A Realistic Dividend Valuation Model

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A new family of dividend valuation models assumes that the discount rate is fixed and models the pattern of dividend payments as a Markov process. The basic model is binomial. It assumes that, in each period, the firm will either keep its dividend payment the same or increase it. A slightly more complex trinomial process assumes the firm can raise the dividend, keep it the same or go bankrupt, paying no dividend.

This approach permits analysts to undertake systematic sensitivity analyses that incorporate their own judgments.

Common stocks are usually valued on the basis of either a firm's expected profitability (earnings or cash flow) or its expected distribution of profits to shareholders (dividends). Williams's seminal dividend discount model tells us that:[1]

$$V_0 = \sum_t \frac{D_t}{(1+k_t)^t} \tag{1}$$

where V_0 is the estimate of value, D_t is the dividend in period t, and k_t is the discount rate applicable in period t.

As general as Equation 1 is, it is also intractable, given the infinite number of parameters that have to be estimated. Among the various attempts to simplify the dividend discount model, the most notable is Gordon's:[2]

$$V_0 = \frac{D_1}{k-g} \tag{2}$$

where D_1 is the dividend expected in one period's time, g the expected growth rate in dividends for the indefinite future, and k and g are intertemporally constant. Although widely used, the Gordon model is criticized for its assumptions, especially the assumption that growth is both geometric and indefinite.

A variety of multistage growth rate models have attempted to model the development of a firm's growth over time.[3] The typical pattern of dividends does not generally coincide with any of these model's assumptions, however. A company will typically maintain its dividend level fairly constant, increasing it only if there is a great deal of confidence that the firm can maintain the higher level and decreasing it only as a last resort.[4] Figure A graphs a typical company's dividend distribution behavior; it resembles a step function, with a flat payment stream interrupted periodically by discrete jumps.

A MARKOV MODEL OF VALUE

This article presents a variation of deterministic dividend valuation models. We give the dividend stream a Markov property in the following sense. We assume that, in each period, a firm will either increase its dividend with a positive probability p or keep it the same with probability $1 - p$. Over time, this results in a step pattern of dividend payments (Figure A), which resembles real-world patterns.

We term the dividend stream generated by this process a "Markov dividend stream." We discount this stream to get an estimate of value. Because we assume the interperiod random variables (those that determine whether the dividend increases) are independant, we are able to derive simple closed-form solutions for value.

We consider two ways in which dividends increase. In the *geometric model*, the dividend, if it increases, increases by a constant percentage amount, much like in the Gordon model. In the *additive model*, the dividend increases by a fixed amount.

Each of these models gives an estimate of a stock's value. In addition, we calculate a lower bound for each of these estimates, as follows. We assume that, each period, there is a small positive probability that the firm will go bankrupt. Thus

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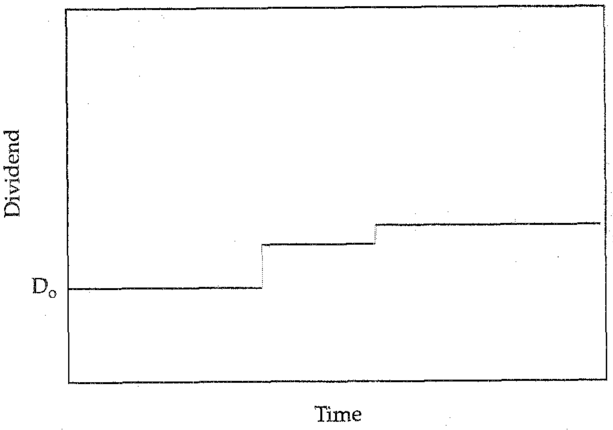
Figure A. A Step Function Dividend Pattern



there are three possibilities for each time period—(1) the dividend increases; (2) the dividend stays the same; or (3) the firm goes bankrupt. There is obviously an intermediate state where the firm does not go bankrupt but does temporarily reduce or suspend its dividend payments. The value of the firm's stock in this case will lie between the estimated value and the lower bound on value.

Before proceeding, we want to clarify how our approach differs from binomial modelling approaches. In our view, these models generally make a binomial assumption about the way interest rates (discount rates) evolve. Black, Derman and Toy, for instance, use such a process to value Treasury bond options.[5] In contrast, we first fix the discount rate, then model the security's cash flow stream (in our case, dividends) as a Markov process. Moreover, we use the term "Markov" generically to encompass not only binomial and trinomial models, but any model in which the security's value depends not on the historical cash flow stream, but on the future cash flow stream. In other words, we need to know that a firm's current dividend is D_0; we do not need to know the pattern of dividend payments that leads to D_0.

The Additive Model

Assume that, in period t, a firm's dividend, D_t, either stays the same with probability $1 - p$ or increases by an amount Δ with probability p:

$$\tilde{D}_{t+1} = \begin{cases} \tilde{D}_t + \Delta & \text{with prob } p \\ \tilde{D}_t & \text{with prob } 1-p \end{cases} \qquad \text{for } t = 1, 2, \ldots \tag{3a}$$

We term this assumption the *additive Markov process* assumption and the resulting dividend stream an *additive Markov dividend stream.*

Given this process, we construct two values for a firm's stock—an *expected value,* V_A, and a *lower bound,* L_A on this expected value. To construct the lower bound, we revise the random process, making the assumption that the firm can (1) raise the dividend with probability p; (2) go bankrupt with probability p_B; or (3) keep the dividend the same with probability $1 - p - p_B$. Letting firm value in period t be V_t, we have:

$$\tilde{V}_t = \begin{cases} \tilde{D}_t + \Delta + V_{t+1}(\tilde{D}_t + \Delta)/(1+k) & \text{with prob } p \\ \tilde{D}_t + V_{t+1}(\tilde{D}_t)/(1+k) & \text{with prob } 1 - p - p_B \\ 0 & \text{with prob } p_B \end{cases} \qquad \text{for } t = 1, 2, \ldots \tag{3b}$$

The following proposition gives closed-form solutions for V_A and L_A under the additive Markov process:

Proposition 1a

$$V_A = \frac{D_0}{k} = \left[\frac{1}{k} + \frac{1}{k^2}\right]\Delta p, \tag{4a}$$

and

$$L_A = \frac{D_0(1 - p_B)}{k + p_B} + \left[\frac{1}{k + p_B} + \frac{1}{(k + p_B)^2}\right]\Delta p, \tag{4b}$$

where k is the return an investor must earn to hold the stock. See the appendix for the proof. Note that, when $p_B = 0$, $L_A = V_A$.

Table 1 shows values for the parameter set $D_0 = 2.50$, $\Delta = 0.25$, $p = 0.25$, and k and p_B of various values. Note that, as the discount rate increases, the range (L_A, V_A) tightens, reflecting the lower possibility of a future bankruptcy due to the time value of money.

Table 1. L_A and V_A for Various Values of k and p_B

	p_B			
k	0.01	0.02	0.03	V_A
0.10	20.97	19.03	17.41	23.33
0.12	19.52	17.90	16.51	21.42
0.14	18.61	17.17	15.92	20.27
0.16	18.00	16.67	15.50	19.52
0.18	17.57	16.31	15.19	19.01
0.20	17.26	16.04	14.97	18.65

The Geometric Model

In the geometric model, the stochastic process is described by:

$$\tilde{D}_{t+1} = \begin{cases} \tilde{D}_t(1+g) & \text{with prob } p \\ \tilde{D}_t & \text{with prob } 1-p \end{cases} \qquad \text{for } t = 1, 2, \ldots \tag{5}$$

We term this process the *geometric Markov process,* and we term the resulting dividend stream a geometric *Markov dividend stream.* We make the same adjustment here as in the previous section for computing the lower bound: In addition to the two outcomes where there are dividends, the firm can go bankrupt, paying no dividends, with probability p_B. Under the multiplicative Markov process:

Proposition 2

$$V_G = \frac{D_0(1 + pg)}{k - pg} \tag{6a}$$

and

$$L_G = D_0\left[\frac{1 + pg - p_B}{k - (pg - p_B)}\right]. \tag{6b}$$

The proof is straightforward, following the same logic as the proof of Proposition 1.

Note that the adjustment to the Gordon model is a simple/ one. In the case where there is no possibility of bankruptcy, we replace the growth rate of the Gordon model, g, with an expected growth rate, pg. In the case where there is the possibility of bankruptcy, we replace the Gordon growth rate by $pg - p_B$. Although the additive case more closely models actual dividend payment streams, the multiplicative case is closer to the way in which other dividend streams are commonly valued.

Table 2 gives values for the parameter set $D_0 = 2.50$, $g = 0.05$, $p = 0.25$ and k and p_B of various values. Again, note that the range (L_G, V_G) tightens as the discount rate increases.

Table 2. L_G and V_G for Various Values of k and p_B

	p_B			
k	0.01	0.02	0.03	V_G
0.10	25.71	23.08	20.90	28.93
0.12	21.33	19.46	17.86	23.55
0.14	18.23	16.82	15.60	19.85
0.16	15.91	14.81	13.84	17.16
0.18	14.12	13.23	12.44	15.11
0.20	12.69	11.96	11.29	13.50

PRACTICAL APPLICATIONS

Dividend valuation models are obviously most useful for those firms with a systematic pattern of dividend payout. To test our models, we selected three telephone utilities, on the not unlikely assumption that they would have regular dividend payments. The three companies are Bell Atlantic, Bell South and Cincinnati Bell. Table 3 presents dividend data for the three firms; note that the data for Bell Atlantic and Bell South begin in 1984, after the ATT breakup. All data are taken from Value Line.

Table 3. Dividend History of Selected Firms

Year	Bell Atlantic	Bell South	Cincinnati Bell
1977			0.22
1978			0.27
1979			0.30
1980			0.32
1981			0.33
1982			0.34
1983			0.35
1984	1.60	1.72	0.37
1985	1.70	1.88	0.42
1986	1.80	2.04	0.44
1987	1.92	2.20	0.48
1988	2.04	2.36	0.56
1989	2.20	2.52	0.68
1990	2.36	2.68	0.76
1991	2.52	2.76	0.80
1992	2.60	2.76	0.80
1993	2.68	2.76	0.80
1994	2.80	2.88	0.84
Beta	0.90	0.80	0.95

We first computed the values of the three stocks using the Gordon model. Growth was estimated as the compound rate of growth of the actual dividend history. The discount rate, k, was estimated using CAPM with Value Line betas, a market risk premium of 5% and the contemporaneous Treasury yield of 6%.

Our valuation exercise is clearly "rough and ready"; we have simply extrapolated the past into the future. Analysts would likely have made appropriate adjustments to the data to suit their own expectations.[6] It is surprising, then, how closely the valuation exercise conforms to the actual prices (see Table 4), especially for Bell Atlantic and Bell South.

Table 4. Gordon Model Valuations

Company	Estimated Price	Actual Price
Bell Atlantic	\$61.30	\$61
Bell South	\$62.61	\$60
Cincinnati Bell	\$35.74	\$22

The reason for the accuracy of the Gordon model for Bell Atlantic and Bell South, and not for

Cincinnati Bell, is evident from an examination of Table 3. For the first two companies (especially Bell Atlantic, which increased dividends each year), the pattern of dividend payments is pretty consistent with the Gordon model's assumptions. With Cincinnati Bell, the pattern is more erratic; to the extent this pattern will continue in the future, the Gordon model gives an upwardly biased estimate of value.

To test our models on Cincinnati Bell, we must first estimate the probability that the dividend will not increase (i.e., $1 - p$). (Note that, for Bell South and Bell Atlantic, our models collapse to the Gordon model, because $p = 1$.) A simplistic backcasting method uses the historical pattern as a proxy for the future. (Again, analysts would doubtless apply more judgment to the process.) Table 3 indicates that dividends increased in 14 of the 16 years, which gives estimates for p of 0.875 and for $(1 - p)$ of 0.125.

We first test the additive model. We estimate Δ from the past pattern of dividend increases as the simple arithmetic average increase, or $0.036. Substituting into Equation 4a gives a price estimate of $10.46. The geometric model, Equation 6a, gives an estimate of $25.25. This latter estimate is much closer than either the Gordon model or the additive Markov model, and is consistent with the company's actual pattern of dividend payments. For other companies with different patterns of dividend payments, the additive model may be superior.

Whether the analyst should select an additive or a geometric model depends on his or her forecast of the behavior of dividend payments. The additive model is likely to prove preferable for firms with erratic dividend patterns, while the geometric model may be preferable for more stable income stocks.

CONCLUSION

Our approach provides the analyst with another way to conduct sensitivity analysis. It is similar to a heuristic adjustment to the growth rate. The advantage is that the analyst is able to make specific adjustments to the growth rate, hence is better able to incorporate his or her judgment.

APPENDIX

To prove Equation 4a, we use a conditioning argument. If the dividend goes up in period 1, the investor receives the dividend and the value of the stock is $V_A(D_0 + \Delta)$. If the dividend stays the same, the investor receives D_0 and the value of the stock is $V_A(D_0)$. Thus, with discounting, we have:

$$V_A(D_0) = p\left[\frac{D_0 + \Delta + V_A(D_0 + \Delta)}{1+k}\right] + (1-p)\left[\frac{D_0 + V_A(D_0)}{1+k}\right]. \quad \text{(A1)}$$

Simplification of Equation A1 gives a difference equation of the form:

$$V(x) = b_0 + ax + bV(x + \Delta), \quad \text{(A2)}$$

where

$$x = D_0 \quad \text{(A3a)}$$

$$b_0 = \frac{p\Delta}{k+p}, \quad \text{(A3b)}$$

$$a = \frac{1}{k+p} \quad \text{(A3c)}$$

and

$$b = \frac{p}{k+p}. \quad \text{(A3d)}$$

To solve Equation A2, we proceed as follows. Evaluating A2 at $x + \Delta$, we have:

$$V(x + \Delta) = b_0 + a(x + \Delta) + bV(x + 2\Delta). \quad \text{(A4)}$$

Substituting this back into Equation A2 for $V(x + \Delta)$ gives:

$$V(x) = (b_0 + ax)(1 + b) + ba\Delta + b^2V(x + 2\Delta). \quad \text{(A5)}$$

Repeating this evaluation and substitution process $n - 2$ more times gives:

$$\begin{aligned} V(x) = {} & [b_0 + ax](1 + b + b^2 + \ldots + b^{n-1}) \\ & + a\Delta(b + 2b^2 + \ldots + (n-1)b^{n-1}) \\ & + b^nV(x + n\Delta). \end{aligned} \quad \text{(A6)}$$

Taking the limit of the right-hand side of Equation A6 as n approaches infinity, we obtain:

$$V(x) = \frac{b_0 + ax}{1-b} + \frac{ab\Delta}{(1-b)^2}, \quad \text{(A7)}$$

and upon substituting for a, b and b_0, we obtain Equation 4a. Equation 4b is obtained in a similar way, and the proof is complete.

FOOTNOTES

1. J. B. Williams, *The Theory of Investment Value* (Cambridge, MA: Harvard University Press, 1938).
2. M. J. Gordon, *The Investment, Financing and Valuation of the Corporation* (Homewood, IL: Richard D. Irwin, 1962).
3. For a review of some of the prominent models in this family, see E. Sorenson and D. Williamson, "Some Evidence on the Value of Dividend Discount Models," *Financial Analysts Journal,* November/December 1985, and F. K. Reilly, *Investment Analysis and Portfolio Management* (Chicago: Dryden Press, 1989). The difficulty with these models is the sensitivity of stock price to the values of the estimated inputs. On this last point, see R. Michaud, "A Scenario-Dependant Dividend Discount Model: Bridging the Gap Between Top-Down Investment Information and Bottom-Up Forecasts," *Financial Analysts Journal,* November/December 1985.
4. See J. Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings and Taxes," *American Economic Review* 46 (1956), 97–113.
5. F. Black, E. Derman and W. Toy, "A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options," *Financial Analysts Journal,* January/February 1990.
6. See, for example, Reilly, *Investment Analysis, op. cit.,* Chapter 15, for a discussion of conventional approaches.