Climate Scenarios with Probabilities via Maximum Entropy and Indirect Elicitation

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Abstract

What are the probabilities of a 2°C, 3°C or 4°C world? In the last decades, researchers and policymakers have worked with wide range of future climate scenarios, but the likelihood of their realisations has never been quantified. We propose two ways to obtain a probability distribution for the aggressiveness of emission abatement policies: one method is based on the elicitation from economists of estimates of the optimal social cost of carbon; the second on the principle of maximum entropy. In both cases we make use of information on the technological and fiscal feasibility of various policies to bound the distribution, and of the observed difference between average recommended and implemented policies to adjust its mean. We find that the results are robust to methodological choices. They suggest that the likelihood of achieving the Paris Agreement target is small; that there is a significant probability of an end-of-century temperature anomalies above 3°C; that the much-studied 8.5 W/m^2 forcing has low probability, but should not be neglected, at least in tail-event studies. If one considers, as we do, the distribution of end-of-century temperatures we obtain as 'too dangerous', the disconnect between economists' recommendations and policy action should be substantially reduced.

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1 Introduction

The unprecedented nature of climate change requires timely and appropriate planning, hedging and allocation of resources. These responses can only be put in place if planners and investors know what can happen, and the degree of uncertainty associated with the various climate projections. To address these needs, a substantial amount of work has been devoted by the academic community to the elaboration of reference climate scenarios.[1] The most widely used scenarios have been constructed under the direction of Intergovernmental Panel on Climate Change (IPCC) by combining six Shared Socioeconomic Pathways (SSPs, see O'Neill, Kriegler, Ebi, Kemp-Benedict, Riahi, Rothman, Van Ruijven, Van Vuuren, Birkmann, Kok, et al. (2017)) and six Representative Carbon Pathways (RCPs, see van Vuuren, Kriegler, Neil, Rihai, Carter, Edmonds, Allegatte, Kram, Mathur, and Winkler (2014)).[2]

Much has been written on the construction and features of these scenarios,[3] but, for the purpose of our discussion, their two most important features are i) that for each choice

[1]The Scenario Model Intercomparison Project (Scenario MIP), which is the primary activity within each phase of the Coupled Model Intercomparison Project (CMIP), consolidates multi-model climate projections based on combinations of different SSP and RCP scenarios.

[2]The scenarios created by the Network for the Greening of the Financial System, NGFS (2022) have also been widely used for policy and investment purposes. They still belong to the SSP/RCP family, because most of them take one particular 'narrative' (the SSP2 'Middle of the Road' scenario) as their point of departure. In all these approaches, the socioeconomic narratives and the carbon pathways are conjoined using a set of approved Process-Based Integrated Assessment Models (PB IAMs).The internal degrees of freedom of the PB IAMs are calibrated as to reproduce one of the six socioeconomic narratives (each of which covers technological, demographic and economic developments). Given this SSP narrative and one of the six carbon pathways the chosen PB IAM then works out by cost-minimization the social cost of carbon that is compatible with the SSP and RCP.

[3]See, eg, van Vuuren, den Elzen, Lucas, Eickhout, Strengers, van Ruijven, Wonink, and van Houdt (2007), van Vuuren, Kriegler, Neil, Rihai, Carter, Edmonds, Allegatte, Kram, Mathur, and Winkler (2014) and O'Neill, Kriegler, Ebi, Kemp-Benedict, Riahi, Rothman, Van Ruijven, Van Vuuren, Birkmann, Kok, et al. (2017))

of SSP they centre their estimates of climate outcomes on the most likely combination of technological, economic and demographic development associated with that narrative (and hence do not provide a *distribution* of outcomes); and ii) that by design, they have not been associated with any probabilistic estimate. With many commentators we argue that, especially given the huge uncertainty in climate outcomes, decision-makers must be provided with a number of possible outcomes, not just with the most likely one.[4] However – and in this we depart from what appears to be the consensus practice – we also claim that, for these scenarios to be of use, at least an order-of-magnitude estimate of their likelihood must be provided. This is self-evident if we want to estimate the impact of climate change on asset valuation: prices, after all, are the sum of *expected* discounted cashflows, and, without probabilities, expectations cannot be estimated. But, unless one adopts an extreme version of the precautionary principle (on the limitations of this approach see, eg, Sunstein (2005)), also policymakers need to know, at least approximately, on which outcomes they should focus their attention. The goal of this paper is to show how these approximate probabilities can be arrived at.

Two sets of objections are normally raised against the feasibility of, or even the need for, a probabilistic characterization of scenarios. The first points to the difficulty in assigning probabilities to quantities (such as the temperature anomaly) that depend, among other things, on difficult-to-quantify policy choices. The second is based on the observation that financial scenarios are often prescribed without any probabilities attached to them, and that climate scenarios as well can therefore be formulated in this probability-agnostic manner. Regarding the first objection, we are aware of, and discuss below, the great challenges that come with a probabilistic quantification of policy choices. We find it difficult to accept, however, that a totally diffuse prior – one that effectively assigns identical likelihood to *any* climate outcome – is the best description of our state of knowledge about how the

[4]The UK Pension Regulator, among others, has argued that "[t]he challenge now is to ensure the models used and the scenario analysis addresses a fuller range of real-world risks and uncertainties" – see Hill (2023).

climate/economy system will evolve. We will return to this point later, but we point out as an example that, in a widely quoted paper, Hausfather and Peters (2020) argue that the SSP5-RCP8.5 should never be used because it refers to an almost impossible world.[5] We do not enter on the merit of this claim here, but we note that the SSP5-RCP8.5 has been one of the most intensely studied scenarios.[6] If indeed it refers to an almost-zero-probability world, this would indicate a large misallocation of analytical resources.

As mentioned, the second objection to estimating probabilities for climate outcomes points to the fact that financial scenarios are often used without any probabilities attached to them. This is true, but it must be stressed that climate and financial scenarios are intrinsically different: when we assign a market or credit scenario, we can, formally or informally, rely on at least a hundred-year history of financial crises, changing economic regimes and various combinations of financial and economic occurrences. Thanks to this wealth of data, a formal probabilistic assessment of the severity of a given financial scenario *can* be obtained using traditional econometric techniques. Even when this is not done, thanks to a body of background expert knowledge an informal mental assessments of the scenario likelihood is implicitly carried out by the professional users of market scenarios.[7]

[5]The SSP5-RCP8.5 corresponds to the 90th percentile level of baseline scenarios and depicts a world "with low income, high population and high energy demand". The "business as usual" descriptor for SSP5-RCP8.5 has been criticized in Hausfather and Peters (2020). The SSP5-RCP8.5 is described in et al (2011). We note that the position by Hausfather and Peters (2020) has been challenged in Schwalm, Gelndon, and Duffy (2020).

[6]To give but one examples, all the scenario-related figures reported in the paper by Burke, Hsiang, and Miguel (2015) that has set the standard of modern climate damage analysis refer to the SSP5-RCP8.5 pathways.

[7]Griffiths and Tennenbaum (2005) studied the accuracy of probability assessments in a variety of situations, and report that 'informal' predictions can come remarkably close to Bayesian normative standards in a surprisingly wide range of forecasting settings. What is particularly important for us is that, when failures were observed, *'these could be explained by imperfect factual knowledge, rather than by deficiency in reasoning'* . (emphasis added) See also Rebonato (2012) for a discussion of this point.

The situation is radically different in the case of climate scenarios, because, when it comes to *climate* outcomes, we simply do not have the wealth of information that has been collected in the financial domain – in the language of work by Griffiths and Tennenbaum (2005) mentioned in footnote 7, we do not have the *factual knowledge* that would assist us in reaching a probabilistic assessment. This, of course, is because so far we have only experienced a modest average temperature anomaly between 1.1 and 1.4 ° C, and the associated damages have been limited and manageable. We expect these temperature increases to be greatly exceeded by the end of the century, and perhaps much earlier,[8] but do not have even an 'intuitive feel' for the relative probability of the different temperature outcomes. To bring the point home forcefully, the human species, let alone financial markets, have never experienced temperature anomalies of 3°C, and there are many emission trajectories that bring us close or beyond this temperature. What cannot currently be answered is whether we should really worry about such an unprecedented climate outcome.

In our paper we suggest how probabilities can be assigned to abatement policies. Uncertainty in abatement policies is one of four fundamental sources of uncertainty in the modelling of the climate-economy system – the others being the uncertainty in economic growth; the uncertainty in the damage function; and the uncertainty in the climate physics. Each of the latter three poses huge problems, and has spawned a correspondingly large literature – (see for a discussion, eg, Rising, Tedesco, Piontek, and Stainforth (2022)).[9]

[8]In 2018, the IPCC (2018) estimated that an average temperature anomaly of 1.5 °C would probably be reached at some point between 2030 and 2052. By 2022 (IPCC (2022)), using a different methodology, the crossing point was revised to the early 2030s. In 2023 the authoritative journal *Nature* suggested that a single year 'will probably cross the line much sooner'. In early 2025, the authoritative journal *Nature* (Tollefson (2025)) acknowledged that 'Earth's average temperature climbed to more than 1.5 °C above pre-industrial levels for the first time in 2024'.

[9]See for instance Jensen and Traeger (2014) regarding the uncertainty in economic growth, Keen, Lenton, Giodin, Yilmaz, Grasselli, and Garret (2021) regarding the uncertainty in the damage function, and Hausfather, Marvel, Schmidt, Nielsen-Gammon, and Zelinka (2022) regarding the uncertainty in the climate physics.

However, challenging as they are, the non-policy types of uncertainty *can* lend themselves to some sort of probabilistic treatment. Broadly speaking, a probabilistic assessment can be arrived at following at least two routes. The first is via a structural model with in-built stochasticity – this is the case, for instance, of the Bansal and Yaron (2004) model, where the stochasticity of the process for consumption is a key part of the model. Alternatively, an uncertainty in outcomes can arise from model risk, ie, from the fact that several plausible but imperfect competing models exist, each giving rise to different projected outcomes. In this case metastudies of the different estimates allow one to gauge the degree of attending uncertainty.

Conditional on an emission policy, these empirical or theoretical studies can therefore produce probability distributions for climate outcomes. What we focus on in this paper is the most intractable source of stochasticity, ie, the uncertainty about climate policy itself. The *unconditional* distribution that we strive to obtain can therefore be seen as the probability-weighted convex combination of the conditional distributions that can be obtained from the available models of the economy, of damages, or of the climate (where we capture under the spacious tent of the word 'model' econometric analyses as well).

How do we propose to achieve this goal? We take a two-pronged approach. The first is based on the principle of maximum entropy, which has widely and successfully been employed in fields as diverse as biological systems (see, eg, DeMartino and DeMartino (2018)), natural language processing (see, eg, Berger, DellaPietra, and DellaPietra (1996)), statistical physics (see, eg, Jaynes (1957)), and many others. This approach is the least committal, and finds the maximum-entropy distribution consistent with some fiscal, monetary and technological boundaries and with the observable degree of actual climate action.

If one believed that the bounds and the expectation of the target distribution is all we can say about the problem at hand, then the principle of maximum entropy would provide the least-committal distribution consistent with this information. Our second approach starts from the observation that additional information *can* be injected into the problem. This comes from the extensive meta-studies that have been carried out of what professional economists believe the optimal social cost of carbon should be, from which one can build a distribution of estimates for this quantity. The difficulty is that no direct metastudy can be carried out of future abatement policies. Luckily, we find that a surprising, but very robust, regularity offers a way out of this impasse: we find that we can build a robust bridge between the quantity for which metastudies are available (the optimal social cost of carbon) and the quantity of interest, ie, the aggressiveness of future policy actions. Indeed, our first important finding is that there is an unexpectedly close relationship between this quantity and the 'effective' abatement speed that we define in Equation 10. Simplifying a bit, first, we characterize the aggressiveness of an abatement policy by a synthetic statistic, the 'effective' abatement speed. Next, we find that by varying the preference parameters, the physics-model parameters, the damage function, the type of utility function (time-separable or Epstein and Zin (1989)-type recursive) and other model configurations obviously produces very different optimal abatement speeds and social costs of carbon. However, we find that these two quantities – the social cost of carbon and the effective abatement aggressiveness – plot on an extremely simple and smooth curve. Put differently: indeed, the optimal social cost of carbon strongly depends on the model specification; however, once the effective abatement speed is assigned, we find that the residual dependence of the optimal social cost of carbon on the model specification is almost zero.

Our strategy is therefore to start from the distribution of social costs of carbon as produced by the metastudy by Tol (2023), and, thanks to the *monotonic* relationship between the effective abatement speed and optimal social cost of carbon, to obtain a distribution of abatement speeds – and, therefore, by assigning probabilities to policies, to obtain an unconditional distribution of climate outcomes.

The leap from the distribution of elicited values for the SCC estimated by economists to the distribution of policy aggressiveness of politicians is admittedly considerable. To some extent, it can be justified by observing that a policymaker is unlikely to have a precise idea about a quantity as technically complex as the social cost of carbon, and would therefore be well advised to seek expert opinion – an expert opinion that is most likely to come from an economist. Having said this, there are several potential sources of bias in translating from one distribution (the economists') to another (the politicians'). First, we recognize that politicians face the task of 'selling' a carbon tax to an electorate that has so far proven extremely reluctant to accept even modest carbon duties. It is therefore reasonable to expect that the distribution of social costs of carbon proposed by economists should be more 'aggressive' than the distribution of associated abatement policies enacted by the politicians. Second, Havranek, Irsova, Janda, and Zilberman (2015) examine potential selective reporting (also known as publication bias) in the literature on the social cost of carbon. They find that 'estimates for which the 95% confidence interval includes zero are less likely to be reported than estimates excluding negative values of the SCC, *which might create an upward bias in the literature*' (emphasis added). Third, we concur with Dietz and Stern (2015) that many estimates of the optimal social cost of carbon do not include technological constraints or adaptation costs, and therefore recommend unrealistically high carbon taxes.

All these biases 'pull in the same direction', and drive a wedge between the economists' distribution (that we can estimate) and the politicians' (in which we are really interested). We try to correct for this bias by observing the difference between the recommended and currently implemented speeds of abatement, as proxied by the current cost of carbon. On the basis of this discrepancy, we shift the distribution of equivalent abatement speeds proposed by economists to recover the observed cost of carbon.

As we shall see, one of the recurrent themes of our analysis will be that the raw data collated in Tol (2023) must be 'curated' in several dimensions before it can be sensibly used for our (or, indeed, for *any*) purpose. There are many (at first blush equally appealing) ways in which the data can be cleaned, interpolated, made fit for purpose and put on a comparable footing. A particularly important aspect of our study will therefore be the robustness analysis of our results, and this is why we have explored several alternative analytical approaches in addition to what reported in Tol (2023): we have done so, that is, not necessarily because we believe that our choices are 'better' than Tol's, but simply to check that equally defensible modelling choices should yield not-too-different (and reasonable) results.

We discuss our results in detail in Section 6, but, in a nutshell, they can be summarized as follows. First, we find that, even under conservative assumptions, the likelihood of limiting end-of-century temperature increases to 1.5°C is very small: the exact value depends on the modelling choices, as discussed in Section 6, but these probabilities are never larger than a few percentage points. We stress that the goal is technologically achievable, but it would require a major and sudden redirection of actual abatement policy towards *and beyond* the consensus (median) recommendations of economists. Since economists have put forth these abatement recommendations for the best part of half a century, and their suggestions have gone largely unheeded, our method finds that the probability of an imminent correction of the politician/economist disconnect is very low. We also note that, despite the fact that we are looking at end-of-century temperature anomalies, the policy realignment would have to occur very soon, because the 1.5°C-compatible carbon budget is likely to be exhausted in the next decade or so.[10]

Next, we find that the median 2100-temperature anomaly (around 2.85°C) is well above the 2.0°C end-of-century target, and that there is a significant probability (around 35-

[10]In their study of the uncertainties surrounding the estimates of the carbon budgets, Lamboll, Nicholls, Smith, Kikstra, Byers, and Rogelj (2023) conclude that 'the RCB [Remaining Carbon Budget] for a 50% chance of keeping warming to 1.5°C is around 250 GtCO2 as of January 2023, equal to around six years of current CO2 emissions.'

40%) that the temperature will exceed 3°C (again, its precise value depends on the specific modelling choices, which are discussed in Section 6). We would therefore enter uncharted territory, and the possibility of triggering tipping points would greatly increase. The precise threshold for the many possible climate tipping points – which the National Research Council (2002) calls 'Inevitable Surprises' – is not known to any degree of accuracy, and neither is the magnitude of the economic losses associated with their triggering (see Lenton, Held, Kriegler, Hall, Lucht, Rahmstorf, and Schellnhuber (2008) for a discussion of these aspects). However, the speed of the rate of temperature change experienced relatively recently (during the exit from the last Ice Age, when locally the temperature rose by as much as 10°C over one or two decades – see Alley and et al (2003)[11]) would make adaptation virtually impossible. This would in turn imply very severe physical climate damages, beyond what can be estimated even with the best (but backward-looking) climate-damage models.

Finally, we note that the expected temperature associated with the SSP5-RCP8.5 scenario (well above 3.75 °C) is certainly far from the body of the distribution, but it is not a 'far-tail event' either, as we find its probability to be at least 5%. We therefore reconcile to some extent the contrasting views of Hausfather and Peters (2020) and Schwalm, Glendon, and Duffy (2020) on whether the SSP5-RCP8.5 pathways should be 'taken seriously'.

As Section 6 shows, our projections assign significant probabilities to relatively high temperature anomalies (where by 'relatively high' we mean temperature anomalies well above the 1.5-2 °C Paris-Agreement targets). High temperature anomalies mean low transition costs. However, recent state-of-the-art assessments of the impact of climate change on economic output (see, Burke, Hsiang, and Miguel (2015), Kotz, Leverman, and Wenz (2024)) have substantially increased the early estimates of climate damages. These findings

[11]'Large, abrupt, and widespread climate changes with major impacts have occurred repeatedly in the past, when the Earth system was forced across thresholds. Although abrupt climate changes can occur for many reasons, it is conceivable that human forcing of climate change is increasing the probability of large, abrupt events.' page 2005

imply that the high temperatures we project give high probability to much higher physical damage costs than early studies had suggested. We analyze critically the validity of our conclusions in Section 6.1, but, if we are correct, our results suggest that prudential regulators, policymakers and investors should devote at least as much attention to physical damages as to transition risk.

2 The Data

2.1 The Input Data

We make use for our analysis of the comprehensive metastudy of estimates of the social cost of carbon (SCC) carried out by Tol (2023), who has surveyed 207 papers, for a total of 5,905 estimates.[12] To be precise, the quantity surveyed by Tol (2023) is the social cost of carbon 'today' (in addition, of the 207 surveyed papers, 94 also provided an estimate of the growth rate of the SCC). Given a utility function that returns today's welfare, $W(0)$, as a function of preferences and of a future, possibly state-dependent consumption stream, the social cost of carbon at time 0, $SCC(0,t)$, of time-t emissions is defined as

$$SCC(0,t) = \frac{\partial W(0)/\partial E(t)}{\partial W(0)/\partial C(t)} \tag{1}$$

where $C(t)$ and $E(t)$ are the consumption and the emissions at time t, respectively. It gives the ratio between change in time-0 welfare for a change in time-t *carbon* emissions to the change in time-0 welfare for a change in time-t consumption. It is expressed in \$/ton of carbon (\$/ton C). (When SCC for CO_2 is provided, the SCC estimate must be adjusted by the factor of 3.66, to reflects the approximate ratio of the weight of a CO_2 molecule to

[12]A note about units. All the social-cost-of-carbon data in Tol (2023) are expressed in 2010-equivalent \$ per metric ton of carbon (C), not of carbon dioxide. When discussing the social cost of carbon, for direct comparability we use consistently the same units. However, the prices of traded carbon permits are universally expressed in \$ per ton of carbon *dioxide* (CO_2). Since these prices are well known and widely quoted, in that context, and only in that context, we also use \$ per ton of carbon dioxide. When SCC for CO_2 is provided, the SCC estimate must be adjusted by the factor of 3.66, to reflects the approximate ratio of the weight of a CO_2 molecule to a C atom.

a C atom.) We note that, despite the fact that in the text of his paper Tol (2023) refers to the social cost of carbon as \$/ton CO_2,[13] all the numbers reported in the tables of his metastudy refer to prices for ton of carbon (not carbon dioxide) emitted. Unless otherwise stated, all prices in the rest of the paper will therefore refer to the cost in 2010 dollars per metric ton of C.

Since the estimates collated and studied in Tol (2023) have been produced over the last 40 years, for comparability the different estimates are grown or deflated to obtain 2010-equivalent values using the growth rate of the SCC, and they are also adjusted for inflation. To ensure consistency, we have reimplemented and reproduced Tol (2023)'s analysis.

The first observation about the distribution of these SCC estimates is that they cover an extremely wide range: after rounding to the nearest power of ten, from $-\$1,000$ to $+\$110 \cdot 10^6$ /ton C. Descriptive statistics of the raw SCC data are provided in Tab 1.

Quantity	Mean	Std dev	Min	Max	5th perc	95th perc
SCC	106,881	2,165,172	−770.0	1.1×10^8	8.7	7,409.0

Table 1: Descriptive statistics of the raw SCC data. The entries are in 2010-equivalent \$ / metric ton C.

Given the extremely fat-tailed nature of the distribution of elicited SCC estimates, and its rather irregular shape, fits of the raw data to parametric distributions provide a poor description of the data. Kernel-based fits are therefore more appropriate, and this is indeed the approach taken in Tol (2023). However, as Tol notices, a fit using Gaussian kernel density estimation can be inappropriate, as it implies a distribution unbounded from below. Tol therefore proposes the use of the Weibull / Gumbel (fat tailed) kernels as these guarantee zero weight outside a user specified support. To test the sensitivity of the final results to

[13]'These are estimates of the social cost of carbon for carbon dioxide emitted', page 2.

the fitting procedure, we have also used a different approach: we transform (either from $[0, \infty)$ or from (a, b)), using a log-normal or a logistic transformation; we use a Gaussian kernel on the transformed data; and we transform back to the original space. When we do so, we find that the final results, discussed in Section 6, depend little on the fitting choice.

Whatever procedure is used to fit a distribution to the raw data, all the problems with the extremely wide range of values still remain. The raw data therefore needs to be 'curated' in order to produce reasonable results. In particular, sensible cut-offs must be chosen (for instance, given current emissions, the estimate of an SCC of $\$110 \cdot 10^6$/ton C would imply a carbon tax far greater than the world GDP). As we shall see, reasonable choices of cut-off levels produce 'optically' rather similar distributions of elicited SCC; however, they can produce rather different estimates for the average value of SCC. Since this quantity will play an important role in converting from the economists' to the politicians' distribution, we must look at this aspect in some detail.

2.2 Setting Lower and Upper Limits for the SCC Distribution

In order to deal with the problem of unreliable and extreme estimate, Tol (2023) applies three different types of weights to the raw data: i) the 'paper weights' equally split the weight across the multiple quotes in each of the surveyed papers; ii) the 'author weights' are hand-collected from the papers and overweight the estimates that the authors of each paper prefer (for instance their baseline vs robustness checks); iii) the 'quality weights' mix 'author weights' with a composite indicator for the quality of the study. The probability density function for the social cost of carbon after applying the three different types of weight is shown in Fig 2. It is reassuring to find that estimates down-weighted on the basis of one criterion tend to be similarly down-weighted when the other criteria are taken into account. We therefore conduct the rest of our analysis using the average of the three differently-weighted distributions shown in Fig 2.

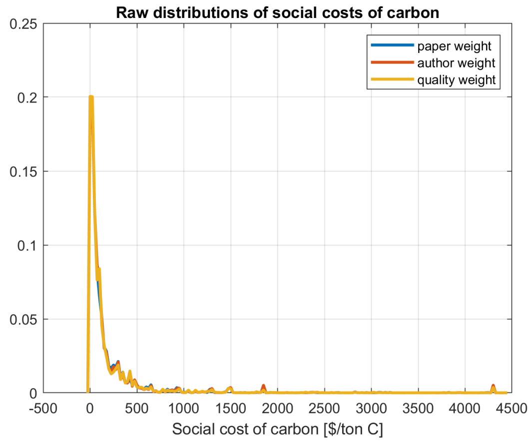

Figure 2: The probability density function for the social cost of carbon after applying Tol (2023)'s author, paper or quality weights to the raw data. Social cost of carbon in 2010 \$/ton C.

The weights modify most substantially the upper and lower section of the distribution. For the lower cut-off, we note that, among the negative estimates in Tol's data, there is one reported value (−\$770.8) which is a marked outlier (the second most negative estimate is −\$17.77). To appreciate how extreme this value is, we point out that a social cost carbon of −\$770.8 would imply that approximately 10% of world GDP should be spent in carbon *subsidies*. This is more than the world currently spends on education and defence *combined*. After applying the weights, this estimate disappears (is assigned a weight of zero). We can justify its exclusion without recourse to weights based on reasonable, but ultimately subjective, criteria such as 'paper quality': when we fit an extreme value distribution[14] (see Fig 3) to the negative estimates of the social cost of carbon *excluding the value of* −\$770.8 (there are 69 such estimates), we find a probability for −\$770.8 more than ten orders of magnitude smaller than the probability for the second smallest estimate. We therefore concur with Tol's weights, and exclude in further analysis the estimate of −\$770.8.

[14]We use the threshold exceedance method, as described in McNeil, Frey, and Embrechts (2015), Chapter 5, Section 2.

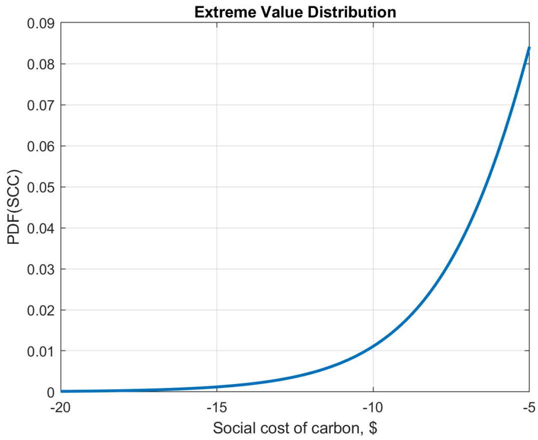

Figure 3: Extreme-value probability density obtained from the lowest 69 estimates of the social cost of carbon.

For the upper cut-off, an extremely conservative value is obtained by capping the total carbon tax to the world GDP. Using 2010 values expressed in \$-equivalents (as in the analysis in Tol (2023)), we have a total GDP of $\$64.48 \cdot 10^{12}$, and global CO_2 emissions of $\approx 33.0 \cdot 10^9$ tons, or $\approx 50.2 \cdot 10^9$ tons of all greenhouse gases expressed in CO_2 equivalent terms. This would give an upper bound for the SCC of \$4,700/ton C if all greenhouse-gas emissions are taken into account, or of \$7,200/ton C if only carbon from CO_2 emissions are considered (as done in Tol (2023)).

Setting the cut-off for the total carbon tax equal to the world GDP is clearly unrealistic. Tol (2023) argues that the total amount of tax that governments can raise is limited, and caps the additional tax at 15%. This choice is plausible, but in is not justified in Tol (2023). To test the sensitivity of the final results to this choice for the cut-off, we have also used an alternative procedure. Since global healthcare (which adds up to more than the combined expenditure for education and defence) is one of the largest items of government expenditure, we have made the assumption that the probability of a global carbon tax exceeding the global healthcare expenditure should fall off very quickly. As Fig 4 shows, there is a strong dependence of the *fraction* of GDP devoted to healthcare on GDP/person. The same figure also shows that for the richest countries the fraction of GDP devoted to healthcare is not far from the 15% one-size-fits-all cut-off used in Tol (2023).[15] With this choice, the lower limit for the SCC becomes \$770 or \$1080 /ton C, depending on whether CO_2-equivalent or CO_2 only emissions are considered.

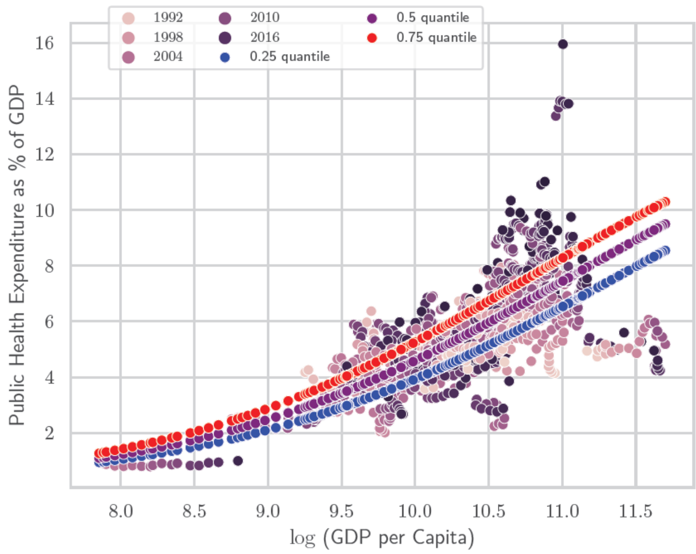

Figure 4: Public Health expenditure as a fraction of GDP per country from 1990 to 2020.

It is difficult, but not impossible, to devote to a project that is considered of high, and perhaps of vital, importance a higher percentage of GDP than what is spent on healthcare. To put things in perspective, the fraction of GDP devoted to the war effort in World War 2 by the US reached 37% of GDP; in the UK, the fraction of GDP devoted to rearmament grew from 2.2% to 6.9% by the start of the war, and climbed to close to 50% in 1943-45.[16] The healthcare-expenditure taxation level should therefore be seen as a 'soft' limit, beyond which the likelihood of a higher carbon tax declines, but does not fall abruptly to zero. As in the work by Tol (2023), in our study we therefore assign additional weights, $w(SCC)$,

[15]The percentiles for the tax revenue data and the attending figures were obtained by Kainth (personal communication) using logistic quantile regression, and assuming a maximum tax burden of 70% of GDP.

[16]Sources: Mason (2024) for the UK data, and Norwich University, *The Cost of US Wars Then and Now*, available at https://online.norwich.edu/online/about/resource-library/cost-us-wars-then-and-now for the US data.

to SCC estimates, such that for $SCC < RUL$ the weight is 1, and for $SCC > TUL$ the weight is 0. For intermediate values, the weights are then interpolated between RUL and TUL. How the interpolation is carried out is called 'tapering' in Tol (2023). Tol uses linear tapering, which introduces discontinuities in the first derivative of the weight function to be interpolated. When we impose continuity of the first derivative at the realistic and theoretical upper limits and we minimize the curvature of the interpolating function, we end up with cubic tapering (in essence a cubic spline).[17] The resulting weights between the realistic and theoretical upper bounds are shown in Fig 5. We find that the cubic tapering we use does not change the dispersion significantly when compared to linear tapering, but lowers somewhat (by approximately 5%) the mean of the SCC values. Given the level of precision we strive to attain, we consider the linear and the cubic methods equivalent, and present in the following our results only for the cubic-tapering choice.

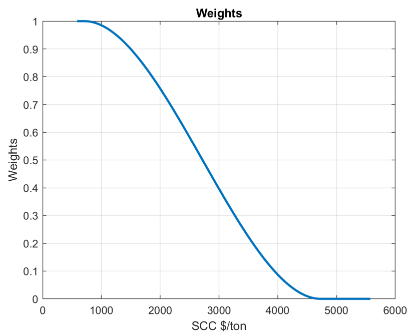

Figure 5: Interpolation of the weights between the 'realistic' and 'upper' values for the SCC. The units on the x axis are thousands of 2010 US dollars.

In sum: whichever method is used, we shall assume in what follows that satisfactory estimates have been carried out

1. for the lower limit;

[17]A smooth interpolant is defined in Handscomb (1966) as one which minimizes the arc integral of the squared curvature ('roughness'), $R = \int_a^b \frac{y''(x)^2}{(1+y'(x))^{5/2}} dx$. To simplify this non-linear problem, it is customary (see Newbery and Garett (1991)) to define the roughness as $R = \int_a^b y''(x)^2 dx$. As proven in Greville (1969), this gives rise to the natural cubic spline.

2. for a 'realistic upper limit', RUL (a flat 15 % for Tol, or the healthcare-expenditure-related value as we discussed in our case);

3. for a 'theoretical upper limit', TUL (the whole GDP).

We have then interpolated smoothly the weights between RUL and TUL using the cubic tapering described above.

In Tol (2023)'s raw distribution there is an area of slightly less of 5% assigned to negative social costs of carbon, which shrinks to less than 1% when the weights and the lower limits are applied. This still leaves the question of how to handle these negative values. As we discuss in Section 4, there is a close positive link between the SCC and the emission intensity (emissions per unit of GDP). A negative SCC would mean that deliberate efforts are made to *increase*, rather than reduce, the amounts of emissions per unit of GDP. Alternatively, a negative SCC could be interpreted as a costly measure – say, a subsidy – to inject *more* CO_2 in the atmosphere than the industrial processes require. Currently, consumer and producer fossil-fuel subsidies are admittedly important (IMF (2022) estimates *direct* subsidies of around 1% of World GDP in 2022). However, throughout this paper we make the consistent choice of skewing our results towards the 'conservative' side (ie, towards projections of lower temperature). Therefore we choose to exclude from our analysis negative estimates of the social cost of carbon.

Given this choice, Fig 6 shows that an excellent fit to the 'curated', weighted empirical distribution for values of SCC greater than 0 can be obtained as a mixture (convex combination) of a truncated Gaussian (TN) and a lognormal (LN) distribution:

$$\phi_{emp} = A \cdot [w \cdot TN(\mu_1, \sigma_1) + (1 - w) \cdot LN(\mu_2, \sigma_2)] \tag{2}$$

with $\mu_1 = 4.9638, \sigma_1 = 9.1582, \mu_2 = 4.3797, \sigma_2 = 1.3206, w = 0.1940$ and the constant $A = 1.0602$ takes care of the normalization. Since the first moment of the distribution will play an important role in the analysis to follow, we determine the coefficients of the fitted density in Eq 2 in such a way to minimize both the squared differences between the fitted and empirical densities, and the squared difference between the sample mean and model expectation. The sample average is \$157.29/ton C and we obtain \$157.19/ton C for the corresponding expectation from the fitted distribution.

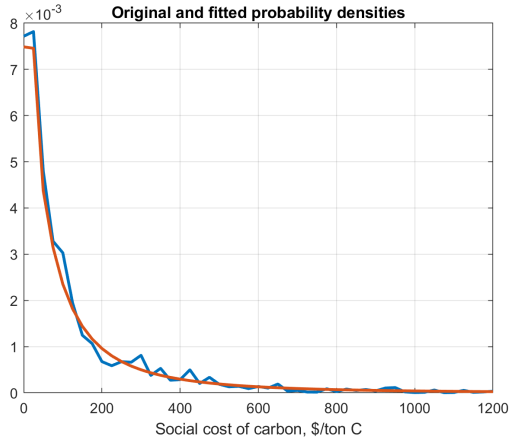

Figure 6: The fit to the empirical probability density (blue line) obtained using a truncated Gaussian and a lognormal distribution (red line). Social cost of carbon in 2010 USD/ton C on the x axis.

3 Transforming the Economists' Distribution Into the Politicians' Distribution

As stated in the introduction, the distribution of social costs of carbon estimated by economists is likely to be biased (upwards) with respect to the politicians' distribution. Making this adjustment is important: in a recent report (WorldBank (2024)), the World Bank comments that '[p]rice levels continue to fall short of the ambition needed to achieve the Paris Agreement goals.' Since the average cost of carbon from the economists' distribution, \$157/ton C (\$ 42.8/ton CO_2), is not far from achieving at least the upper Paris-Agreement target, there is a sizeable discrepancy between the actual total cost of carbon and what economists say this price should be. This is what we are trying to correct for. Since both politicians' actions and economists' recommendations have been visible for decades, and actual policies have consistently lagged behind the economists' recommendations, this (possibly time-dependent) bias constitutes a robust feature of the two distributions. We choose to estimate the magnitude of this bias by imposing that the first moment of the politicians distribution of SCC should match the current observed carbon cost. We therefore modify the curated economists' distribution of SCC in such a way that its mean coincides with the empirically observed carbon tax today.

By doing so, we have assumed that the traded price of emission targets can be taken as a proxy of the social cost of carbon. Is this reasonable? The total carbon price is made up of direct carbon taxation (a small percentage of the total price) plus emission trading, plus net fuel taxes minus net fuel subsidies. From 2015 to 2021 the global total carbon price has traded in a range between \$31 and \$48 per ton of CO_2. The increasing trend from 2017 to 2020 has recently been reversed. From a recent comprehensive study of carbon pricing by the WorldBank (2024), we estimate a global carbon tax of approximately \$30/ton CO_2 (\$ 110.1/ton C) and \$50/ton CO_2 (\$ 183.5/ton C). For comparability with the values in Tol (2023) (which are expressed in 2010 dollars per metric ton of carbon, not of carbon dioxide), the latest values must first be reduced by a factor of 1.25 to account for inflation, and multiplied by the ratio of the weights of CO_2 and C, which is $44/12 \approxeq 3.667$. Furthermore, only a fraction of current emissions comes under some form of emission-trading-permit scheme, as the 36 carbon trading systems in operation around the world by early 2024 cover approximately 20 % of global emissions.[18] When these adjustments are applied to the prices of traded emission permits, the current social cost of carbon, expressed in 2010 dollars per ton of carbon is in the range \$20/ton C to \$40/ton C. Since, as stated, we want to produce conservative temperature projections, we use the average of these estimates and take a forward-looking approach by assuming emission coverage of 30% or 40%, corresponding, after rounding, to effective carbon costs in the range of \$50 /ton C to \$70/ton C.

[18](Source: International Carbon Action Partnership, 2024)

Modifying a full distribution to match a single moment can, of course, be done in an infinity of ways. We want to achieve this goal by retaining as much as possible the shape of the economists' distribution. To do so we proceed in two ways. With the first, we minimize the Kullback-Leibler (KL) deviation from the unshifted density and a density with the assigned first moment (see Kullback and Leibler (1951)). The derivation is presented in Appendix D, where we show that, if $\Phi(x)$ is the unshifted density, the moment-matching, minimum-KL-deviation density, $Q(x)$, is given by

$$Q(x) = \frac{\Phi(x)}{\lambda_1 + \lambda_2 \cdot x} \tag{3}$$

The Lagrange multipliers, λ_1 and λ_2, which are associated with the normalization and the moment-matching condition, are also derived in Appendix D.

The second approach (which we call the shifting method, see Appendix B for details) does not have the theoretical foundations of the Kullback-Leibler method, but is constructed to retain as much as possible the shape of the original, unshifted distribution. If $\Phi_i(SCC) \cdot \Delta SCC_i$ is the probability for the unshifted distribution in the interval $[SCC_i, SCC_{i+1}]$, then we impose that the same probabilities will apply for new values of the social cost of carbon, *scc*, given by $scc = R \cdot SCC$, where R is the ratio between the expected value of the social cost of carbon with the shifted distribution to the same quantity for the unshifted distribution. By so doing, the correct expectation is recovered, and, modulo a 'stretching', the shape of the original distribution is preserved. Fig 7 shows the original (blue line), shifted (yellow line) and Kullback-Leibler (red line) probability densities for the social cost of carbon for the case $< SCC >=$ \$70/ton C (left panel) and $< SCC >=$ \$50/ton C (right panel).

The approach presented so far has been based on the assumption that the distribution of estimates for the optimal SCC produced by economists contains some useful information about the aggressiveness of future abatement policies. One can take a more agnostic approach, and assume that we only know the range and the expectation of the social cost

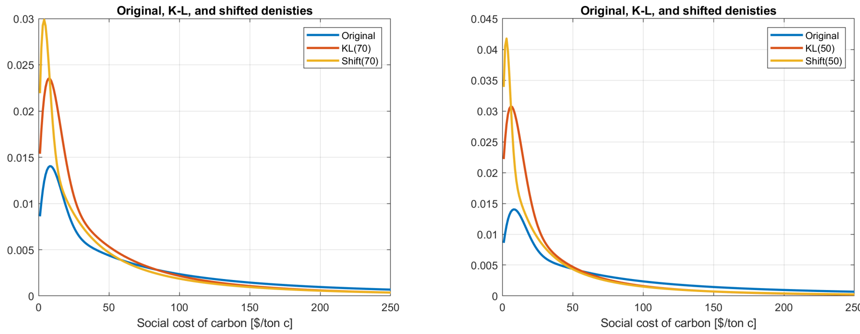

Figure 7: The original (blue line), shifted (yellow line) and Kullback-Leibler (red line) probability densities for the social cost of carbon for the case $< SCC >=$ \$70 (left panel) and $< SCC >=$ \$50 (right panel). Social cost of carbon in 2010 USD per ton C on the x axis.

of carbon. If this is the case, the maximum-entropy distribution, $\phi(SCC)_{ME}$, for the SCC between lower and upper values a and b is derived in Appendix C, where we show that it is given by[19]

$$\phi(SCC)_{ME} = C_\lambda \exp(\lambda \cdot SCC) \tag{4}$$

for $a \leq SCC \leq b$ and zero otherwise. The normalization constant is given by

$$C_\lambda = \frac{\lambda}{\exp(\lambda \cdot b) - \exp(\lambda \cdot a)} \ . \tag{5}$$

The constant λ is obtained by imposing for the expected value of SCC, $< SCC >$,

$$< SCC >= \frac{b \cdot \exp(\lambda \cdot b) - a \cdot \exp(\lambda \cdot a)}{\exp(\lambda \cdot b) - \exp(\lambda \cdot a)} - \frac{1}{\lambda} \ . \tag{6}$$

The resulting maximum-entropy distribution is also displayed in Fig 7. Not surprisingly, the maximum-entropy distribution gives more weight to low values of the SCC: despite its bias, the politician's distribution does convey the information that very low values of the SCC are not very likely. By assuming that we know absolutely nothing about which abatement

[19] We have chosen for the upper value, b, the weighted average of the SCC between RUL and TUL, with the weights the same as those used for the cubic tapering. We have tested that different choices for b between RUL and TUL make little difference to the distribution of end-of-century temperature that we discuss in the Section 6. We thank Dr Dherminder Kainth who pointed out a mistake in a previous derivation, and provided us with the correct result.

policies are more likely, we are neglecting this information, and we are left with assigning relatively high probabilities to arguably implausibly slow abatement policies. We quantify this observation below.

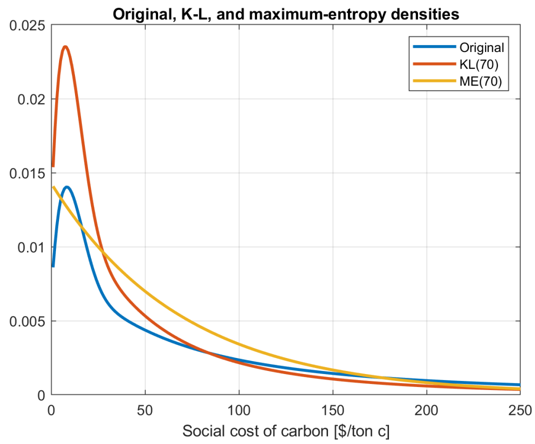

Figure 8: The original (blue line), Kullback-Leibler (red line) and maximum-entropy probability densities for the social cost of carbon for the case $< SCC >= \$70$. Social cost of carbon in 2010 USD per ton C on the x axis.

Since both the informative (shifted) and the maximum-entropy distributions have, by construction, the same mean, and are proper probability densities, any distribution built as a convex combination of the two would also be an acceptable distribution with mean μ. For robustness tests, one can therefore explore how the distribution of derived quantities, such as the distribution for the-end-of-century temperature anomaly, varies as the parameter ρ varies from 0 to 1. One could even consider ρ as a metaparameter, with a uniform distribution between 0 and 1, and obtain the resulting mixture distribution as the weighted average of all the possible combination:

$$\Psi(x) = \int_0^1 \Phi(x;\rho)u(\rho)d\rho = \int_0^1 \Phi(x;\rho)d\rho \tag{7}$$

where $u(\rho)$ is the uniform distribution of ρ between 0 and 1, and the last equality follows because, over the interval $[0,1]$ $u(\rho) = 1$. This would be the optimal choice for someone who had no *a priori* reasons to prefer either the informative or the maximum-entropy distribution. Fig 9 shows several functions $\Phi(x, \rho)$ for $\rho = 0.9, 0.7, 0.5, 0.3, 0.1$ (left panel), and the function $\Psi(x) = \int_0^1 \Phi(x; \rho)d\rho$ (right panel). In Section 6 we present our results for the limiting cases of the pure maximum-entropy or pure informative distributions ($\rho = 1$ and $\rho = 0$, respectively).

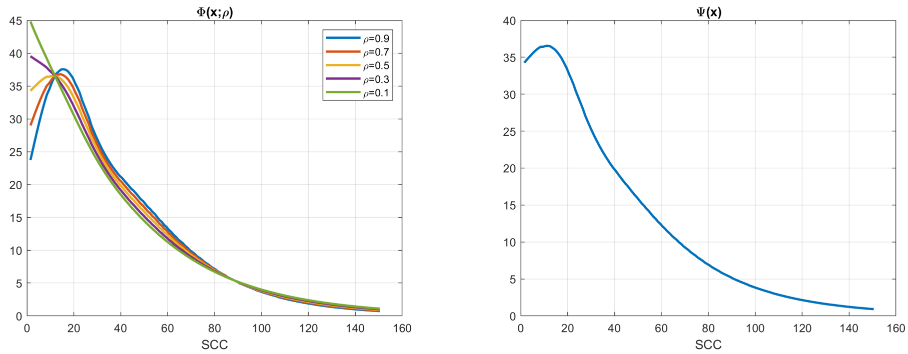

Figure 9: Functions $\Phi(x, \rho)$ for $\rho = 0.9, 0.7, 0.5, 0.3, 0.1$ (left panel) and the function $\Psi(x) = \int_0^1 \Phi(x; \rho)d\rho$ (right panel). Social cost of carbon in 2010 USD on the x axis.

4 The Relationship Between SCC and Abatement

We will assume from now on that we have at our disposal an acceptable curated *politicians'* distribution for the social cost of carbon. Our next goal is to obtain an associated distribution of abatement policies. The task will be particularly simple if the aggressiveness of an abatement policy can be synthetically described by a one-parameter function, and if we can establish a monotonic relationship between this policy-aggressiveness parameter and today's social cost of carbon. If this is the case, from the distribution of social costs of carbon that we have obtained we can derive the corresponding distribution for the parameter that characterizes the aggressiveness of the abatement policy. So, our next steps are: i) to show that a strong relationship does exist between the social cost of carbon and the pace of abatement; and ii) to obtain the distribution for the 'aggressiveness' parameter.

We take as our starting point to characterize the aggressiveness of an abatement policy what Nordhaus and Sztorc (2013) call the abatement function, $\mu(t)$, implicitly defined by

$$e_{ind}(t) = \sigma(t) \cdot (1 - \mu(t)) \cdot y_{gross}(t) \ , \tag{8}$$

where $\sigma(t)$, $y_{gross}(t)$ and $e_{ind}(t)$ are the no-controls emission intensity, the gross economic output and the industrial emissions, all evaluated at time t, respectively. The interpretation of the key quantity $\mu(t)$ is the following: in the absence of any abatement efforts, industrial emissions are linked to gross economic output via the relationship

$$e_{ind}(t) = \sigma(t) \cdot y_{gross}(t) \ . \tag{9}$$

(It is for this reason that Nordhaus and Sztorc (2013) call $\sigma(t)$ the 'no-controls' emission intensity.) In the presence of abatement efforts the link between economic output and industrial emissions is progressively weakened, and the emission intensity becomes $\sigma'(t) = \sigma(t) \cdot (1 - \mu(t))$. Therefore $(1 - \mu(t))$ is the ratio between the no-controls emission intensity and the emission intensity in the presence of abatement efforts.

We assign to the abatement function, $\mu(t)$, the following functional form:[20]

$$\mu(t) = \mu_0 \cdot \exp[-\kappa \cdot t] + (1 - \exp[-\kappa \cdot t]) \tag{10}$$

This function is characterized by a single free parameter, κ, which describes the speed of the abatement policy. The additional parameter of the distribution, μ_0, is calibrated so as to recover today's abatement, $\mu_0 = 0.05$. Fig 10 displays the single-parameter function $\mu(t; \kappa)$ for different values of the abatement speed, κ.

The simple parametric form of Equation 10 allows the user to carry out some useful sanity checks: for instance, combining the information in the lines in Fig 10 with the information conveyed in Fig 7 one can see that a social cost of carbon of, say, \$ 250/ton C, corresponds to an abatement speed of approximately $\kappa = 0.1125\ y^{-1}$, which (from Fig 10)

[20]The functional form in Eq 10 is the expectation of a mean-reverting Ornstein-Uhlenbeck process with reversion speed κ and reversion level of 1.

in turn implies that in just 10 years' time 70% of the economy will be decarbonized. Going back to Fig 7, this implausibly aggressive abatement policy would have a significant probability of occurrence for the unshifted (economists') distribution, but becomes very unlikely both for the shifted (politicians') or for the maximum-entropy distribution. This stresses again the importance of adjusting the economists' distribution in order to obtain reasonable results.

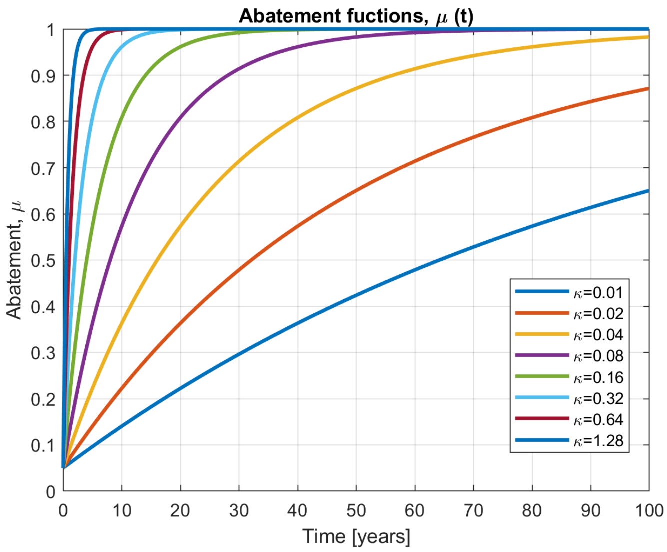

Figure 10: The abatement function, $\mu(t)$, for different abatement speeds, κ. Time in years on the x axis.

In general, the abatement policy can be a very complex function of time and of the state of the world. The one-parameter specific choice of abatement function in Equation 10 may therefore seem unduly restrictive. However, given a horizon of interest (say, the end of the century), as Hassler, Krussel, and Olovsson (2024) point out based on work by Allen, Frame, and et al (2009), 'global warming is approximately proportional to the cumulative emissions of CO_2 in both the short and long run.' So, for a given emission pattern, it matters little 'how we get there'. Given this insight, we show in Appendix E that, to a very good approximation, all policies that give rise to the same emission-weighted average abatement produce the same temperature at a chosen horizon. This means that we could have chosen a number of different functions $\mu(t)$, (as we show in Appendix E, in the limit, even a constant function $\mu(t) = \overline{\mu}$) with the same emission-weighted average, with little effect on the terminal temperature. Apart from analytical tractability, we have chosen the particular functional form in Equation 10, because of an important and surprisingly tight empirical relationship that we establish between the abatement speed, κ, and today's social cost of carbon. This relationship is at the heart of our procedure. The functional form we have chosen can therefore be regarded as one convenient place-holder among many – a placeholder chosen because of a useful relationship we can establish between the quantity for which we have determined a probability distribution (the social cost of carbon 'today'), and the quantity (the policy aggressiveness) for which we seek a distribution. The relationship comes about as follows.

First, we use the stylized one-parameter abatement function of the form in Equation 10 in the rich modification of the DICE-like model described in Rebonato, Kainth, Melin, and O'Kane (2024) to obtain the optimal abatement schedule. We remind the reader that with the Nordhaus and Sztorc (2013) model savings and the abatement function are the control variables used to optimize the welfare of a utility-optimizing agent who is exposed to climate damages to economic output. These damages can be mitigated through abatement whose cost increases exponentially with the degree of abatement, as captured by the function $\mu(t)$. We therefore optimize over the savings ratio and the time-dependent abatement function, and we calculate the associated optimal social cost of carbon today. We repeat this optimization exercise using

- deterministic or stochastic (à la Jensen and Traeger (2014)) processes for the total factor of production;
- widely different assumptions about the best climate models;
- widely different assumptions about the damage function (ranging from the 'tame' Nordhaus and Sztorc (2013) quadratic formulation to the 'aggressive' Howard and

Sterner (2017) with-tipping-point[21] parametrization);

- values for the utility discount rate ranging from 0.0010 (the value used in the Stern (2007) review) to 0.035 (more than double the value used in the DICE model);
- values for the elasticity of intertemporal substitution ranging from 0.5 (close to the value used in the DICE model) to 1.7 (close to the value used in the Bansal and Yaron (2004) approach);
- both time-separable and recursive (Epstein and Zin (1989)-type) utility functions;
- following Dietz and Stern (2015), damages that affect not just economic output, but also capital.

Unsurprisingly, each different choice for the preference, physics or economics parameters produces very different estimates for the optimal $SCC(0)$ and for the optimal abatement schedule, $\mu(t)$. However, empirically we find not only that higher values of the SCC are always associated with more aggressive abatement policies, but also that there is a surprisingly tight relationship between the characteristic abatement speed, κ, and the optimal cost of carbon, as vividly shown in Fig 11.[22]

The relationship in Fig 11 can be excellently approximated by the following empirical fit (with social cost of carbon expressed in \$/ton CO_2):

$$\kappa = 0.000428 \cdot SCC \text{ for } SCC \leq 200 \tag{11}$$

$$\kappa = 0.1656 - 0.00115 \cdot SCC + 0.00000375 \cdot SCC^2 \text{ for } SCC > 200 \tag{12}$$

This equation gives us the monotonic relationship between the social cost of carbon and the abatement speed that we need to determine the probability density of the abatement-speed parameter, κ. These probability distributions (shown in the Fig 12 for the original,

[21] Howard and Sterner (2017) refer to this parametrization as the 'catastrophic' one.

[22] For comparability with results quoted using DICE-family models. In Fig 11, and unlike the rest of the paper, the social cost of carbon is reported in \$/ton CO_2, not \$/ton C.

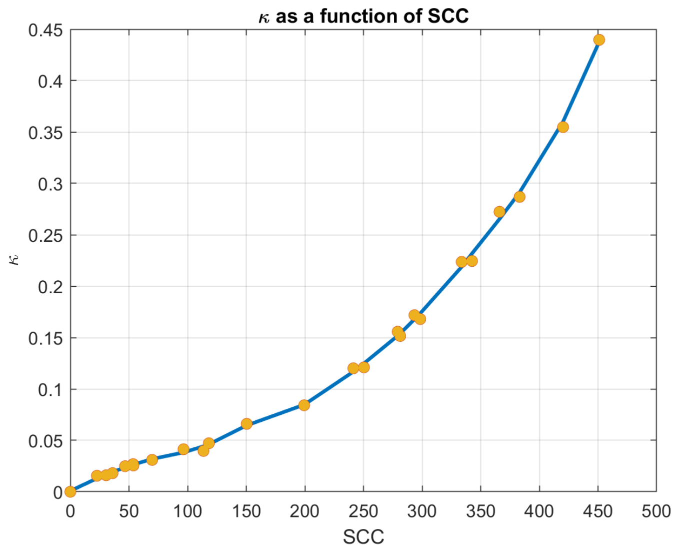

Figure 11: The abatement speed, κ (years^{-1}, y axis) as a function of the optimal social cost of carbon (\$/ton CO_2, x axis. The continuous curve is a LOWESS (Cleveland (1979)) quadratic smooth fit to the calculated points, shown as filled dots.

shape-preserving shifted, Kullback-Leibler shifted and maximum-entropy cases) allow us to assign different probabilities to abatement policies of different aggressiveness. These is the goal we set out to achieve. The following section shows how this information can be put to use.

5 Non-Policy Methodological Choices

The unconditional temperature distribution that we seek to obtain depends not only on the abatement policy, but also on the path of economic growth, on the uncertainty about the correct temperature model, and on the speed of no-controls decarbonization of the economy. We briefly review in this section our non-policy modelling choices, deferring a discussion of the model parameters until Section 6.1, where we provide a critical analysis of our results.

Industrial emissions are closely linked to economic output – in the DICE model a specific link is assigned through the no-controls emission intensity function, $\sigma(t)$, and the abatement

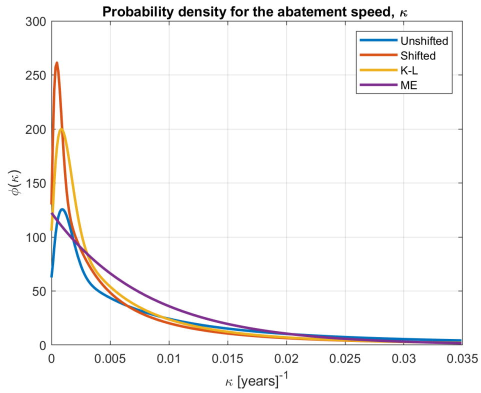

Figure 12: The probability density of the effective abatement speed, κ, for the unshifted distribution (blue line, labelled 'Unshifted'), the shape-preserving shifted distribution (red line, labelled 'Shifted'), the minimum-divergence K-L shifted distribution (yellow line, labelled 'K-L'), and the Maximum-Entropy distribution, (blue line, labelled 'ME').

function, $\mu(t)$, but the relationship is general. Recall then that, when it comes to future CO_2 concentrations, one of our main sources of uncertainty comes from the evolution of economic output. Following the adaptation for the DICE model by Jensen and Traeger (2014) of the seminal work by Bansal and Yaron (2004), we therefore capture this source of uncertainty by simulating as in Rebonato, Kainth, Melin, and O'Kane (2024) a distribution of economic outcomes at several horizons. This is a key quantity that characterizes the SSP narratives that underpin much of current scenario analysis. We note our model of economic growth nests many simpler models as special cases, and that the envelope of the economic growth paths produced by the economic model we use easily contains all the SSP growth trajectories. Does our distribution assign the correct probability weights to them? We cannot know for sure, but at least we note that the Bansal and Yaron (2004) distribution, coupled with Epstein and Zin (1989) utility functions, can recover both the equity risk premium *and* the level of real rates. So, our distribution 'contains' all the SSP growth trajectories, and ensures the recovery of important stylized facts in financial economics, suggesting that the implicit weights may be reasonably assigned. We discuss in Section 6.1 the parametrization of the model.

All modelling approaches used in the climate-economics literature assume that, independently of any explicit decarbonization efforts, the world economy will progressively decarbonize as GDP per capita increases – a form of the environmental Kuznets hypothesis discussed, eg, in Leal and Marques (2022). This is a common feature of the SSP/RCP approach, of the Oxford Economics model, of the NGFS model, and of the DICE model. In the latter both the no-controls emission intensity, $\sigma(t)$, and the GDP per capita, $h(t)$, are described by deterministic and monotonic functions of time (see the discussion in Nordhaus and Sztorc (2013)). We therefore eliminate the time parameter between the functions $\sigma(t)$ and $h(t)$, and establish a functional relationship between the two. We add 'noise' to this relationship, where the magnitude of the noise is estimated as the residual from a non-linear regression between the empirically observed functions $\sigma(t)$ and $h(t)$.[23] Also in this case we discuss in Section 6.1 the parametrization of the model.

Finally, as for the uncertainty about the correct temperature model (which is substantial), we rely on the analysis carried out in the IPCC (2019) report within the Coupled Model Intercomparison Phase VI (CMIP6). We simply note that we have taken care of excluding the 'hot-world' climate models criticized in Hausfather, Marvel, Schmidt, Nielsen-Gammon, and Zelinka (2022) – see Appendix F for a discussion.

6 Results and Discussion

The emissions associated with the different paths of economic output produced by the Jensen and Traeger (2014) model (traced from Bansal and Yaron (2004)) will be abated to different extents, depending on the abatement policy undertaken. We gauge the likelihood of different policies from the probability density shown in Fig 12 and, for every distribution, we run our simulations for each of the percentiles (from the 2.5th to the 97.5th, in increments of 5). We repeat this exercise using all the non-rejected reduced-form climate models

[23]Source: Our World in Data.

calibrated from the Coupled Model Intercomparison Phase VI (CMIP6). To be clear: we run a full simulation of economic outcomes for each combination of abatement speed and climate model, thereby producing a different probability-weighted CO_2 concentrations at different horizons. After averaging over the climate models, this procedure therefore gives twenty (one for each percentile) distributions of terminal CO_2 concentrations, each conditioned on a particular abatement speed. The corresponding unconditional distribution is obtained as the equal-weight average of the twenty conditional ones.

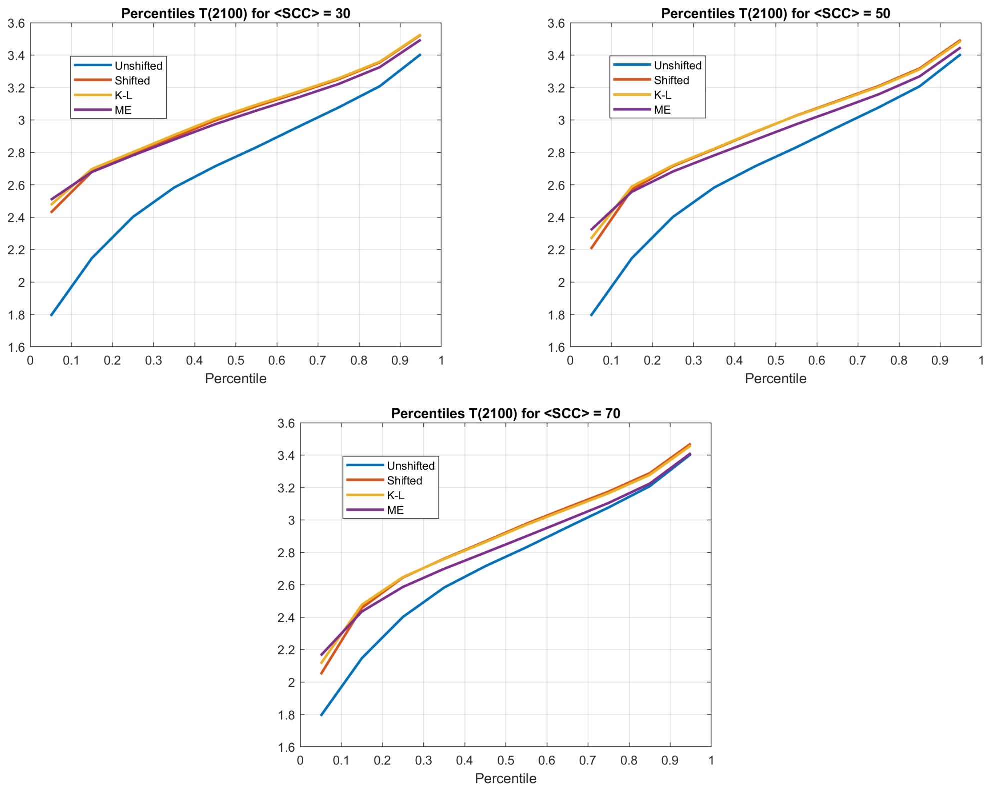

Figure 13: Percentiles of the distribution of the 2100 temperature anomaly for the unshifted distribution (blue line, labelled 'Unshifted'), the shape-preserving shifted distribution (red line, labelled 'Shifted'), the minimum-divergence K-L shifted distribution (yellow line, labelled 'K-L'), and the Maximum-Entropy distribution, (purple line, labelled 'ME'), for the case when the expectation of the social cost of carbon, $< SCC >$, is \$30/ton C, \$50/ton C and \$70/ton C (top left and right, and bottom panels, respectively).

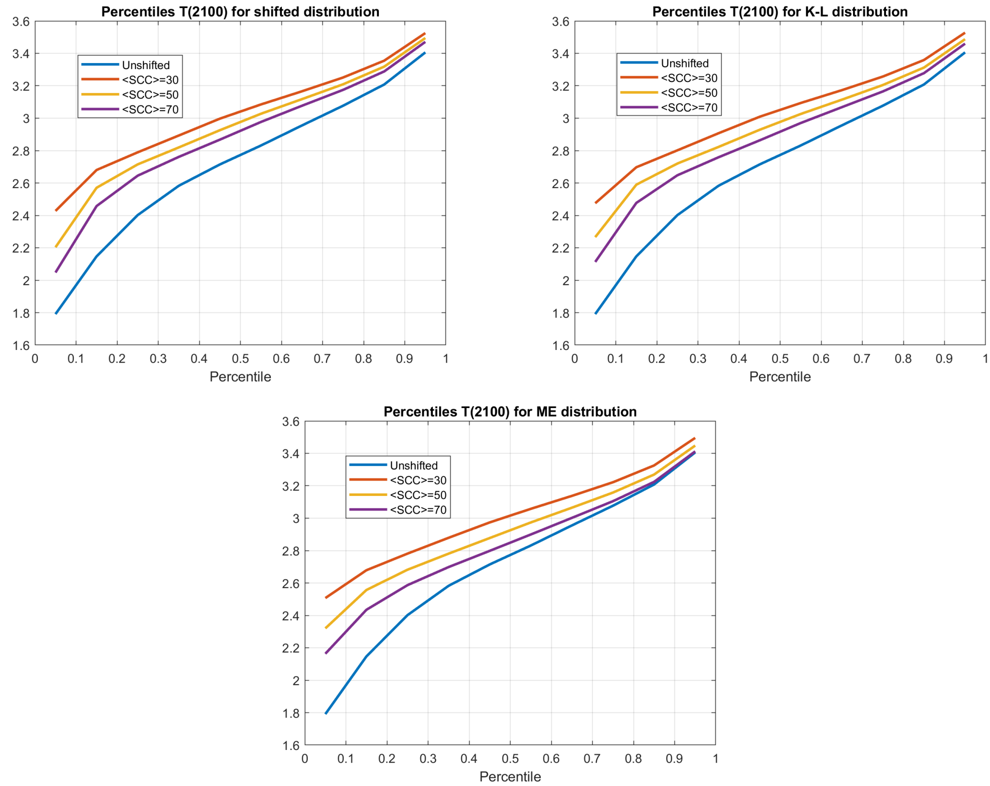

Figure 14: Percentiles of the distribution of the 2100 temperature anomaly as a function of the expectation of the social cost of carbon, $< SCC >$ (values of \$30, \$50 and \$70) for the unshifted distribution (blue line, labelled 'Unshifted'), the shape-preserving shifted distribution (red line, labelled 'Shifted'), the minimum-divergence K-L shifted distribution (yellow line, labelled 'K-L'), and the Maximum-Entropy distribution, (purple line, labelled 'ME'), displayed in the top left top right and bottom panels, respectively.

The results that we obtain using this approach are synthetically displayed in Figs 13 and 14: Fig 13 displays the percentiles of the distribution of the 2100 temperature anomaly for the unshifted distribution (blue line, labelled 'Unshifted'), the shape-preserving shifted distribution (red line, labelled 'Shifted'), the minimum-divergence K-L shifted distribution (yellow line, labelled 'K-L'), and the Maximum-Entropy distribution, (purple line, labelled 'ME'), for the case when the expectation of the social cost of carbon, $< SCC >$, is \$30/ton C, \$50/ton C, and \$70/ton C, (top left, top right and bottom panels, respectively). Fig 14 displays the same information, but each panel shows the effect of imposing different expectations of the social cost of carbon for the shifted, K-L and Maximum-Entropy distributions (again, top left, top right and bottom panels, respectively).

The first observation is that, while choosing different expected values for the expectation of the social cost of carbon, $< SCC >$, gives significantly different percentiles, for a given expectation the different ways to shift the economists' distribution yield remarkably similar values – see Fig 13. This suggests that the overall methodology we have proposed is very robust. This is all the more remarkable, given that the Maximum-Entropy procedure completely by-passes the elicitation of the economists' views about the optimal cost of carbon, yet recovers very similar temperature distributions once the same first-moment constraint has been imposed.

Next, we observe that the greatest difference in percentiles between the economists' distribution and all the other distributions, howsoever obtained, is for the 'left tail' (low-temperature) end of the distribution: the 5th percentile of the unshifted 2100 distribution is between 0.65 and 0.30 °C lower than the shifted distributions, but the 95th percentiles only differ by about a tenth of degree centigrade: so, all approaches roughly agree on how high the temperatures can get, but disagree on how low they may be contained. Different expected carbon taxes, $< SCC >$, affect the low percentiles much more than the high ones: see Fig 14.

As for the most likely levels of the temperature anomaly, we find values between 2.9 and 3.0 °C for the median value for the shifted distributions, and 2.8 °C for the unshifted distribution. If these estimates are correct, they should give food for serious thought, as the human species has never experienced temperatures 3.0 °C higher than pre-industrial levels. Given their importance, we critically discuss these results in Section 6.1, but, at this stage we note that, for all distributions (including the unshifted one), the lower Paris-Agreement target of 1.5 °C is virtually unachievable, and even the achievement of the 2.0 °C target is a very-low-probability event. Since these are the end-of-century temperatures whose attainment would require the most significant transition costs, we find that the probability of the economy suffering large transition costs is low, but the associated physical damages can be very high (see, in this respect, the recent analyses in Burke, Hsiang, and Miguel (2015) and Kotz, Leverman, and Wenz (2024)). Given the policy relevance of these conclusions, we provide in the next section a critical analysis of our results.

6.1 Critical Discussion of the Results

Our estimates give a significant probability to high temperature anomalies by the end of the century (with a median between 2.8 and 3.0 °C). These relatively high temperature values are in turn associated with low abatement efforts. If we are correct, this would mean that the current predominant focus on transition costs rather than physical damages is misplaced (the IPCC SSP-RCP scenarios do not even consider physical damages; the NGFS scenarios do allow for physical damages, but they are mainly articulated around different degrees of abatement around the 'Middle of the Road' (SSP2) narrative). How confident are we about our conclusions? We have discussed at length how we have arrived at a distribution of abatement speeds. Here we would like to assess critically the other methodological assumptions that affect our results. Our purpose in this section is to show that our modelling choices are in line with standard assumptions made in the climate-scenario literature (and, when not, more conservative), and that the high temperature distribution we obtain is therefore not a spurious feature of modelling aspects other than the speed of abatement.

Since reabsorption by natural mechanisms plays a limited role by the end of the century (and therefore concentrations to a given horizon depend on the total emissions to that horizon), and since the temperature anomaly is to a good approximation proportional to concentrations, we examine in detail what our model assumes for the components of the emission function, $e(t)$, which we present again here for ease of reference:

$$e(t) = \sigma(t) \cdot (1 - \mu(t)) \cdot y_g(t) \tag{13}$$

where the symbols have the usual meaning. Therefore, apart from the function $\mu(t)$, that depends on the abatement speed κ, and has been the focus of our discussion so far, the other key 'ingredients' to arrive at a temperature distribution are the no-controls emission intensity (the function $\sigma(t)$), and the process for economic growth (that determines economic output, $y_g(t)$). We examine these two quantities in turn.[24]

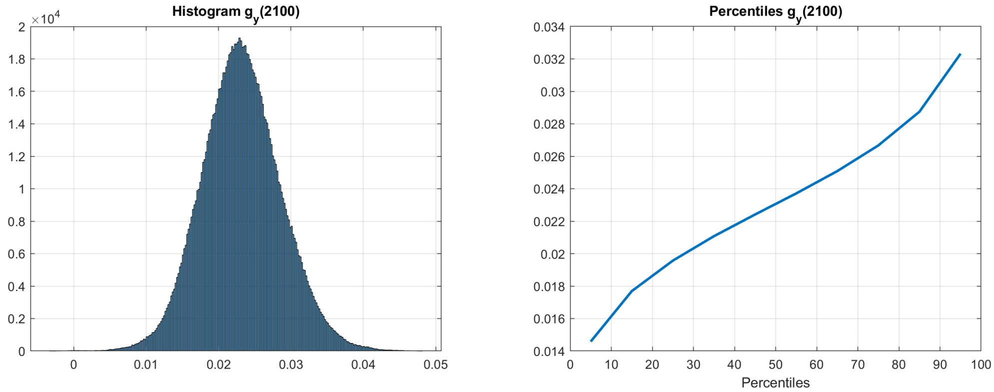

Figure 15: Histogram and percentiles of the distribution of economic growth from 2025 to 2100 obtained with the Maximum-Entropy distribution.

Starting from the distribution of economic growth (displayed in Fig 15), we note that the average economic growth to the end of the century assumed in the SSP scenarios is 2.00%, and 2.23% in our scenario. The lowest and highest levels of economic growth in the SSP scenarios are 1.28% and 2.87%, which correspond to the 3rd and 83rd percentiles of the distribution we obtain. The (deterministic) end-of-century growth in the Nordhaus and Sztorc (2013) DICE model (2.42%) is slightly above our average. As of 2024, the scenarios provided by Oxford Economics project economic growths (to 2050) ranging between 1.65% and 2.93%, with an average of 2.30%, slightly above our average value. We can therefore conclude that our assumed growth in economic output is in line with prevalent estimates,

[24]The temperature distribution, of course, also depends (and greatly so) on the climate models, and on how uncertain they are. As discussed above, we have used the set of reduced-form climate emulators that have passed the IPCC scrutiny, and in this respect, our modelling choices are therefore wholly 'standard'.

and that the relatively high weight to higher temperatures implied by our distribution is not due to an excessively high economic growth.

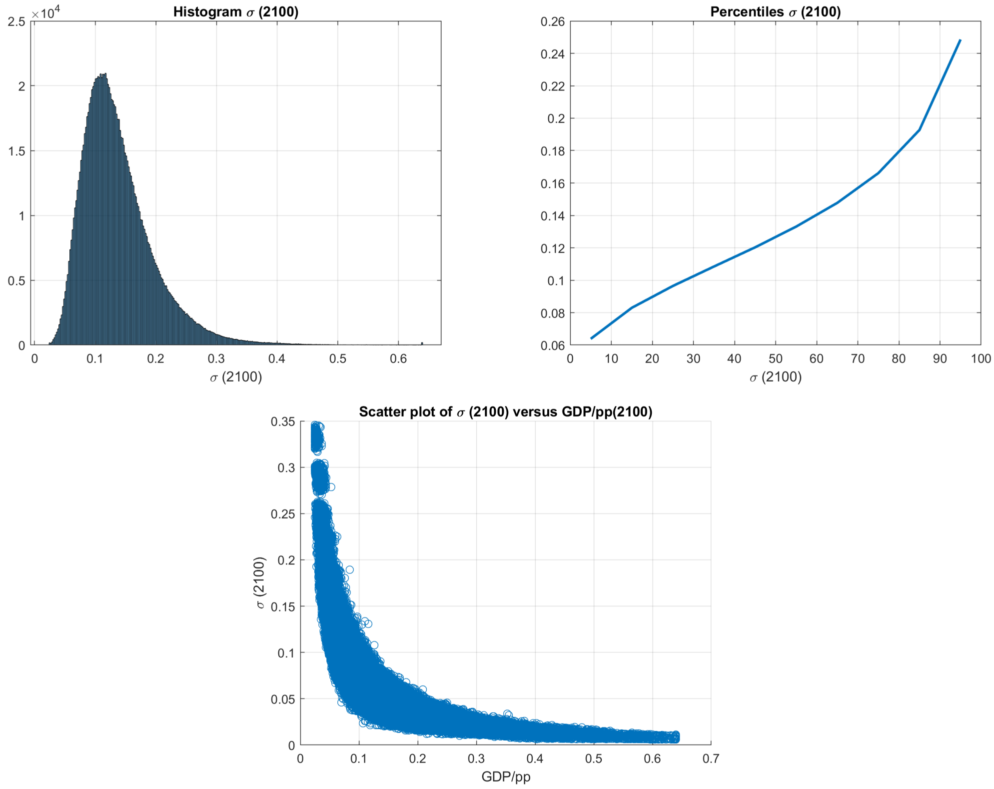

Figure 16: Histogram and percentiles of the distribution of the 2100 no-controls emission intensity, $\sigma(2100)$, obtained with the Maximum-Entropy distribution (top left and right panels, respectively), and the relationship between the no-controls emission intensity in the year 2100, $\sigma(2100)$, and GDP per person in the same year (bottom panel).

Moving to the no-controls emission intensity, $\sigma(t)$, in Fig 16 we show for this quantity the histogram and percentiles as obtained from our simulations (top left and right panel, respectively), and the implied relationship between the no-controls emission intensity and GDP per capita as produced by our model. We then show in Fig 17 the emission intensity, σ, for the five IPCC-endorsed SSPs (left panel), and as a function of GDP per person (right panel). Both our model and the SSP scenarios clearly display an inverse relationship between emission intensity and GDP per capita (a version of the environmental Kuznets curve). Apart from the qualitative similarity, the two figures show that our function is considerably more aggressive than what assumed by the IPCC-SSP scenarios (its expectation actually tracks very well the Nordhaus and Sztorc (2013) DICE deterministic function). A more aggressive no-controls decarbonization of the economy implies lower emissions for a given abatement function, $\mu(t)$. Lower 'no-controls' emissions in our model than in the current benchmark scenario models imply *lower* end-of-century concentrations implied by our model. We can therefore conclude that the high emissions, concentrations and hence temperatures that we find are not due to a rate of 'Kuznets decarbonization' slower than what is assumed in current scenario and optimization models.

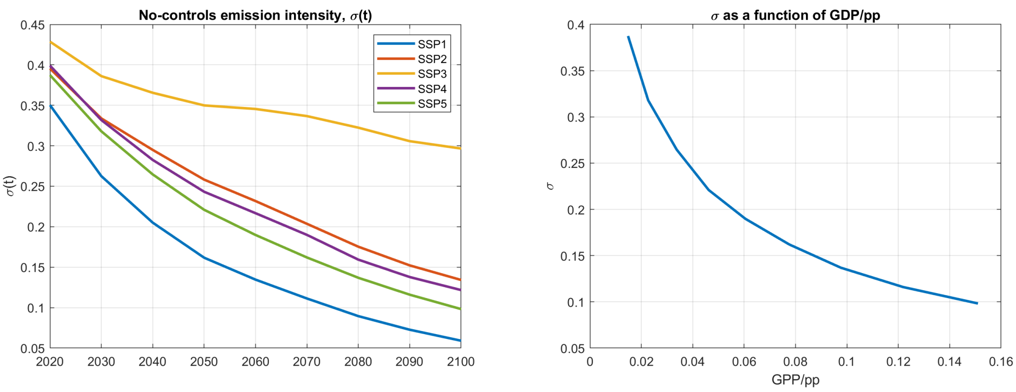

Figure 17: The no-controls emission intensity, σ, for the five IPCC-endorsed SSPs (left panel), and as a function of GDP per person (right panel).

In our search for an explanation of the relatively high end-of-century temperatures that we find, this therefore leaves us with the abatement function, $\mu(t)$. How realistic is our modelling of the distribution of this key quantity? Since there is a one-to-one correspondence between the effective abatement speed, κ, and the function, $\mu(t, \kappa)$, one can easily obtain the percentiles of this function at different horizons. These are displayed in Fig 18 for the year 2060 (left panel) and 2100 (right panel), for the case of $< SCC >=$ \$70/ton C. The 50th percentile of $\mu(2060)$ is no higher than 15-25% even for the case of $< SCC >=$ \$70/ton

C, and only climbs to 25 to 40% by the end of the century. As the right panel shows, a 90% decarbonization of the 2100 economy is only reached in 5% of our simulations (15% for the unshifted distribution).

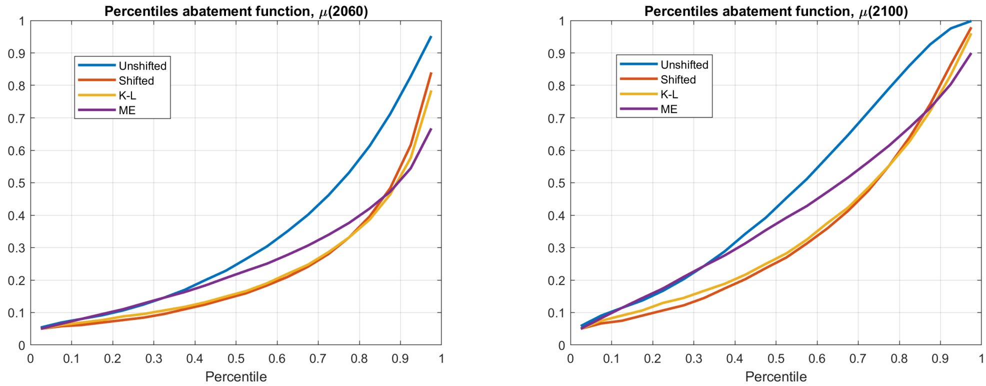

Figure 18: The percentiles of the abatement function, $\mu(t)$, in the year 2060 (left panel) and 2100 (right panel), for the case of $< SCC >= 70$.

It is this slow projected pace of decarbonization that is therefore at the heart of our results. How does this policy aggressiveness compare with the current pledges and actions? In its Climate Pledges Explorer,[25] which has been produced following the 2006 IPCC Guidelines, the International Energy Agency reports the emissions compatible with subsequent versions of Nationally Determined Contributions (NDCs) and Long-Term Low Emission Development Strategies (LT-LEDS) submitted to the UNFCCC. For the near-term horizon of 2030, if current NDCs are followed, emissions would have to fall globally by 9.7%, reversing the historical rate of change of greenhouse gas emissions from +297.8 Mt of CO_2-equivalent for the period 2010-2020 to −447.9 Mt of CO_2-equivalent for the period 2022-2030. When we estimate the percentiles of the abatement function, μ, for 2030, we find decarbonization rates *for this horizon* ranging from 7.5% to 10% (for $< SCC >=$ \$70/ton C), ie, values roughly in line with the current decarbonization pledges.

[25]available at https://www.iea.org/data-and-statistics/data-tools/climate-pledges-explorer , last updated 26 November 2024.

The major divergence between net-zero targets and our projections occurs after 2030. Indeed, the current net-zero-by-2050 plans imply a major acceleration of the rate of decarbonization between 2030 and 2050. This acceleration, however, is an aspiration: 75% of global emissions are covered by 'inadequate target design', 47% by 'poor design', and an additional 5% have no targets at all. Only 7% of global emissions are covered by 'acceptable' targets.[26] According to the United Nations assessment of the existing climate plans,[27] if the NDC commitments were respected, 2050 emissions would have to be roughly 65% lower than in 2020. With our projections, we find emission reductions by mid-century between 15% and 20%. Our values may seem unduly low, but we note that the United Nations updated NDC report concludes that 'for all available NDCs of all 192 Parties taken together, a sizeable increase, of about 16%, in global GHG emissions in 2030 compared to 2010 is anticipated. Comparison to the latest findings by the Intergovernmental Panel on Climate Change shows that such an increase, unless changed quickly, *may lead to a temperature rise of about 2.7 °C by the end of the century.*'[28] This is only a couple of tenths of °C lower than our median projections, despite assuming that the current NDCs will in reality be respected.

In sum: our projected emission reductions are roughly in line with current pledges only up to approximately 2030. After that, our projections and climate pledges substantially diverge. However, the temperatures associated with our projections only appear high when compared with the ambitious decarbonization *targets*, not with actual practices – in other words, we focus, as Schwalm, Glendon, and Duffy (2020) do, on the disconnect between actions and recommendations or pledges, rather than on the pledges themselves. When looked at in this light, our projections appear (unfortunately) very justifiable.

[26] Source: Climate Action Tracker, https://climateactiontracker.org/global/cat-net-zero-target-evaluations/, accessed January 2024.

[27] *Climate Plans Remain Insufficient: More Ambitious Action Needed Now*, October 2022, available at https://unfccc.int/news/climate-plans-remain-insufficient-more-ambitious-action-needed-now .

[28] Available at https://unfccc.int/news/updated-ndc-synthesis-report-worrying-trends-confirmed, accessed January 2024 (emphasis added).

7 Conclusions and Limitations

Borrowing an expression from price theory, the results we have presented can be looked at from the perspective of a 'climate taker' (someone who has to handle climate outcomes, set beyond her control)' or a 'climate setter' (someone who can influence climate outcomes).

From a climate-taker's perspective, the results we present suggest that there is a substantial probability of end-of-century temperatures well above the Paris Agreement upper target. As for the probability of remaining under 1.5°C, we find that it is extremely low – much lower than the probability to end up well above 3°C. The non-negligible probabilities of reaching these high temperatures, when translated into probabilities of end-of-century forcings suggest that the claim by Hausfather and Peters (2020) that scenarios with a 8.5 W/m^2 forcing (probably the forcing most analysed in climate studies) should be ignored because well-nigh-impossible is probably too strong. Indeed, we find that there is merit in Schwalm, Gelndon, and Duffy (2020) counterclaim in this respect that current abatement trajectories (not pledges) align well with the RCP 8.5 scenario. If our analysis is correct, our findings reinforce the conclusions by economists such as Pindyck (2022) that we should devote substantial resources to adaptation. At the same time, the RCP 8.5-based scenarios are *not* the most likely ones, and focussing predominantly on these, as often done in the literature, is indeed a poor use of analytical resources.

From a climate-setter's perspective, if one considers, as we do, the 2100-temperature distribution in Figs 13 and 14 as 'too hot for comfort',[29] the results we present recommend

[29] Whether the high temperatures in Figs 13 and 14 with non-negligible probabilities are indeed 'too high for comfort' brings us into the enormous topic of the damage function. Without entering the topic, we simply point out that paleoclimatic evidence suggests that large, relatively sudden, increases in temperature ('tipping points') have occurred many times in the past – with the most recent instance in the exiting from the last ice age, when locally temperature increased by 7°C or more in a few decades (see section 2.4.3 of the IPCC report – *Climate Change 2021: The Physical Science Basis*). Under such conditions, the scope for adaptation would be very limited. The location of the threshold for tipping points, and the associated

a decisive change in the pricing of carbon emissions. Recall that a key ingredient of our approach is the wedge between what (economists say) we should do, and what we actually do – as proxied by the difference between the average economist-recommended and the actually implemented cost of carbon. Since this disconnect has been observed for decades now, the working assumption of our approach is that this bias is stable, and will remain a feature of future policy/recommendation interactions. Admittedly, there is no law that stipulates that this should be the case. However, we don't see a way to achieve substantially lower temperatures unless politicians (and the electorate) become willing to bear the cost of carbon emissions that economists on average estimate.

This does not mean that a literal carbon tax is the solution – there are many ways to impose a cost on emissions, and their relative merits (theoretical and practical) are still hotly debated (see, eg, Hassler, Krussel, and Olovsson (2024), Acemoglu, Aghion, Barrage, and Hemus (2024), WorldBank (2024) for recent perspectives). Whatever forms the 'carbon tax' will assume, our study suggest that substantially greater resources than currently devoted should be directed towards abatement if we want the end-of-century temperature to be close to the Paris targets: in our terminology, ideally, there should be no need to shift the economists' distribution; in practice, there still is, and the shift is large.

Another important conclusions that our results suggest is that, policies with an aggressive (and hence costly) abatement path have low probability. The probability of high transition costs borne by the economy as a whole is therefore also small. The verso of this coin is that the probability of high physical damages is significant. With Rebonato, Melin, and Zhang (2024) we therefore also conclude that policymakers and investors would be ill advised to assume that effects of physical climate damages on macrofinancial variables will be either minor (because of effective and prompt abatement), or 'discounted away'.

increase in temperature, are not known with any degree of certainty (see Lenton, Held, Kriegler, Hall, Lucht, Rahmstorf, and Schellnhuber (2008) for a discussion), but the National Research Council (see(NRC, 2002)) refers to tipping points as 'inevitable surprises'.

Finally, the fiscal (tax and public debt) and technological bounds that we have identified also caution against waiting too long, because, as Dietz and Stern (2015) point out, the abatement efforts cannot be ramped up too quickly – a fact often ignored by theoretical estimates of optimal costs of carbon, but at least approximately captured by our upper bounds.

APPENDICES

A Modelling Uncertainty in Economic Growth

We model uncertainty in economic growth using the Jensen and Traeger (2014) modification of the influential 'long-term growth' Bansal and Yaron (2004), Bansal and Shaliastovich (2012) model. In this approach, the variability in economic outcomes arises from uncertainty in the growth process for the total factor productivity (TFP), $A(t)$, denoted by $g_A(t)$. This is given by:

$$A(t + \Delta t) = A(t) \exp\left(g_A(t)\Delta t \ + z(t)\Delta t\right) \tag{14}$$

where $g_A^{\text{det}}(t)$ is the deterministic growth trend and $z(t)$ is a random growth shock. The deterministic component of the technology process is assumed to decay with time following the Nordhaus specification:

$$g_A^{\text{det}}(t) = g_A^{\text{det}}(0)\exp(\delta_a t), \qquad \delta_a = -0.005\text{yr}^{-1}, g_A^{\text{det}}(0) = 0.076 \tag{15}$$

To capture the empirically observed strong time persistence of TFP, growth shocks are assumed to consist of two uncorrelated components

$$z(t) = x(t) + w(t) \tag{16}$$

where $x(t)$ is assumed to follow a Wiener process and $w(t)$ follows an Ornstein-Uhlenbeck process with reversion level of 0:

$$\begin{aligned} x(t + \Delta t) &= x(t) + \mu_x \Delta t + \sigma_x dZ_t^x \\ w(t + \Delta t) &= \mu_w(1 - e^{-\theta\Delta t}) + w(t)e^{-\theta\Delta t} + dZ_t^w \end{aligned} \tag{17}$$

Here $dZ_t^x \sim N(0,1)$ and $dZ_t^w \sim N\left(0, \frac{\sigma_w^2}{2\theta}(1 - e^{-2\theta\Delta t})\right)$ and $\mathbb{E}(dZ_t^x, dZ_t^w) = 0$.

When simulating the model, we discretise assuming a finite time step, $\Delta t = 5$ years. When discretised, the Ornstein-Uhlenbeck process becomes an AR(1) process:

$$w_{t+1} = \mu_\epsilon + \gamma w_t + \epsilon_t$$

where the properties of γ, μ_ϵ and ϵ_t are readily developed from the corresponding values of θ and Δt:

$$\begin{aligned}
\gamma &= e^{-\theta\Delta t} \\
\mu_\epsilon &= \mu_w(1-e^{-\theta\Delta t}) \\
\epsilon_t &\sim N(0,\sigma_\epsilon^2),\ \sigma_\epsilon^2 = \frac{\sigma_w^2}{2\theta}(1-e^{-2\theta\Delta t})
\end{aligned} \tag{18}$$

The volatilities, σ_x and σ_ϵ and the autocorrelation parameter γ are estimated by Jensen and Traeger (2014) so that $A(t)$ is consistent with empirical long run US data on the total factor productivity and consistent with Bansal and Yaron (2004). Both volatilities are set at 1.9%, while γ is determined for a 5-year interval and is set at 0.5. The drift terms, μ_x and μ_ϵ, are developed by requiring that the overall mean of the growth rate of the TFP, $g_{A,t}$, should match the deterministic growth rate component, ie, we require that:

$$\mathbb{E}_t\left[A(t+\Delta t)\right] = A(t)\exp\left(g_A^{\text{det}}(t)\Delta t\right)$$

We can use this to show that $\mu_x = -\frac{\sigma_x^2}{2}$ and:

$$\mu_\epsilon = -\frac{\sigma_\epsilon^2}{2}\frac{\sum_{p=0}^{T-1}\left(\frac{1-\gamma^{T-p}}{1-\gamma}\right)^2}{\sum_{p=0}^{T-1}\left(\frac{1-\gamma^{T-p}}{1-\gamma}\right)} \tag{19}$$

Here T is the (finite) long horizon over which we simulate the discrete model (typically $T = 100$)[30].

[30] This is a correction to the result presented in Jensen and Traeger (2014)

B Shape-Preserving Shift of the Distribution of the Social Cost of Carbon

In this Appendix we show how the distribution for the social cost of carbon can be shifted as to recover an exogenously assigned first moment in such a way as to retain the shape of the original (unshifted) distribution as much as possible.

We start from a distribution, $\phi'(x)$, about which we only know the normalization constraint, $\int_{\Omega'} \phi'(x) \cdot dx = 1$ (where Ω' denotes the finite support of $\phi'(x)$). We want to create a distribution, $\phi(x)$, such that, in addition to the normalization constraint, also the condition $\int_{\Omega} \phi(x) \cdot x \cdot dx = \mu$ should be satisfied. (Here μ denotes the observed cost of traded carbon permits, and Ω is a different, but still finite, support.) Of course, this additional constraint can be satisfied in an infinity of ways. Since we believe that there is information on the economists' distribution, we want to preserve its shape as much as possible. To do so, we discretize the integral above (and, for economy of notation, we denote the independent variable, the social cost of carbon, by x). We have

$$\int_{\Omega} \phi'(x) ds \approxeq \sum_i \phi'(x_i) \cdot \Delta x_i \tag{20}$$

The probability, $P(x_i)$, of the SCC having values between x_i and x_{i+1} is given by $P(x_i) = \phi'(x_i) \cdot \Delta x_i$. We impose that the same probability will apply for values of x in a different range, given by $R \cdot [x_i, x_{i+1}]$, with $R > 0$. To preserve the probability of the interval, the density function, $\phi'(x)$, will have to be changed to

$$\phi'(x_i) \to \phi(x) = \frac{\phi'(x_i)}{R} \tag{21}$$

When we do this, the normalization will be preserved by construction, and, if we define $\int_{\Omega'} \phi'(x) \cdot x \cdot dx = \mu'$, the ratio R that ensures that $\int_{\Omega} \phi(x) \cdot x \cdot dx = \mu$ is simply given by $R = \frac{\mu}{\mu'}$. The construction ensures that, apart from a 'stretching' of the x axis, the shape of the original distribution – which we believe to convey useful information – is preserved.

C Derivation of the Maximum-Entropy Distribution

For a continuous random variable, X, with a strictly positive probability density function $\phi(x)$ with support Ω, the entropy of the distribution, $H(\phi)$, is defined as

$$H(\phi) = -\int_{\Omega} \phi(x) \cdot \log \phi(x) \cdot dx \tag{22}$$

Let

$$\int_{\Omega} f_i(x) \cdot \phi(x) \cdot dx = c_i \tag{23}$$

be a number of constraints ($i = 1, 2, ..., n$) on the moments or range of the distribution. Then there are strong information-theoretical reason to argue that the least-committal distribution that reflects these constraints is the distribution that maximizes the entropy defined above, subject to the chosen constraints. The problem can therefore be solved by means of Lagrange multipliers, by creating a Lagrangean function, $L(\phi, \lambda_i)$, given by

$$L(\phi, \lambda_i) = H(\phi) + \sum_i \lambda_i \left(c_i - \int_{\Omega} f_i(x) \cdot \phi(x) \cdot dx \right) \tag{24}$$

To obtain an extremum of the Lagrangean, we differentiate with respect to the Lagrange multipliers, λ_i, perform a variational differentiation with respect to $\phi(\cdot)$, and set all the derivatives to zero.

For the problem at hand, we want to ensure i) that the density of SCC integrates to 1 between 0 and TUL; and ii) that the expectation of this distribution is equal to μ. The constraints therefore are

$$\int_0^{TUL} \phi(x) \cdot dx = 1 \tag{25}$$

$$\int_0^{TUL} x \cdot \phi(x) \cdot dx = \mu \tag{26}$$

and the Lagrangean is given by

$$L(\phi, \lambda_1, \lambda_2) = -\int_0^{TUL} \phi(x) \cdot \log \phi(x) \cdot dx - \lambda_1 \left(1 - \int_0^{TUL} \phi(x) \cdot dx \right) - \lambda_2 \left(\mu - \int_0^{TUL} x \cdot \phi(x) \cdot dx \right) \tag{27}$$

Setting to zero the functional derivative with respect to $\phi(\cdot)$ gives

$$\log \phi(x) = -1 + \lambda_1 + \lambda_2 \cdot x \to \phi(x) = \exp(\lambda_1 - 1) \exp(\lambda_2 \cdot x) = K \exp(\lambda_2 \cdot x) \tag{28}$$

with K a normalization constant (a function of λ_2). Imposing

$$\int_0^{TUL} K_{\lambda_2} \exp(\lambda_2 \cdot x) = 1 \tag{29}$$

gives

$$K_{\lambda_2} = \frac{\lambda_2}{\exp(\lambda_2 \cdot TUL) - 1} \tag{30}$$

The value for λ_2 is then obtained by imposing the expectation constraint,

$$\mu = \int_0^{TUL} x \cdot K_{\lambda_2} \cdot \exp(\lambda_2 x) \cdot dx = \frac{TUL \cdot \exp(TUL \cdot \lambda_2)}{\exp(TUL \cdot \lambda_2) - 1} - \frac{1}{\lambda_2} \tag{31}$$

and solving for λ_2.

D Derivation of the Minimum K-L-Deviation Distribution

For a continuous random variable, X, with a strictly positive probability density function $\phi(x)$ with support Ω, and a 'reference' distribution, Q, the Kullback-Leibler (KL) deviation of the distribution, $KL(\phi, Q)$, is defined as

$$KL(\phi, Q) = -\int_{\Omega} \phi(x) \cdot \log \frac{\phi(x)}{Q(x)} \cdot dx \tag{32}$$

For the derivation below we re-write this as

$$KL(\phi, Q) = -\left[\int_{\Omega} \phi(x) \cdot \log \phi(x) \cdot dx - \int_{\Omega} \phi(x) \cdot \log Q(x) \cdot dx\right] \tag{33}$$

Proceeding as in Appendix C, we want to determine a density function, $Q(x)$, such that i) it integrates to 1 between 0 and TUL; ii) its expectation is equal to μ; iii) it is equivalent to $\phi(x)$ (it shares the same null set); and iv) *the KL divergence $KL(\phi, Q)$ is minimized.* The constraints therefore are

$$\int_0^{TUL} Q(x) \cdot dx = 1 \tag{34}$$

$$\int_0^{TUL} x \cdot Q(x) \cdot dx = \mu \tag{35}$$

and (neglecting the irrelevant term $\int_0^{TUL} \phi(x) \cdot \log \phi(x) \cdot dx$) the Lagrangean is now given by

$$L(\phi, \lambda_1, \lambda_2) = -\int_0^{TUL} \phi(x) \cdot \log Q(x) \cdot dx + \lambda_1 \left(1 - \int_0^{TUL} Q(x) \cdot dx\right) + \lambda_2 \left(\mu - \int_0^{TUL} x \cdot Q(x) \cdot dx\right) \tag{36}$$

Setting to zero the functional derivative with respect to $Q(\cdot)$ gives

$$Q(x) = \frac{\phi(x)}{\lambda_1 + \lambda_2 \cdot x} \tag{37}$$

By imposing the normalization and first moment conditions the constants λ_1 and λ_2 can be numerically determined.

E The Emission-Weighted Average Abatement Function

We start from

$$\Delta C(T) = \int_0^T e_{ind}(s)ds \tag{38}$$

and

$$e_{ind}(t) = \sigma(t) \cdot (1 - \mu(t)) \cdot y(t) \tag{39}$$

where we have dropped the subscript 'gross' for gross economic output, $y(t)$. These two equations can be combined to give

$$\Delta C(T) = \int_0^T \sigma_s \cdot y_s \cdot ds - \int_0^T \sigma_s \cdot y_s \cdot \mu_s \cdot ds \tag{40}$$

Define

$$w_t \equiv \sigma_t \cdot y_t \tag{41}$$

and rewrite Eq 40 as

$$\Delta C(T) = \int_0^T w_s \cdot ds - \int_0^T w_s \cdot \mu_s \cdot ds \tag{42}$$

Dividing both sides by $\int_0^T w_s \cdot ds$ one gets

$$\frac{\int_0^T w_s \cdot \mu_s \cdot ds}{\int_0^T w_s \cdot ds} = 1 - \frac{\Delta C}{\int_0^T w_s \cdot ds} \tag{43}$$

We recognize the LHS as the weighted average, $< \mu >$, of the abatement function μ_t, with weights given by the function w_t. We therefore write

$$< \mu >= 1 - \frac{\Delta C}{\int_0^T w_s \cdot ds} \tag{44}$$

Since the RHS does not depend on the abatement function, we conclude that, for given paths of economic output and no-control emission intensity, all the abatement functions with the same weighted average give rise to the same change in CO_2 concentration.

We have chosen the symbol w_t for the product $\sigma_t \cdot y_t$ to emphasize the role of weighting function played by this quantity. We can gain further insight by defining

$$e_s^* \equiv \sigma_t \cdot y_t \tag{45}$$

$$\Delta e_s \equiv \mu_t \cdot \sigma_t \cdot y_t \tag{46}$$

We recognize e^*_t as the emissions that would occur for a given level of economic output and a given level of emission intensity, σ_t, if no active decarbonization steps were taken, ie, if all the energy needed for production were obtained from burning fossil fuels. (The emission intensity, σ_t, may in general decline over time as the economy moves from manufacturing to services (the environmental Kuznets effect, as reviewed in Leal and Marques (2022)), but this is independent of any additional decarbonization effort.) With Nordhaus and Sztorc (2013), we call this a 'no-controls' situation. The quantity

$$\Delta C^* = \int_0^T e^*_s ds \tag{47}$$

is therefore the increase in concentration from time 0 to time T that would occur under no controls. Equation 48 can therefore be rewritten as

$$< \mu >= 1 - \frac{\Delta C}{\Delta C^*} \tag{48}$$

showing that the weighted average abatement function is a function of the ratio of concentrations with the abatement schedule μ_t to the concentration that would obtain under no controls.

Define next

$$H(T) = \int_0^T \Delta e_s \cdot ds \tag{49}$$

Then we can write

$$\Delta C(T) = \int_0^T \sigma_s \cdot y_s \cdot ds - \int_0^T \sigma_s \cdot y_s \cdot \mu_s \cdot ds = \Delta C^*(T) - H(T) \tag{50}$$

and therefore $H(T)$ can be interpreted as the change in time T concentration because of abatement efforts, over and above the decarbonization brought about by the declining function σ_t: $H(T) = \Delta C^*(T) - \Delta C(T)$. Therefore the weighted average abatement speed can be written as

$$< \mu >= \frac{H}{\Delta C^*} \tag{51}$$

ie, as the ratio of the abatement-induced concentration reduction (H) to the no-controls concentration (ΔC^*).

Let $M(\Delta C)$ denote the set of all the abatement functions that, for given paths of economic output and no-control emission intensity, have the same weighted average, and let $m_s^1, m_s^2 \in M$. Then we have

$$\int_0^T w_s \cdot m_s^1 \cdot ds = \int_0^T w_s \cdot m_s^2 \cdot ds \tag{52}$$

This equation holds in particular for the constant function, $\overline{m}$, and, denoting by m_s the generic element of the set M, we have

$$\overline{m} = \frac{\int_0^T w_s \cdot m_s \cdot ds}{\int_0^T w_s \cdot ds} =< \mu > \tag{53}$$

ie, there exists a constant abatement function, $\overline{m}$, that produces the desired change in concentration and this constant is equal to the weighted average abatement function.

A final comment about negative emissions. Negative emissions at one point in time can be modelled by a function $\mu_t > 1$. Even in this case, however, as long as the weighted average of the abatement function *over the whole period* is smaller than 1, a function, μ_s, that is smaller than 1 everywhere can produce the correct time-T concentration. It is only if the weighted average concentration were greater than 1 that an instantaneous function, μ_t greater than 1 at least in some parts of the interval $[0\ ,\ T]$ is needed. Since this is most unlikely to be the case, the fact that our abatement functions cannot exceed 1 does not pose a serious limitation.

F CMIP6 climate models

Our list of reference models is based on Hausfather, Marvel, Schmidt, Nielsen-Gammon, and Zelinka (2022) recommended procedure of excluding models with Transient Climate Response (TCR) and Equilibrium Climate Sensitivity (ECS) outside 'likely' ranges (1.4-2.2°C and 2.5-4°C respectively). These are the models associated with the so-called 'hot-world problem' identified in the last generation of climate model simulations in CMIP6, ie, the models that, on the basis of alternative evidence, appear to overestimate how much the planet will warm in response to a sudden (ECS) or gradual (TCR) increase in CO_2 concentration.[31] When we exclude the hot-world models, we are left with 16 models, which we report below. Tab 19 lists the models used in our simulations. After the exclusions, the average, maximum and minimum 2100 temperatures are 2.85, 3.28 and 2.45 °C, with a standard deviation of 0.26 °C.

[31] The Equilibrium Climate Sensitivity is the equilibrium change in temperature corresponding to an instantaneous doubling of the CO_2 concentration. The Transient Climate Response (TCR) is the amount of warming that might occur at the time when CO2 doubles, having increased gradually by 1% each year.

Model	ID
BCC-ESM1	1
GISS-E2-1-G	2
MPI-ESM1-2-LR	3
ACCESS-ESM1-5	4
CAMS-CSM1-0	5
GISS-E2-1-H	6
MIROC6	7
MRI-ESM2-0	8
AWI-CM-1-1-MR	9
GISS-E2-2-G	10
MIROC-ES2L	11
NorCPM1	12
BCC-CSM2-MR	13
FGOALS-g3	14
MPI-ESM1-2-HR	15

Table 19: List of the models used in our study.

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