# Climate Risk and Asset Pricing

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# Contents

**1 States of the World and Asset Pricing** **1**
- 1.1 Utility and Asset Pricing . . . 1
- 1.2 Risk Valuation . . . 3
- 1.3 Risk and Beta . . . 5
  - 1.3.1 Expected Excess Returns and Covariance . . . 5
  - 1.3.2 Betas Formulation . . . 5
  - 1.3.3 Interpreting Excess Return and Beta Relationship . . . 6
- 1.4 CAPM and Multifactor models . . . 8
- 1.5 Conclusion . . . 9

**2 Climate Risk** **11**
- 2.1 Brown / Green Assets . . . 12
- 2.2 Climate Risk and Utility . . . 12
- 2.3 Climate Betas and Expected Returns . . . 12

**3 Risk Factors: Empirical Methods** **13**

**4 The Green Factor** **15**

**5 Time-Varying Risk Premia** **17**

**6 Chasing the Climate Risk Factor** **19**

# Chapter 1

# States of the World and Asset Pricing

Assets give a *payoff* $x_{t+1}$. In our focus on stocks, $x_{t+1} = p_{t+1} + d_{t+1}$, where $p_{t+1}$ is the price of the stock at time $t+1$ and $d_{t+1}$ is the dividend paid at time $t+1$. $x_{t+1}$ is a random variable, like a coin-flip - we don't know at $t$ what it will be at $t+1$. But we can assign probabilities to the possible outcomes of $x_{t+1}$. We can think of the *randomness* of $x_{t+1}$ as being due to the randomness of the *state of the world* at $t+1$. $x_{t+1}$ takes on different values in different *states of the world*. We have:

$$E(x_{t+1}) = \sum_s \pi(s)x(s) \tag{1.1}$$

where $E(x_{t+1})$ is the expected value of $x_{t+1}$, $\pi(s)$ is the probability of state $s$, and $x(s)$ is the value of $x_{t+1}$ in state $s$.

The question we are trying to answer here is: what is the price or value $p_t$ of the payoff $x_{t+1}$ at time $t$?

## 1.1 Utility and Asset Pricing

We want to find the value of the payoff $x_{t+1}$ at time $t$ to an investor. We describe the investor's preferences using an *utility function*:

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}) \tag{1.2}$$

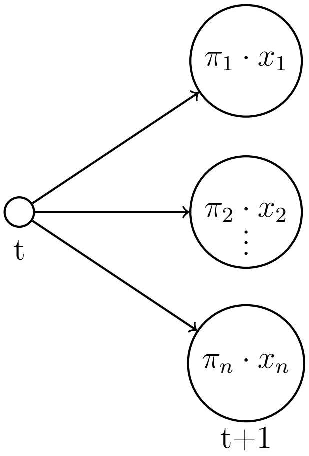

Figure 1.1: States of the world at time $t + 1$

where $c_t$ is consumption at time $t$, $c_{t+1}$ is consumption at time $t + 1$, $u(c_t)$ is the utility of consumption at time $t$. $\beta$ is the *discount factor* that measures how much the investor values consumption at time $t + 1$ relative to consumption at time $t$.

The point of the utility function is to capture the investor's aversion to *risk* and *delay*, and discounting prices accordingly. Asset pricing depends on what are people willing to pay for. It depends on how impatient and risk averse they are. An example is the logarithmic utility function. In that case, if $u(c) = \log(c)$, then $u'(c) = 1/c$.

$u(c)$ is the level of utility from consumption. It can be described as the level of happiness of the investor. $u'(c)$ is the marginal utility of consumption. It can be roughly described as hunger of the investor. If $u'(c) > 0$ when $u(c)$ rises, it means that people always want more consumption. If $u'(c) < 0$ when $u(c)$ rises, then $u(c)$ is concave, and hunger decrease as you eat more. $\beta$ is a number, typically 0.95. People prefer money now than later, they dislike delay. $\beta$ captures their impatience. $c_{t+1}$ is random. You don't know at time $t$ how things will turn out, what $c_{t+1}$ will be.

**Example X**

xxx

## 1.2 Risk Valuation

To apply what we have learned so far to risk valuation, we can start from the definition of the covariance:

$$\mathrm{Cov}(m,x) = E(mx) - E(m)E(x) \tag{1.3}$$

Thus,

$$p = E(mx) = \mathrm{Cov}(m,x) + E(m)E(x) \tag{1.4}$$

$$p = \frac{1}{R^f}E(x) + \mathrm{Cov}(m,x) \tag{1.5}$$

with the approximation

$$m_{t+1} \approx 1 - \delta - \gamma \Delta c_{t+1} \tag{1.6}$$

we have

$$p_t^i \approx \frac{1}{R^f}E(x_{t+1}^i) + \mathrm{Cov}(x_{t+1}^i, \Delta c_{t+1}) \tag{1.7}$$

That is, the price is lower if the asset do well when consumption is low, and vice versa. Prices are higher for assets that provide insurance against consumption risks. Prices are low for assets that, if you buy them, make your consumption more risky.

In risk valuation, the covariance term really matters. The same $m$ and the same $x$ can have different prices depending on the covariance between $m$ and $x$.

**Example 1**

Suppose there are two states $u$ and $d$ tomorrow, with probability 1/2 each. In state $u$, consumption is high, and in state $d$, consumption is low.

$$p_t = E(m, x) = \frac{1}{2} m_u x_u + \frac{1}{2} m_d x_d \tag{1.8}$$

Suppose $x$ pays off well in good times if $x_u = 2$ and $x_d = 1$. Suppose also that $m_u = 0.5$ and $m_d = 1$. Then,

$$p_t = \frac{1}{2} \times 0.5 \times 2 + \frac{1}{2} \times 1 \times 1 = 1 \tag{1.9}$$

Now suppose we keep the same volatility but $x$ pays off well in bad times and badly in good times, with $x_u = 1$ and $x_d = 2$.
Then,

$$p_t = \frac{1}{2} \times 0.5 \times 1 + \frac{1}{2} \times 1 \times 2 = 1.25 \tag{1.10}$$

The payoff is worth more in the second case because it pays off more in bad times, that is when $m$ is high (hungry) rather than $m$ is low (full). $m$ acts like a price: it says that payoffs delivered in the bad state of nature $d$ worth more than payoffs delivered in the good state of nature $u$.

In the previous part, we had the discount factor of the form:

$$p_t^i = \frac{E(x_{t+1}^i)}{ER^i} \tag{1.11}$$

where $ER^i$ is the expected return on the asset. Our new version is:

$$p_t^i = E(m_{t+1} x_{t+1}^i) \tag{1.12}$$

where $m_{t+1}$ is the stochastic discount factor. It is stochastic in the sense that it is unknown at time $t$, and therefore is inside the expectation. It is the same for all assets. Different covariance of $m_{t+1}$ with $x_{t+1}^i$ gives different risk adjustments for different assets.

## 1.3 Risk and Beta

In this section, we will see that $p = E(mx)$ implies:

$$E(R^{ei}) = -R^f \mathrm{Cov}(R^{ei}, m) = \beta_{R^{ei},m}\lambda_m \tag{1.13}$$

where $\beta_{R^{ei},m}$ is the beta of the asset $i$ with respect to $m$.

### 1.3.1 Expected Excess Returns and Covariance

We start with the fact that if *excess* returns or zero-cost portfolios have price 0, then, when excess returns $R^e = R^i - R^f$ is the payoff, $p = E(mx)$ implies:

$$0 = E_t(m_{t+1}R^e_{t+1}) \tag{1.14}$$

Using again the definition of the covariance and looking to obtain the betas:

$$\mathrm{Cov}(m, x) = E(mx) - E(m)E(x) \tag{1.15}$$

$$E(mx) = \mathrm{Cov}(m, x) + E(m)E(x) \tag{1.16}$$

Then

$$0 = E(mR^e) = E(m)E(R^e) + \mathrm{Cov}(m, R^e) \tag{1.17}$$

$$E(m)E(R^e) = -\mathrm{Cov}(m, R^e) \tag{1.18}$$

$$E(R^e) = -R^f \mathrm{Cov}(m, R^e) \tag{1.19}$$

### 1.3.2 Betas Formulation

We can reformulate our previous result in terms of betas:

$$E(R^e) = \frac{\mathrm{Cov}(m, R^e)}{var(m)}[-R^f var(m)] \tag{1.20}$$

$$E(R^e) = \beta_{R^e,m}\lambda_m \tag{1.21}$$

We can now link it directly to consumption. If $m_{t+1} = a - bf_{t+1}$ then:

$$E(R^e) = -R^f \text{Cov}(R^e, m) = R^f \times b \times cov(R^e, f) \tag{1.22}$$

$$E(R^e) = \frac{\text{Cov}(R^e, f)}{var(f)}[R^f \times b \times var(f)] = \beta_{R^e,f}\lambda_f \tag{1.23}$$

So, using $m_{t+1} \approx 1 - \delta - \gamma\Delta c_{t+1}$, we have:

$$E(R^e) \approx -\text{Cov}(1 - \delta - \gamma\Delta c_{t+1}, R^e) \approx \gamma\text{Cov}(\Delta c_{t+1}, R^e) \tag{1.24}$$

where we also have assumed that $R^f \approx 1$. We finally have:

$$E(R^e) \approx \beta_{R^e,\Delta c} \times \lambda_c \tag{1.25}$$

### 1.3.3 Interpreting Excess Return and Beta Relationship

In regression terms, it means that we can run time series regressions to find betas:

$$R^i_{t+1} = \alpha_i + \beta_{i,\Delta c}\Delta c_{t+1} + \epsilon^i_{t+1} \tag{1.26}$$

The average returns should indeed be linearly related to betas:

$$E(R^i) = R^f + \beta_{i,\Delta c}\lambda_c \tag{1.27}$$

where $\beta$ is the right hand variable (the usual $x$) and $\lambda$ is the slope (usually the $\beta$).

The superscript $i$ emphasizes that this is about why average returns of one asset are higher than of another (cross section). This is not about the fluctuation in ex-post return or predicting returns. $E(R^i)$ is the reward, $\beta_i$ the quantity of risk that varies across assets $i$, and $\lambda$ is the price of risk (it is common to all assets).

An asset must offer high $E(R^i)$ (reward) to compensate the investors for high $\beta_i$ (risk). Assets that covary negatively with $m$, hence positively with consumption growth, must pay a higher average return.

**Example X**

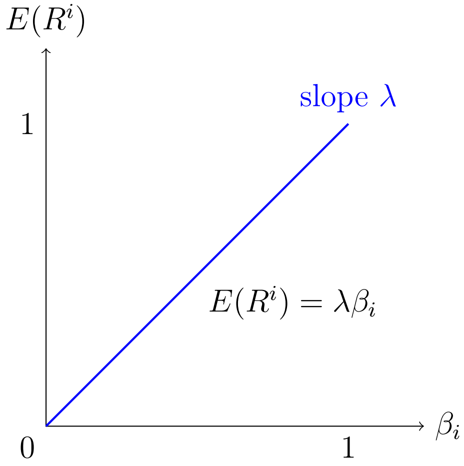

Figure 1.2: Expected Return and Beta Relationship

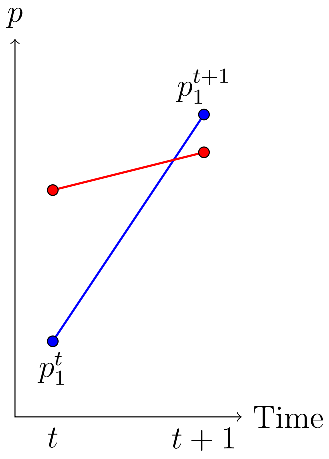

Figure 1.3: High $E(R^i)$ = Low $p^i$

Given a certain amount of volatility (price must move at some point), price (risk-discount) depends on when good/bad performance occurs. Average returns are high if beta on $m$ or $\Delta c$ is large. Stocks must pay high returns if they tend to go down in bad times. Price is depressed if a payoff is low in bad times, when the investor is hungry (high $m$, low $\Delta c$), therefore we have high $E(R^i)$. Price is high if a payoff is high in bad times, when the investor is full (low $m$, high $\Delta c$). Therefore we have low $E(R^i)$. Higher $\gamma$ (risk aversion) implies larger price effects.

The variance $\sigma(R^e)$ of an individual asset does not matter, only its covariance with $m$ (*e.g.* consumption growth) matters. Recall that we have the following:

$$R^{ei} = \beta_{i,m} m + \epsilon^i \tag{1.28}$$

Therefore the variance is:

$$var(R^{ei}) = \beta_{i,m}^2 \sigma_m^2 + \sigma_{\epsilon^i}^2 \tag{1.29}$$

and the second component of variance has no effect on mean returns.

## 1.4 CAPM and Multifactor models

We can relate what we have seen so far to the CAPM and Fama-French model. This underlines the economic rationale for $E(hml)$, $E(smb)$, and so forth.

To do so, we start from:

$$E(R^{ei}) = R^f \mathrm{Cov}(R^{ei}, m) = \beta_{R^{ei},m} \lambda_m \tag{1.30}$$

We find reasons to say:

$$m = a - b \times f \tag{1.31}$$

to get

$$E(R^{ei}) = \beta_{R^{ei},f} \lambda_f \tag{1.32}$$

with the full algebra to be:

$$E(R^{ei}) = R^f \mathrm{Cov}(R^{ei}, f \times b) = R^f \mathrm{Cov}(R^{ei}, f) \times b = \frac{\mathrm{Cov}(R^{ei}, f)}{var(f)} \times [R^f \times b \times var(f)] \tag{1.33}$$

The intuition is to say that if low $f$ indicates bad times, when people are hungry, then assets which pay off badly in times of low $f$ must have low prices and deliver high expected returns.

## 1.5 Conclusion

What we have seen here is the beginning of all asset pricing models. In the empirical version, we'll see that we use risk factors instead of $\Delta c$.

# Chapter 2

# Climate Risk

Pastor *et al.* extends their theoretical framework by allowing climate to enter investors' utility. Expected returns then depend not only on market betas but also on climate betas, which measure firms' exposure to climate shocks. According to the authors, evidence suggests that brown assets have higher climate betas than green assets. This difference pushes up brown assets' expected returns in PST model. The idea is that investors dislike unexpected deteriorations in the climate. If the climate worsens unexpectedly, brown assets lose value relative to green assets. Because brown firms lose values in state of the world investors dislike, they are riskier, so they must offer higher expected returns to compensate investors for bearing this risk.

## 2.1 Brown / Green Assets

## 2.2 Climate Risk and Utility

## 2.3 Climate Betas and Expected Returns

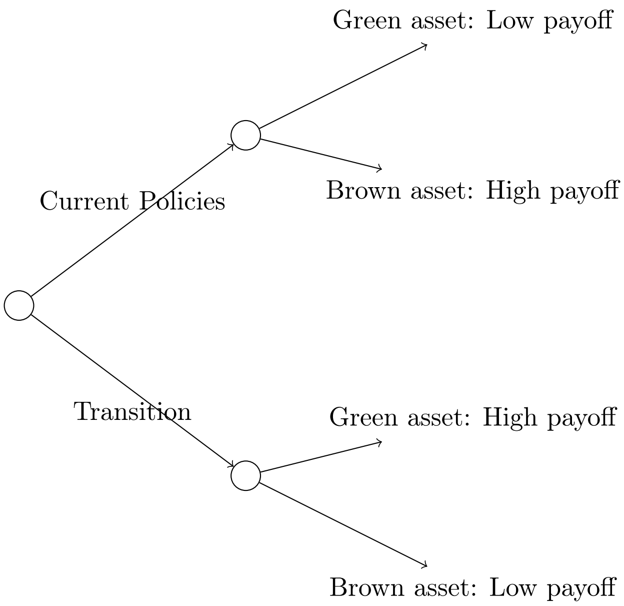
Green asset: Low payoff
Brown asset: High payoff
Current Policies
Transition
Green asset: High payoff
Brown asset: Low payoff

Figure 2.1: Climate States of the World

# Chapter 3

# Risk Factors: Empirical Methods

# Chapter 4

# The Green Factor

# Chapter 5

# Time-Varying Risk Premia

In a dynamic setting, we may have $E_t(R^e_{t+1})$ varying over time. We have:

$$E_t(R^e_{t+1}) \approx \gamma Cov_t(R^e_{t+1}, \Delta c_{t+1}) \tag{5.1}$$

$$\approx \gamma \sigma_t(R^e_{t+1}) \sigma_t(\Delta c_{t+1}) \rho_t(R, \Delta c_{t+1}) \tag{5.2}$$

Therefore, expected returns may vary over time if $\sigma_t(R^e_{t+1})$, $\sigma_t(\Delta c_{t+1})$ or $\rho_t(R, \Delta c_{t+1})$ vary over time.

# Chapter 6

# Chasing the Climate Risk Factor

ESG Factor