Climate Risk Hedging

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May 31, 2024

Contents

Introduction

Chapter 1

Factor Mimicking Portfolios

1.1 Risk is Innovation

We have a vector of K factors of risks F_{t+h}, with h the forecast horizon. Investors form expectations of these factors at time $t-1$ and adjust their expectations at time t based on new information. The change in expectations is given by:

$$\tilde{F}_{t+h} = F_{t+h} - \phi \tag{1.1}$$

where $\tilde{F}_{t+h}$ is the *innovation* in the factors of risks.

1.2 Innovation and Unexpected Returns

On the other hand, we have the *unexpected* returns $\tilde{R}_t$:

$$\tilde{R}_t = R_t - \mu \tag{1.2}$$

The main assumption behind factor mimicking portfolios is that the innovation $\tilde{F_{t+h}}$ is reflected in the unexpected returns $\tilde{R}_t$:

$$\tilde{R}_t = B\tilde{F}_{t+h} + \varepsilon_t \tag{1.3}$$

where B is a $N \times K$ matrix of factor loadings, ε_t is a $N \times 1$ vector of mean zero disturbances.

It means that investors reprice assets (unexpected returns $\tilde{R}_t$) based on the arrival of new information on the factors of risks (innovation $\tilde{F}_{t+h}$).

1.3 Linear Factor Model

If:

$$R_t = \mu + \tilde{R}_t \tag{1.4}$$

Then, substituting $\tilde{R}_t$, we have the following factor model:

$$R_t = \mu + B\tilde{F}_{t+h} + \varepsilon_t \tag{1.5}$$

with R_t a $N \times 1$ vector of asset returns, μ a $N \times 1$ vector of expected returns, B a $N \times K$ matrix of factor loadings, F_{t+h} a $K \times 1$ vector of factor innovations and ε_t a $N \times 1$ vector of mean zero disturbances.

1.4 Factor Mimicking

The vector of weights w_k is the solution to the following optimization problem:

$$\begin{aligned} \min_{w_k} \quad & \frac{1}{2} w_k^T \Sigma w_k \\ \text{subject to} \quad & B^T w_k = \beta_k \end{aligned} \tag{1.6}$$

where B is the $N \times K$ matrix of factor loadings, β_k is the $K \times 1$ vector of factor exposures, with the k-th element equal to 1 and the other elements equal to $\beta_{k,l}$, and Σ is the $N \times N$ covariance matrix of asset returns.

We can form the Lagrangian:

$$\mathcal{L}(w_k, \lambda) = \frac{1}{2} w_k^T \Sigma w_k - \lambda_k^T (B^T w_k - \beta_k) \tag{1.7}$$

where λ_k is the $K \times 1$ vector of Lagrange multipliers.

The first order condition is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_k} &= \Sigma w_k - B\lambda = 0 \\ \Rightarrow w_k &= \Sigma^{-1} B \lambda_k \end{aligned} \tag{1.8}$$

Substituting w_k in the constraint, we have:

$$\begin{aligned} B^T w_k &= \beta_k \\ B^T \Sigma^{-1} B \lambda_k &= \beta_k \\ \Rightarrow \lambda_k &= (B^T \Sigma^{-1} B)^{-1} \beta_k \end{aligned} \tag{1.9}$$

Substituting λ_k in w_k, we finally have the solution to the optimization problem:

$$w_k^* = \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} \beta_k \tag{1.10}$$

Taking together all the K factors, we have the matrix of weights W:

$$W = \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} B_K \tag{1.11}$$

where B_K is the $K \times K$ matrix with the k-th column equal to β_k and the other columns equal to $\beta_{k,l}$.

1.5 Risk Premia

1.6 Conclusion

In what follows, we will focus on the case of climate risk factors. First stage is to identify how to measure climate risk factors.

Chapter 2

Climate Risk Mimicking Porfolios

Two main approaches of FMPs have been proposed in the literature: (i) the two-pass cross-sectional regression (Fama and MacBeth, 1973) and (ii) the maximum correlation portfolio (MCP) (Huberman et al, 1987).

It is possible to recover both approaches with the equation in the chapter 1:

$$W = \Sigma^{-1}B(B^T\Sigma^{-1}B)^{-1}B_K \tag{2.1}$$

2.1 Two-Pass Fama-MacBeth

In the case of the two-pass Fama-MacBeth, assets are uncorrelated and have constant variance.

$$\Sigma = \sigma^2 I_N \tag{2.2}$$

where σ^2 is the variance of the asset returns.

B is multivariate (i.e., $K > 1$) and the target exposure is:

$$B_K = I_K \tag{2.3}$$

That is, we have a *beta* of one to the k-th factor and zero to the others.

Substituting Σ and B_K in the equation (3.1), we have:

WHY $\sigma^2 I_N$ and I_K cancels out?

$$\begin{aligned} W &= \sigma^2 I_N B(B^T B)^{-1} I_K \\ &= B(B^T B)^{-1} \end{aligned} \tag{2.4}$$

FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD

2.2 Maximum Correlation Portfolio

We have the Target-Beta MCP, where B is univariate (i.e., $K = 1$) and the target exposure is:

$$B_K = B^T \Sigma^{-1} B \tag{2.5}$$

Substituting B_K in the equation (3.1), we have:
FIND THE INTERMEDIARY STEPS

$$\begin{aligned} W &= \Sigma^{-1} B (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} B \\ &= \Sigma^{-1} B \end{aligned} \tag{2.6}$$

FIGURE 2 IN JURCENZKO MACRO FACTORS WITH THIS METHOD

Chapter 3

Practical Use: Hedging Climate Risk for a Mutual Fund

3.1 Hedging an Existing Fund

3.2 Backtesting a Climate Risk Hedging Strategy

Chapter 4

Conclusion

More generally can be applied to other ESG risks. See biodiversity risk from Giglio et al.