

Climate Risk Hedging

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Contents

Introduction	v
1 Risk Premium	1
1.1 Capital Asset Pricing Model (CAPM)	1
1.2 ESG Preferences	1
1.3 Climate Risk	6
2 Sources of Risk	7
2.1 Market Risk	7
2.2 ESG Factor Risk	7
2.3 Climate Risk	7

Introduction

Chapter 1

Risk Premium

1.1 Capital Asset Pricing Model (CAPM)

1.2 ESG Preferences

1.2.1 Expected Utility and Optimal Portfolio

Setting the Investor's Expected Utility

Let's assume a single period model, from $t = 0$ to $t = 1$. We have N stocks.

We have a $N \times 1$ vector of returns \tilde{r}_1 at period 1, assumed to be normally distributed:

$$\tilde{r}_1 = \mu + \tilde{\epsilon}_1 \quad (1.1)$$

with μ the equilibrium expected excess returns and $\tilde{\epsilon}_1$ the random component of the returns $\tilde{\epsilon}_1 \sim N(0, \Sigma)$.

The investor i has an exponential CARA utility function, with $\tilde{W}_{1,i}$ the wealth at period 1, and X_i the $N \times 1$ vector of portfolio weights.

$$V(\tilde{W}_{1,i}, X_i) = -\exp(-A_i \tilde{W}_{1,i} - b_i^T X_i) \quad (1.2)$$

with A_i agent's absolute risk aversion, b_i an $N \times 1$ vector of nonpecuniary benefits.

PLACEHOLDER

Figure 1.1: Efficient Frontier

$$b_i = d_i g \quad (1.3)$$

with g an $N \times 1$ vector and $d_i \geq 0$ a scalar measuring the agent's taste for the nonpecuniary benefits.

The expectation of agent i 's in period 0 are:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = E_0(-\exp(-A_i \tilde{W}_{1,i} - b_i^T X_i)) \quad (1.4)$$

We can replace $\tilde{W}_{1,i}$ by the relation $\tilde{W}_{1,i} = W_{0,i}(1 + r_f + X_i^T \tilde{r}_1)$ and define $a_i := A_i W_{0,i}$. The idea is to make out from the expectation the terms that we know about (in period 0), and reexpress the terms within the expectation as a function of the portfolio weights X_i . The last two steps use the fact that \tilde{r}_1 is normally distributed with mean μ and variance Σ .

$$\begin{aligned} E_0(V(\tilde{W}_{1,i}, X_i)) &= E_0(-\exp(-A_i W_{0,i}(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)) \\ &= E_0(-\exp(-a_i(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)) \\ &= E_0(-\exp(-a_i(1 + r_f) - a_i X_i^T \tilde{r}_1 - b_i^T X_i)) \\ &= -\exp(-a_i(1 + r_f)) E_0(-\exp(-a_i X_i^T \tilde{r}_1 - b_i^T X_i)) \\ &= -\exp(-a_i(1 + r_f)) E_0(-\exp(-a_i X_i^T (\tilde{r}_1 + \frac{b_i}{a_i}))) \quad (1.5) \\ &= -\exp(-a_i(1 + r_f)) \exp(-a_i X_i^T (E_0(\tilde{r}_1) + \frac{b_i}{a_i}) + \frac{1}{2} a_i^2 X_i^T \text{Var}(\tilde{r}_1) X_i) \\ &= -\exp(-a_i(1 + r_f)) \exp(-a_i X_i^T (\mu + \frac{b_i}{a_i}) + \frac{1}{2} a_i^2 X_i^T \Sigma X_i) \end{aligned}$$

Solving for the Investor's Optimal Portfolio

The investors choose their optimal portfolios at time 0. The optimal portfolio X_i is the one that maximizes the expected utility. To find it, we differentiate the expected utility with respect to X_i and set it to zero, to obtain the first-order condition.

We are going to do it step by step:

1. Combine the Exponential Terms:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp(-a_i(1 + r_f) - a_i X_i^T (\mu + \frac{b_i}{a_i}) + \frac{1}{2} a_i^2 X_i^T \Sigma X_i) \quad (1.6)$$

and let $f(X_i)$ be the exponent:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp f(X_i) \quad (1.7)$$

2. Differentiate $f(X_i)$ with respect to X_i . We have the chain rule:

$$\frac{\partial h}{\partial X_i} = \frac{\partial h}{\partial f} \frac{\partial f}{\partial X_i} \quad (1.8)$$

If $h = -\exp(f)$, then $\frac{\partial h}{\partial f} = -\exp(f)$. Therefore we have:

$$\frac{\partial h}{\partial X_i} = -\exp(f) \frac{\partial f}{\partial X_i} \quad (1.9)$$

To tackle the derivative of $f(X_i)$, we use two rules. First $\frac{\partial x^T b}{\partial x} = b$ and $\frac{\partial x^T A x}{\partial x} = 2Ax$ if A is symmetric. We have:

$$\frac{\partial f}{\partial X_i} = -a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i \quad (1.10)$$

Combining:

$$\frac{\partial h}{\partial X_i} = -\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i) \quad (1.11)$$

3. Set the derivative to zero:

$$\begin{aligned} -\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i) &= 0 \\ -a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i &= 0 \end{aligned} \quad (1.12)$$

where the exponential term is always positive, so we can drop it.

4. Rearrange and solve for X_i :

$$\begin{aligned} a_i^2 \Sigma X_i &= a_i(\mu + \frac{b_i}{a_i}) \\ a_i \Sigma X_i &= \mu + \frac{b_i}{a_i} \\ \Sigma X_i &= \frac{1}{a_i}(\mu + \frac{b_i}{a_i}) \\ X_i &= \frac{1}{a_i} \Sigma^{-1}(\mu + \frac{b_i}{a_i}) \end{aligned} \quad (1.13)$$

PLACEHOLDER

Figure 1.2: Efficient Frontier with ESG Preferences

For the sake of simplicity, we assume that $a_i = a$ for all investors. We now have:

$$\begin{aligned} X_i &= \frac{1}{a} \Sigma^{-1} \left(\mu + \frac{b_i}{a} \right) \\ &= \frac{1}{a} \Sigma^{-1} \left(\mu + \frac{d_i}{a} g \right) \end{aligned} \quad (1.14)$$

Therefore, the optimal portfolio differs across investors due to the ESG characteristics g of the stocks and the investors' taste for nonpecuniary benefits d_i .

1.2.2 Heterogeneous Investors and Expected Returns

Heterogeneous Market

The n th element of investor i 's portfolio weight vector X_i is:

$$X_{i,n} = \frac{W_{0,i,n}}{W_{0,i}} \quad (1.15)$$

with $W_{0,i,n}$ the wealth invested in stock n by investor i at time 0.

The total wealth invested in stock n at time 0 is:

$$W_{0,n} := \int_i W_{0,i,n} di \quad (1.16)$$

The n th element of the market-weight vector w_m is:

$$w_{m,n} = \frac{W_{0,n}}{W_0} \quad (1.17)$$

We can now express $W_{0,n}$ in terms of individual investors' wealths by using the definition of $W_{0,n}$:

$$w_{m,n} = \frac{1}{W_0} \int_i W_{0,i,n} di \quad (1.18)$$

We now that $W_{0,i,n} = W_{0,i} X_{i,n}$, so we can rewrite the equation:

$$w_{m,n} = \frac{1}{W_0} \int_i W_{0,i} X_{i,n} di \quad (1.19)$$

Defining $\omega_i = \frac{W_{0,i}}{W_0}$, we have:

$$\begin{aligned} w_{m,n} &= \int_i \frac{W_{0,i}}{W_0} X_{i,n} di \\ &= \int_i \omega_i X_{i,n} di \end{aligned} \quad (1.20)$$

We can now plug in X_i to obtain w_m the vector of market weights:

$$\begin{aligned} w_m &= \int_i \omega_i X_i di \\ &= \int_i \omega_i \frac{1}{a} \Sigma^{-1} \left(\mu + \frac{d_i}{a} g \right)_n di \\ &= \frac{1}{a} \Sigma^{-1} \mu \left(\int_i \omega_i di \right) + \frac{1}{a^2} \Sigma^{-1} g \left(\int_i \omega_i d_i di \right) \end{aligned} \quad (1.21)$$

We have $\int_i \omega_i di = 1$ and we define $\bar{d} := \int_i d_i di \geq 0$, the wealth-weighted mean of ESG tastes d_i across agents. Therefore:

$$w_m = \frac{1}{a} \Sigma^{-1} \mu + \frac{1}{a^2} \Sigma^{-1} g \bar{d} \quad (1.22)$$

Expected Returns

Starting from the the vector of market weights w_m , we now can solve for μ the vector of expected returns. We have:

PLACEHOLDER

Figure 1.3: μ_m and $w_m^T g$ relationship.

$$\begin{aligned}
w_m &= \frac{1}{a} \Sigma^{-1} \mu + \frac{1}{a^2} \Sigma^{-1} g \bar{d} \\
aw_m &= \Sigma^{-1} \mu + \frac{1}{a} \Sigma^{-1} g \bar{d} \\
aw_m - \frac{1}{a} \Sigma^{-1} g \bar{d} &= \Sigma^{-1} \mu \\
\Sigma(aw_m - \frac{1}{a} \Sigma^{-1} g \bar{d}) &= \mu \\
\mu &= a \Sigma w_m - \frac{1}{a} \Sigma \Sigma^{-1} g \bar{d} \\
\mu &= a \Sigma w_m - \frac{1}{a} g \bar{d}
\end{aligned} \tag{1.23}$$

Multiplying by w_m , we find the market equity premium $\mu_m = w_m^T \mu$:

$$\begin{aligned}
\mu_m &= aw_m^T \Sigma w_m - \frac{\bar{d}}{a} w_m^T g \\
&= a \sigma_m^2 - \frac{\bar{d}}{a} w_m^T g
\end{aligned} \tag{1.24}$$

where $\sigma_m^2 = w_m^T \Sigma w_m$ is the market return variance.

The equity premium μ_m depends on the average of ESG tastes, \bar{d} , through the "greenness" of the market portfolio $w_m^T g$. If the market is net green (i.e., $w_m^T g > 0$), then stronger ESG tastes (higher \bar{d}) lead to lower equity premium.

Conversely, if the market is net "brown" ($w_m^T g < 0$), then stronger ESG tastes lead to higher equity premium as investors demand compensation for holding brown stocks.

Expected Excess Returns

1.2.3 ESG Portfolio

Portfolio Tilts

Factor Pricing with the ESG Portfolio

1.3 Climate Risk

Chapter 2

Sources of Risk

2.1 Market Risk

2.2 ESG Factor Risk

2.3 Climate Risk

