mC}^2}{\sigma_m^2}\psi \\ &= \mu_m\beta_m - \frac{\bar{d}}{a}g + \bar{c}(1-\rho_{mC}^2)\psi\end{aligned} \tag{2.27}$$

recalling that $\sigma_C = 1$.

We have our next proposition:

Proposition X. *Expected excess returns in equilibrium are given by:*

$$\mu = \mu_m\beta_m - \frac{\bar{d}}{a}g + \bar{c}(1-\rho_{mC}^2)\psi \tag{2.28}$$

where ψ is the $N \times 1$ vector of climate betas (slope coefficients on $\tilde{C}_1$ in a multivariate regressions of $\tilde{\epsilon}_1$ on $\tilde{\epsilon}_m$ and $\tilde{C}_1$), and ρ_{mC} is the correlation between $\tilde{\epsilon}_m$ and $\tilde{C}_1$.

Expected returns depend on climate betas, ψ, which represent firms' exposures to non-market climate risk. Recall $\tilde{\epsilon}_1$ is the vector of unexpected returns, and $\tilde{\epsilon}_m$ is the market unexpected return. A firm climate beta ψ_n therefore measures its loading on $\tilde{C}_1$, after controlling for the market return.

Corrolary X. *Stock n's climate beta ψ_n enter expected returns positively. Thus, a stock with a negative ψ_n that provides investors with a climate-risk hedge, has a lower expected return than it would in the absence of climate risk. Vice versa, a stock with a positive ψ_n, which performs particularly poorly when the climate worsens unexpectedly, has a higher expected return.*

Corrolary X. *Because the vector of stocks' CAPM alphas is defined as $\alpha := \mu - \mu_m\beta_m$, we have:*

$$\alpha_n = -\frac{\bar{d}}{a}g + \bar{c}(1-\rho_{mC}^2)\psi \tag{2.29}$$

With the assumption that $\bar{c} > 0$, stocks with positive ψ_n have positive alphas, and stocks with negative ψ_n have negative alphas.

2.3 Climate Risk Hedging Portfolio

2.3.1 Hedging Portfolio Tilt

We now want to reexpress the investor's optimal portfolio weights X_i in terms of the ESG characteristics and climate betas.

We can plug excess returns $\mu = \mu_m\beta_m - \frac{\bar{d}}{a}g + \bar{c}(1-\rho^2_{mC})\psi$ into the portfolio weight:

$$
\begin{aligned}
X_i &= \frac{1}{a}\Sigma^{-1}(\mu + \frac{b_i}{a} - c_i\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}) \\
&= \frac{1}{a}\Sigma^{-1}(\mu_m\beta_m - \frac{\bar{d}}{a}g + \bar{c}(\sigma_{mC} - \frac{1}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1,m}\sigma_{m,C})\psi + \frac{b_i}{a} - c_i\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}) \\
&= \frac{\mu_m}{a}\Sigma^{-1}\beta_m - \frac{\bar{d}}{a^2}\Sigma^{-1}g + \frac{\bar{c}}{a}\Sigma^{-1}(\sigma_{mC} - \frac{1}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1,m}\sigma_{m,C})\psi + \frac{d_i}{a^2}g\Sigma^{-1} - \frac{c_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\
&= \frac{\mu_m}{a}\Sigma^{-1}\beta_m - \frac{1}{a}\Sigma^{-1}\bar{c}\frac{\sigma_{mC}}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1 m} + \frac{1}{a}\Sigma^{-1}(\frac{d_i}{a}g - \frac{\bar{d}}{a}g) - \frac{1}{a}\Sigma^{-1}(c_i - \bar{c})\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\
&= \frac{\mu_m}{a}\Sigma^{-1}\beta_m - \frac{1}{a}\Sigma^{-1}\bar{c}\frac{\sigma_{mC}}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1 m} + \frac{1}{a}\Sigma^{-1}\frac{\delta_i}{a}g - \frac{c_i - \bar{c}}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\
&= \frac{\mu_m}{a}\Sigma^{-1}\beta_m - \frac{1}{a}\Sigma^{-1}\bar{c}\frac{\sigma_{mC}}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1 m} + \frac{1}{a}\Sigma^{-1}\frac{\delta_i}{a}g - \frac{\gamma_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}
\end{aligned}
\tag{2.30}
$$

with $\gamma_i = c_i - \bar{c}$ and $\delta_i = d_i - \bar{d}$.

From the market expected return $\mu_m = a\sigma^2_m + \bar{c}\sigma_{mC}$, we note that $\bar{c}\sigma_{mC} = \mu_m - a\sigma^2_m$. We also note that $\beta_m = \frac{1}{\sigma^2_m}\sigma_{\tilde{\epsilon}_1,m} = \frac{1}{\sigma^2_m}\Sigma w_m$. We can rewrite the portfolio weight as:

$$
\begin{aligned}
X_i = \frac{\mu_m}{a}\Sigma^{-1}\beta_m - \frac{1}{a}\Sigma^{-1}(\mu_m - a\sigma^2_m)\beta_m + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{\gamma_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\
= \sigma^2_m\Sigma_{-1}\beta_m + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{\gamma_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \\
= w_m + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{\gamma_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}
\end{aligned}
\tag{2.31}
$$

which leads to our next proposition:

Proposition X. *Investor i's equilibrium portfolio weights on the N stocks are given by:*

$$X_i = w_m + \frac{\delta_i}{a^2}\Sigma^{-1}g - \frac{\gamma_i}{a}\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1} \tag{2.32}$$

where $\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}$ is the vector of covariances between unexpected returns and climate risk.

The fourth fund is now a climate risk hedging portfolio, whose weights are proportional to $\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}$. Investors with $\delta_i > 0$, whose climate risk expectation is higher than the market average, will short the hedging portfolio, whereas investors with $\delta_i < 0$ will go long on the hedging portfolio.

The climate hedging portfolio, $\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}$, is a natural mimicking portfolio for $\tilde{C}_1$. Indeed, note that the N elements of $\Sigma^{-1}\sigma_{\tilde{\epsilon}_1,\tilde{C}_1}$ are the slopes of the multivariate regression of $\tilde{C}_1$ on $\tilde{\epsilon}_1$. Therefore, the return on the hedging portfolio has the highest correlation with $\tilde{C}_1$, among all portfolios of the N stocks. Investors in this model hold this maximum-correlation portfolio, to various degree, determined by their γ_i, to hedge climate risk.