Climate Risk Hedging

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Introduction

Chapter 1

Factor Mimicking Portfolios

Ross (1976) [?] introduced the concept of *arbitrage pricing theory* (APT). In this model, the expected return of an asset is a linear function of a set of risk factors. Famous examples of risk factors are the *Fama-French factors* (see Fama and French (1993) [?]). Those factors are the excess return of the market, the excess return of small cap stocks over big cap stocks and the excess return of high book-to-market stocks over low book-to-market stocks:

$$E(R_i) = \beta_m R_m + \beta_{smb} R_{smb} + \beta_{hml} R_{hml} \tag{1.1}$$

with $E(R_i)$ the expected return of asset i, R_m the excess return of the market, R_{smb} the excess return of small cap stocks over big cap stocks, R_{hml} the excess return of high book-to-market stocks over low book-to-market stocks, β_m the market beta of asset i, β_{smb} the size beta of asset i and β_{hml} the value beta of asset i. Those factors are tradable, as they are directly traded in financial markets (you can buy the market, small cap stocks and high book-to-market stocks and short sell the opposite side of the trade).

Macroeconomic factors are examples of *non-tradable factors* (think about inflation, industrial growth, *etc*). Economic conditions have pervasive effects on asset returns (see Flannery and Protopapadakis (2002) [?]). A standard way to tackle the problem of non-tradable factors is to use factor mimicking portfolios (FMPs), such as in Jurczenko and Teiletche (2022) [?]. That is, to construct a portfolio of tradable assets that mimics the behavior of non-tradable factors.

In some sense, climate risks are non-tradable factors, as they are not directly traded in financial markets (see Jurczenko and Teiletche (2023) [?]).

We can use the same approach of FMPs to construct a portfolio of tradable assets that mimics the behavior of climate risks.

1.1 Climate News

1.1.1 Climate Risks

A first step is to identify the factors of climate risks. If we go back to the example of macroeconomic factors as non-tradable factors, we can think about expectations of inflation, industrial growth, *etc.* Why a non-tradable factor could have an impact on asset prices?

To get an intuition, we can start from the definition of returns in a one period (*ie.* asset lasts only one period):

$$R_{t+1} = \frac{D_{t+1}}{P_t} \tag{1.2}$$

with R_{t+1} the return of the asset from time t to time $t+1$, D_{t+1} the dividend paid at time $t+1$ and P_t the price of the asset at time t. There is no P_{t+1} in the equation, as we are in a one period model and the stock doesn't exist anymore at time $t+1$. Take the expectations:

$$E_t(R_{t+1}) = \frac{E_t(D_{t+1})}{P_t} \tag{1.3}$$

And solve for P_t:

$$P_t = \frac{E_t(D_{t+1})}{E_t(R_{t+1})} \tag{1.4}$$

These formula represents the price of the asset at time t as the discounted value of future dividends. Dividends are used as a proxy for future cash flows. The idea is: asset prices are determined by expectations of future cash flows or discount rate. Therefore, if a non-tradable factor affects expectations of future cash flows or discount rate, it will have an impact on asset prices.

The idea is quite immediately intuitive for macro factors: it is easy to see how industrial growth could affect future cash flows or discount rate. But what about climate risks?

1.1.2 Measuring Climate Risks

1.1.3 Innovation in Climate Risks

We have a vector of K *non-tradable* factors F_{t+h}, with h the forecast horizon. Investors form expectations of these factors adjust their expectations through time, based on new information or "surprise":

$$\tilde{F}_{t+h} = F_{t+h} - \phi \tag{1.5}$$

with ϕ the $K \times 1$ vector of expected factors. $\tilde{F}_{t+h}$ is the *innovation* in the factors of risks.

1.2 Climate News and Unexpected Returns

On the other hand, we have the *unexpected* returns $\tilde{R}_t$:

$$\tilde{R}_t = R_t - \mu \tag{1.6}$$

The main assumption behind factor mimicking portfolios is that the innovation $\tilde{F_{t+h}}$ is reflected in the unexpected returns $\tilde{R}_t$:

$$\tilde{R}_t = B\tilde{F}_{t+h} + \varepsilon_t \tag{1.7}$$

where B is a $N \times K$ matrix of factor loadings, ε_t is a $N \times 1$ vector of mean zero disturbances.

It means that investors reprice assets (unexpected returns $\tilde{R}_t$) based on the arrival of new information on the factors of risks (innovation $\tilde{F}_{t+h}$).

1.3 Linear Factor Model

If:

$$R_t = \mu + \tilde{R}_t \tag{1.8}$$

Then, substituting $\tilde{R}_t$, we have the following factor model:

$$R_t = \mu + B\tilde{F}_{t+h} + \varepsilon_t \tag{1.9}$$

with R_t a $N \times 1$ vector of asset returns, μ a $N \times 1$ vector of expected returns, B a $N \times K$ matrix of factor loadings, F_{t+h} a $K \times 1$ vector of factor innovations and ε_t a $N \times 1$ vector of mean zero disturbances.

1.4 Factor Mimicking

The vector of weights w_k is the solution to the following optimization problem:

$$\begin{aligned} \min_{w_k} \quad & \frac{1}{2} w_k^T \Sigma w_k \\ \text{subject to} \quad & B^T w_k = \beta_k \end{aligned} \tag{1.10}$$

where B is the $N \times K$ matrix of factor loadings, β_k is the $K \times 1$ vector of factor exposures, with the k-th element equal to 1 and the other elements equal to $\beta_{k,l}$, and Σ is the $N \times N$ covariance matrix of asset returns.

We can form the Lagrangian:

$$\mathcal{L}(w_k, \lambda) = \frac{1}{2} w_k^T \Sigma w_k - \lambda_k^T (B^T w_k - \beta_k) \tag{1.11}$$

where λ_k is the $K \times 1$ vector of Lagrange multipliers.

The first order condition is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_k} &= \Sigma w_k - B\lambda = 0 \\ \Rightarrow w_k &= \Sigma^{-1} B \lambda_k \end{aligned} \tag{1.12}$$

Substituting w_k in the constraint, we have:

$$\begin{aligned} B^T w_k &= \beta_k \\ B^T \Sigma^{-1} B \lambda_k &= \beta_k \\ \Rightarrow \lambda_k &= (B^T \Sigma^{-1} B)^{-1} \beta_k \end{aligned} \tag{1.13}$$

Substituting λ_k in w_k, we finally have the solution to the optimization problem:

$$w_k^* = \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} \beta_k \tag{1.14}$$

Taking together all the K factors, we have the matrix of weights W:

$$W = \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} B_K \tag{1.15}$$

where B_K is the $K \times K$ matrix with the k-th column equal to β_k and the other columns equal to $\beta_{k,l}$.

1.5 Climate Risk Premia

Careful about conclusion on risk premia based on time average of returns on the hedging portfolio.

1.6 Conclusion

In what follows, we will focus on the case of climate risk factors. First stage is to identify how to measure climate risk factors.

Chapter 2

Climate Risk Mimicking Porfolios

Two main approaches of FMPs have been proposed in the literature: (i) the two-pass cross-sectional regression (Fama and MacBeth, 1973) and (ii) the maximum correlation portfolio (MCP) (Huberman et al, 1987).

It is possible to recover both approaches with the equation in the chapter 1:

$$W = \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} B_K \tag{2.1}$$

2.1 Two-Pass Fama-MacBeth

In the case of the two-pass Fama-MacBeth, assets are uncorrelated and have constant variance.

$$\Sigma = \sigma^2 I_N \tag{2.2}$$

where σ^2 is the variance of the asset returns.

B is multivariate (i.e., $K > 1$) and the target exposure is:

$$B_K = I_K \tag{2.3}$$

That is, we have a *beta* of one to the k-th factor and zero to the others.

Substituting Σ and B_K in the equation (3.1), we have:

WHY $\sigma^2 I_N$ and I_K cancels out?

$$\begin{aligned} W &= \sigma^2 I_N B (B^T B)^{-1} I_K \\ &= B (B^T B)^{-1} \end{aligned} \tag{2.4}$$

FMP composition as estimated by different methods
FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD

2.2 Maximum Correlation Portfolio

We have the Target-Beta MCP, where B is univariate (i.e., $K = 1$) and the target exposure is:

$$B_K = B^T \Sigma^{-1} B \tag{2.5}$$

Substituting B_K in the equation (3.1), we have:
FIND THE INTERMEDIARY STEPS

$$\begin{aligned} W &= \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} B \\ &= \Sigma^{-1} B \end{aligned} \tag{2.6}$$

FMP composition as estimated by different methods
FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD

Chapter 3

Practical Use: Hedging Climate Risk for a Fund

An investor might be seeking to hedge the climate risks to improve the risk-return profile of a portfolio.

3.1 Hedging a Portfolio with FMPs

A practical way to would be to determine a combination of an existing portfolio p with, climate FMPs that minimizes the variance of the combined portfolio returns.

More precisely, let's assume that the investors determines a vector "tilt" ω that represents the weights of the FMPs in the combined portfolio.

The vector ω would be determined by:

$$\min_{\omega} \quad T^{-1} \sum_{t=1}^{T} (R_t^p - \omega^T FMP_t)^2 \tag{3.1}$$

3.2 Backtesting a Climate Risk Hedging Strategy

Figure 3 – Macro Risk Contributions

Figure 4 – Endowment portfolio and its macro-hedged version: Quarterly returns and Maximum Drawdowns

Chapter 4

Conclusion

More generally can be applied to other ESG risks. See biodiversity risk from Giglio et al.