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Exploring time variability of AGN with the CARMA models

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and

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CARMA introduction

CARMA(p, q)

Continuous time autoregressive moving average process (Kelly et al. 2014, ApJ, 788, 33)

A zero-mean CARMA process of order (p, q) is defined according to the stochastic differential equation:

autoregressive coefficients

AR

$$\frac{d^p y(t)}{dt^p} + \alpha_{p-1}\frac{d^{p-1} y(t)}{dt^{p-1}} + \ldots + \alpha_0 y(t) = \beta_q \frac{d^q \epsilon(t)}{dt^q} + \beta_{q-1}\frac{d^{q-1}\epsilon(t)}{dt^{q-1}} + \ldots + \epsilon(t).$$

MA

moving average coefficients

Gaussian white noise process with zero mean and variance σ^2

CARMA process is **stationary** when
- q < p, and
- roots of the AR polynomial have negative real parts

$$A(z) = \sum_{k=0}^{p} \alpha_k z^k$$

CARMA introduction

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Continuous time autoregressive moving average process (Kelly et al. 2014, ApJ, 788, 33)

$\boxed{\text{CARMA(p=1, q=0) = CAR(1)}}$

$$\frac{d^p y(t)}{dt^p} + \alpha_{p-1} \cancel{\frac{d^{p-1} y(t)}{dt^{p-1}}} + \ldots + \alpha_0 y(t) =$$

$$\beta_q \cancel{\frac{d^q \epsilon(t)}{dt^q}} + \beta_{q-1} \cancel{\frac{d^{q-1} \epsilon(t)}{dt^{q-1}}} + \ldots + \epsilon(t).$$

CAR(1) process:
Kelly et al. 2009

Superposition of CAR(1) processes:
Kelly et al. 2011
Sobolewska et al. 2014

CARMA introduction

$$A(z) = \sum_{k=0}^{p} \alpha_k z^k$$

Autocovariance function at lag τ

$$R(\tau) = \sigma^2 \boxed{\sum_{k=1}^{p}} \frac{\left[\sum_{l=0}^{q} \beta_l r_k^l\right]\left[\sum_{l=0}^{q} \beta_l (-r_k)^l\right] \boxed{\exp(r_k \tau)}}{-2\,\mathrm{Re}(r_k) \prod_{l=1, l\neq k}^{p} (r_l - r_k)(r_l^* + r_k)}$$

- weighted sum of *p* exponential functions
- weights are functions of MA coefficients, β
- arguments depend on the roots of AR polynomial that might be complex-valued (exponentially damped sinusoids for complex roots, exponential decays for real roots)
- PSD of a CARMA process can be expressed as a weighted sum of **Lorentzian functions**

see e.g. Nowak 2000, Belloni 2010, McHardy 2007 for observed X-ray PSDs of X-ray binaries and AGN

CARMA introduction

carma_pack is available from GitHub

https://github.com/brandonckelly/carma_pack

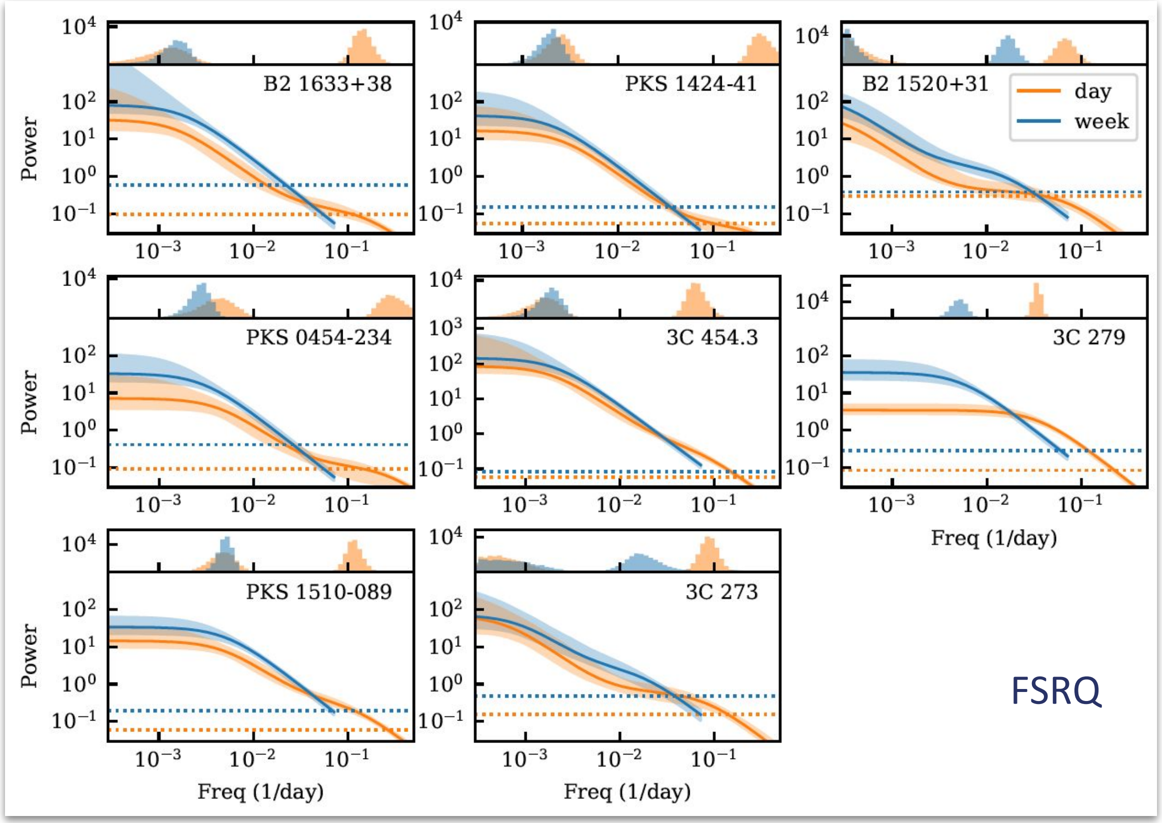
Extensive tutorial is included with the **carma_pack**

examples/carma_pack_guide.ipynb

Edited tutorial containing material for this session

https://github.com/malgosias/carma_tutorial

Gamma-ray variability of Fermi/LAT blazars



Ryan et al. 2019, ApJ, submitted

- 13 blazars (8 FSRQ + 5 BL LACs)
- PSDs computed using **daily** and **weekly** binned Fermi/LAT lightcurves

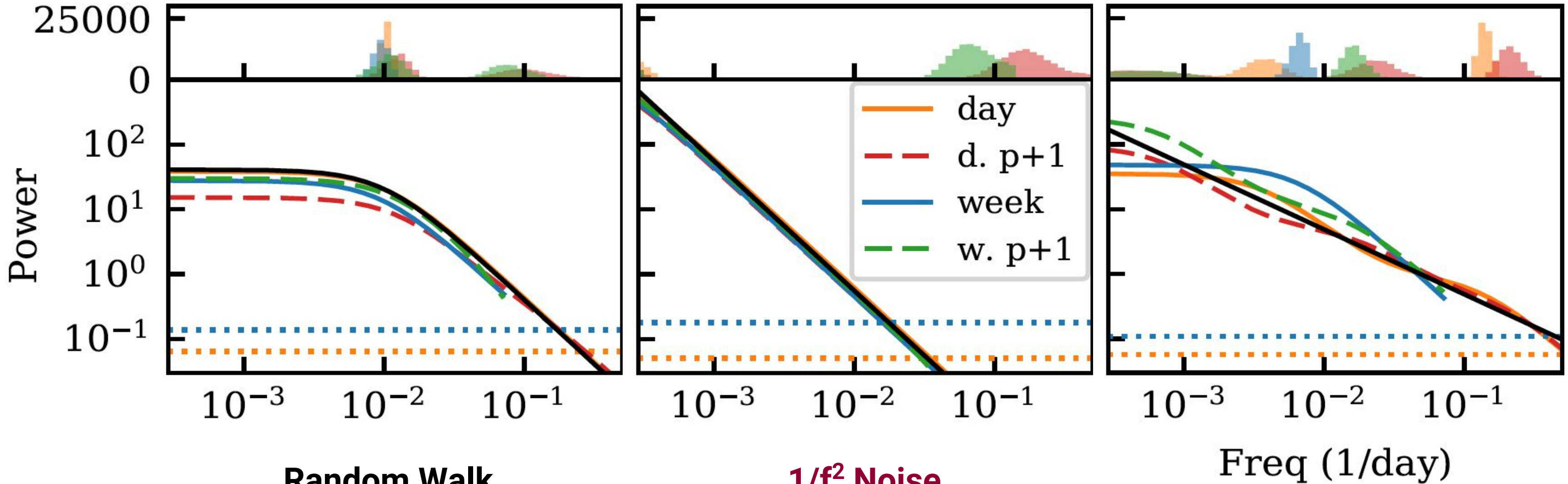
  (difference in S/N and number of ``missing`` measurements due to non-detections)

Gamma-ray variability of Fermi/LAT blazars

Ryan et al. 2019 (ApJ submitted):
True PSD and PSDs recovered with CARMA plotted for two different orders of CARMA models applied to simulated lightcurves with different time bin sizes

Spurious breaks (?) with location changing depending on the order of a CARMA model

1/f Noise



Random Walk

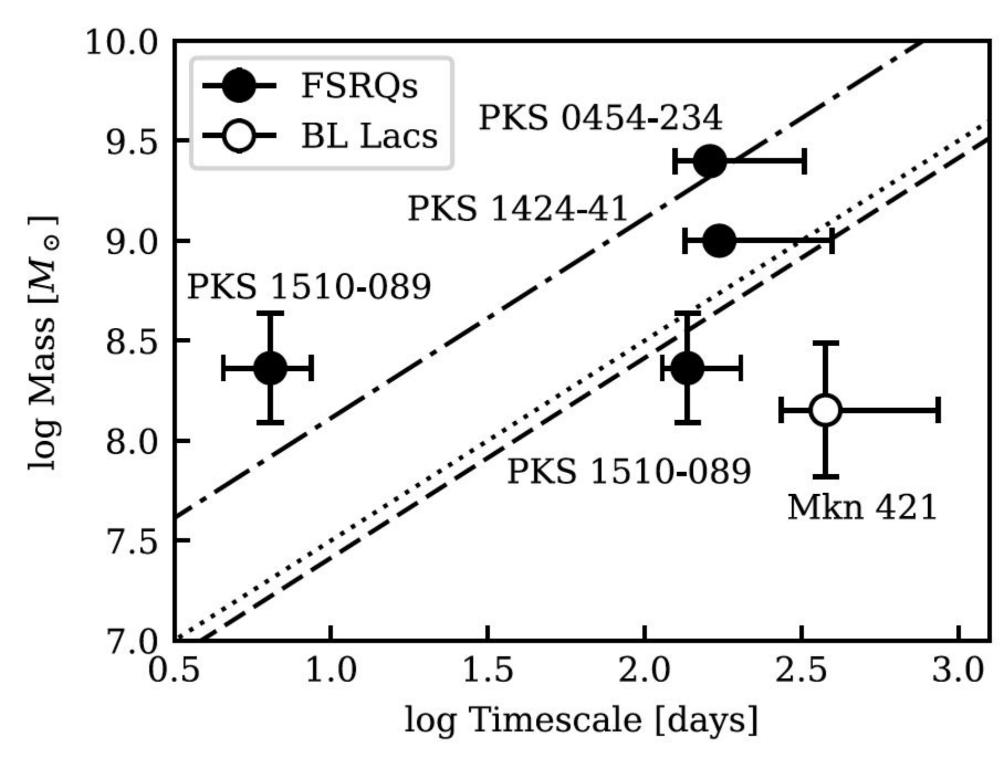
$1/f^2$ Noise

Location of the break **does not** dependent on the CARMA order

Unconstrained breaks

Gamma-ray variability of Fermi/LAT blazars

Ryan et al. 2019, ApJ, submitted



Constraints from X-ray variability:

. . . . . . Seyfert 1s, Markovitz+ 2003

- - - - - Cyg X-1, low state, McHardy+ 2004

_ . _ Cyg X-1, high state, McHardy+ 2004

No apparent correlation with black hole mass

Different origin of gamma-ray and X-ray variability?

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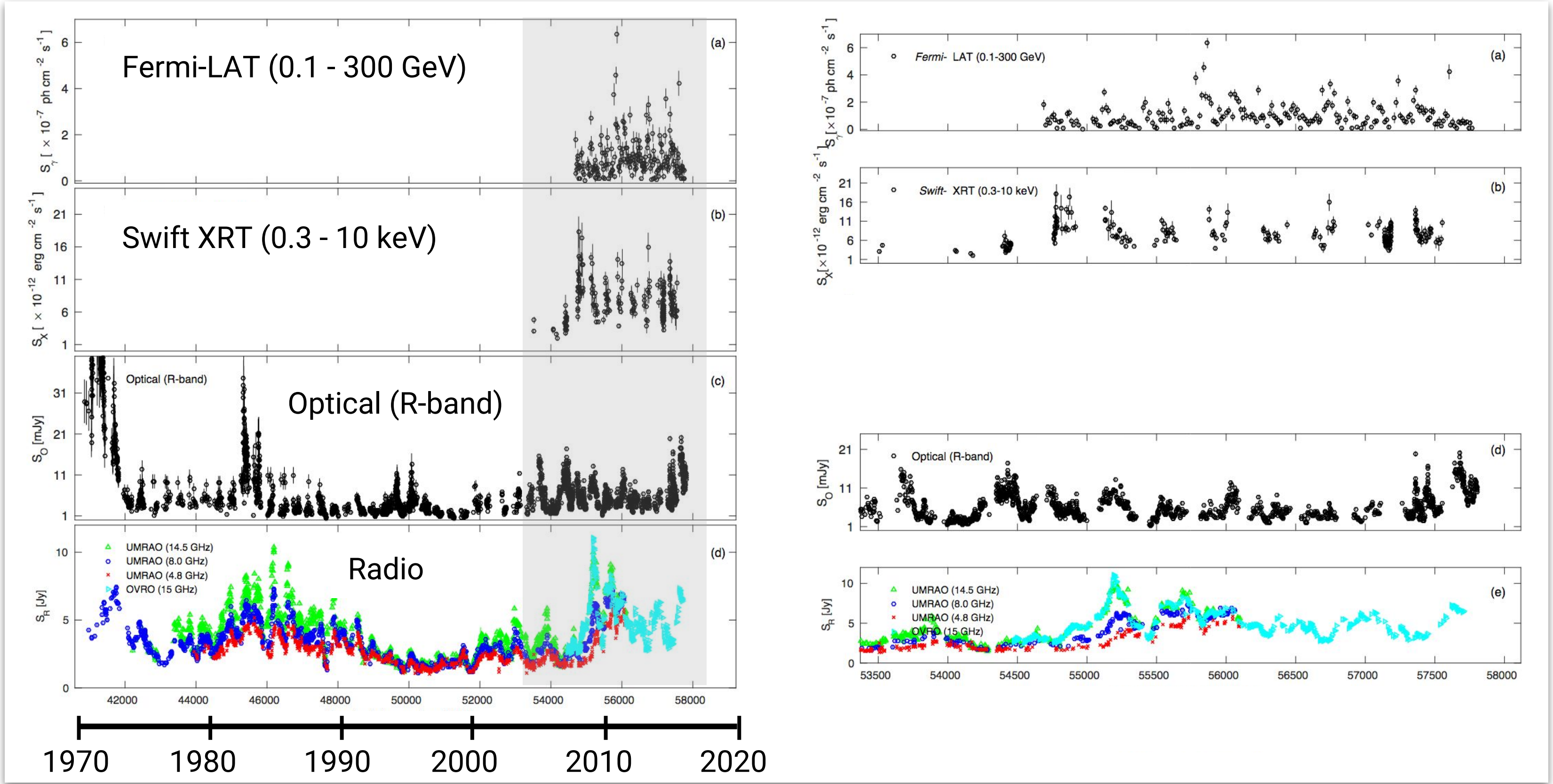
Multi-wavelength variability of a BL LAC, OJ 208

Why is this an interesting source in the context of variability study?

Goyal et al. 2018, ApJ, 863, 175

- A supermassive black hole binary was claimed in the system, based on the evidence for a ~12 yr periodicity in its optical and radio light curves
  (Sillanpaa et al. 1996; Valtonen et al. 2016; Valtaoja et al. 2000)
- Hints for a quasi-periodicity, with a characteristic timescale of ~400–800 days reported in the decade-long optical/near-infrared and gamma-ray light curves
  (Sandrinelli et al. 2016, Bhatta et al. 2016, and references therein)
- One of a few blazars for which good-quality, long-duration optical monitoring dating back to circa 1896
- One of a few blazars that have been observed by the Kepler satellite
- Monitored in the radio domain with a number of telescopes, in X-rays by the Swift's XRT, and in the high-energy gamma-ray range with the Fermi/LAT

Multi-wavelength variability of a BL LAC, OJ 208



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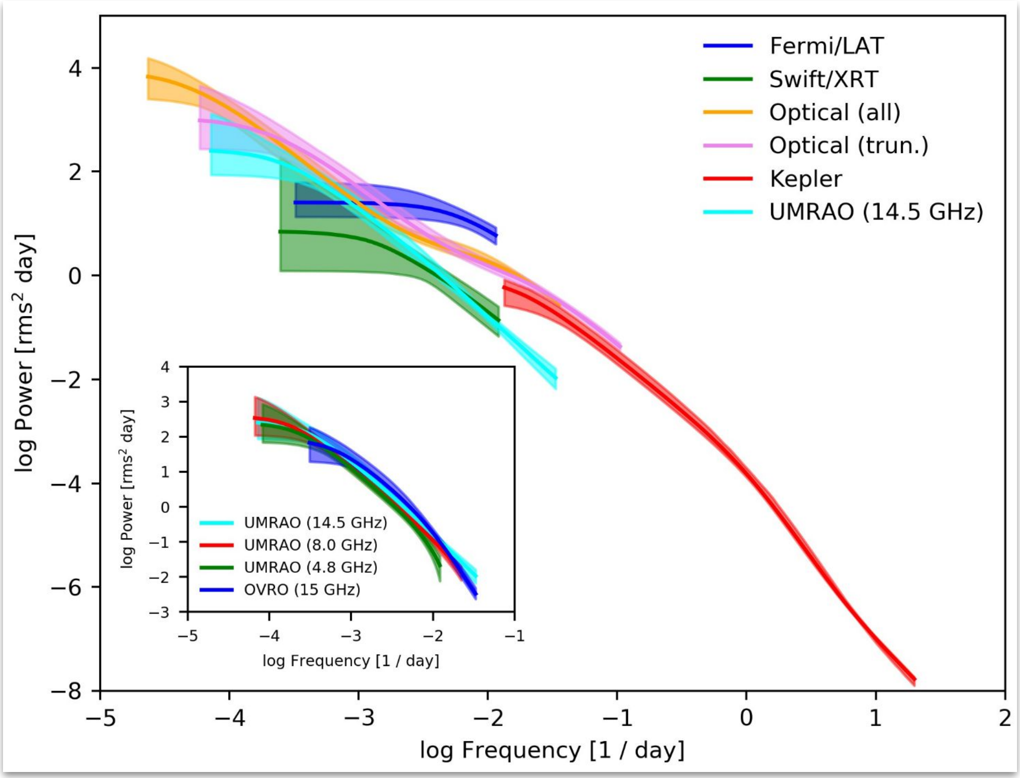

- **Optical PSD** constructed over 6 orders of magnitude in variability timescales (decades to hours)
- PSDs in the **radio** (inset) and **X-ray** bands have similar shapes (timescales: from a year down to months/weeks)
- **Gamma-ray** PSD is noticeably **flatter** than the optical and radio/X-ray PSDs
- **Gamma-ray** PSD has a relaxation timescale of about 150 days
- (Quasi-)periodicities not detected

Goyal et al. 2018, ApJ, 863, 175

CARMA - Summary

Main strengths

- Bayesian method for Gaussian CARMA models via Markov Chain Monte Carlo (MCMC) sampling, to infer the distribution of power spectral parameters given the measured lightcurve

- Gaussian measurement noise is naturally incorporated into the analysis

- CARMA models have a very flexible parametric form for their power spectrum and autocorrelation function, and because of this they are able to model a broad range of non-deterministic time series

- flexible framework for modeling irregularly-sampled gappy time series

- many of the computations involved with fitting, interpolating, and forecasting can be efficiently performed using the Kalman Filter

Selected publications utilizing CARMA

e.g. Edelson et al. 2014; Davenport et al. 2015; Graham et al. 2015; Simm et al. 2016; Kasliwal et al. 2017; Sanchez et al. 2017; Goyal et al. 2018; Alston et al. 2019; Ryan et al. 2019