Climate Risk Hedging

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Chapter 2

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2.1 Regression and Few Statistics

We will run *regression* to estimate the *beta coefficient*. For example, we can regress a return on the market return:

$$R_t = \alpha + \beta R_{m,t} + \epsilon_t \tag{2.1}$$

We can also do it with the returns of several other portfolios:

$$R_t = \alpha + \beta_m R_{m,t} + \gamma R_{p,t} + \epsilon_t \tag{2.2}$$

The *population value* of the beta coefficient with the single factor model is:

$$\beta = \frac{Cov(R, R_m)}{Var(R_m)} \tag{2.3}$$

The regression recovers the *true* (*i.e.* unbiased) β only if the *error term* ϵ is *uncorrelated* with the right hand variables (R_m in our first case).

In the multiple regression example, β_m captures the effect on the return of only movements in the market portfolio that are not correlated with movement in the other portfolio p.

2.2 Simple Factor Mimicking Portfolio

Factor mimicking is a useful practice in finance. The core idea is to replace some variables with a linear combination of other variables. More specifically, **some variable of interest can be written as a portfolio of *tradable assets***. It can be used to proxy economic variable that are not directly observable with tradable assets.

This is the idea from Lamont (2001): we can construct, from financial assets, a "matching portfolio" of some economic factor that is not directly observable (in the case of Lamont, the idea was to forecast variables).

Say you want to estimate current (this is *nowcasting*) GDP or inflation. You can construct the portfolio of assets that best mimics the movements of GDP or inflation. Once you've run your regression, you can use your estimate of returns to predict the macro variable.

Let's say we don't have individual stock returns and we want to estimate the market return. All what we have is the return of K industry portfolios. We can estimate the market return as a linear combination of the industry returns:

$$R_{m,t} = \beta_{\text{energy}} R_{\text{energy},t} + \beta_{\text{financials}} R_{\text{financials},t} + \ldots + \beta_k R_{k,t} + \epsilon_t \tag{2.4}$$

It looks very much like a portfolio, with the estimated β as the weights of the assets.

2.3 Mimicking Factor Signal

Now, let's say we want to form a portfolio that mimics the behavior of a *factor signal* y. Specifically, our target is "news" or *innovations* about the signal, defined as the difference between the current expectation and the previous expectation:

$$\Delta E_t(y_{t+1}) := E_t(y_{t+1}) - E_{t-1}(y_{t+1}) \tag{2.5}$$

In can be for example the news that the market learns about the industrial production in May about the industrial production in June.

The mimicking portfolio portfolio returns are:

$$R_{y,t} = w^T R_t \tag{2.6}$$

where R_t is the vector of returns of the tradable assets. You construct the mimicking portfolio with *unexpected returns* of the tradable assets. Unexpected returns are actual returns minus expected returns:

$$\tilde{R}_t := R_t - E_{t-1}(R_t) \tag{2.7}$$

with the assumption that **the expected returns are a linear function of factors F_t**:

$$E_{t-1}(R_t) = \gamma^T F_t \tag{2.8}$$

The porfolio weights of the mimicking portfolio of y are chosen so that $\tilde{R}_{y,t}$ is as close as possible to $\Delta E_t(y_{t+1})$ (maximally correlated).

To do this, the key assumption is that **innovations in returns (unexpected returns) reflect innovations in expectations about the factor signal**, such that:

$$\Delta E_t(y_{t+1}) = \beta^T \tilde{R}_t + \epsilon_t \tag{2.9}$$

If the factor signal y is correlated with future cash flows and discount rates, then we may find something in the β, relating news reflected in the unexpected returns. Again, this is based on the assumption that the unexpected returns reflect news about the future cash flows and discount rates (*i.e.* about $\Delta E_t(y_{t+1})$).

Recalling that the returns are $R_t = E_{t-1}(R_t) + \tilde{R}$, we can therefore rewrite it with the factors:

$$R_t = \gamma^T F_t + \tilde{R} \tag{2.10}$$

and then includes the innovations in the factor signal:

$$R_t = \gamma^T F_t + \eta \Delta E_t(y_{t+1}) + u_t \tag{2.11}$$

What we have here? **The returns of any asset can be written as a function of its expected returns ($\gamma^T F_t$) and the unexpected returns. The unexpected returns are decomposed into the news about the factor signal $\Delta E_t(y_{t+1})$ and uncorrelated errors (u_t).**

2.4 Climate Factor Signal

In the case of climate risk, we have:

$$\Delta E_t(CC_{t+k}) = E_t(CC_{t+k}) - E_{t-1}(CC_{t+k}) \tag{2.12}$$

with CC_{t+k} the climate risk at an undefined horizon k.

We have seen in the chapter 1 how to use text to proxy for the market expectation of the climate risk $E_t(CC_{t+k})$.

Therefore, we can estimate:

$$R_{i,t} = \beta_i \Delta E_t(CC_{t+k}) + \gamma_i^T \text{Factors}_t + \epsilon_{i,t} \tag{2.13}$$

The β_i for each portfolio is the signal upon which we want to derive our mimicking portfolio. The higher is β_i, the higher will be the weight of the portfolio in the mimicking portfolio.

However, if β_i is negative, it means that we should short the portfolio. In practice, it is less common to short portfolios, so we can set $\beta_i = 0$ if it is negative. In that case, the mimicking portfolio would be a *long-only* portfolio.

A simple method to go from the β_i to the weights of the mimicking portfolio is to normalize it:

$$w_i = \frac{\tilde{\beta}_i}{\sum_{i=1}^{N} \tilde{\beta}_i} \tag{2.14}$$

with $\tilde{\beta}_i = \max(\beta_i, 0)$.

The vector of weights w is already an intuitive long-only mimicking portfolio: it simply **weights the portfolios based on their positive β_i on the climate innovation signal $\Delta E_t(CC_{t+k})$ we wish to mimic**.

Chapter 3

Efficient Mimicking Porfolio

We now want to improve upon this first portfolio by including more sophisticated optimized mimicking portfolio construction, by **taking into account the information on the covariance between the assets**. Indeed, even though the mimicking portfolio we have seen in the previous chapter is very intuitive, it can be **suboptimal depending on the risk appetite of the investor**.

3.1 Portfolio Optimization

Let's consider an universe of N assets. We have $w = \begin{bmatrix} w_1 & w_2 & \dots & w_N \end{bmatrix}^T$ the vector of weights in the portfolio. We suppose the portfolio is fully invested, i.e. $w^T\mathbf{1} = 1$. We have $R = \begin{bmatrix} R_1 & R_2 & \dots & R_N \end{bmatrix}^T$ the vector of returns of the assets.

The return of the portfolio is given by:

$$R_p = \sum_{i=1}^{N} w_i R_i = w^T R \tag{3.1}$$

The vector of expected asset returns is denoted by $\mu = E(R)$. The covariance matrix of the asset returns is denoted by Σ:

$$\Sigma = E[(R - \mu)(R - \mu)^T] \tag{3.2}$$

The variance of the portfolio is given by:

$$\begin{aligned}
\sigma_p^2 &= E[(R_p - E(R_p))^2] \\
&= E[(w^T R - w^T \mu)^2] \\
&= E[(w^T (R - \mu))^2] \\
&= E[w^T (R - \mu)(R - \mu)^T w] \\
&= w^T E[(R - \mu)(R - \mu)^T] w \\
&= w^T \Sigma w
\end{aligned} \tag{3.3}$$

The problem of the investor can be formulated as:

1. **Maximize the expected return of the portfolio under a volatility constraint ($\sigma_p \leq \sigma^*$)**

2. **Minimize the volatility of the portfolio under a return constraint ($\mu_p \geq R^*$)**

The key idea of Markowitz (1956) is to combine the two objectives into a single quadratic optimization problem:

$$\begin{aligned}
\min_w \quad & \frac{1}{2} w^T \Sigma w - \lambda w^T \mu \\
\text{subject to} \quad & w^T \mathbf{1} = 1
\end{aligned} \tag{3.4}$$

where λ is a parameter that allows to trade-off between the two objectives (this is the risk appetite in this case).

If we set $\lambda = 0$, we are minimizing the volatility of the portfolio and obtain the minimum variance portfolio. If we set $\lambda = \infty$, we are maximizing the return of the portfolio without taking into account the volatility of the portfolio.

The exact value of λ depends on the risk preference of the investor: the higher λ, the more higher the risk appetite.

3.2 Climate Efficient Mimicking Portfolio

We can use the derived signal b to compute a constrained long-only portfolio ($w_i \geq 0$) and fully invested ($w^T \mathbf{1} = 1$) mimicking portfolio. We are now using the covariance matrix Σ of the assets to construct the *efficient mimicking portfolio*, along the signal b.

The optimization problem is:

$$\begin{aligned} \min_{w} \quad & \frac{1}{2} w^T \Sigma w - \lambda w^T b \\ \text{subject to} \quad & w^T \mathbf{1} = 1 \\ & w_i \geq 0 \end{aligned} \tag{3.5}$$

For the sake of simplicity, let's define $\Sigma = I$ (the identity matrix) and $\lambda = 2$. The minimization function becomes $w^T w - 2w^T b$. The solution of this problem is given by:

$$2w - 2b = 0 \Rightarrow w^* = b \tag{3.6}$$

This case corresponds to the intuitive weighting scheme we have seen in the previous chapter, with the weights defined as the signal b. Therefore, our previous case is a special case of the efficient mimicking portfolio when the covariance matrix is the identity matrix. In other words, **the signal b is an optimal weighting scheme only if the assets are uncorrelated** (*i.e.* the covariance matrix is the identity matrix).

Another interesting case is when $\lambda = 0$ (and $\Sigma = I$). In that case, the solution becomes $w^* = 1/N$. Therefore, **if the investor has absolutely no risk appetite, the equal-weighted portfolio is the optimal portfolio, regardless of the signal b.**

3.3 Performance Measures