

# Period Detection with Stingray

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**These materials:**

<https://github.com/tloredo/AAS237-TimeSeries>

**Documentation:**

<https://stingray.readthedocs.io/en/latest/index.html>

**Tutorials:**

<https://github.com/stingraysoftware/notebooks>

**Iconic 60s Theme Song:**

<https://youtu.be/sgkk-MMif-4?t=50>

# Terminology:

**“energy”** = photon energy (i.e. photon wavelength)

**“frequency”** = Fourier (temporal) frequency

# Terminology\*:

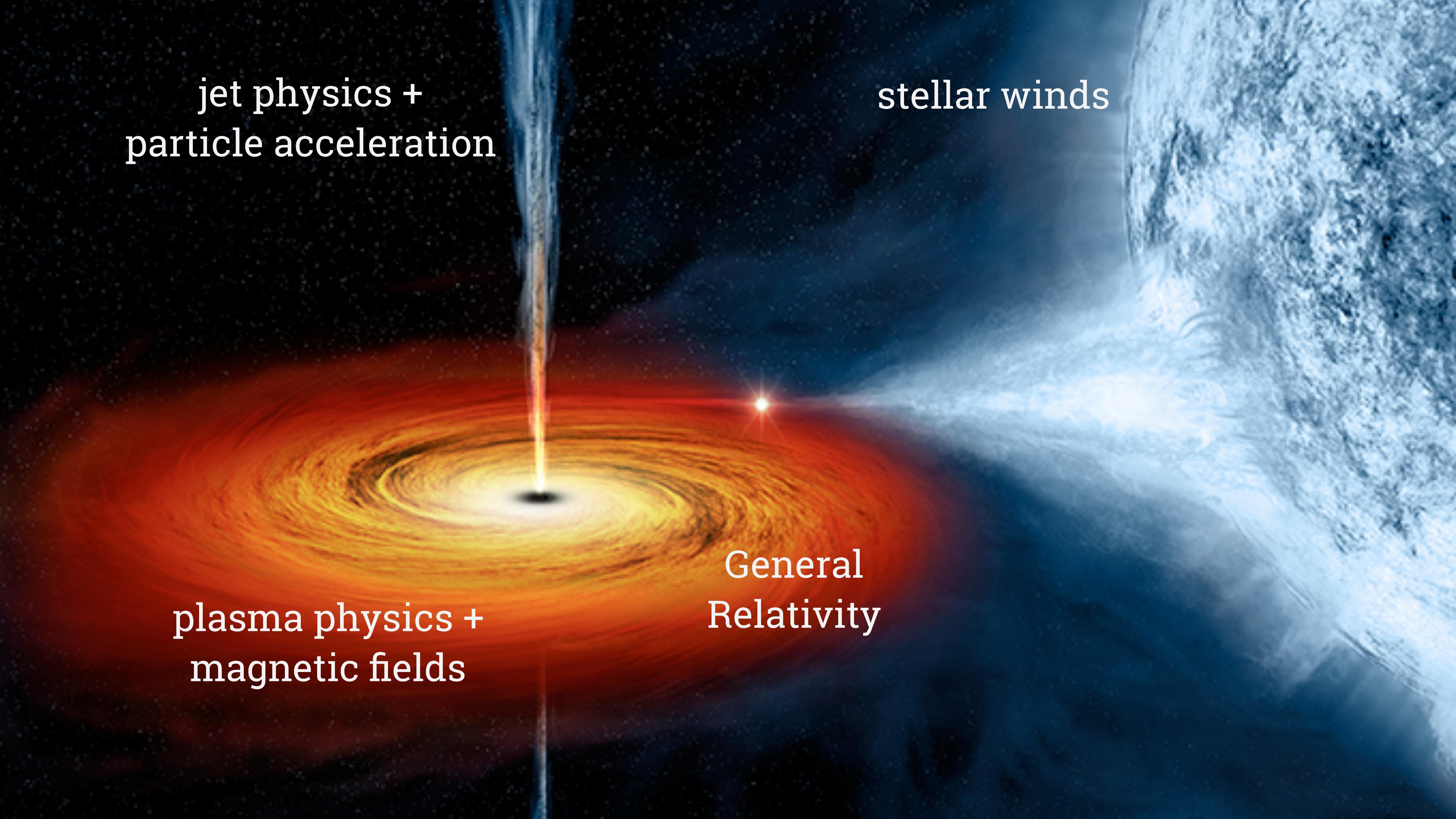
“**periodogram**” = an observed random realization of a stochastic process

“**power spectrum**” = the underlying physical process that generated the observation

\*except in **Stingray**

# Some Scientific Examples

**How do black holes accrete  
matter?**



jet physics +  
particle acceleration

stellar winds

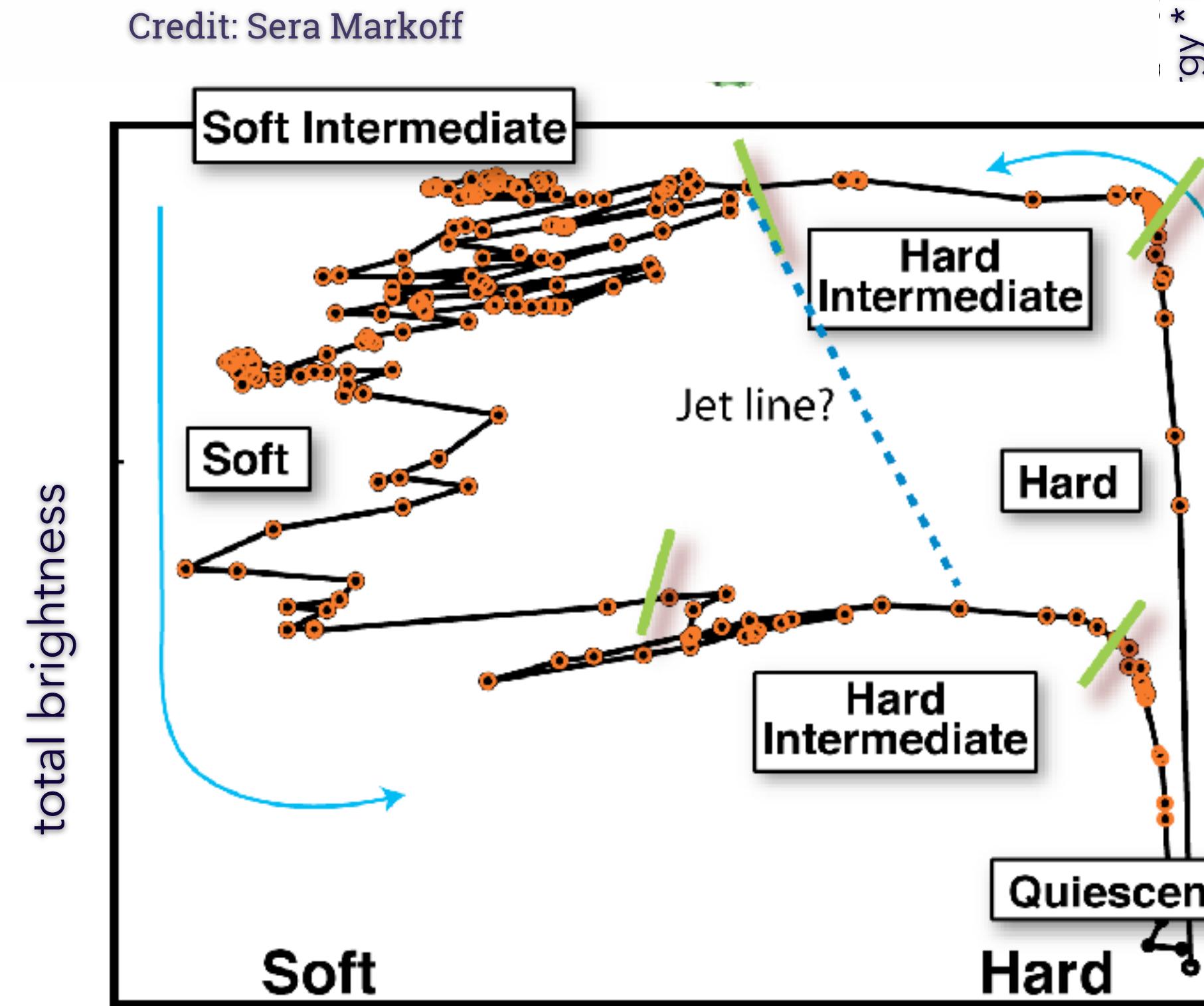
plasma physics +  
magnetic fields

General  
Relativity

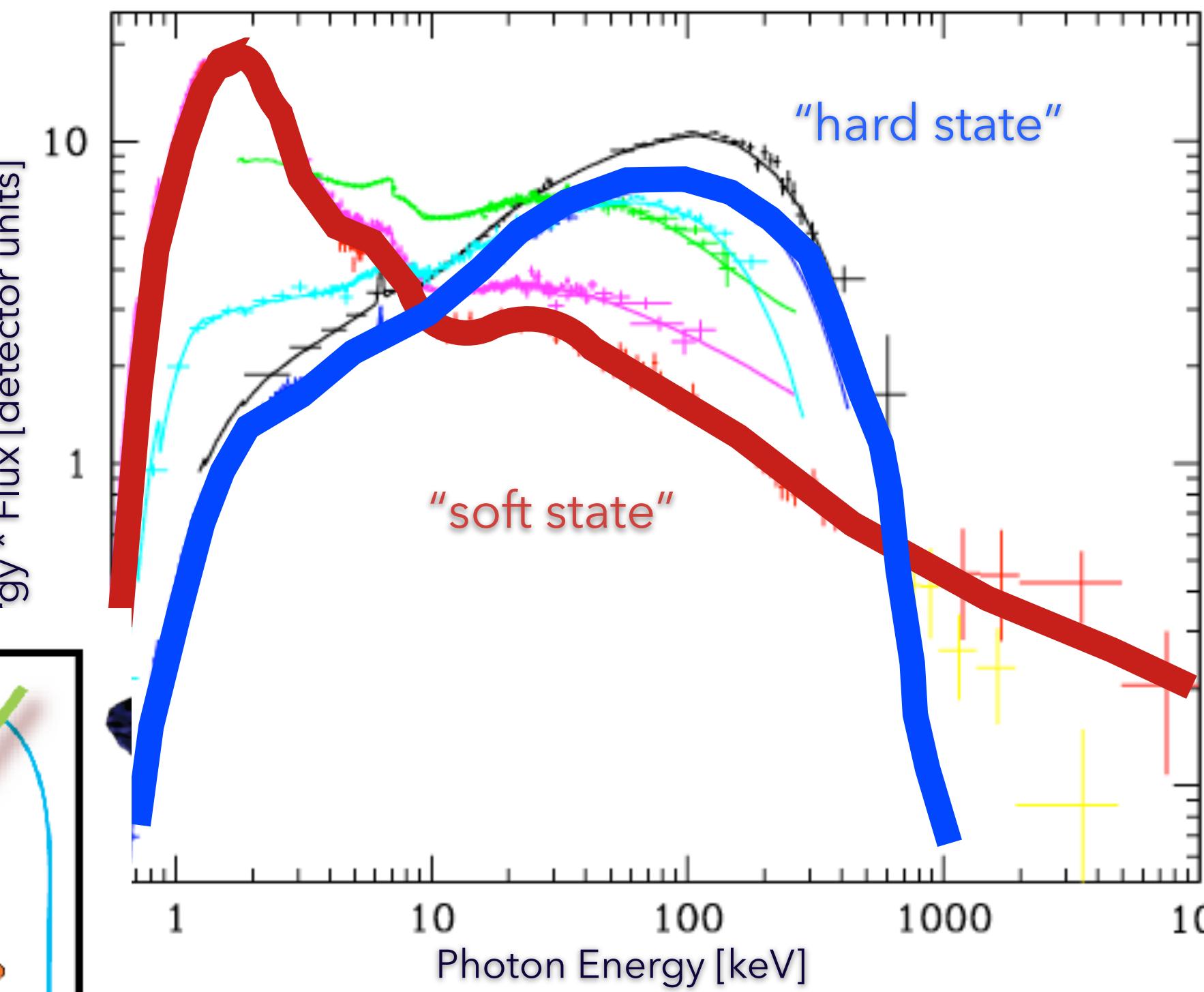
# Spectral States



Malzac 2008

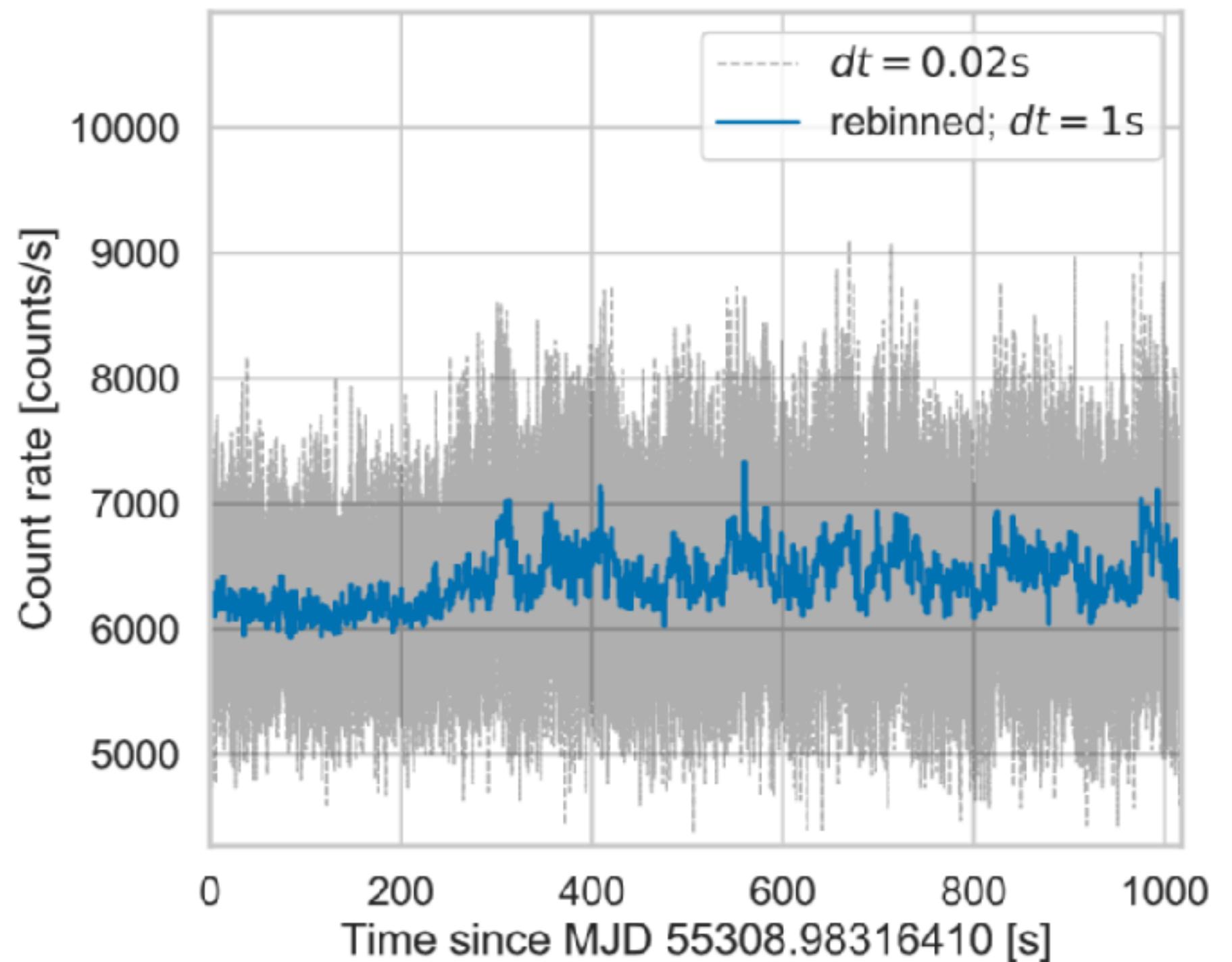


high-frequency/low-frequency X-ray brightness



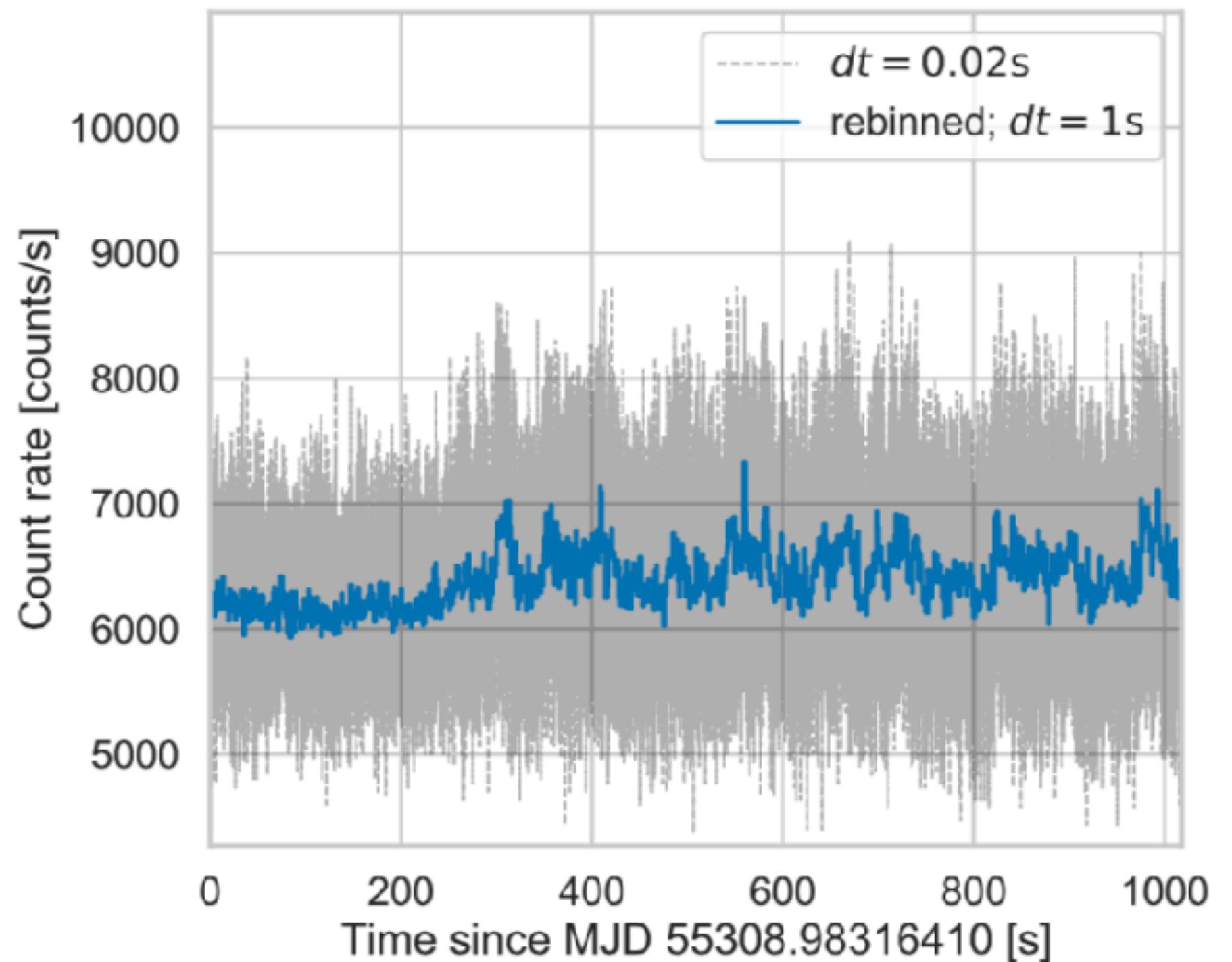
# Variability: GX 339-4

Yamaoka et al (2010);  
Huppenkothen et al. (2019)



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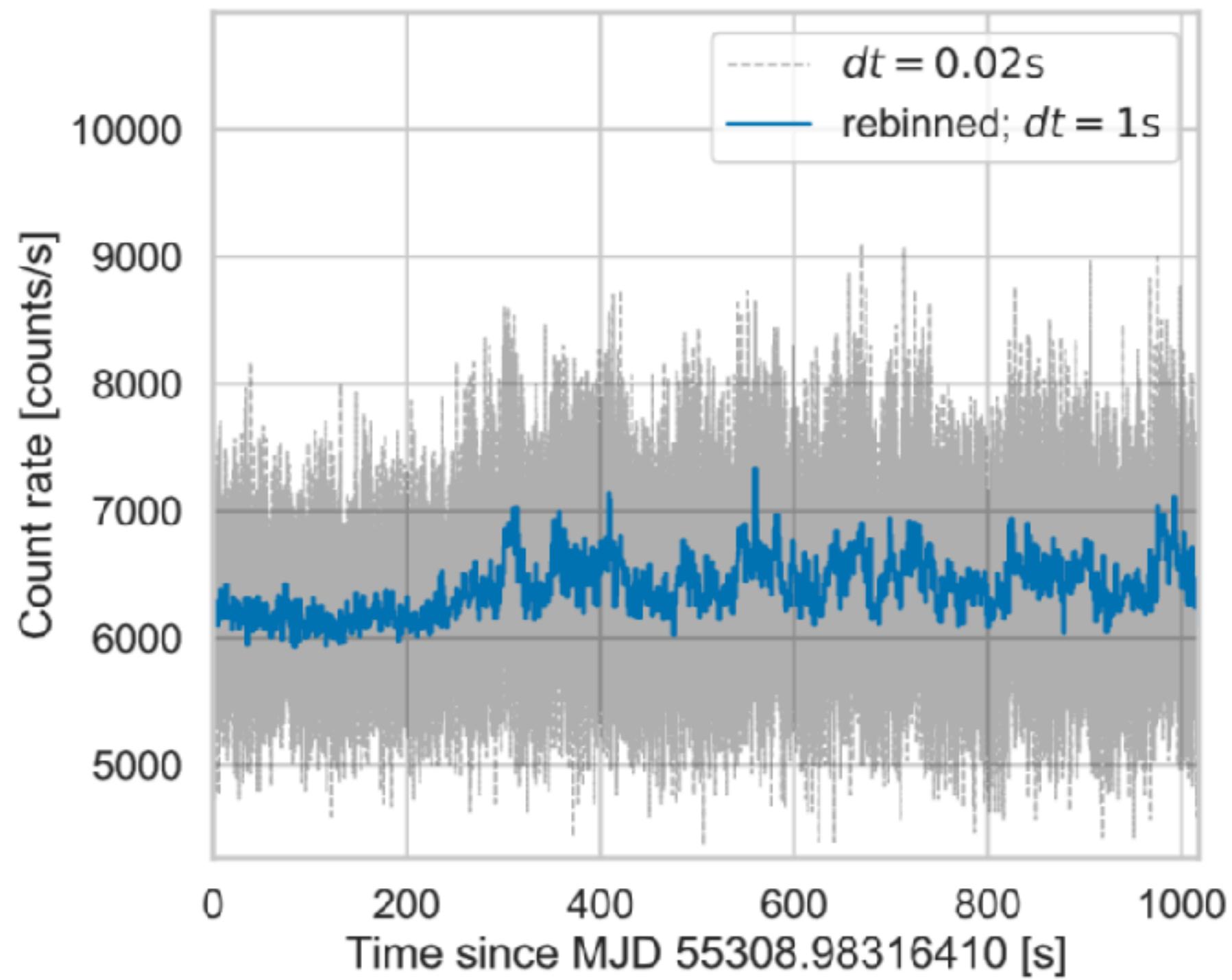
Yamaoka et al (2010);  
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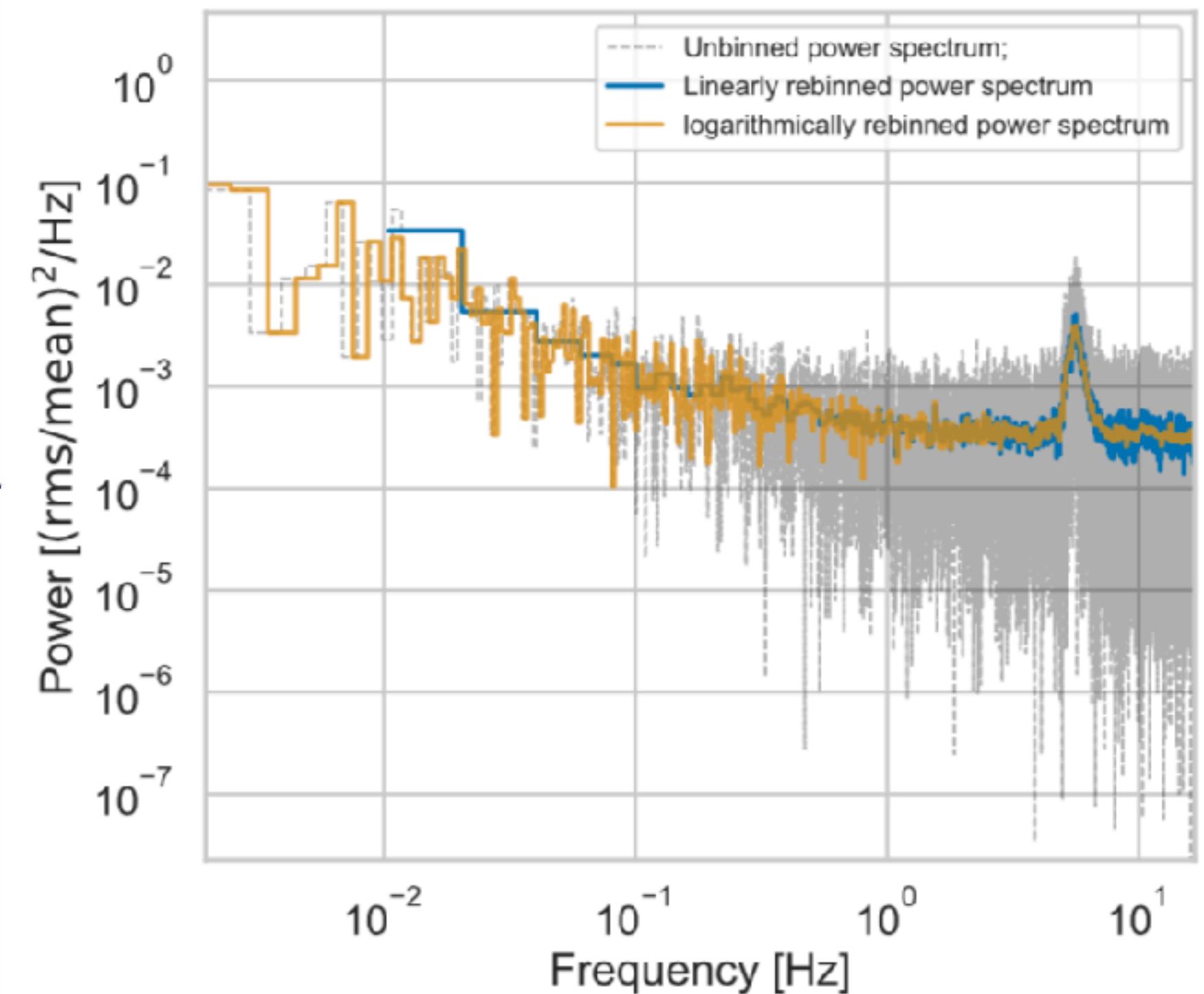
Fast Fourier  
Transform

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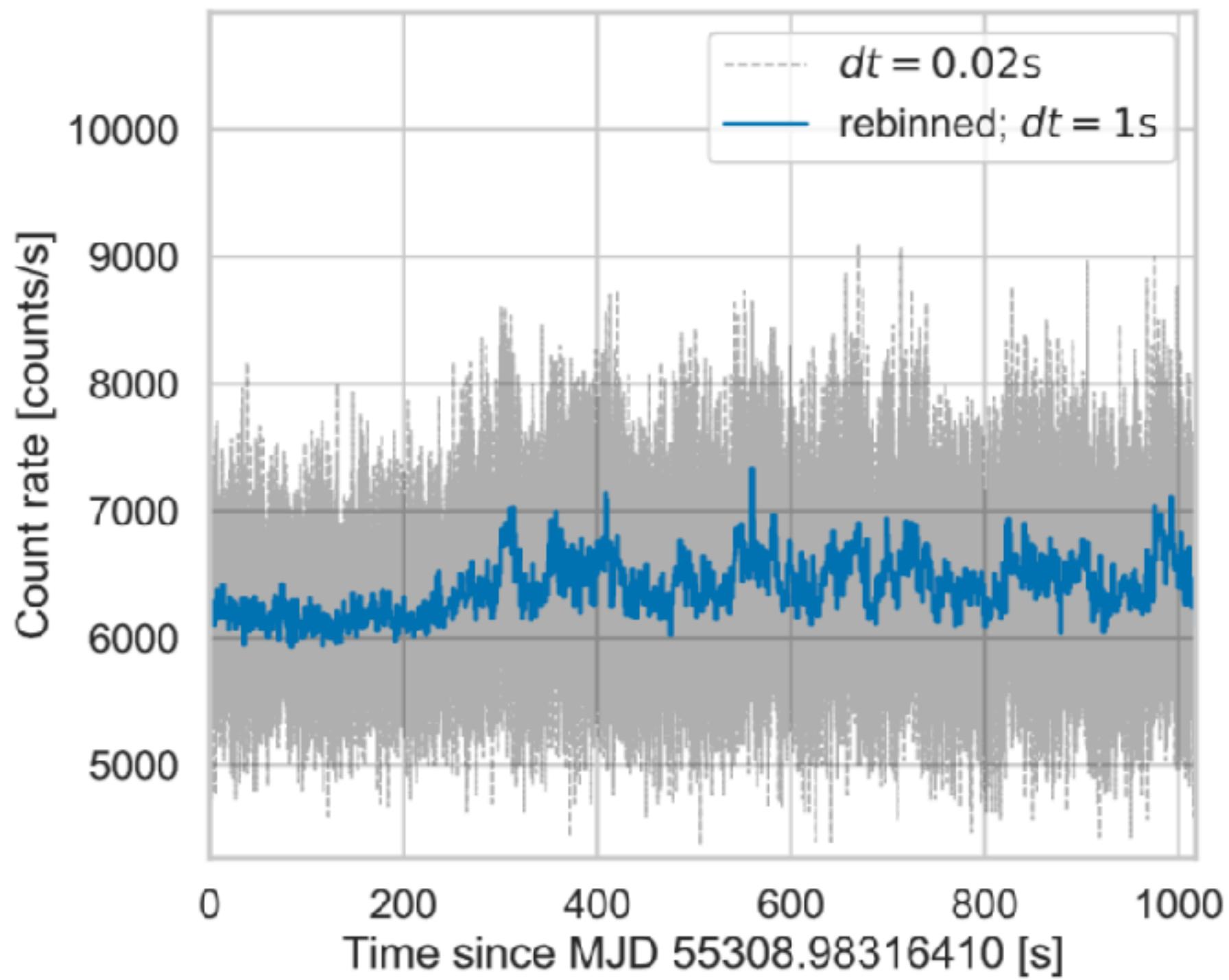


Fast Fourier  
Transform

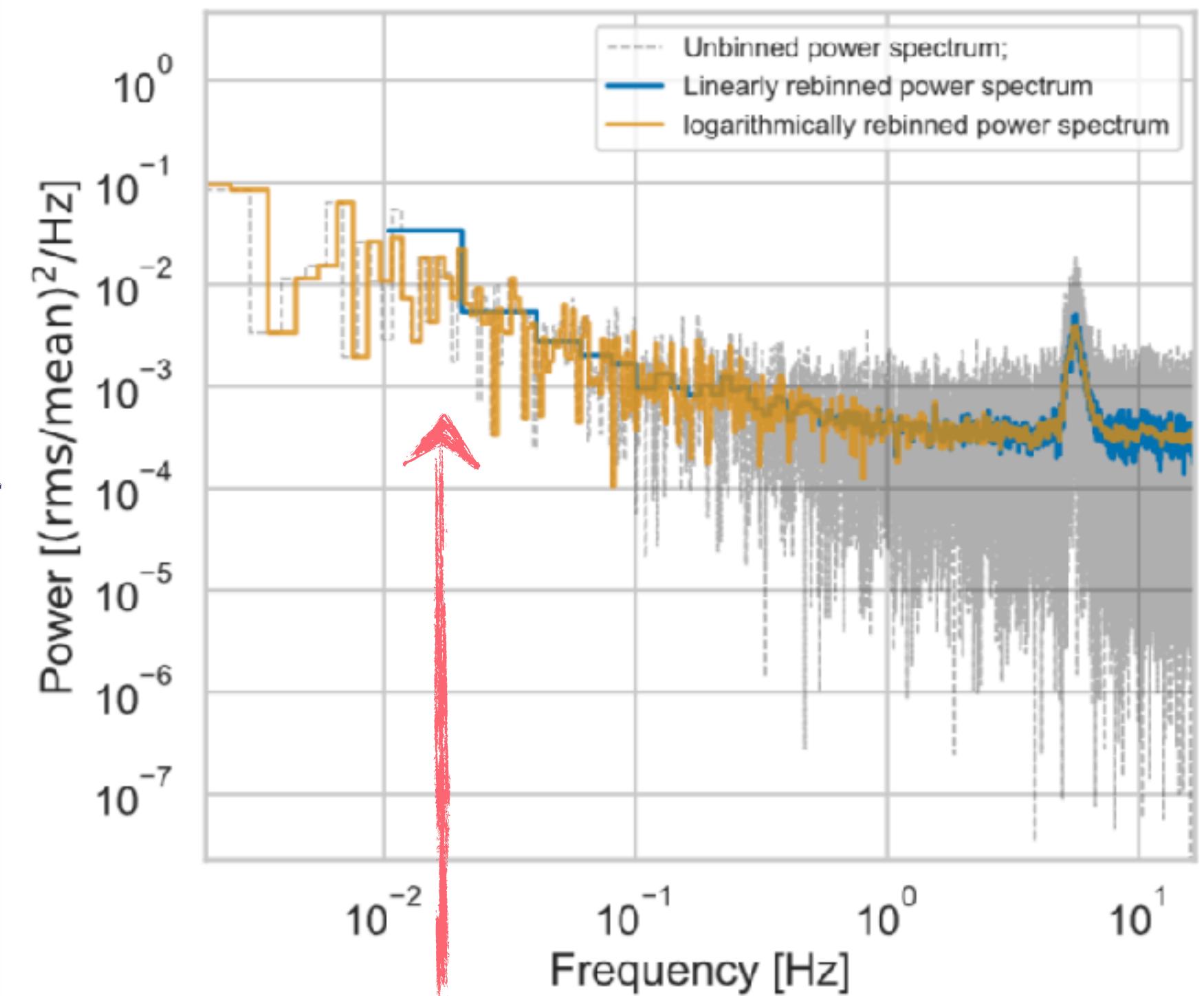


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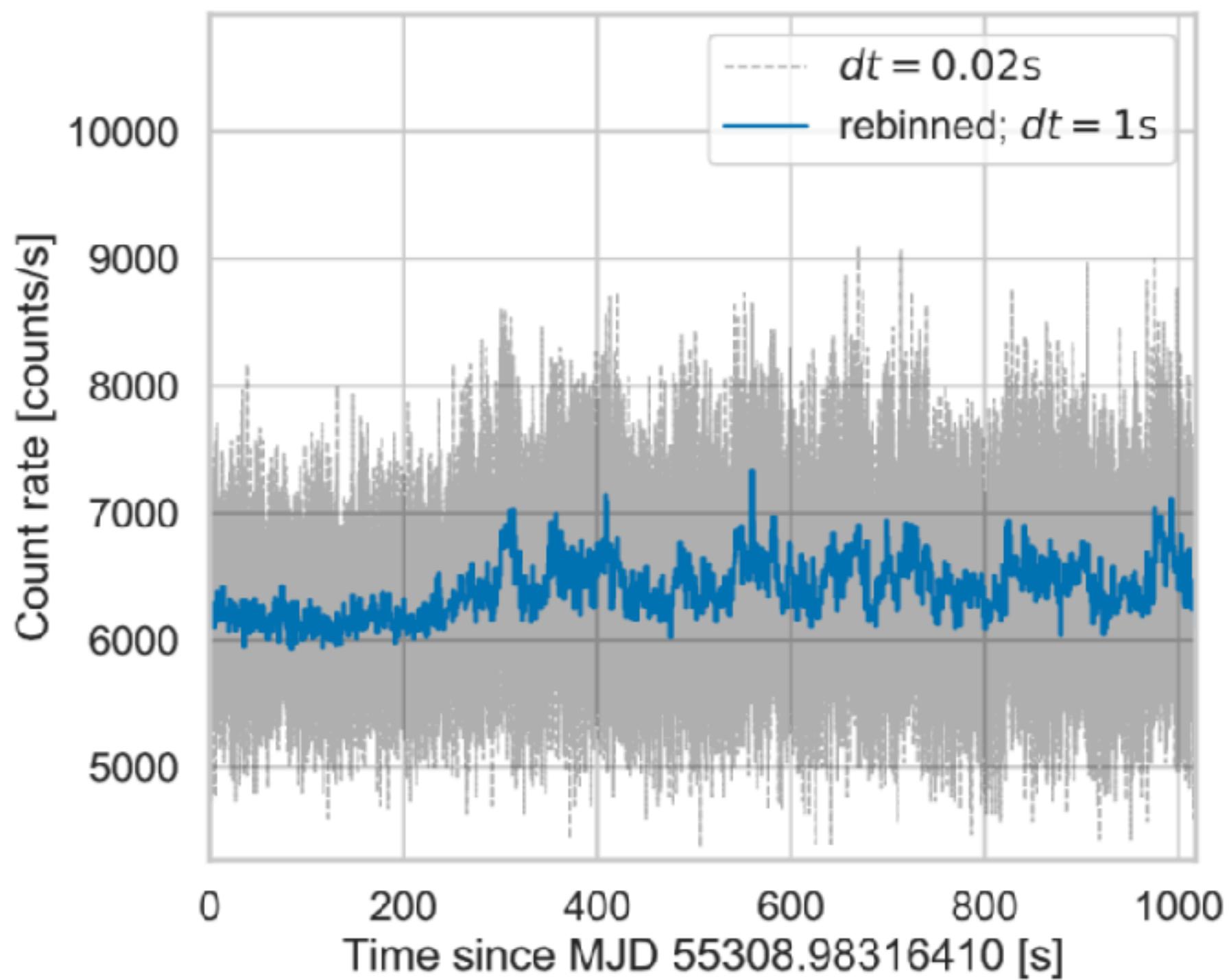
Fast Fourier  
Transform



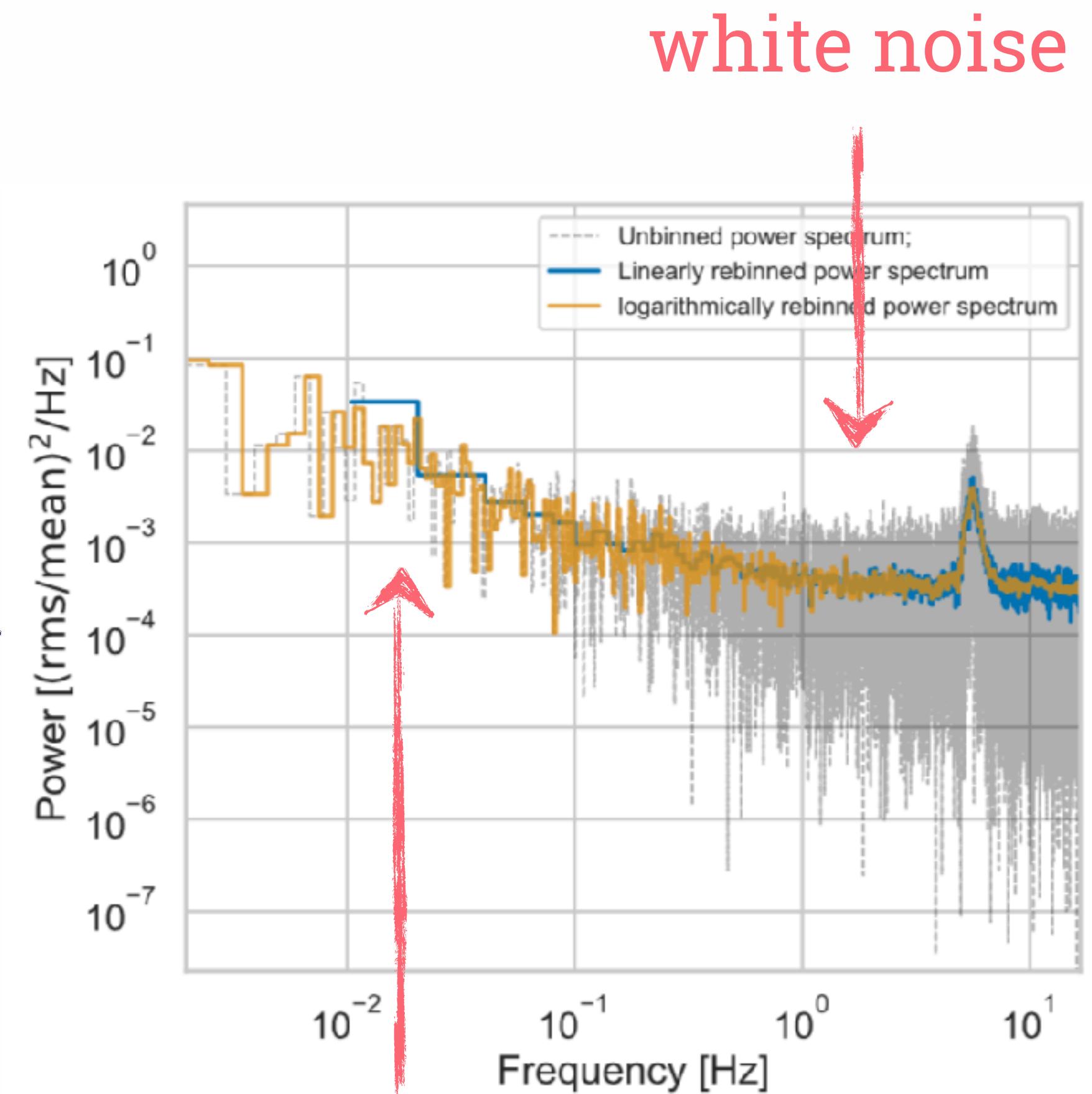
stochastic  
variability  
(red noise)

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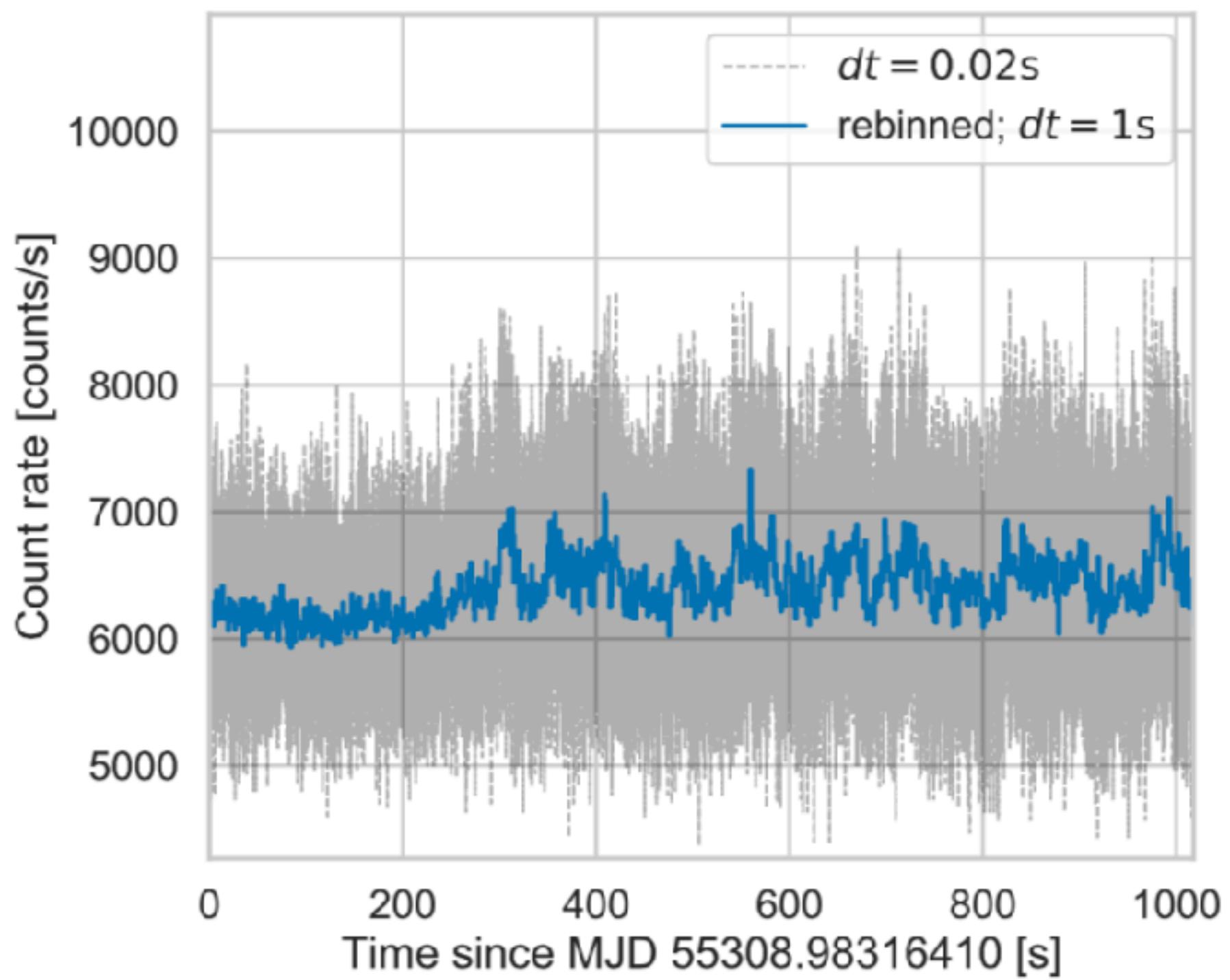
Fast Fourier  
Transform



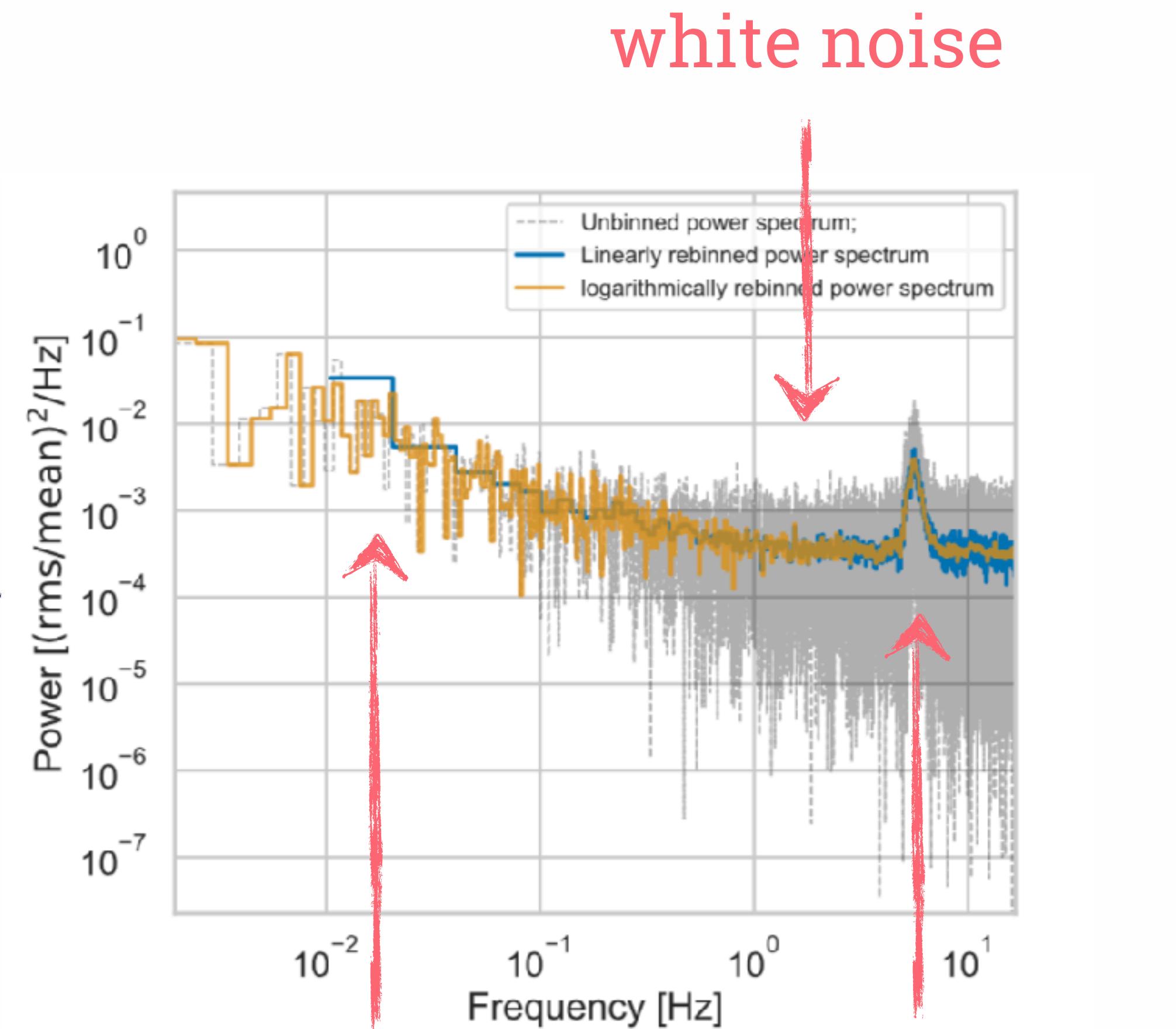
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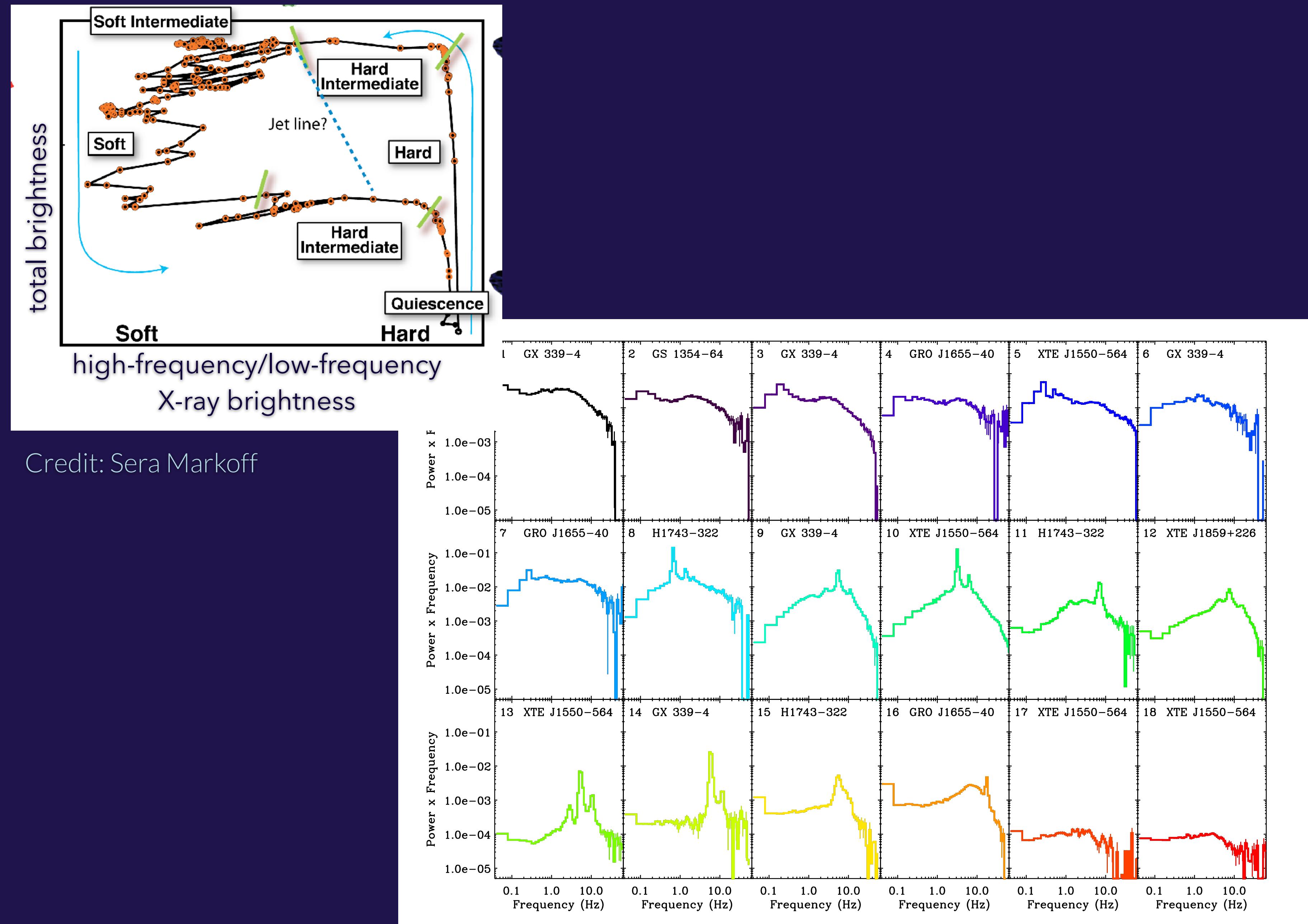
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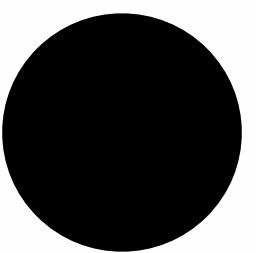
Fast Fourier  
Transform



white noise  
stochastic variability (red noise)  
quasi-periodic oscillation (QPO)

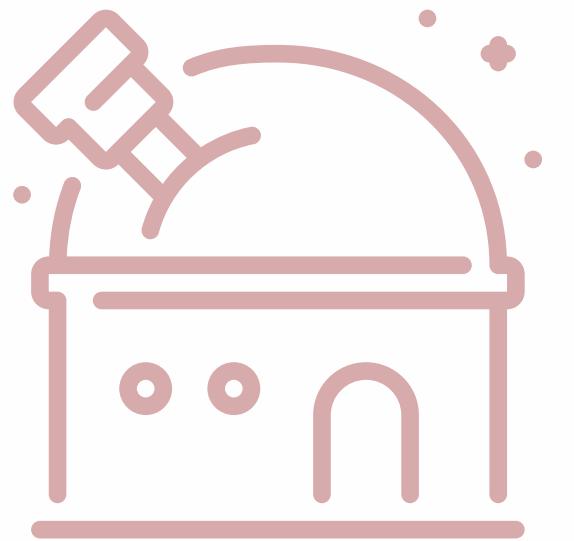


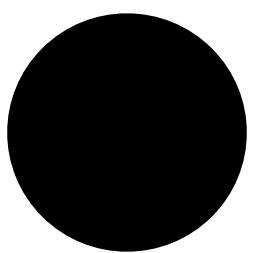
Credit: Sera Markoff



hot plasma

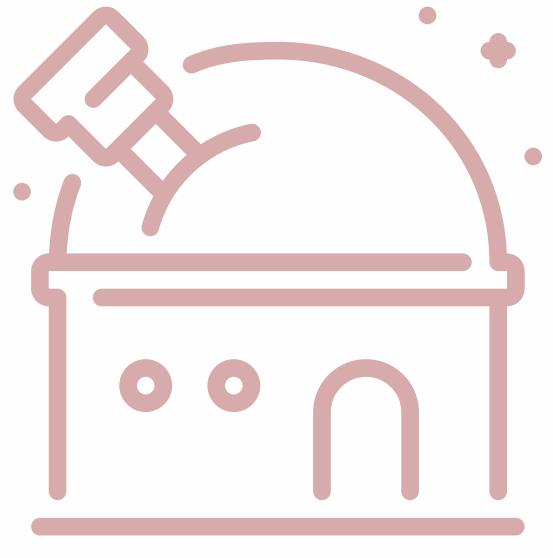
cold plasma

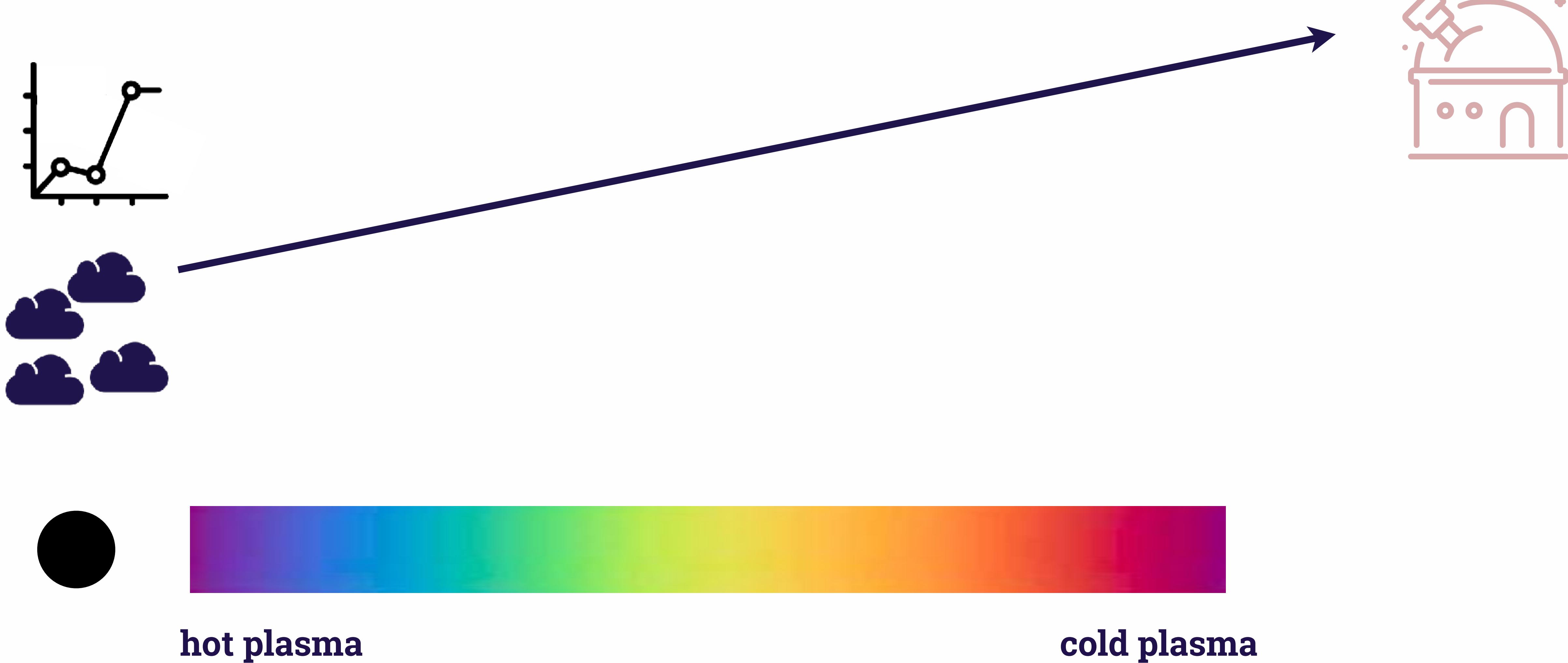


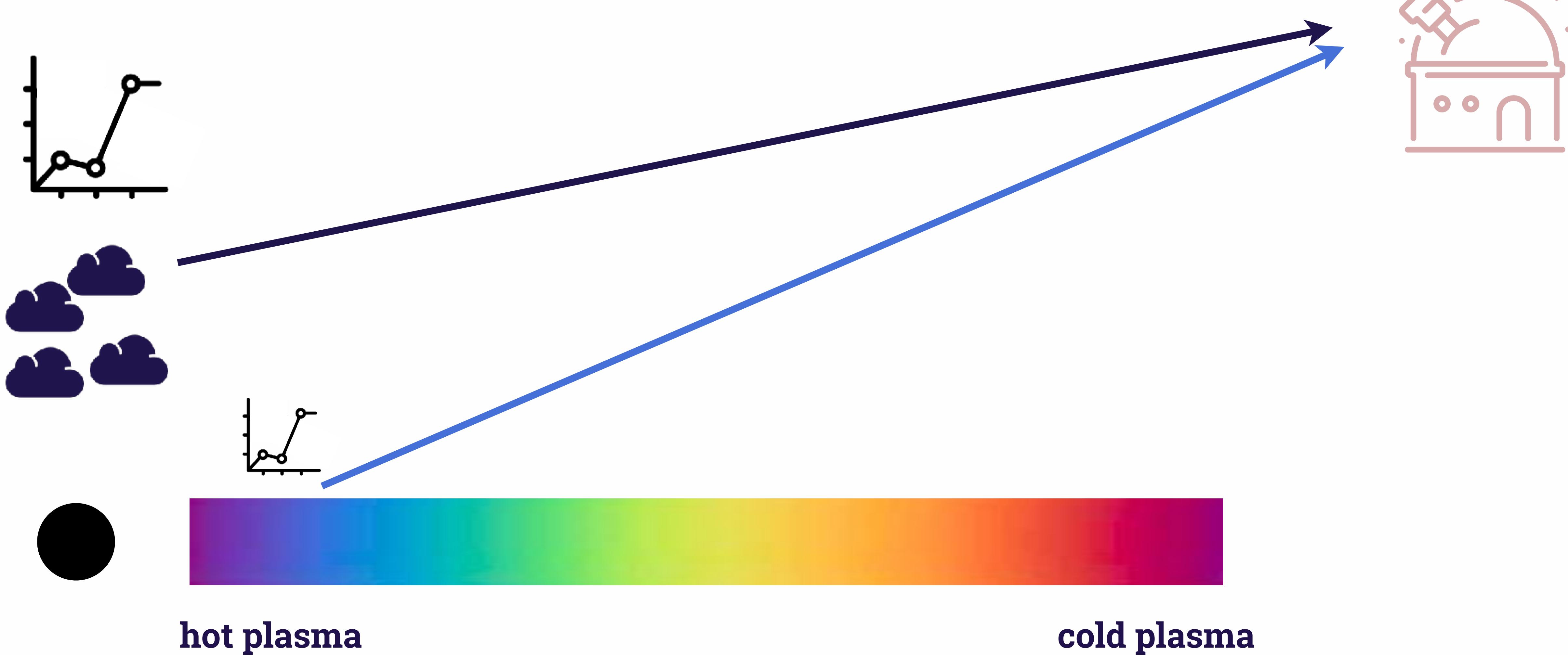


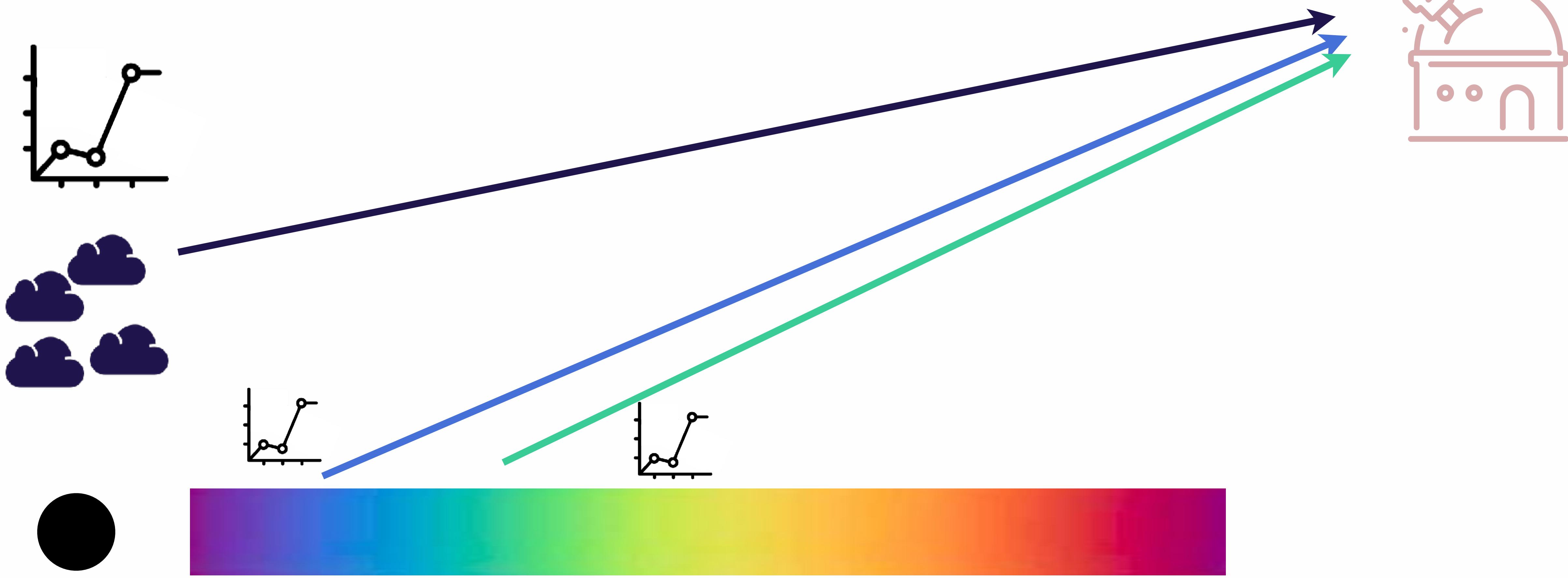
hot plasma

cold plasma



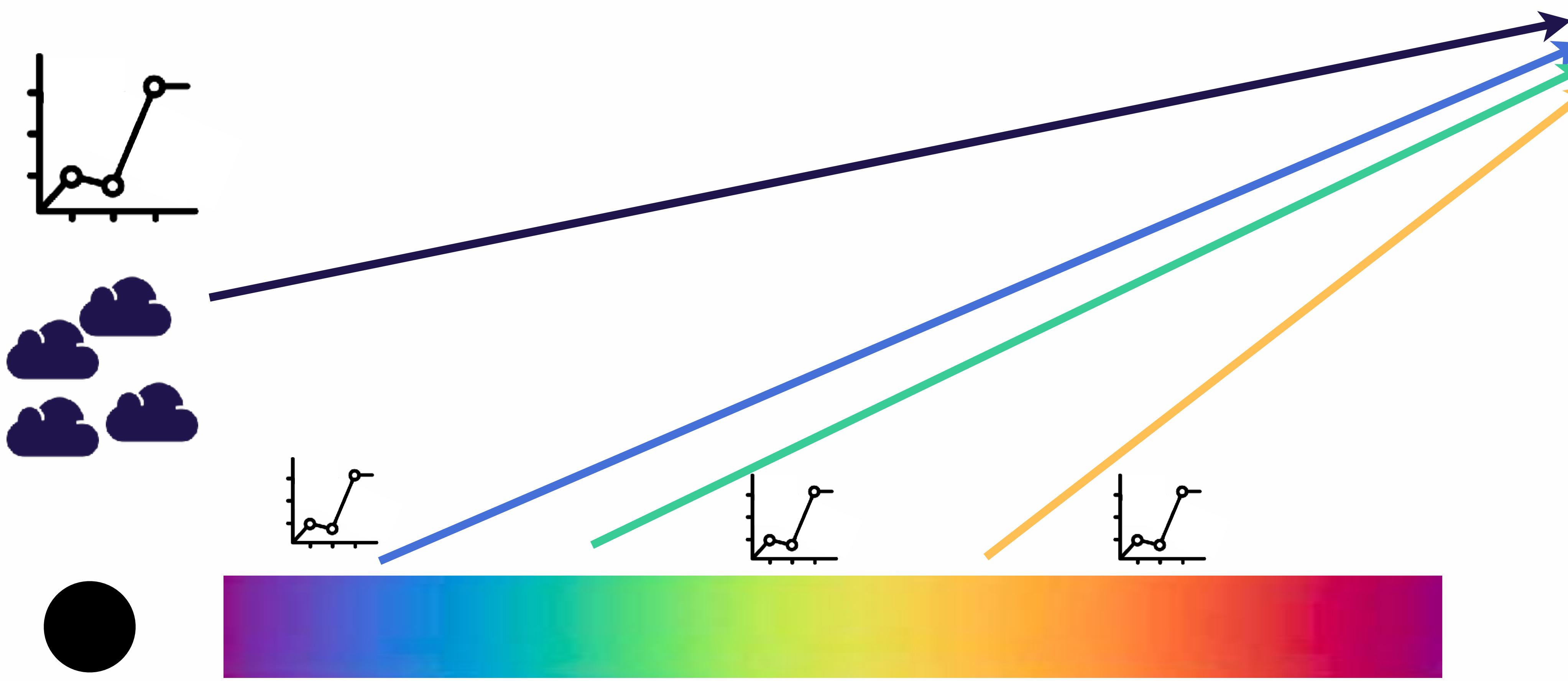






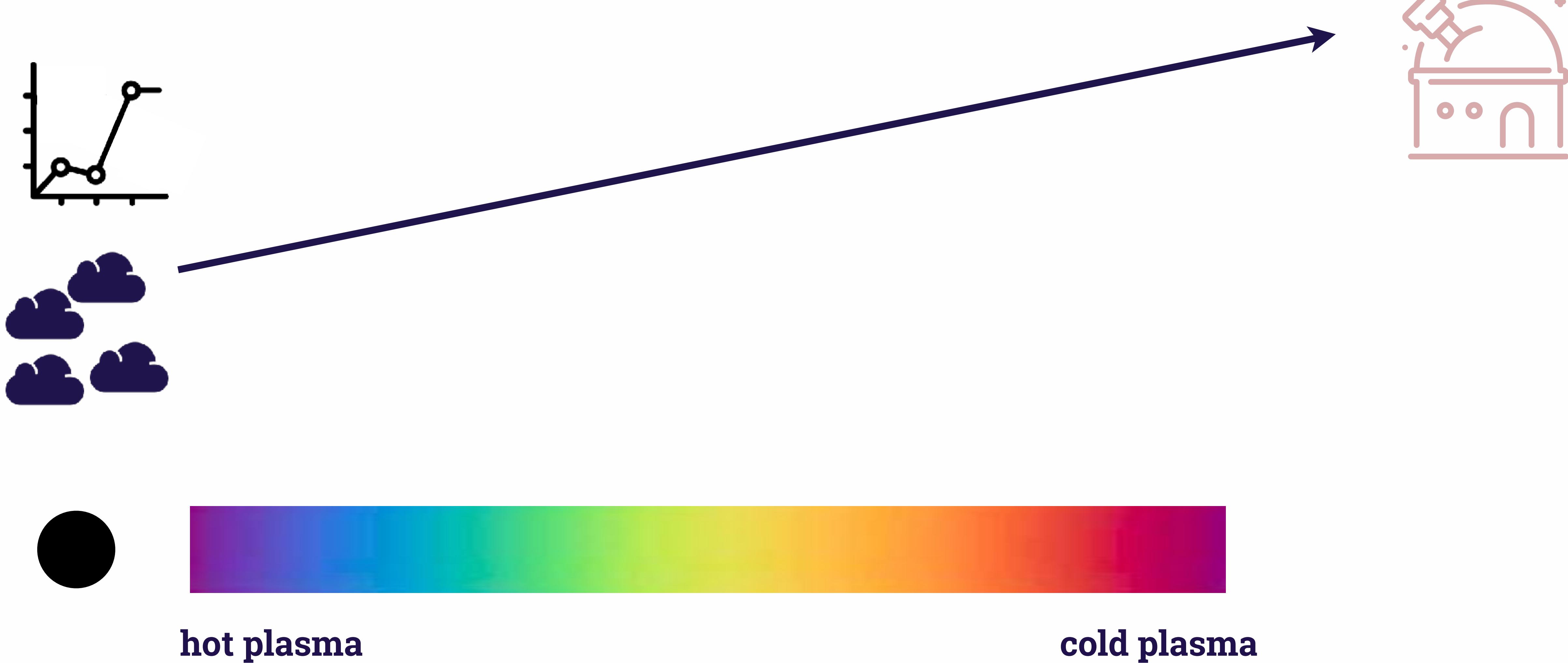
hot plasma

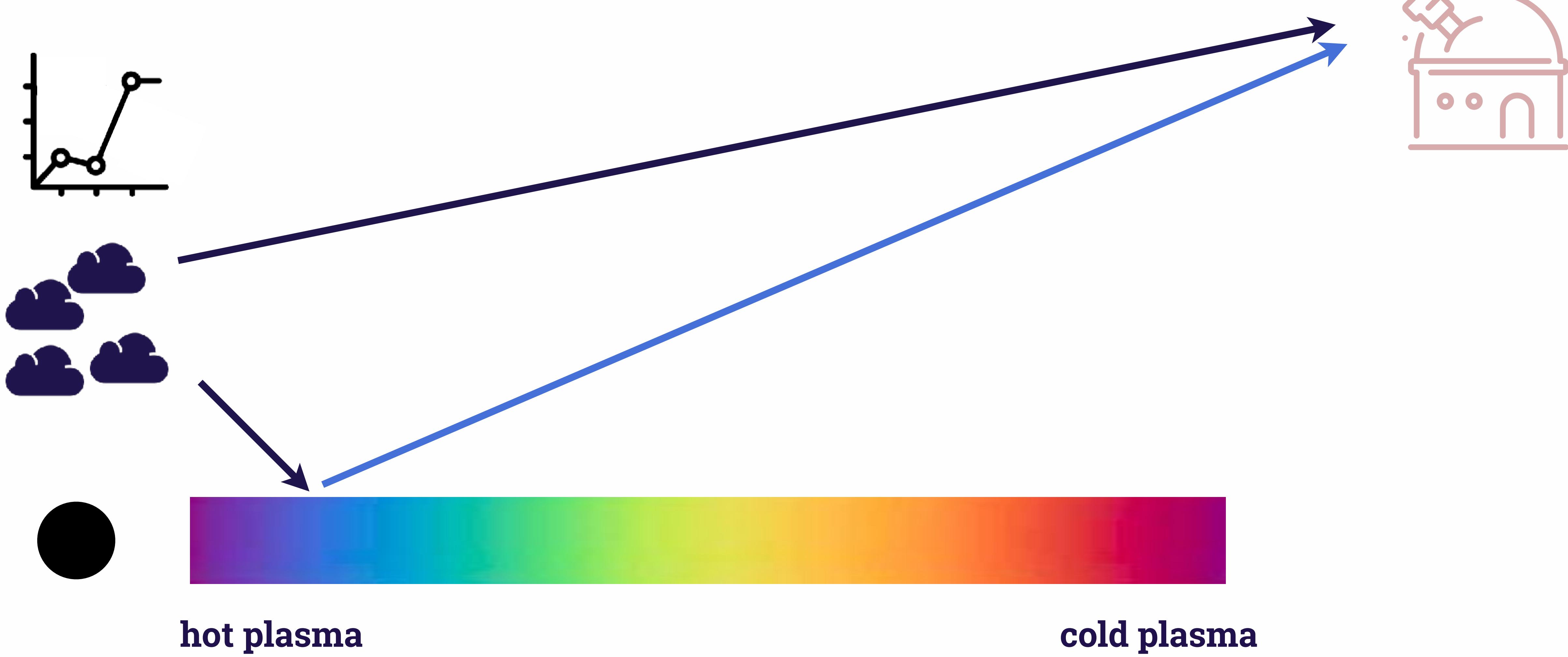
cold plasma

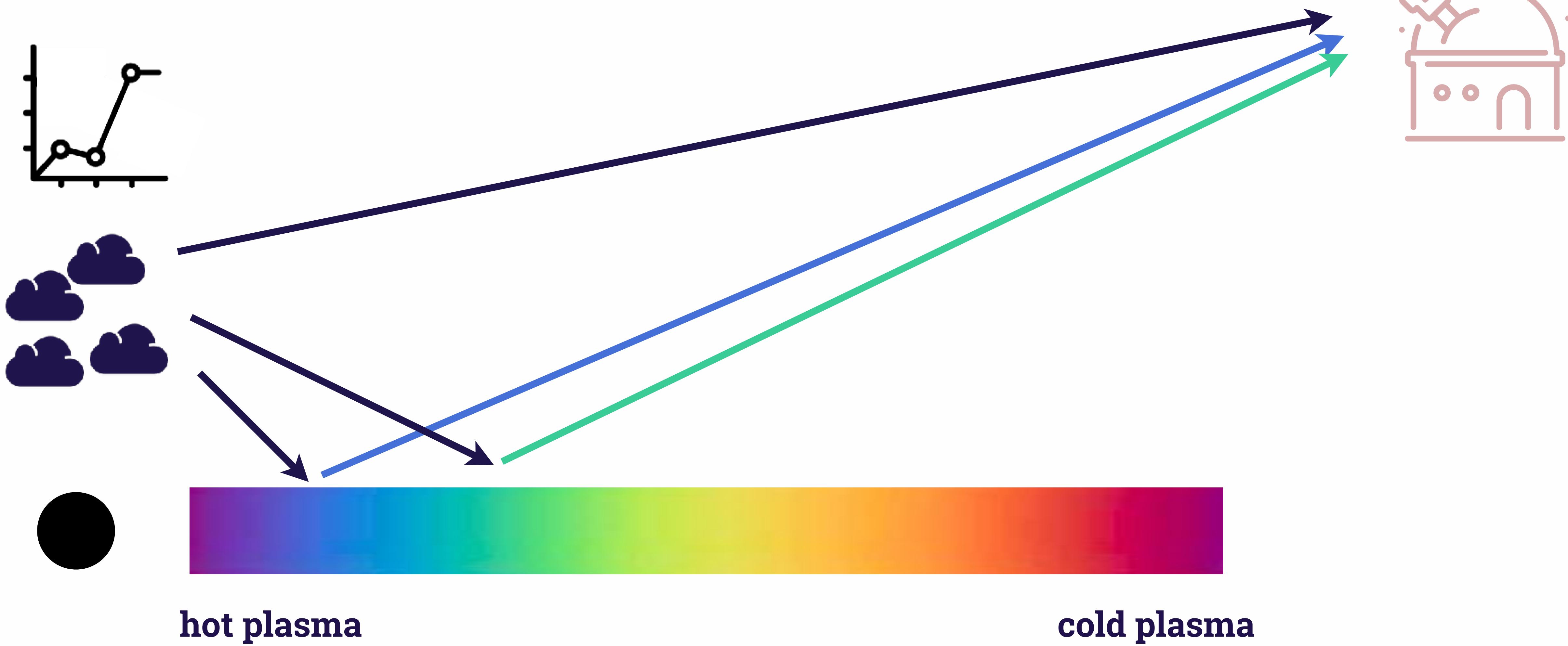


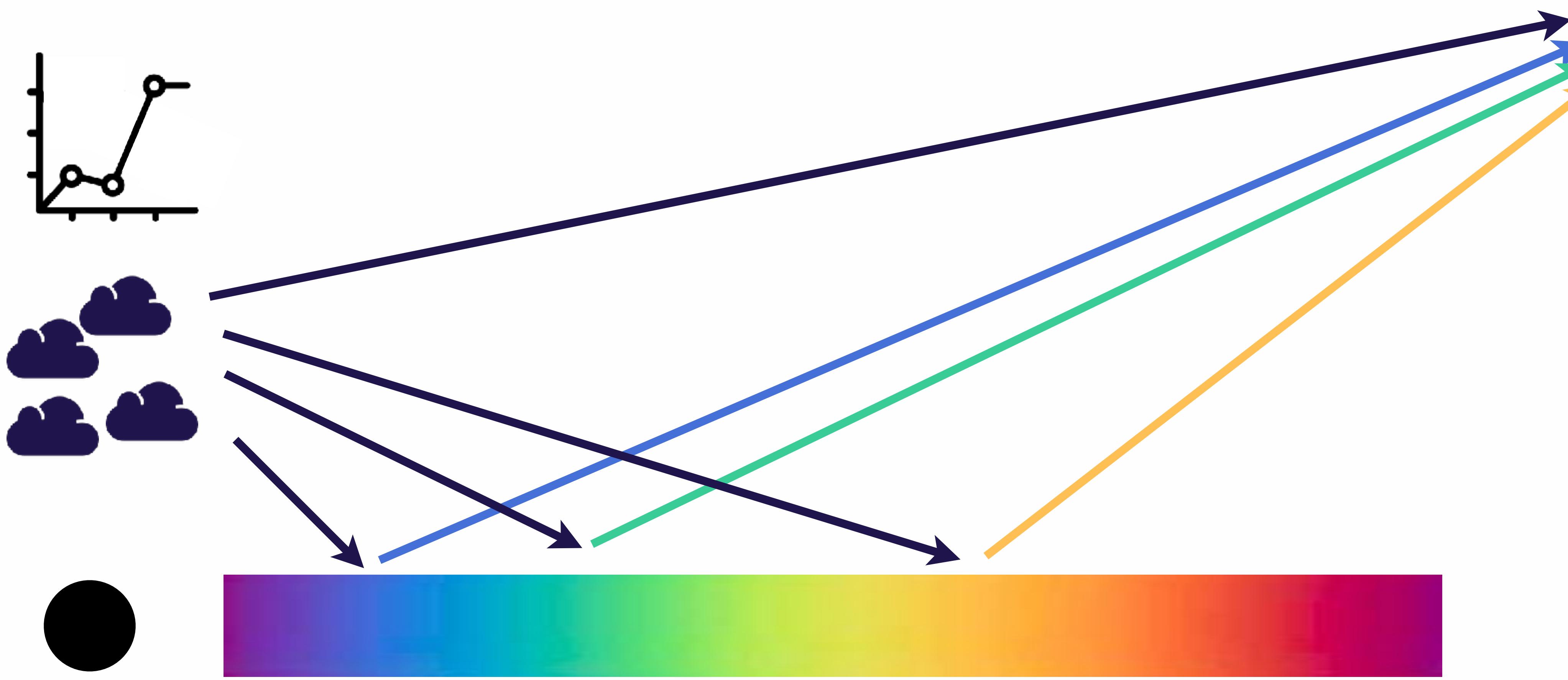
hot plasma

cold plasma



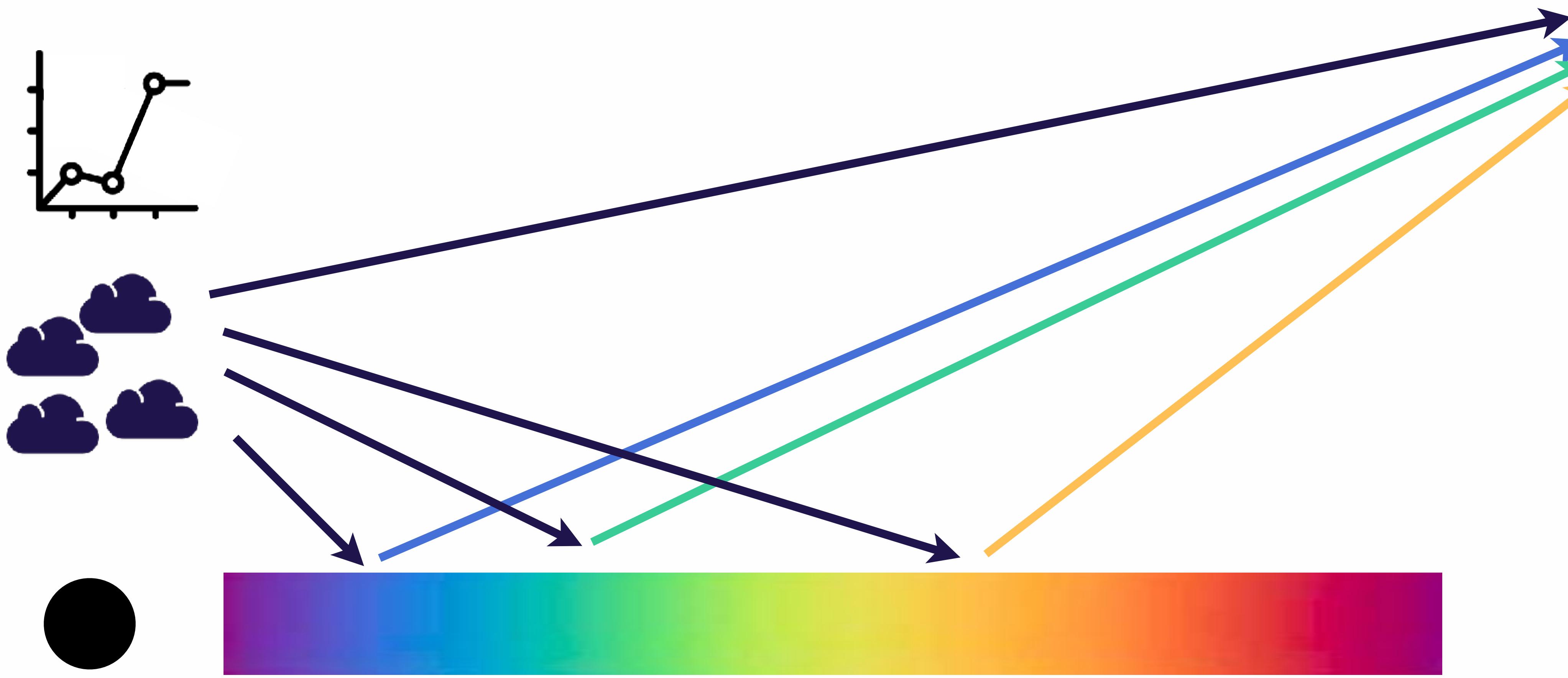






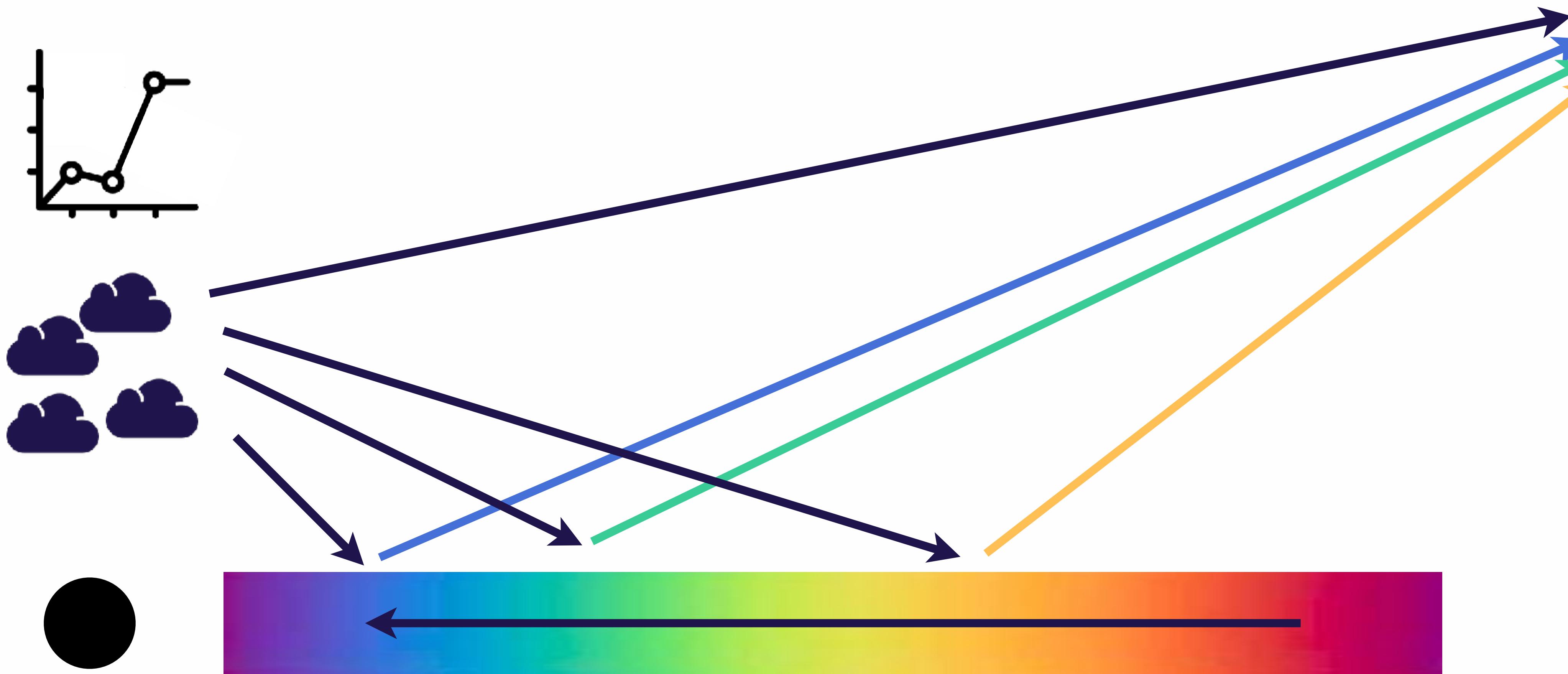
hot plasma

cold plasma



hot plasma

cold plasma



hot plasma

cold plasma

How can we use variability and  
energy to uncover the underlying  
physical processes?

# Fourier Transforms

$$a_j = \sum_{k=0}^{N-1} x_k e^{2\pi i j k / N} \quad j = -\frac{N}{2}, \dots, \frac{N}{2} - 1$$
$$x_k = \frac{1}{N} \sum_{j=-N/2}^{N/2-1} a_j e^{-2\pi i j k / N} \quad k = 0, \dots, N - 1.$$

**Complex Fourier Amplitude**

**Frequency step:**  $\delta\nu = 1/T$ .

**Nyquist frequency:**  $\nu_{N/2} = \frac{1}{2}N/T$

# Fourier Transforms

$$a_j = \sum_{k=0}^{N-1} x_k e^{2\pi i j k / N} \quad j = -\frac{N}{2}, \dots, \frac{N}{2} - 1 \quad \text{Complex Fourier Amplitude}$$
$$x_k = \frac{1}{N} \sum_{j=-N/2}^{N/2-1} a_j e^{-2\pi i j k / N} \quad k = 0, \dots, N - 1.$$

**Frequency step:**  $\delta\nu = 1/T$ .

**Nyquist frequency:**  $\nu_{N/2} = \frac{1}{2}N/T$

Earlier in the  
workshop ...

# Fourier Transforms

$$\sum_{k=0}^{N-1} |x_k|^2 = \frac{1}{N} \sum_{j=-N/2}^{N/2-1} |a_j|^2.$$

**Parseval's Theorem**

$$\text{Var}(x_k) = \frac{1}{N} \sum_{\substack{j=-N/2 \\ j \neq 0}}^{N/2-1} |a_j|^2.$$

# Fourier Transforms

**Leahy normalization:**

$$P_j \equiv \frac{2}{N_{ph}} |a_j|^2 \quad j = 0, \dots, \frac{N}{2},$$

use for period searches

**Fractional RMS normalization:**

$$P_j = \frac{2T|a_j|^2}{N^2\mu^2}$$

use for characterizing  
quasi-periodic signals

**Absolute RMS normalization:**

$$P_j = \frac{2|a_j|^2}{T}$$

# The Cross Spectrum

$$\mathcal{F}_x(j)\mathcal{F}_y^*(j) = \frac{1}{2} \{(A_{xj}A_{yj} + B_{xj}B_{yj}) + i(A_{xj}B_{yj} - A_{yj}B_{xj})\}$$

$$C_{XY,j} = C_{x,j}C_{y,j} \exp[i\phi_j] \quad (\text{same in complex polar coordinates})$$

amplitude

phase

**Note: be aware of your denominator  
for the rms normalization!**

$$P_j = \frac{2T|C_{X,j}C_{Y,j}|^2}{N^2\langle x \rangle \langle y \rangle}$$

# The Cross Spectrum

$$\mathcal{F}_x(j)\mathcal{F}_y^*(j) = \frac{1}{N} \{(A_{xj}A_{uj} + B_{xj}B_{uj}) + i(A_{xj}B_{uj} - A_{uj}B_{xj})\}$$

In practice, you should almost  
always average across  
frequencies/segments!

$$C_{XY,j} = C$$

ampl

Note: be aware of your denominator  
for the rms normalization!

$$P_j = \frac{2T|C_{X,j}C_{Y,j}|^2}{N^2\langle x \rangle \langle y \rangle}$$

**What do I mean by “**different light curves**”:**

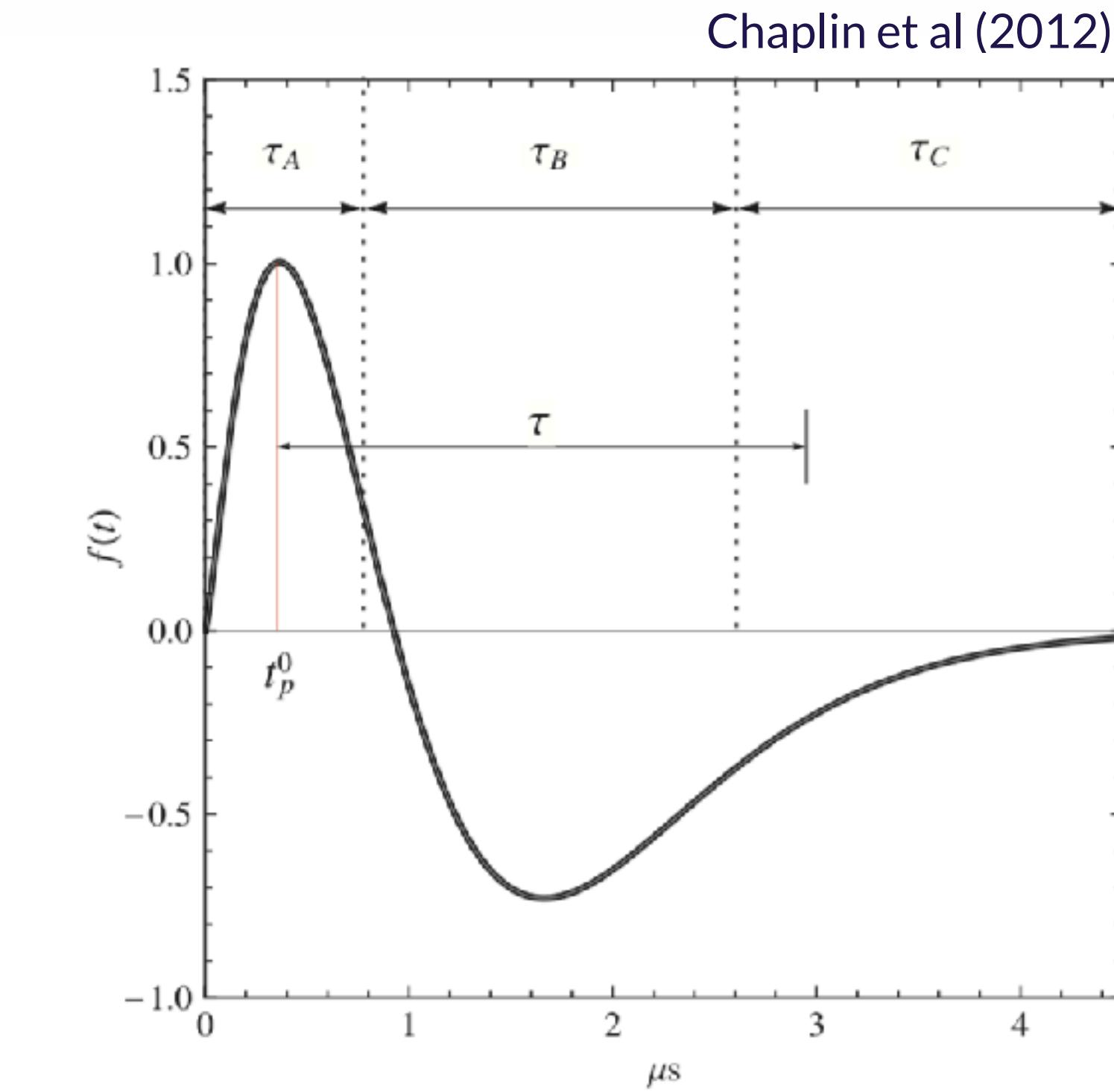
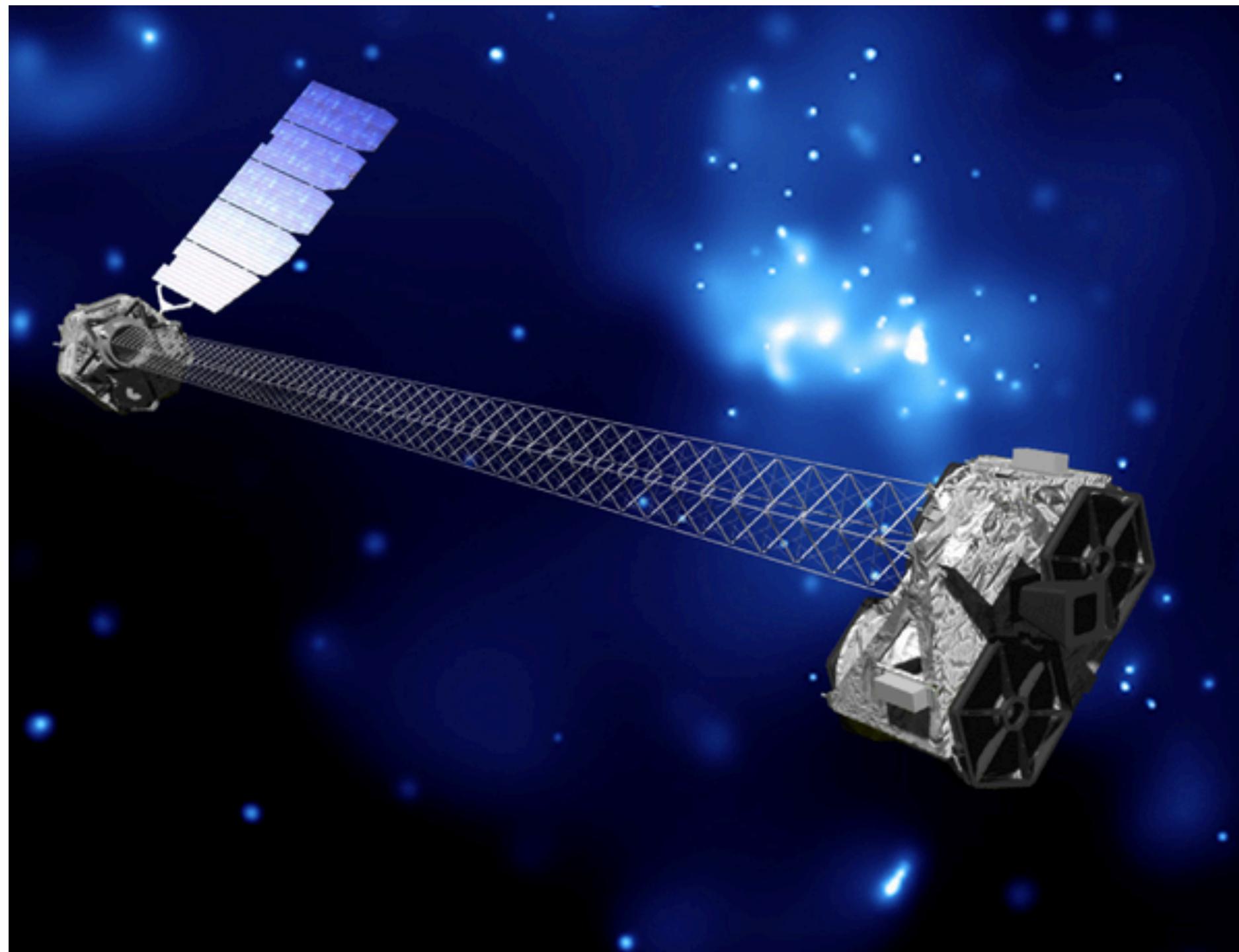
- **same energy range; independent, but identical detectors**
- **different energy ranges, same detector**

# The Cospectrum

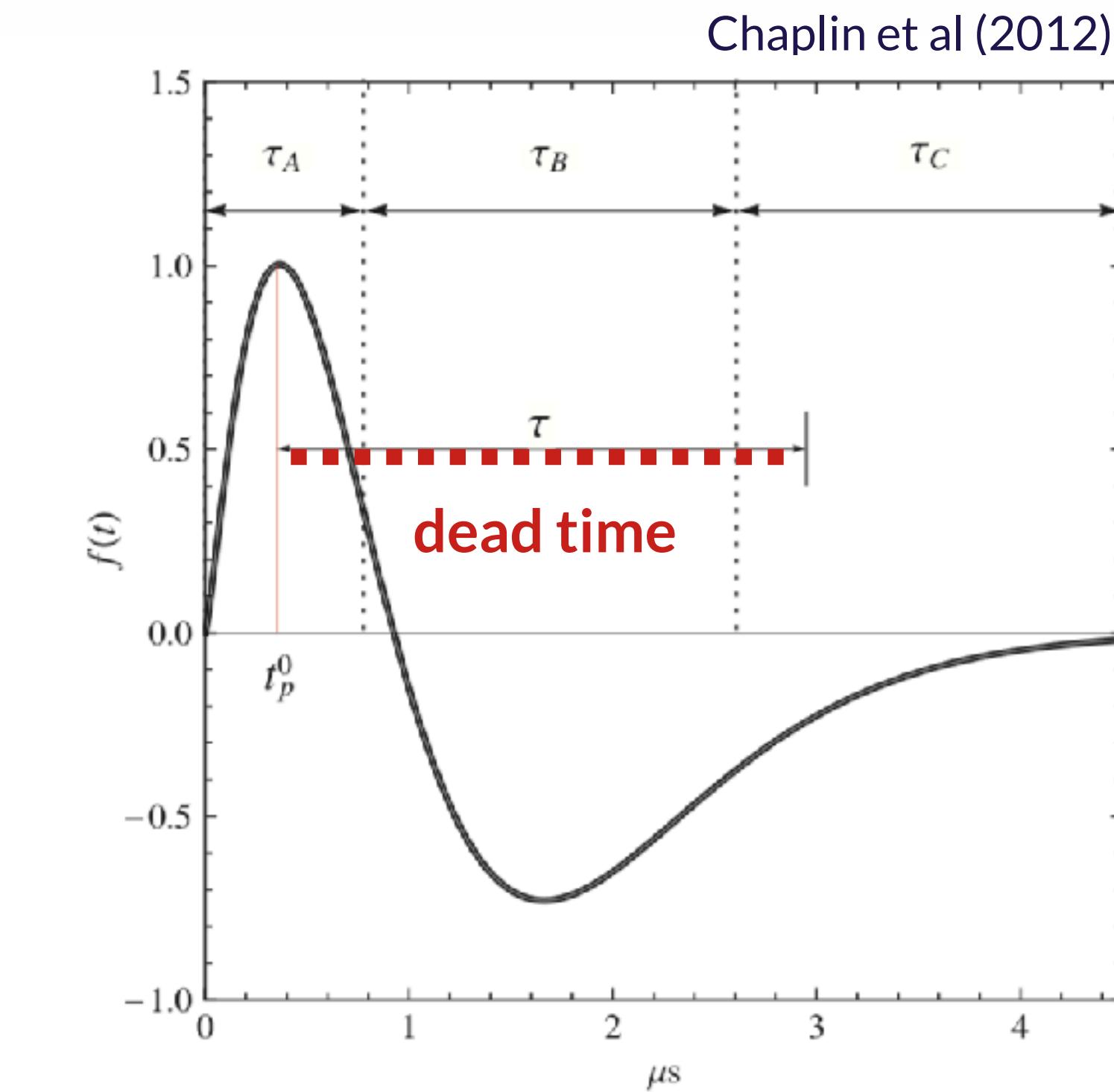
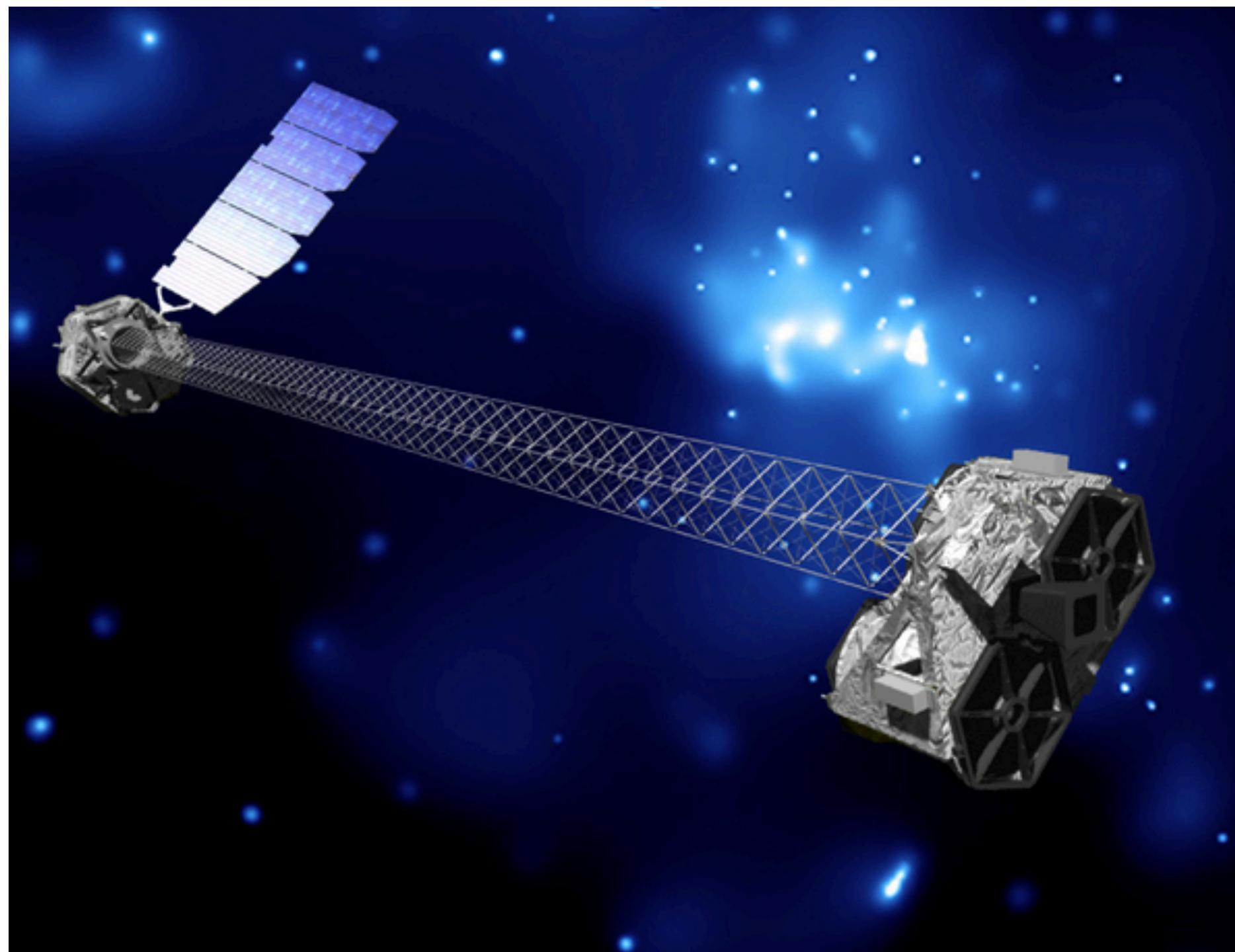
$$C_j = \frac{1}{2}(A_{xj}A_{yj} + B_{xj}B_{yj}).$$

**Be careful:** interpretation of time lags for phases close to  $\pi$  or  $-\pi$  can be difficult due to phase wrapping:

# NuSTAR

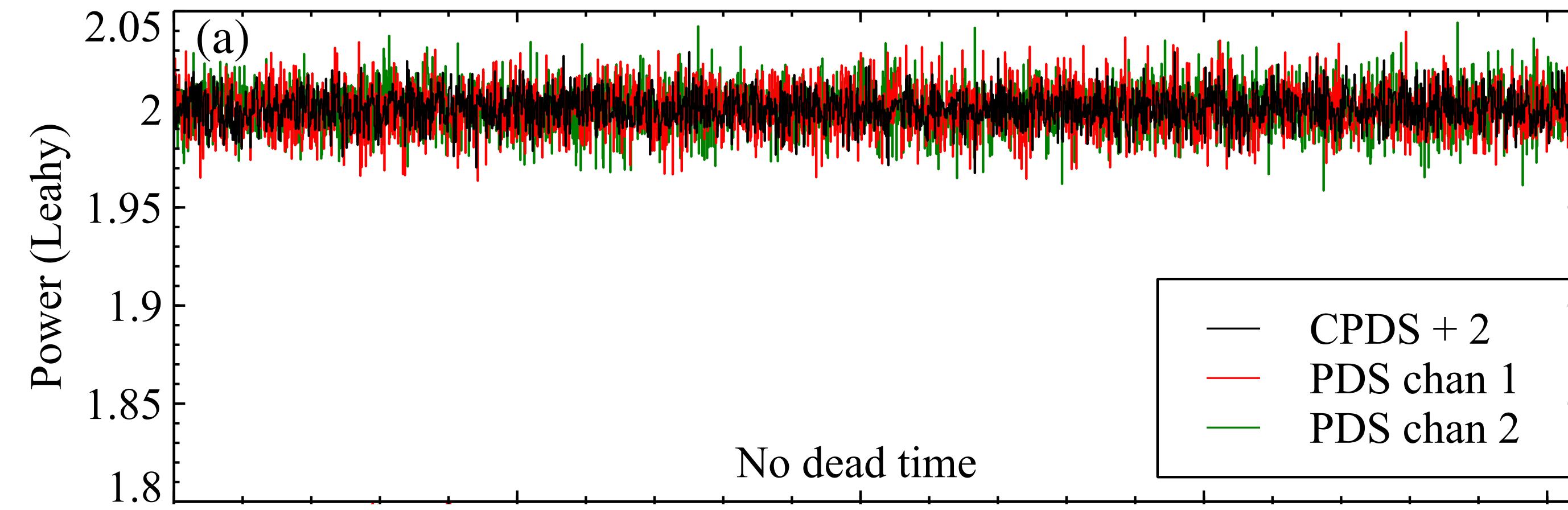


# NuSTAR

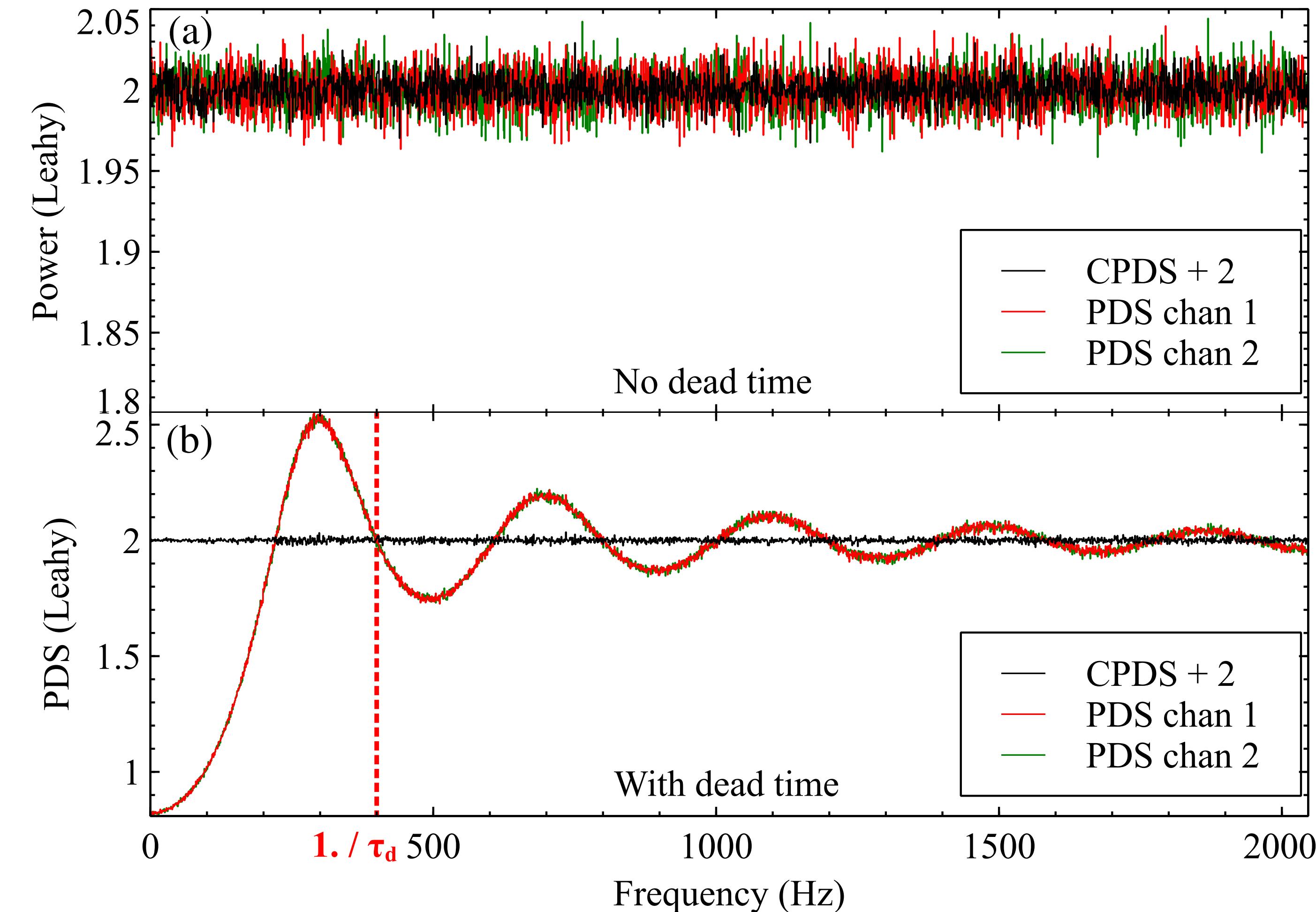


# Dead Time

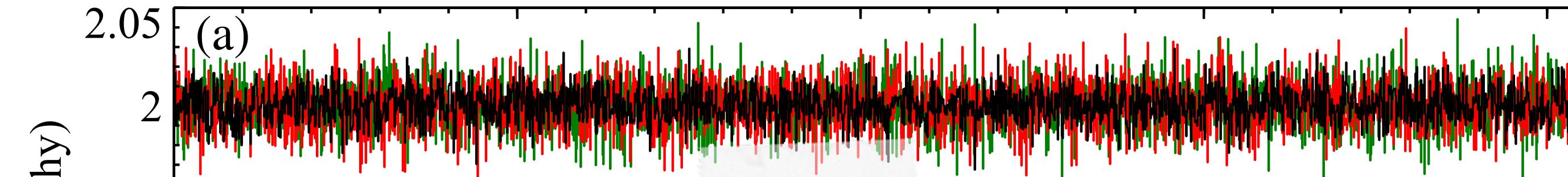
# Dead Time



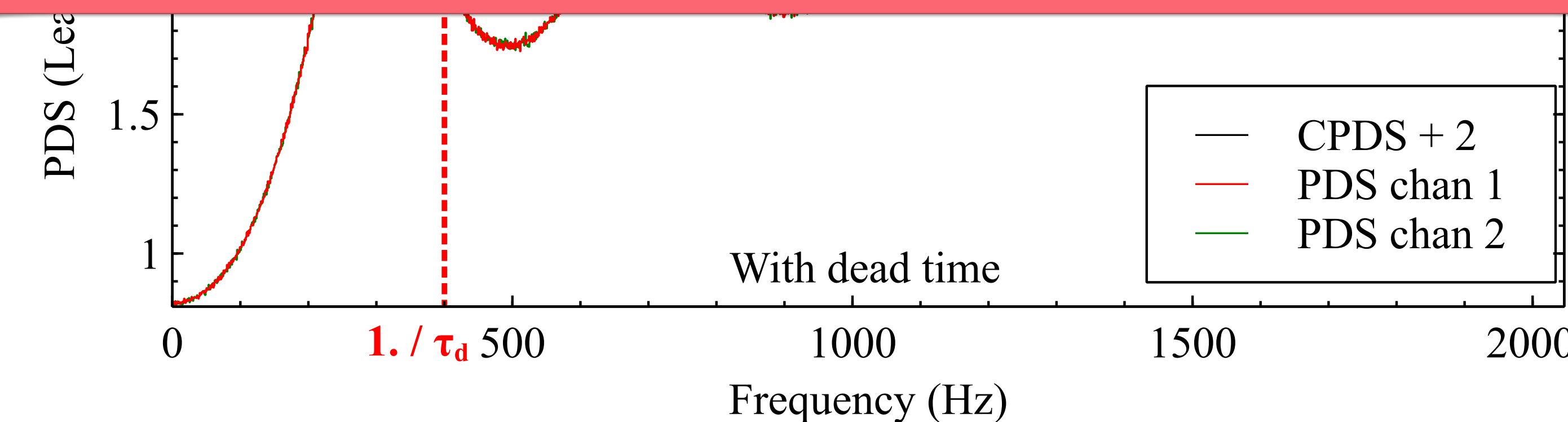
# Dead Time



# Dead Time



**Caution! The power spectrum and the cospectrum do not have the same statistical distribution!**



# The Time Lag

$$\tau_j = \frac{\phi_j}{2\pi\nu_j}$$

**Be careful:** interpretation of time lags for phases close to  $\pi$  or  $-\pi$  can be difficult due to phase wrapping.

# The Coherence

$$\gamma^2(\nu_j) = \frac{|\bar{C}_{XY}(\nu_j)|^2 - n^2}{\bar{P}_X(\nu_j)\bar{P}_Y(\nu_j)}$$

where

$$n^2 = \frac{(\bar{P}_X(\nu_j) - P_{X,\text{noise}})P_{Y,\text{noise}} + (\bar{P}_Y(\nu_j) - P_{Y,\text{noise}})P_{X,\text{noise}} + P_{X,\text{noise}}P_{Y,\text{noise}}}{KM}$$

**normalization term to account for the contribution of  
Poisson noise to the cross spectrum term**

# Interpretation

**fraction of variance in both light curves that can be predicted via a linear transformation between them**

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**fraction of variance in both light curves that can be predicted via a linear transformation between them**

**indication of scatter on the cross spectrum vector caused by incoherent components**

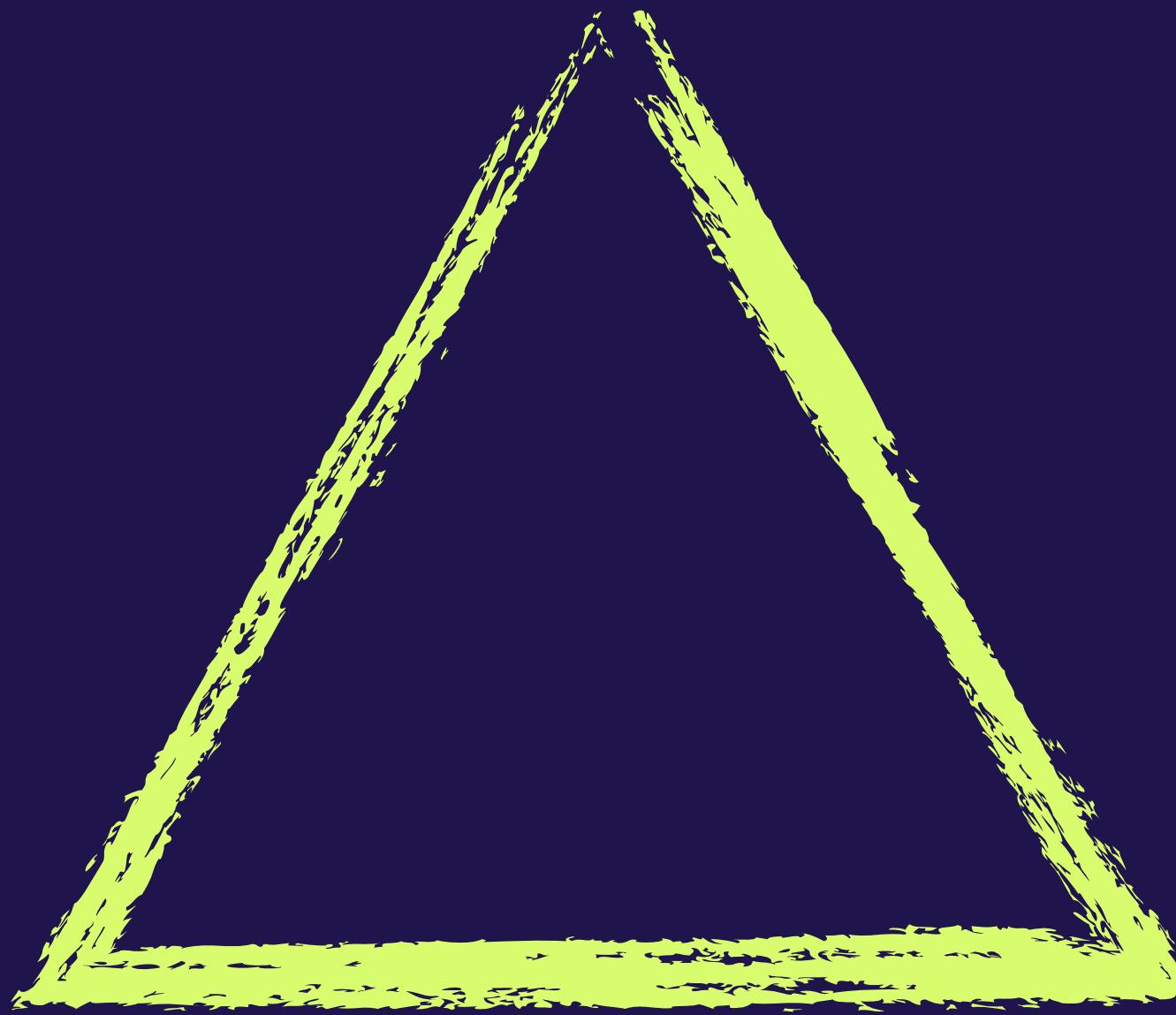
# Error of the Phase Lag

$$\Delta\phi(\nu_j) = \sqrt{\frac{1 - \gamma^2(\nu_j)}{2\gamma^2(\nu_j)KM}}$$

**Caution:** uses raw coherence (Poisson noise not subtracted from periodograms)

# Practical Applications

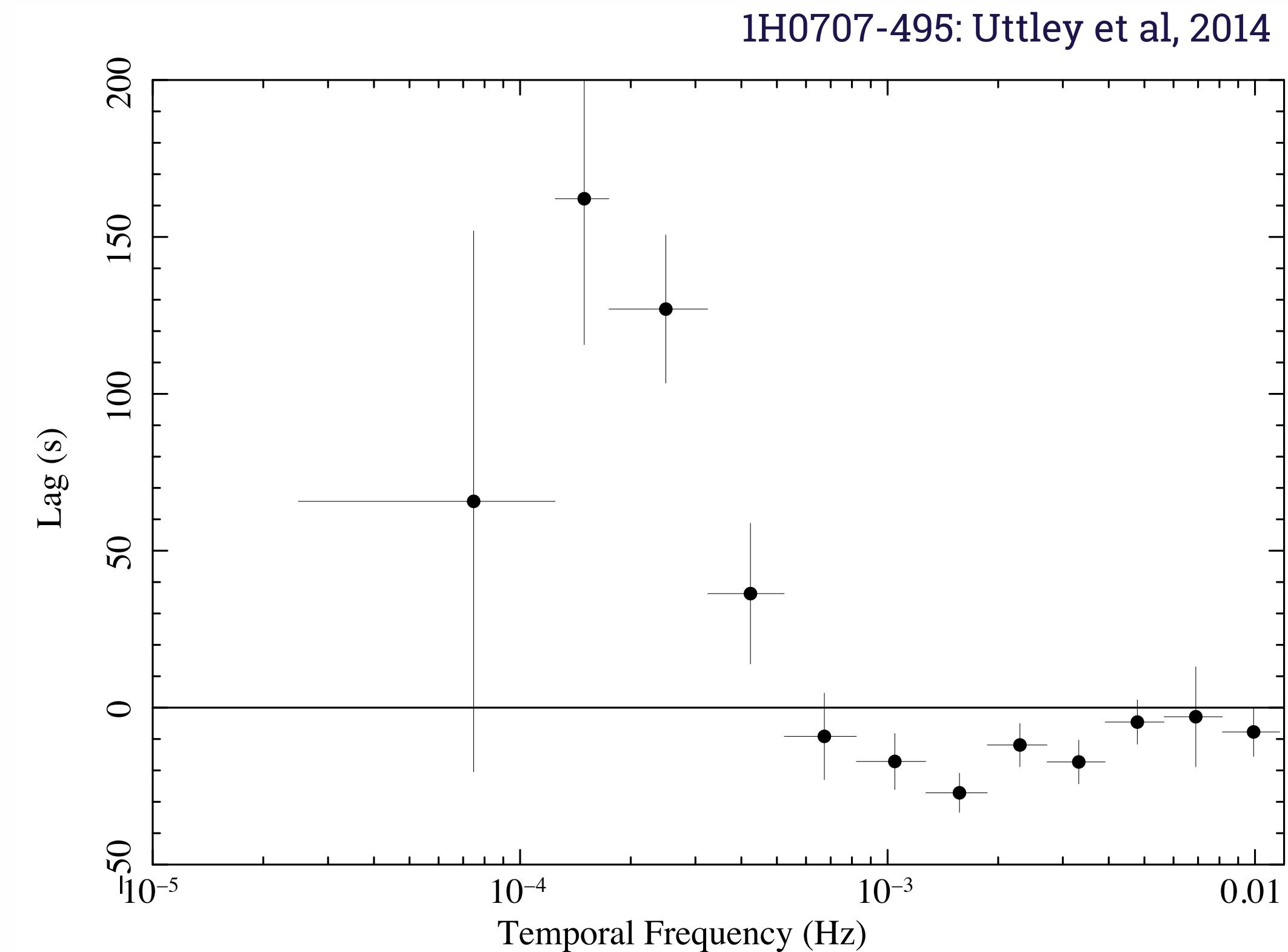
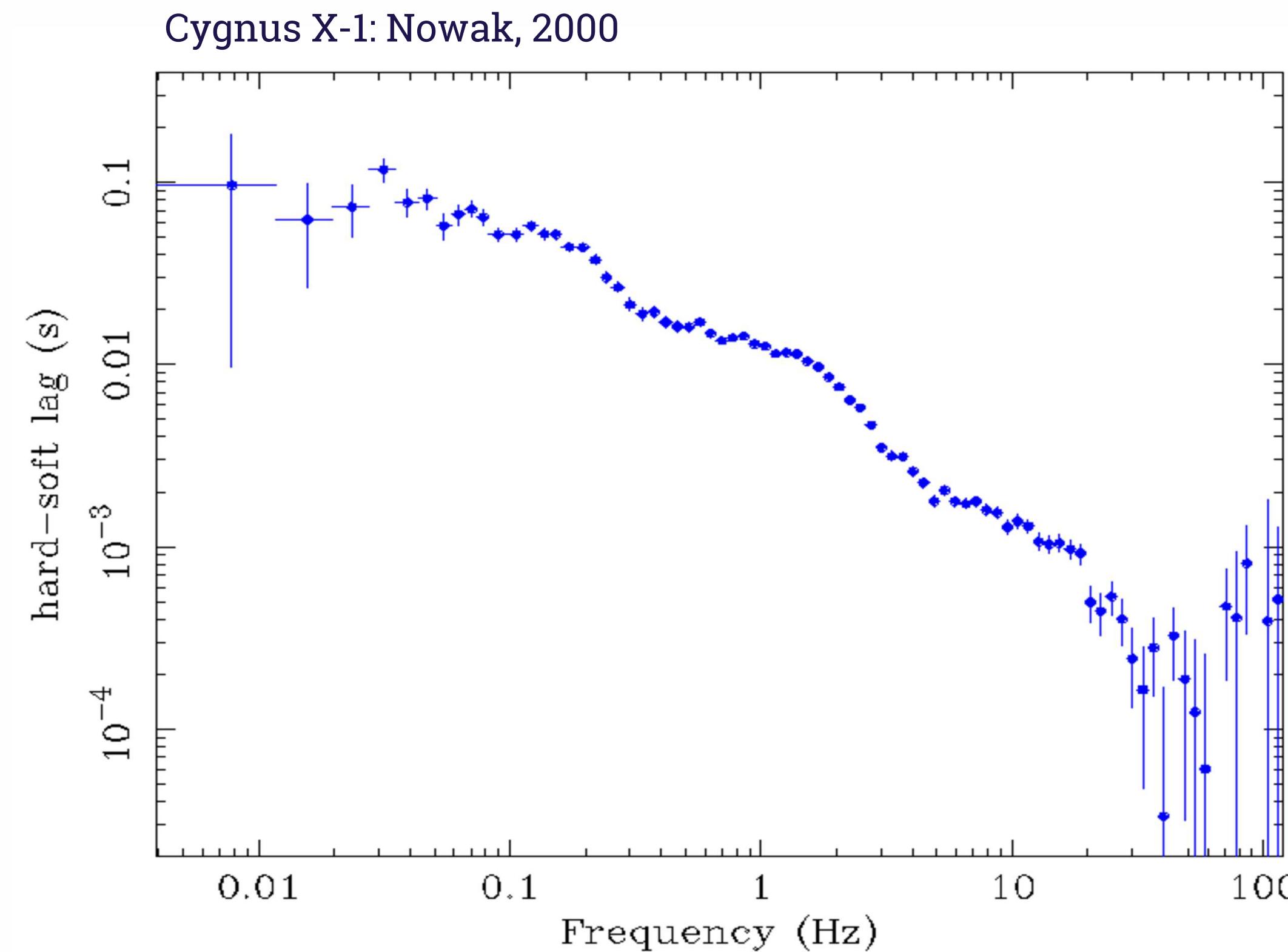
Fourier  
product



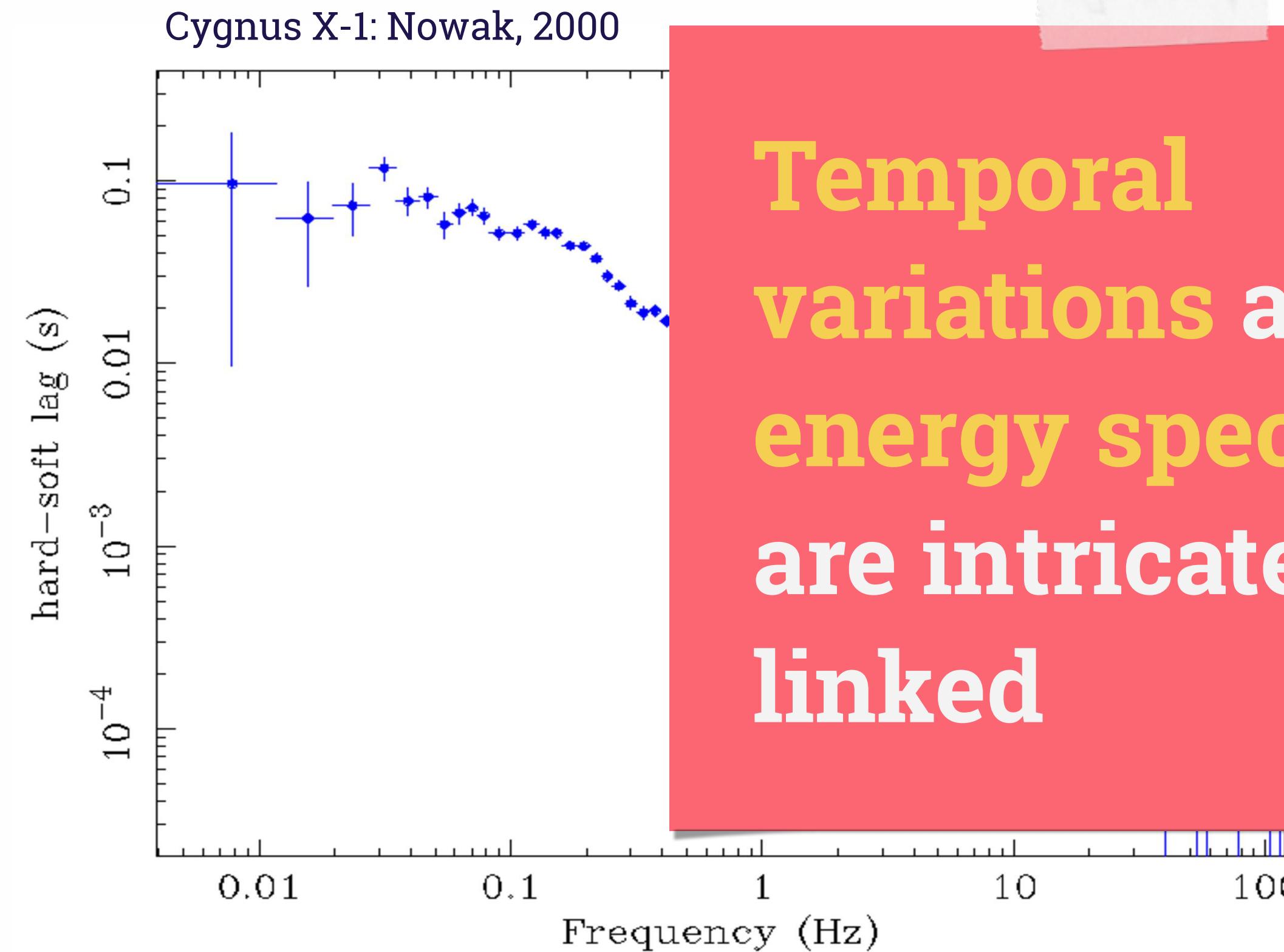
energy

frequency

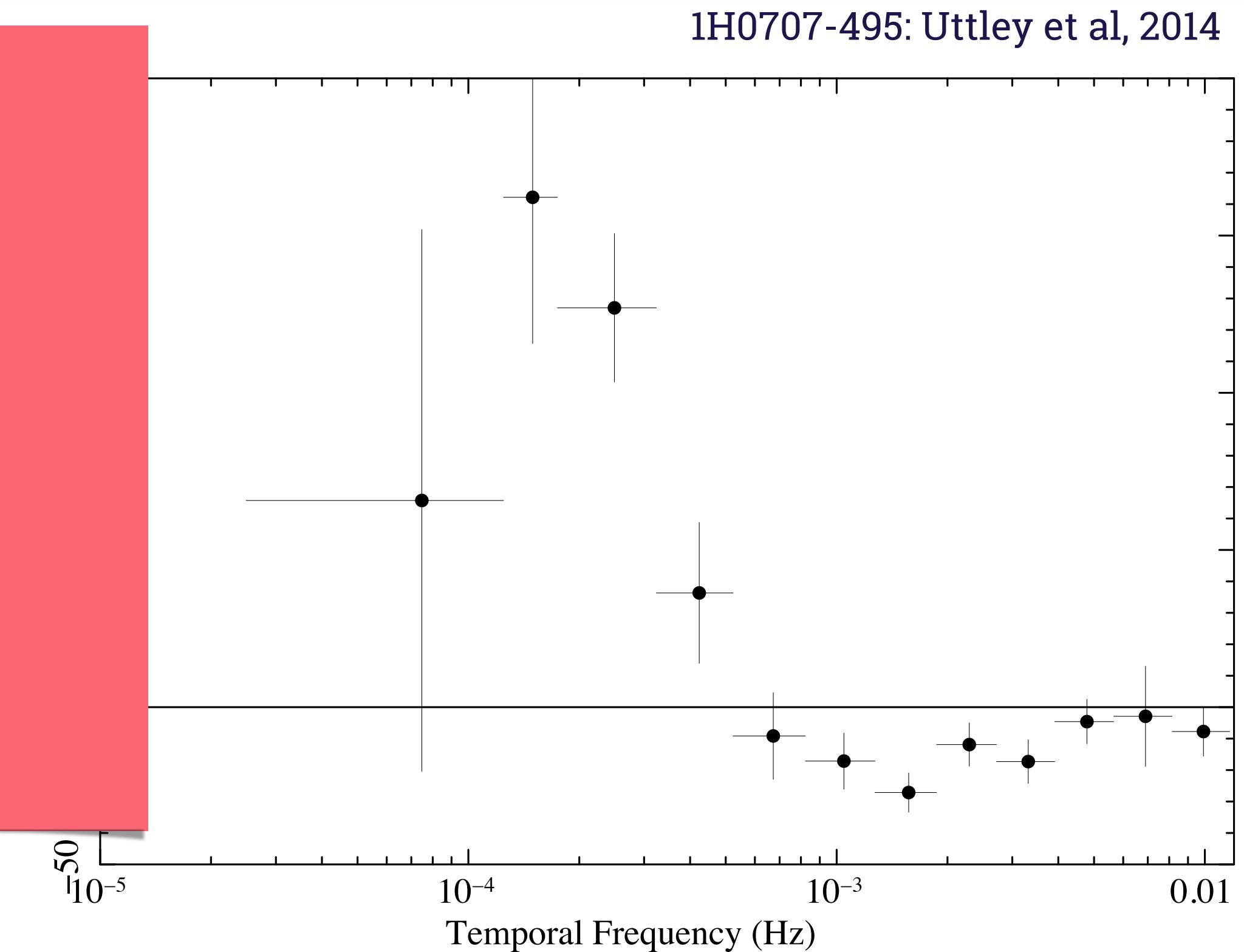
# Lag-Frequency Spectrum



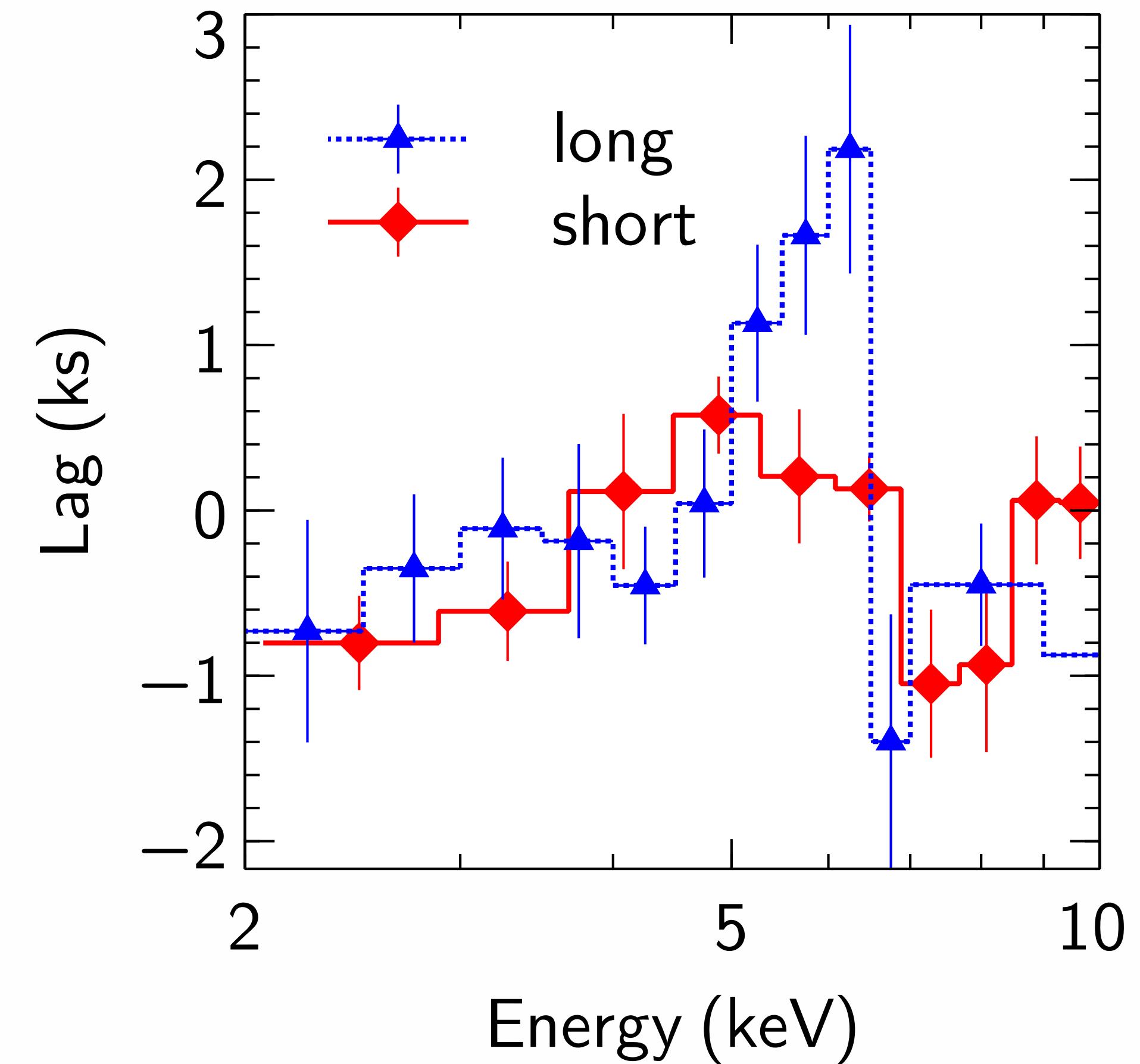
# Lag-Frequency Spectrum



Temporal  
variations and  
energy spectra  
are intricately  
linked



# Lag-Energy Spectrum



**Let's do this ourselves!**