Climate Risk Hedging

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Introduction

Chapter 1

ESG Taste and Climate Risk Premia

1.1 ESG Preferences

1.1.1 Expected Utility and Optimal Portfolio

Let's assume a single period model, from $t = 0$ to $t = 1$. We have N stocks.

The investor i has an exponential CARA utility function, with $\tilde{W}_{1,i}$ the wealth at period 1, and X_i the $N \times 1$ vector of portfolio weights.

$$V(\tilde{W}_{1,i}, X_i) = -\exp\left(-A_i\tilde{W}_{1,i} - b_i^T X_i\right) \tag{1.1}$$

with A_i agent's absolute risk aversion, b_i an $N \times 1$ vector of nonpecuniary benefits.

$$b_i = d_i g \tag{1.2}$$

with g an $N \times 1$ vector and $d_i \geq 0$ a scalar measuring the agent's taste for the nonpecuniary benefits.

The expectation of agent i's in period 0 are:

$$E_0(V(\tilde{W}_{1,i}, X_i)) = E_0(-\exp\left(-A_i\tilde{W}_{1,i} - b_i^T X_i\right)) \tag{1.3}$$

We can replace $\tilde{W}_{1,i}$ by the relation $\tilde{W}_{1,i} = W_{0,i}(1 + r_f + X_i^T \tilde{r}_1)$ and define $a_i := A_i W_{0,i}$. The idea is to make out from the expectation the terms that we know about (in period 0), and reexpress the terms within the expectation

as a function of the portfolio weights X_i. The last two steps use the fact that $\tilde{r}_1 \sim N(\mu, \Sigma)$.

$$
\begin{aligned}
E_0(V(\tilde{W}_{1,i}, X_i)) &= E_0(-\exp{(-A_i W_{0,i}(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)}) \\
&= E_0(-\exp{(-a_i(1 + r_f + X_i^T \tilde{r}_1) - b_i^T X_i)}) \\
&= E_0(-\exp{(-a_i(1 + r_f) - a_i X_i^T \tilde{r}_1 - b_i^T X_i)}) \\
&= -\exp{(-a_i(1 + r_f))} E_0(-\exp{(-a_i X_i^T \tilde{r}_1 - b_i^T X_i)}) \\
&= -\exp{(-a_i(1 + r_f))} E_0(-\exp{(-a_i X_i^T(\tilde{r}_1 + \frac{b_i}{a_i})))} \quad (1.4) \\
&= -\exp{(-a_i(1 + r_f))}\exp{(-a_i X_i^T(E_0(\tilde{r}_1) + \frac{b_i}{a_i}) + \frac{1}{2}a_i^2 X_i^T \mathrm{Var}(\tilde{r}_1) X_i)} \\
&= -\exp{(-a_i(1 + r_f))}\exp{(-a_i X_i^T(\mu + \frac{b_i}{a_i}) + \frac{1}{2}a_i^2 X_i^T \Sigma X_i)}
\end{aligned}
$$

The investors choose their optimal portfolios at time 0. The optimal portfolio X_i is the one that maximizes the expected utility. To find it, we differentiate the expected utility with respect to X_i and set it to zero, to obtain the first-order condition.

We are going to do it step by step:

1. Combine the Exponential Terms:

$$
E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp{(-a_i(1 + r_f) - a_i X_i^T(\mu + \frac{b_i}{a_i}) + \frac{1}{2}a_i^2 X_i^T \Sigma X_i)}
$$
$$(1.5)$$

and let $f(X_i)$ be the exponent:

$$
E_0(V(\tilde{W}_{1,i}, X_i)) = -\exp f(X_i) \quad (1.6)
$$

2. Differentiate $f(X_i)$ with respect to X_i. We have the chain rule:

$$
\frac{\partial h}{\partial X_i} = \frac{\partial h}{\partial f}\frac{\partial f}{\partial X_i} \quad (1.7)
$$

If $h = -\exp{(f)}$, then $\frac{\partial h}{\partial f} = -\exp{(f)}$. Therefore we have:

$$
\frac{\partial h}{\partial X_i} = -\exp{(f)}\frac{\partial f}{\partial X_i} \quad (1.8)
$$

To tackle the derivative of $f(X_i)$, we use two rules. First $\frac{\partial x^T b}{\partial x} = b$ and $\frac{\partial x^T A x}{\partial x} = 2Ax$ if A is symmetric. We have:

$$\frac{\partial f}{\partial X_i} = -a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i \qquad (1.9)$$

Combining:

$$\frac{\partial h}{\partial X_i} = -\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i) \qquad (1.10)$$

3. Set the derivative to zero:

$$-\exp(f)(-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i) = 0$$
$$-a_i(\mu + \frac{b_i}{a_i}) + a_i^2 \Sigma X_i = 0 \qquad (1.11)$$

where the exponential term is always positive, so we can drop it.

4. Rearrange and solve for X_i:

$$a_i^2 \Sigma X_i = a_i(\mu + \frac{b_i}{a_i})$$
$$a_i \Sigma X_i = \mu + \frac{b_i}{a_i}$$
$$\Sigma X_i = \frac{1}{a_i}(\mu + \frac{b_i}{a_i}) \qquad (1.12)$$
$$X_i = \frac{1}{a_i}\Sigma^{-1}(\mu + \frac{b_i}{a_i})$$

For the sake of simplicity, we assume that $a_i = a$ for all investors. We now have:

$$X_i = \frac{1}{a}\Sigma^{-1}(\mu + \frac{b_i}{a})$$
$$= \frac{1}{a}\Sigma^{-1}(\mu + \frac{d_i}{a}g) \qquad (1.13)$$

Therefore, the optimal portfolio differs across investors due to the ESG characteristics g of the stocks and the investors' taste for nonpecuniary benefits d_i.

PLACEHOLDER

Figure 1.1: Efficient Frontier with ESG Preferences

1.1.2   Heterogeneous Investors and Expected Returns

The nth element of investor i's portfolio weight vector X_i is:

$$X_{i,n} = \frac{W_{0,i,n}}{W_{0,i}} \tag{1.14}$$

with $W_{0,i,n}$ the wealth invested in stock n by investor i at time 0. We define $W_{0,n} := \int_i W_{0,i,n} di$ the total wealth invested in stock n at time 0 by all investors.

The nth element of the market-weight vector w_m is:

$$\begin{aligned}
w_{m,n} &= \frac{W_{0,n}}{W_0} \\
&= \frac{1}{W_0} \int_i W_{0,i,n} di \\
&= \frac{1}{W_0} \int_i W_{0,i} X_{i,n} di \\
&= \int_i \frac{W_{0,i}}{W_0} X_{i,n} di
\end{aligned} \tag{1.15}$$

Defining $\omega_i = \frac{W_{0,i}}{W_0}$, we have:

$$w_{m,n} = \int_i \omega_i X_{i,n} di \tag{1.16}$$

1.2   Climate Risk

Chapter 2

Sources of ESG Factor and Climate Risk

2.1 ESG Factor Risk

2.2 Climate Risk