Climate Risk Hedging

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Introduction

Chapter 1

ESG Risk Premium

1.1 ESG Risk

Let's assume a single period model, from $t = 0$ to $t = 1$. We have N stocks.

The investor i has an exponential CARA utility function, with $\tilde{W}_{1,i}$ the wealth at period 1, and X_i the $N \times 1$ vector of portfolio weights.

$$V(\tilde{W}_{1,i}, X_i) = -\exp\left(-A_i \tilde{W}_{1,i} - b_i^T X_i\right) \tag{1.1}$$

with A_i agent's absolute risk aversion, b_i an $N \times 1$ vector of nonpecuniary benefits.

$$b_i = d_i g \tag{1.2}$$

with g an $N \times 1$ vector and $d_i \geq 0$ a scalar measuring the agent's taste for the nonpecuniary benefits.

To derive the first-order condition for X_i, we compute the expectation of agent i's in period 0:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = E_0(-\exp\left(-A_i \tilde{W}_{1,i} - b_i^T X_i\right)) \tag{1.3}$$

We can replace $\tilde{W}_{1,i}$ by the relation $\tilde{W}_{1,i} = W_{0,i}(1 + r_f + X_i^T \tilde{r}_1)$ and define $a_i := A_i W_{0,i}$. The idea is to make out from the expectation the terms that we know about (in period 0), and reexpress the terms within the expectation as a function of the portfolio weights X_i. The last two steps use the fact that $\tilde{r}_1 \sim N(\mu, \Sigma)$.

$$\begin{aligned}
E_0(V(\tilde{W}_{1,i}, X_i)) &= E_0(-\exp(-A_iW_{0,i}(1+r_f+X_i^T\tilde{r}_1) - b_i^TX_i)) \\
&= E_0(-\exp(-a_i(1+r_f+X_i^T\tilde{r}_1) - b_i^TX_i)) \\
&= E_0(-\exp(-a_i(1+r_f) - a_iX_i^T\tilde{r}_1 - b_i^TX_i)) \\
&= -\exp(-a_i(1+r_f))E_0(-\exp(-a_iX_i^T\tilde{r}_1 - b_i^TX_i)) \\
&= -\exp(-a_i(1+r_f))E_0(-\exp(-a_iX_i^T(\tilde{r}_1 + \frac{b_i}{a_i}))) \\
&= -\exp(-a_i(1+r_f))\exp(-a_iX_i^T(E_0(\tilde{r}_1) + \frac{b_i}{a_i}) + \frac{1}{2}a_i^2X_i^T\mathrm{Var}(\tilde{r})X_i) \\
&= -\exp(-a_i(1+r_f))\exp(-a_iX_i^T(\mu + \frac{b_i}{a_i}) + \frac{1}{2}a_i^2X_i^T\Sigma X_i)
\end{aligned} \tag{1.4}$$

1.2 Climate Risk