Prediction Markets: Rewarding Expertise and Enhancing Forecasting Accuracy

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Abstract

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1 Introduction

General intro predictive markets.

Prediction market is generally implemented as a wager (or contract) that pays off if a particular outcome, taking a particular value y, occurs. Assuming that both the efficient markets hypothesis holds, and that the market acts as a risk-neutral representative trader, the price of the contract will be the best estimate of various parameters tied to the probability of that outcome [**?**].

Table 1: Contract Types: Estimating Uncertain Quantities or Probabilities

Contract	**Example**	**Details**	**Reveals market expectation of...**
Winner-takes-all	Outcome y: Level of initial unemployment claims (in thousands).	Contract costs p. Pays \$1 if and only if event y occurs. Bid according to the value of p.	Probability that outcome y occurs.
Index	Contract pays \$1 for every 1,000 initial unemployment claims.	Contract pays \$ y.	Mean value of outcome y: $E[y]$.
Spread	Contract pays even money if initial unemployment claims are greater than y^*	Contract cost \$ 1. Pays \$2 if $y \geq y^*$. Pays \$ 0 otherwise. Bid according to the value of y^*.	Median value of outcome y.

Notes: Adapted from Table 1 in Wolfers and Zitzewitz (2004).

Predictive markets have demonstrated their ability to aggregate dispersed information and improve forecasting accuracy in various domains. However, in the context of climate science, predictive markets remain underexplored. Traditional climate modeling faces key challenges:

- a lack of systematic assessment of forecasts, leading to information asymmetries, and
- the absence of direct financial incentives for accurate predictions.

This paper proposes climate predictive markets as a mechanism to enhance forecasting quality, incentivize expert participation, and improve societal preparedness for climate-related risks. We examine the potential benefits of climate predictive markets for two key stakeholders:

- Market Makers: What are the financial incentives for creating and maintaining such a market?
- Climate Experts (Smart Money): Can climate specialists leverage their knowledge profitably, and do they require the presence of less-informed participants to realize gains?

2 Litterature review

Most prediction markets, like those available on the industry standard Intrade.com are implemented like equity markets. That is, buyers and sellers interact through a continuous double auction. However, a number of other designs have been suggested and gained traction. The major alternative are market scoring rules ([**?**], [**?**]).

This format starts with a simple scoring rule - which rewards a single person for the accuracy of his or her prediction - and allows other entrants to essentially purchase the right to the reward when they believe they have a more accurate prediction. This format is used largely to reduce speculation and deal with problems that arise from thin markets.

However, like standard scoring rules, market scoring rules require a market maker to subsidize the reward for accurate predictions.

3 Methodology

3.1 Market

Consider a model that generates a forecast for a specific event, with result A or B. Let $s \in \{0, 1\}$ denote the outcome realization, where $s = 1$ if event A occurs and $s = 0$ otherwise. Let p_t denote the probability with which the event $s = 1$ is predicted to occur, according to the model forecast at data t.

Now suppose that there exists a prediction market listing a contract that pays one dollar if event A occurs and zero dollars otherwise. Let q_t denote the price of this contract at date t. This price may differ from the model forecast p_t, in which case a trader who believes the model will recognize a profit opportunity.

To facilitate trading, we introduce an automated market maker who sets the price q_t of the contract. We employ the Logarithmic Market Scoring Rule (LMSR), a market-making algorithm that dynamically adjusts prices based on trader demand while maintaining a bounded cost function. The LMSR mechanism ensures liquitdity at all times and updates the contract price to reflect the collective information of market participants.

The LMSR maintains a market consisting of a single market maker who sets the price of the event-contingent contract. The market maker employs a cost function $C(q_A, q_B)$ that determines the total cost required to purchase a given quantity of contracts. The LMSR cost function is given by:

$$C(q_A, q_B) = b \ln(e^{q_A/b} + e^{q_B/b}) \tag{1}$$

where q_A and q_B are the number of outsanding contracts for outcomes A and B respectively, and b is a liquidity parameter that determines the curvature of the cost function (ie. how quickly prices adjust).

The price of a contract for outcome A, denoted as q_t is derived as:

$$q_t = \frac{e^{q_A/b}}{e^{q_A/b} + e^{q_B/b}} \tag{2}$$

Similarly, the price of a contract for outcome B is given by:

$$1 - q_t = \frac{e^{q_B/b}}{e^{q_A/b} + e^{q_B/b}} \tag{3}$$

As traders buy and sell contracts, the outstanding quantities q_A and q_B are updated, and the price q_t is recalculated. The LMSR guarantees that the market price continuously reflects the aggregate information of traders by ensuring that the price update follows a smooth function of the quantity of contracts traded.

The total cost incurred by a trader purchasing Δq_A additional contracts for outcome A is given by:

$$C(q_A + \Delta q_A, q_B) - C(q_A, q_B) = b \ln(1 + e^{\Delta q_A / b}) \tag{4}$$

This mechanism prevents arbitrage opportunities while ensuring that no individual trader can manipulate prices without incurring increasing costs.

Under the LMSR mechanism, the contract price q_t represents a market-implied probability of the event occuring. If market participants collectively believe the model's probability p_t is incorrect, trading will drive the contract price toward the consensus belief. Consequently, the LMSR algorithm serves as a continuous price discovery mechanism, aligning q_t with the collective expectations of market participants.

3.2 Participants

Suppose that a virtual trader or bot that believes the model is active in the market, and enters period y with a portfolio (y_{t-1}, z_{t-1}) inherited from the previous period. Here, y_{t-1} is the amount of cash held at the start of period t, and z_{t-1} is the number of contracts held. It is possible for z_{t-1} to be nefative, in which case the trader has previously sold (rather than purchased) contracts on balance, and is therefore betting that event A will not occur.

The trader must now decide whether to buy or sell contracts in period t, having observed the model forecast p_t and the market price q_t. Let x_t denote the number of contracts trader in period t. If $x_t > 0$ then contracts are purchased, z_t exceeds z_{t-1} by this amount and y_t is lower than y_{t-1} by an amount equal to the cost $q_t x_t$ of the contracts bought. If $x_t < 0$, then contracts are sold, z_t is lower than z_{t-1} by this amount and y_t is higher than y_{t-1} by an amount equal to the revenue $q_t x_t$ from the sale of the contracts. That is, the portfolio passed on to the subsequent period is given by:

$$\begin{aligned} y_t &= y_{t-1} - q_t x_t \\ z_t &= z_{t-1} + x_t \end{aligned} \tag{5}$$

But how is x_t determined? When all contracts are resolved, the terminal cash value or wealth resulting from portfolio (y_t, z_t) is:

$$W = y_t + s z_t \tag{6}$$

That is, the trader is left with $y_t + z_t$ if $s = 1$ and y_t otherwise. We assume that the trader is risk-averse and maximizes the expected utility of terminal wealth, given by:

$$E(u) = p_t u(y_t + z_t) + (1 - p_t)u(y_t) \tag{7}$$

where u is a concave utility function. This objective function can be written in terms of the decision variable x_t, the interhited portfolio (y_{t-1}, z_{t-1}), the model forecast p_t and the market price q_t by substituting the expressions for y_t and z_t into the expression for $E(u)$:

$$E(u) = p_t u(y_{t-1} - q_t x_t + x_t) + (1 - p_t)u(y_{t-1} - q_t x_t) \tag{8}$$

This is the expected value of terminal wealth given current beliefs, conditional on the portfolio (y_t, z_t), being held to expiration. The trader knows that beliefs are likely to change, but cannot know the extent or direction of these future shifts, and accordingly optimizes based on the model forecast and market price available at t.

A trading bot programmed to execute transactions in accordance with the maximization of expected utility will trade whenever there is a change in the model forecast or the market price.

Any such trades must be consistent with solvency even in the worst case scenario. That is, the trading bot chooses x_t to maximize the expected utility of terminal wealth, subject to the constraint

$$\begin{aligned} y_t &\geq 0 \\ y_t + s z_t &\geq 0 \end{aligned} \tag{9}$$

The first constraint states simply that the cash position at any date cannot become negative, and the second states that the trader must have enough cash to cover all obligations if contract holdings are negative (so the trader is betting against the event A) and it turns out that event A occurs.

We look for itendifying hypothetical trades that would have been executed by the trader on each period, starting with an initial cash position $y_0 = \$1000$ and an initial contract position $z_0 = 0$. The market has two contracts referencing the same event, one for each outcome.

In order to determine the sequence of trades x_t, we need to specify the utility function that represents the preferences of this virtual trader. A useful class of functions for which the degree of risk aversion can be tuned for experimental purposes is that exhibiting constant relative risk aversion (CRRA):

$$u(w) = \frac{1}{1-\rho} w^{1-\rho} \quad \text{for} \quad \rho \geq 0, \rho \neq 1$$
$$u(w) = \ln w \quad \text{for} \quad \rho = 1 \tag{10}$$

where ρ is the coefficient of relative risk aversion. We assume the simple case of $\rho = 1$, corresponding to logarithmic utility.

3.3 Event and Models

We consider a simple time serie X_t that evolves over time according to a stochastic process with a deterministic trend component. The event of intereset is whether this series exceeds a predefined threshold θ at the end of the period T. Formally, we define the event as:

$$s = 1(X_T \geq \theta) \tag{11}$$

where $1(\cdot)$ is the indicator function that takes the value 1 if the condition is met and 0 otherwise.

The probability of this event occuring at each time t is denoted by p_t, which represents the subjective probability assigned by the forecasting model.

We consider two distinct types of markets participants, each with a different approach to estimating p_t. These two participant represent different levels of sophistication in their forecasting methods.

3.3.1 Bayesian Traders (Naive)

The first type of trader follows a simple Bayesian updating rule, treating the most recently observed value X_t as the best available information to update their beliefs about s. This trader assumes that X_t follows a random walk with constant variance and updates their probability estimate as:

$$p_t = P(X_T \geq \theta | X_t) \propto P(X_t \geq \theta | X_t, \sigma^2) \tag{12}$$

where σ^2 represents the perceived variance of future increments.

Under the naive assumption that X_t follwos a normal distribution centered at X_t, the Bayesian trader sets:

$$p_t = 1 - \Phi\left(\frac{\theta - X_t}{\sigma\sqrt{T-t}}\right) \tag{13}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. This trader only reacts to the latest observed value.

3.3.2 Statistical Traders (Sophisticated)

The second type of participant represents an expert trader, who incorportes a simple statistical model to make a more informed forecast. Instead of relying solely on the latest value X_t, this trader fits a linear trend model to the observed series and projects forward to estimate p_t. Assuming a simple trend model:

$$X_t = \alpha + \beta t + \epsilon_t \tag{14}$$

where α and β are the intercept and slope of the trend line, and ϵ_t is a white noise error term. The expert estimates α and β using rolling regression over past observations and then predicts the final value:

$$\hat{X}_T = \hat{\alpha} + \hat{\beta} T \tag{15}$$

The expert then derives their probability estimates as:

$$p_t = 1 - \Phi\left(\frac{\theta - \hat{X}_T}{\sigma\sqrt{T-t}}\right) \tag{16}$$

Since this trader incoporates the estimated trend $\hat{\beta}$, their probability estimates may significantly differ from those of the Bayesian trader, especially in the presence of strong upward or downward trends in X_t.

4 Results

5 Conclusion

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References

[Dong et al.(2024)Dong, Zheng, Li, and Zhou] Dong, L., H. Zheng, L. Li, and C. Zhou. 2024. Distilling wisdom of crowds in online communities: A novel prediction market constructed with comment posters. *Decision Support Systems* 180:114190–.

[Hsu(2011)] Hsu, S.-L. 2011. A prediction market for climate outcomes. *U. Colo. L. Rev.* 83:179–.