

The globular cluster system of the Auriga simulations

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ABSTRACT

We investigate whether the galaxy formation model used for the Auriga simulations can produce a realistic globular cluster (GC) population at redshift zero. We compare properties of the simulated star particles in the Auriga haloes with catalogues of observations of the Milky Way (MW) and Andromeda (M31) globular cluster populations available in the literature. We find that the Auriga simulations do produce sufficient mass, radii and metallicities that are typical for the MW globular cluster system (GCS), although we observe a varying mass-excess for different R_{GC} -[Fe/H] bins. This implies different values for the combined product of the bound cluster formation efficiency and the globular cluster disruption rate. Furthermore we test whether any of the Auriga galaxies has a metallicity and radial distribution that is consistent with the MW or M31 GCS. For any one of the Auriga haloes we reject the null hypothesis that the observed and simulated metallicities of GCs (GC candidates) are drawn from the same distribution at $\geq 98.32\%$ confidence level, for the GCS of the Milky Way as well as that of the Andromeda galaxy. The same holds true for the distribution of galactocentric radius. Overall, the Auriga simulations produce old star particles with higher metallicities than the MW and M31 GCS and at larger radii. The formation efficiency would have to linearly decrease with increasing metallicity for the Auriga GC candidates to be consistent with the MW GCS, if cluster disruption is equally efficient regardless of metallicity. In addition, the number of GCs relative to field stars would have to be smaller for higher metallicity star particles at large radii than for those with lower metallicity at smaller radii, assuming GC disruption is more efficient in the inner regions of the Galaxy.

Key words: methods: numerical – galaxies: formation – galaxies: star clusters: general.

1 INTRODUCTION

Globular clusters (GC)s are old, bright, ubiquitous, and various properties of GC systems show correlations with their host galaxies. GC systems are believed to retain information about the gas conditions at times of formation, thus, could offer unique insight into the (chemodynamical) evolution of their parent galaxies, if the formation and evolution of GCs- and GC systems themselves is adequately understood. However, despite decades of research (the extensive literature on GCs is summarised in several books and review articles, e.g. Harris 1991; Harris 2001; West et al. 2004; Brodie & Strader 2006; Gratton et al. 2012; Kruijssen 2014; Forbes

et al. 2018), consensus on an exhaustive picture of the formation of GCs is yet to be reached.

The literature offers a wealth of formation scenarios. Peebles & Dicke (1968) and Peebles (1984) argue that GCs form as the earliest bound structures in the Universe (i.e. prior to formation of the main galaxy), noting that the Jeans length and mass shortly after recombination is consistent with typical GC masses and sizes. Diemand et al. (2005) and Boley et al. (2009) argue that (the blue subpopulations of) GCs form in biased dark matter halos at high redshift. Other recent hypotheses of GC formation prior to collapse of the proto-galaxy include formation in colliding supershells (Recchi et al. 2017), in supersonically induced gaseous objects (Chiou et al. 2019), or in high-speed collisions of dark matter subhaloes (Madau et al. 2019). Boylan-Kolchin (2017) suggests that (the blue

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subpopulation of) GCs could form in high density regions along the cosmic filament before or during collapse.

Other models also date GCs formation during formation of the proto-galaxy itself, for example as a result of thermal instabilities in hot gas-rich haloes (Fall & Rees 1985, also see the discussions in Kang et al. 1990). Alternative formation triggers are also explored, such as (other causes of) shock compression, or cloud-cloud collisions (e.g. Gunn 1980; Murray & Lin 1992; Harris & Pudritz 1994; Vietri & Pesce 1995; Larson 1996; Cen 2001).

Yet another hypothesis is that star cluster formation is triggered by (major) gas-rich (spiral) galaxy mergers (Schweizer 1987; Ashman & Zepf 1992), which is naturally expected within the framework of hierarchical assembly. One testable prediction of this scenario is the formation of young clusters in interacting and merging galaxies, which has been observed and are found to show remarkable similarities with globulars in the Milky Way (e.g. Whitmore & Schweizer 1995; Holtzman et al. 1996; Zepf et al. 1999; Whitmore et al. 1999). Moreover, modelling efforts of this framework yield GC (sub)populations consistent with various observables (e.g. Muratov & Gnedin 2010; Choksi et al. 2018), and the recent numerical simulation of an isolated dwarf-dwarf merger executed at high resolution (baryonic mass $m_b \sim 4M_\odot$; softening $\epsilon = 0.1$) produce star clusters that could be globular progenitors (Lahén et al. 2019).

As for the formation timeline, the scenarios (and flavours thereof) of Fall & Rees (1985) and Ashman & Zepf (1992) are intertwined because accretion and mergers continuously occurs through out hierarchical build-up of galaxies. Various (other) hierarchical formation channels thus combine different aspects of the aforementioned paradigms, such as GC formation in (small) galactic disks before they are accreted onto an assembling galaxy (e.g. Côté et al. 2000, 2002; Beasley et al. 2002; Gnedin 2003). We refer to Gnedin et al. (2001) for a scoreboard of the aforementioned paradigms compared to observations of the Milky Way globular cluster system.

An idea that has recently been studied in detail is the hypothesis that the physical processes responsible for the formation of young massive clusters (YMCs, see Portegies Zwart et al. 2010 for a review), as observable in detail in the local Universe in general, also governs GC formation at high redshifts. In this framework, differences between both classes of objects are caused by nearly a Hubble time of (dynamical) evolution (e.g. Spitzer 1987). This picture is based on observed similarities between YMCs and GCs (e.g. Holtzman et al. 1992; Whitmore et al. 1999) and strengthened by observations of gravitationally lensed objects at high redshifts ($z = 2 - 6$). These sources have properties reminiscent of (local) YMCs and may be GC progenitors at times of formation (Vanzella et al. 2017; Johnson et al. 2017). The modelling work by Kruijssen et al. (2011, 2012; Kruijssen 2015) is now incorporated into cosmological zoom simulations and shows promising results (Pfeffer et al. 2018; Kruijssen et al. 2019).

Simulation resolutions have reached the mass range populated by GCs, and the gravitational force softening can be as low several parsec. A number of groups can thus incorporate formation of (globular) star clusters into their high-resolution hydrodynamical simulations. Ricotti et al. (2016) run parsec scale simulations of the high-redshift universe prior to reionization (the simulations stop at $z = 9$), Li et al. (2017) implement a new subgrid model for star (cluster) formation and run simulations that reach $z = 3.3$, and the run of Renaud et al. (2017) reaches $z = 0.5$. Kim et al. (2018) find that mergers can push gas to high density that quickly forms clustered stars that end up tightly bound by the end of the simulation. A somewhat different approach couples semi-analytical models to DM-only simulations (Muratov & Gnedin 2010; Li & Gnedin 2014;

Choksi et al. 2018; Choksi & Gnedin 2019b,a), as is done in the work by El-Badry et al. (2019).

In this work we use state of the art simulations that produce realistic spiral galaxies at redshift zero for which several global properties are consistent with the observations. The simulations yield realistic Milky Way analogues, and real galaxies host globular cluster systems. The question thus naturally arises whether the star formation histories of the simulations (also) give rise to a GC system similar to the Milky Way and/or Andromeda globular cluster systems. In particular, we use the Auriga suite of cosmological zoom simulations (Grand et al. 2017, and described in Sec. 2) to investigate whether the star formation model implemented produces metallicity, radial, and metallicity-radial distributions that are consistent with the MW and/or M31 GC systems, and whether the model produces enough stellar mass with the right properties to allow for formation efficiencies lower than unity and the expected (dynamical) mass loss over the cluster lifetime (not modelled explicitly in this study).

The plan of the paper is as follows. We summarise the relevant characteristics of the Auriga simulations in section 2, followed by a summary of the observations of the Milky Way (MW) and Andromeda globular cluster system (GCS) in section 3 that we use to compare our simulations to in section 4. We discuss our findings in section 5 to come to our conclusions in section 6.

2 THE AURIGA SIMULATIONS

We use the Auriga simulations (Grand et al. 2017, hereafter G17), a suite of high-resolution cosmological zoom simulations of Milky Way-mass selected initial conditions. The simulations are performed with the AREPO (Springel 2010; Pakmor et al. 2016) code, which solves the magnetohydrodynamical equations on a moving mesh. The galaxy formation model produces realistic spiral galaxies at redshift $z = 0$.

The interstellar medium is modelled using a sub-grid approach which implements the physical processes most relevant to galaxy formation and evolution. This model was tailored to the AREPO code and calibrated to reproduce key observables of galaxies, such as the history of the cosmic star formation rate density, the stellar mass to halo mass relation, and galaxy luminosity functions.

The sub-grid includes primordial and metal-line cooling with self-shielding corrections. Reionization is completed at redshift six by a time-varying spatially uniform UV background (Faucher-Giguère et al. 2009; Vogelsberger et al. 2013). The interstellar medium is described by an equation of state for a two-phase medium in pressure equilibrium (Springel & Hernquist 2003) with stochastic star formation thermally unstable gas with a density threshold of $n = 0.13 \text{ cm}^{-3}$, and consecutive stellar evolution is accounted for. Stars provide feedback by stellar winds (Marinacci et al. 2014; Grand et al. 2017), and further enrich the ISM with metals from SNIa, SNII, and AGB stars (Vogelsberger et al. 2013). The formation of black holes is modelled which results in feedback from active galactic nuclei (Springel et al. 2005; Marinacci et al. 2014; Grand et al. 2017). Finally, the simulations follow the evolution of a magnetic field of 10^{-14} (comoving) G seeded at $z = 127$ (Pakmor & Springel 2013; Pakmor et al. 2014). See G17 for further details of the numerical setup as well as the galaxy formation model.

The Auriga suite has a fiducial resolution level L4, accompanied by the lower (higher) level L5 (L3) that is available for selected initial conditions. The baryonic mass resolution in order of increasing level is $m_b = [4 \times 10^5, 5 \times 10^4, 6 \times 10^3] M_\odot$ with grav-

itational softening of collisionless particles $\epsilon = [738, 369, 184]$ pc. The mass resolution of the Auriga simulations is thus close to the characteristic peak mass of the lognormal GC mass distribution of $10^5 M_\odot$ (Harris 1991), although the gravitational softening is two orders of magnitudes larger than typical GC radii. High-density gaseous regions are thus not expected to produce surviving stellar clumps with masses and radii consistent with GCs because such objects would numerically disperse, even in the highest-resolution runs. On the other hand, we can investigate (statistical) properties of age-selected GC candidates because each star particle represents a single stellar population with a total mass that could be consistent with one globular cluster. This means that their formation sites and that of a real-world GC may be consistent.

3 OBSERVATIONAL DATA

We describe the observations of the MW GCS in Sec. 3.1, and of the Andromeda (M31) GCS in Sec. 3.2

3.1 Milky Way

Harris (1996, 2010 edition; hereafter H96e10) provides a catalogue¹ of the Milky Way globular cluster system that contains properties of 157 GCs. The authors initially estimated the size of the MW GCS to be 180 ± 10 , thus, their catalogue to be $\sim 85\%$ complete. However, an additional 59 GCs are claimed to have been discovered by various authors. The total number of GCs in the MW might add up to 216 with recent estimates now anticipating an additional thirty GCs yet to be discovered (e.g. Ryu & Lee 2018, and references therein). We still use data from the Harris catalogue, but caution that it may (only) be 53-72% complete. Specifically, the relevant data fields that we use from H96e10 are the metallicity $[\text{Fe}/\text{H}]$, the Galactic distance components X , Y , and Z (in kpc)², and absolute magnitude in the V-band $M_{V,0}$. We use the latter to calculate mass-estimates by assuming $M_{V,0} = 4.83$ and a mass to light ratio $M/L_V = 1.7 M_\odot/L_\odot$, the mean for MW clusters (McLaughlin & van der Marel 2005). We supplement the catalogue with age-estimates from isochrone fits to stars near the main-sequence turnoff in 55 GCs (VandenBerg et al. 2013, hereafter V13).

3.2 Andromeda

The fifth revision of the revised bologna catalogue (RBC 5, last updated August, 2012) is the latest edition of three decades of systematically collecting integrated properties of the globular cluster system of the Andromeda galaxy (Galleti et al. 2004, and references therein). One contribution to RBC 5 is the work by Caldwell et al. (2011, hereafter C11), subsequently updated by Caldwell & Romanowsky (2016, hereafter CR16).

C11 and CR16 present a uniform set of spectroscopic observations calibrated on the Milky Way GCS of the inner 1.6° (~ 21) kpc that is believed to be 94% complete. GCs in the outer stellar halo, up to $R_{\text{proj}} \sim 150$ kpc, are observed in the Pan-Andromeda Archaeological Survey (PAndAS, Huxor et al. 2014, hereafter H14), but see

also Veljanoski et al. (2014) and Mackey et al. (2019). H14 presents the discovery of 59 new GCs and publishes updates to RBC 5. The work of H14 is incorporated in the latest public release of the C11 dataset, further revised by CR16. It seems that CR16 is the most recent aggregated dataset of M31's GCS that contains properties of interest for our study as it contains GCs in the inner region and in the outer halo. The relevant fields in the CR16 dataset that we use are the age, metallicity, and the mass-estimate⁴. Radii are calculated from RA and DEC, further discussed in Sec. 3.4.

3.3 Age-estimates

The top panel of Figure 1 shows a histogram of the age-estimates of the 55 MW GCs in V13 and 88 GCs in M31 for which age-estimates are available in CR16. The mean age of the MW GCs in this data set is 11.9 ± 0.1 Gyr and the dispersion is 0.8 Gyr. Furthermore, only one of the 55 GC age-estimates is below 10 Gyr. The M31 GCS has a mean age of 11.0 ± 0.2 Gyr with a dispersion of 2.2 Gyr, and 24 GCs have age-estimates below 10 Gyr with a minimum age of 4.8 Gyr. Based on these data, we find that the age distributions of the MW and M31 GCS's are not statistically consistent. Moreover, the Milky Way appears to host a globular cluster system that is somewhat older than the Andromeda galaxy, which is somewhat surprising given that M31 is generally considered to be earlier type than the MW meaning that an older stellar population would naively be expected. However, we do caution that both data sets are incomplete and the age measurements have large uncertainties (of 1 – 2 Gyr). On the other hand, the magnitude of the uncertainty is insufficient to explain the low-age tail in M31.

3.4 Radial distribution

The bottom panel of Figure 1 shows the radial distribution of the MW and M31 GCS. Both quantities are readily available in H96e10 (assuming $R_\odot = 8.0$ kpc), but the galactocentric radius of GCs in M31 is not available in CR16. Therefore we follow Wang et al. (2019, Sec. 4.1) to calculate the projected radius R_{proj} from the observed positions, adopting M31's central position from the NASA Extragalactic Database⁵ ($\alpha_0, \delta_0 = (0^h 42^m 44.35^s, +41^\circ 16' 08.63'')$) and distance $D_{\text{M31}} = 780$ kpc (McConnachie et al. 2005; Conn et al. 2012). We calculate R_{GC} as 'average deprojected distance' $R_{\text{GC}} = R_{\text{proj}} \times (4/\pi)$.

The solid lines show the distributions using all available data (because the sky coordinates are known for each GC), while the subset for which age-estimates are available is indicated using dotted lines. The latter shows a narrower range of radii than the full data set: it appears that age-estimates are neither available for the innermost (< 1 kpc) GCs, nor for those beyond roughly twenty kpc (the halo GCs). We compare both distributions of the full data set (solid lines) and find that the Milky Way has more GCs in the range 1 – 4 kpc than M31 (when accounting for the larger number of total GCs in M31). Interestingly, both distributions show a similar trend for $R_{\text{GC}} > 4$ kpc and host a subpopulation of halo GCs. However, both radial distributions are not statistically consistent due to substantial differences at intermediate radii.

¹ See https://www.physics.mcmaster.ca/Fac_Harris/mwgc.dat

² In a Sun-centered coordinate system: X points toward Galactic center, Y in direction of Galactic rotation, and Z toward the North Galactic Pole. We calculate the galactocentric radius $R_{\text{GC}} = \sqrt{(X - R_\odot)^2 + Y^2 + Z^2}$, assuming the solar radius $R_\odot = 8$ kpc.

³ Last revised 23 Sep 2015, see https://www.cfa.harvard.edu/oir/eg/m31clusters/M31_Hectospec.html

⁴ The authors assumed $M/L_V = 2$ independent of $[\text{Fe}/\text{H}]$

⁵ <https://ned.ipac.caltech.edu/>

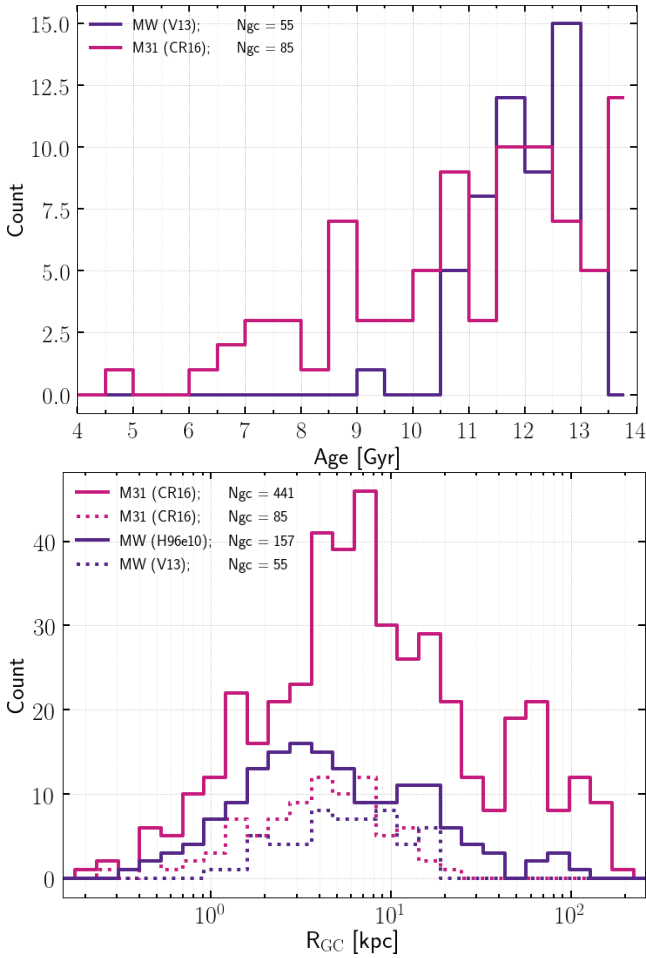


Figure 1. *Top:* Age distribution of 55 GCs in the MW (data from [VandenBerg et al. 2013](#)) and 88 GCs in M31 (data from [Caldwell & Romanowsky 2016](#)). *Bottom:* Distribution of galactocentric radius in the MW and M31. The dotted lines show the subset of data that also have age measurements (i.e. the same sample as used in the top panel).

3.5 Total GC mass in metallicity-radial space

We show the two-dimensional mass-weighted metallicity-radial distribution of the MW (M31) GCS in the top (bottom) panel of Figure 2. The observations indicate that no GCs with high metallicities are to be expected at large radii (the three bins in the upper right corner, both for MW and M31), and relatively few GCs at large radii in general ($R_{GC} > 30$ kpc; right column: 11 GCs or 7.3% in the MW and 17 or 4.6% in M31). Moreover, the M31 hosts more metal-rich ($[Fe/H] > -1$) GCs in each radial bin in comparison to the MW GCSs than what would be expected when accounting for the fact that Andromeda hosts a larger GC system than the MW. Finally, given that both marginalized (i.e. the metallicity and radial) distributions are not statistically consistent, we find that the two-dimensional distributions are also not consistent. More generally, the GC systems of the Milky Way and that of Andromeda both differ significantly. We compare these observations to the Auriga simulations later on in Sec. 4.3.

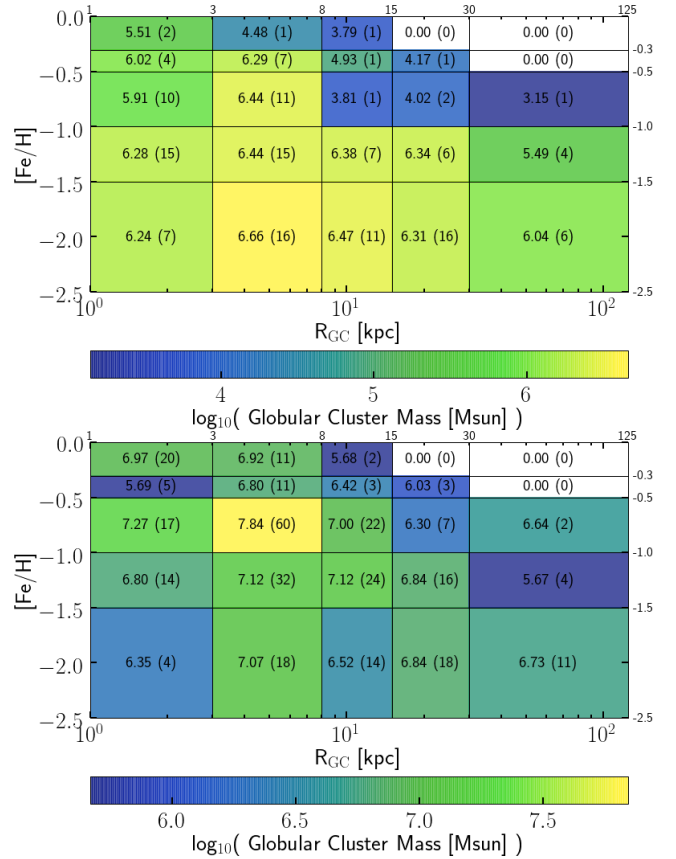


Figure 2. *Top:* Mass-weighted R_{GC} - $[Fe/H]$ distribution of 151 GCs in the MW (data from [Harris 1996, 2010 ed.](#)), which is 98.6 (92.2) % of the total mass (clusters) of the MW GCS in the Harris catalog. *Bottom:* Same for M31, showing 366 GCs and 88.4 (83.9) % of the total mass (clusters) in CR16 (data from [Caldwell & Romanowsky 2016](#)). Note that the range of the colourmap differs in both figures.

4 RESULTS

We define GC candidates in the Auriga simulations as all star particles older than 10 Gyr based on the age distribution of the MW GCS (top panel of Figure 1), and following [Renaud et al. \(2017\)](#).

Troughout this analysis we compare the distributions of three subsets of star particles: *old stars* (age > 10 Gyr, or GC candidates), *old in situ* stars (defined as those bound to the most-massive halo/subhalo in the first snapshot that the particle was recorded), and *old accreted* star particles (those that have formed *ex situ* and are bound to the most-massive halo/subhalo at $z = 0$). For comparison we also include the results for *all stars* (when no additional selection criterion is applied to the star particles). We consider the metallicity distribution in Sec. 4.1, the distribution of galactocentric radii in Sec. 4.2, and the combination of both in Sec. 4.3.

4.1 Metallicity distribution

We investigate whether the star formation model implemented in Auriga produces metallicity distributions consistent with the MW and M31 GC systems, and whether the simulations generate sufficient total mass at metallicities typical for the MW and M31 GCS's. To visually inspect the former we show the normalized metallicity

distribution of three specific Auriga galaxies in Figure 3 in comparison to the MW and M31 [Fe/H] distributions.

We select Au4-10⁶, Au4-21, and Au4-4 to show the distributions of three individual simulation runs that we consider representative for specific behaviour, and to highlight that different runs could give rise to different distributions. We plot the age-selected GC candidates in green, those that are accreted in red, and the *in situ* subpopulation in blue. The top half of the left figure shows the MW (M31) GC system in purple (magenta), where we overplot a double Gaussian for the MW GCS (the purple dashed lines) by adopting literature values of the mean μ and standard deviation σ of the metal-rich and metal-poor subpopulations (Harris 2001, p. 38).

We find that an age cut, on average, lowers the mean metallicity from 0.0 to -0.6 . Furthermore, the old accreted stars generally have lower mean metallicities than the old *in situ* stars with differences of roughly ~ 0.5 for the majority of the simulation runs. This behaviour can be seen in Au4-21, while a slightly larger difference of ~ 0.5 dex is seen for Au4-10 (as plotted), Au4-16, Au4-17, Au4-18 and Au4-22. However, this trend is reversed for Au4-1 and Au4-4 for which the old *in situ* population has a lower mean metallicity instead. We caution that Au4-1 is undergoing a major merger at redshift zero and note that we find $\mu = -1.51$ (-0.74) for the old *in situ* (accreted) although the former consists of only 1019 particles (1.3% of all GC candidates, and with a total mass of $5e7 M_\odot$). For Au4-4, 10.8% of the GC candidates is classified as *in situ* (compared to *in situ* fractions of 40 – 80% for other haloes). After inspection of the same figure for every one of the thirty Auriga L4 haloes we find that the simulations produce (sub)populations of GC candidates that are more metal-rich than the MW and M31 GC systems. Moreover, none of the simulations has a population of GC candidates with a bimodal metallicity distribution (the green curves).

We show the mean metallicity and standard deviation of all thirty Auriga L4 haloes in Figure 4 to show the behaviour for the full set of Auriga galaxies. The green crosses are to be compared to the purple (magenta) cross, which shows the mean value of all MW (M31) GCs. In addition, we show the metal-rich (metal-poor) population of the MW GCS using a solid (open) dot. We caution that these literature values result from individual Gaussian fits to sub sets of the observational data cut at $[\text{Fe}/\text{H}] = -1$. We do not include corresponding data points calculated using such an artificial cut for the simulated star particles. Having said that, we do find it interesting that the mean metallicities of the old *in situ* populations appear roughly consistent with that of the metal-rich population of the MW although the simulations show larger dispersions. The latter could simply be caused by the hard separation of the MW data into two groups, which means the range is smaller and the resulting dispersion lower. With regard to the M31 GCS, we are uncertain whether definitive consensus is reached in the literature concerning uni- bi- or trimodality in the $[\text{Fe}/\text{H}]$ distribution, but CR16 argues that the data, after removal of younger objects due to improved age classification, hint at three populations separated at $[\text{Fe}/\text{H}] > -0.4$ and $[\text{Fe}/\text{H}] < -1.5$. None of the Auriga sub sets has a mean value that offers much hope that the simulated distribution is consistent with the lowest metallicity group in the M31 GCS. This is also true for the metal-poor population of the MW with a mean value of -1.6 . The main take-away from this plot, again, is that all Auriga L4 galaxies have metallicity distributions with (much) larger mean

values than what is observed for the MW and M31 GCS, and that we systematically find lower mean metallicities for the accreted GC candidates than for those that have formed *in situ*.

We now turn to the second goal, which is to see whether sufficient total mass in GC candidates is produced by the Auriga simulations. To answer this question we plot a mass-weighted metallicity distribution in Figure 5. We show the median (coloured lines) for all thirty Auriga L4 haloes with the 1σ interval around it (shaded regions, which shows the scatter between runs that have different initial conditions, thus, have unique merger histories). We chose to aggregate the data to indicate general trends that we find when the GC candidates are split up according to birth location, rather than selecting typical examples of individual (simulated) galaxies. Once again we notice that the peak metallicity shifts down from 0 to -0.6 for old stars (green solid) compared to all stars (green dotted), and we learn that the mass at the peak lowers by roughly one dex. The mass budget of the old stars is dominated by the old *in situ* population (blue solid) below $[\text{Fe}/\text{H}] = -1$, and by the old accreted stars (red solid) above this value. We show the MW (M31) GCS in purple (magenta) and notice that the difference between the MW and M31 distributions is substantially larger than the scatter between different Auriga galaxies, particularly around $[\text{Fe}/\text{H}] = -1$. In addition, Andromeda does host (a hand full) GCs with $[\text{Fe}/\text{H}] < -2.5$ as well as GCs with $[\text{Fe}/\text{H}] > 0$ while the Milky Way does not.

We show the ratio of the simulated to the observed profiles in the middle (bottom) panel. This mass excess can be thought of the ‘mass budget’ that the Auriga GC candidates can ‘afford to lose’ due to a combination of smaller than unity bound cluster formation efficiencies combined with a Hubble time of dynamical evolution, while still producing sufficient mass at the right metallicities. In particular, the cluster formation efficiency would have to decrease with decreasing metallicity for Auriga GC candidates to produce a population of GC candidates that is consistent with the MW. For the GC candidates in M31 we find a constant mass excess up to -0.9 , above which the simulations produce a higher mass excess with increasing metallicity. If dynamical evolution is not expected to more efficiently disrupt GCs of higher metallicity, then we would find that the efficiency to form bound star clusters would have to decrease with increasing metallicity.

4.2 Radial distribution

We repeat the analysis of the previous subsection for the distribution of galactocentric radius instead of metallicity. We look for general trends present in all Auriga L4 galaxies. Figure 6 shows the mean and standard deviation of $\log_{10}(R_{\text{GC}})$ of star particles in all Auriga L4 simulations. The purple (magenta) cross again shows the mean of the MW (M31) GCs (which are to be compared to the green crosses), but solid (open) dot now shows our calculation of the mean value of the metal-rich (metal-poor) population as no literature values were available. We split the radii up into the metal-rich/metal-poor groups by taking a metallicity cut at $[\text{Fe}/\text{H}] = -1$ as is done in the literature.

We notice that the old *in situ* populations are more centrally distributed, whereas the old accreted subsets have a larger radial extent. This is not surprising because the classification of *in situ* requires star particles to have formed within the virial radius, thus they could naturally be expected to end up at small galactocentric radii. Accreted star particles, on the other hand, are formed in another (sub)halo beyond the virial radius, thus, would first have to migrate inwards in order to populate the innermost radii. Moreover, we find that the simulations have a wider dispersion in $\log(R_{\text{GC}})$

⁶ The nomenclature is ‘Au’ for Auriga, followed by the resolution level (4) and halo number (10, indicating which set of initial conditions was used the run).

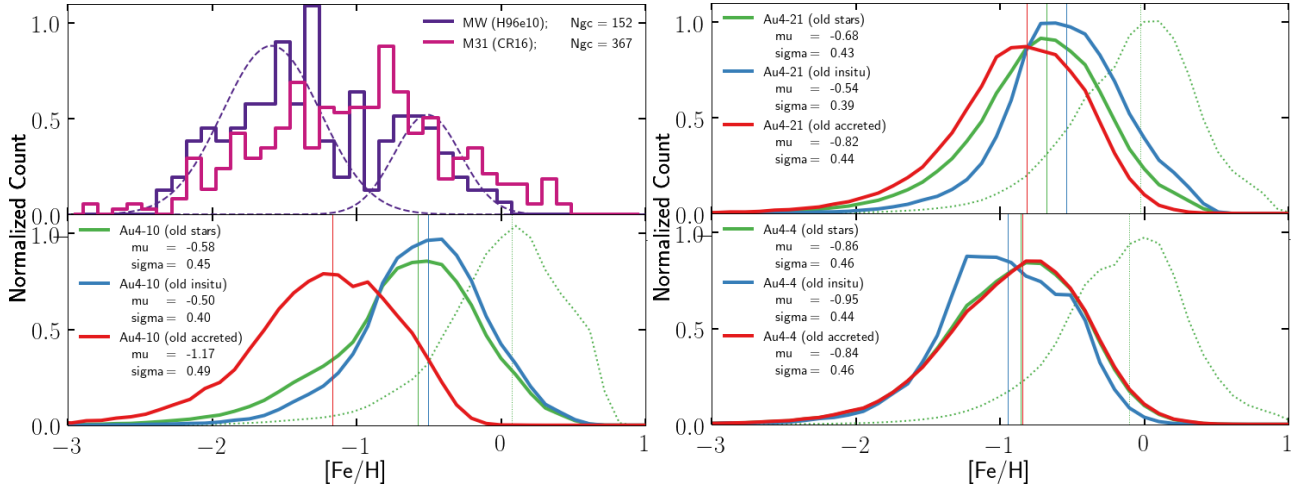


Figure 3. *Left:* Metallicity distribution of Au4-10 (bottom panel). We show the GG candidates in green. We split the GC candidates into two subpopulations, those that have formed *in situ* (blue), and those that have been accreted (red). The dotted green line shows all star particles. The solid purple (magenta) line in the top panel shows the GC system of the MW (M31). *Right:* Au4-4 (bottom), and Au4-21 (top).

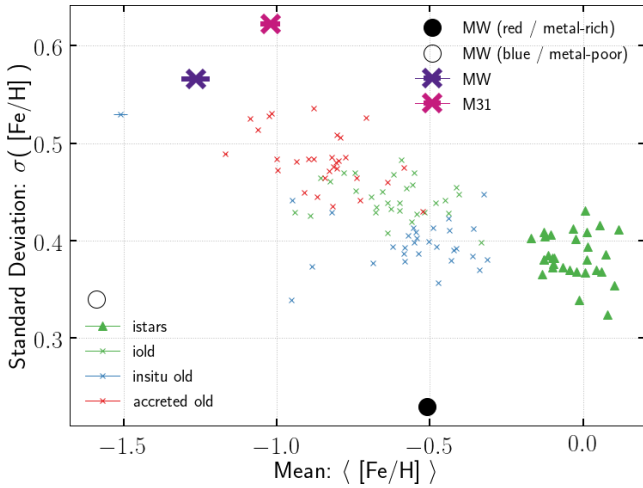


Figure 4. First vs second central moment of the Auriga L4 metallicity distributions. Each cross (for a given colour) represents one simulation. The green (blue) [red] crosses show the values calculated using the old (old *in situ*) [old accreted] star particles. Green triangles indicate that all stars were used. The purple (magenta) cross denotes our calculation using all MW (M31) observations (which would be appropriate for a unimodal distribution). The black solid (open) dots indicate the literature values of a bimodal Gaussian fit to the data (values from [Ashman & Zepf 1998](#), p. 38), showing the metal-rich (metal-poor) component of the MW.

than the MW GCS, while the dispersion of M31 seems to lie within the range of dispersions found in the Auriga galaxies. Furthermore, the mean of the MW GCS lies roughly within the range of values produced by the Auriga simulations, whereas the mean of M31 is somewhat larger and slightly closer to typical mean values of the accreted GC candidates in the Auriga simulations. The larger radial extent of the M31 GCS is generally believed to hint at a more accretion-dominated origin of the GCS, and may reflect a richer accretion history of the galaxy in comparison to the Milky Way.

Figure 7 shows the mass-weighted radial distribution of the Auriga L4 haloes. We notice a subtle peak around 10 kpc for all star particles that is not present for the GC candidates, indicating

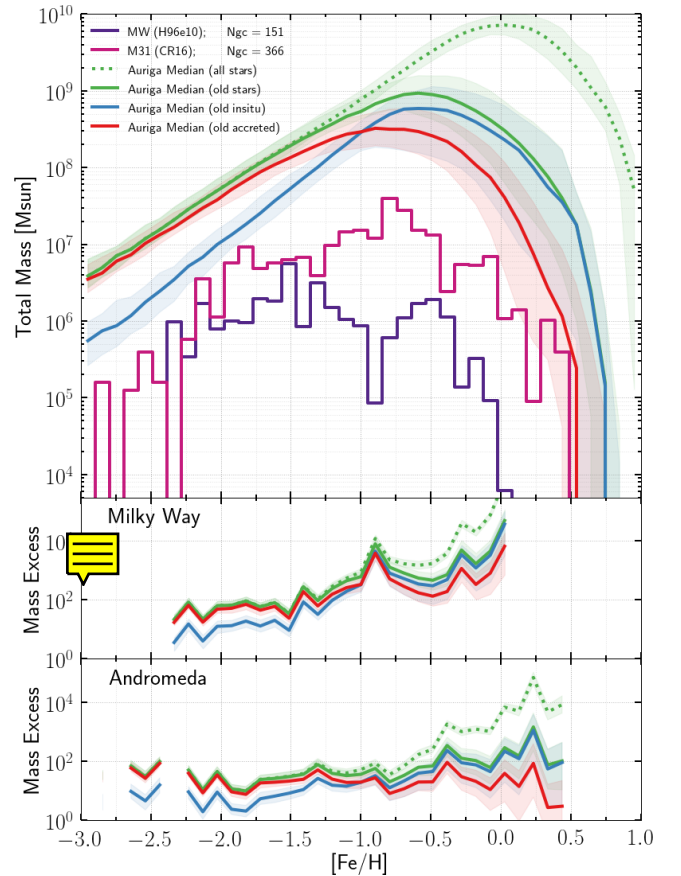


Figure 5. Mass-weighted metallicity distribution of star particles in the Auriga simulations. We show the median value of all Auriga haloes for all stars (green dotted) and globular cluster candidates (i.e. stars with age > 10 Gyr; green solid). The latter sub set is further split up between stars that formed *in situ* (blue solid), and those that were accreted (red solid). Shaded regions indicate the 1σ interval. The MW (M31) GCS is shown in purple (magenta). The middle (bottom) panel shows the ratio of the simulated mass to the mass in the MW (M31) GCS.

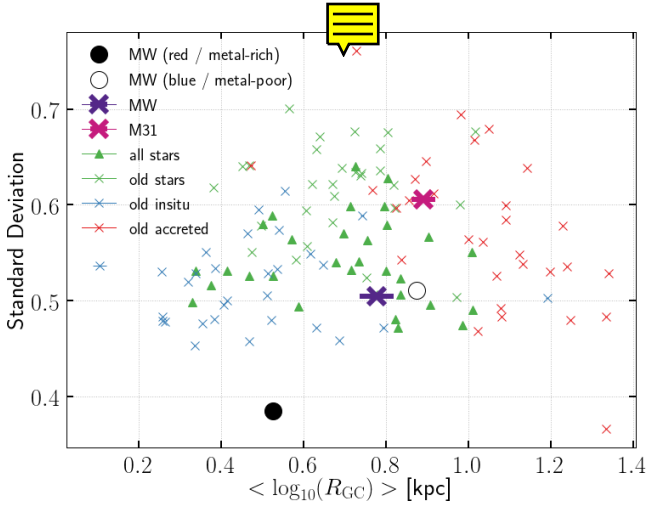


Figure 6. Mean and standard deviation of the logarithm of radial distribution of star particles in each of the thirty Auriga L4 haloes compared to the MW (M31) GCS shown in purple (magenta).

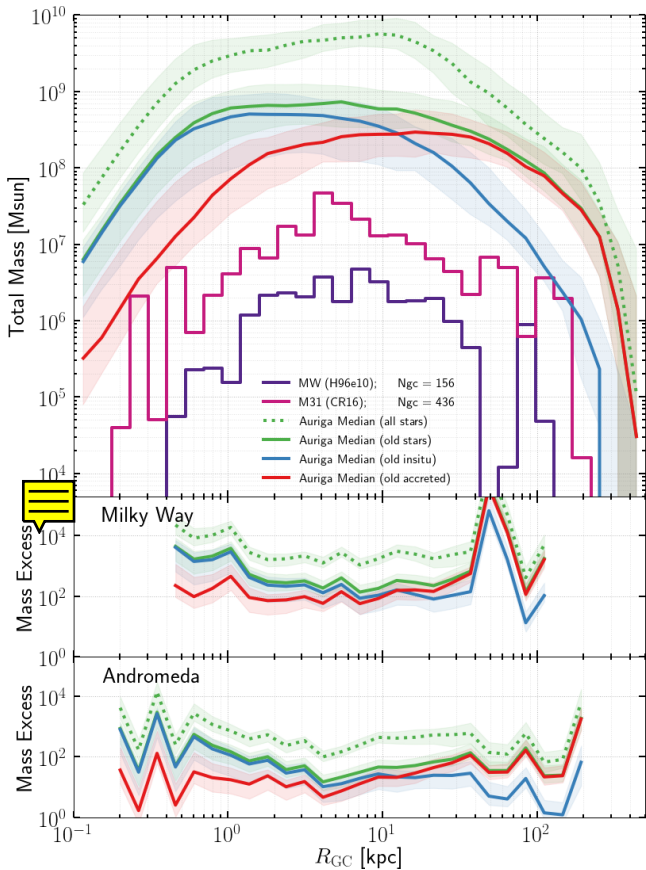


Figure 7. Mass-weighted radial distribution of star particles in the Auriga simulations. We show the median value of all Auriga haloes for all stars (green dotted) and globular cluster candidates (i.e. stars with age > 10 Gyr; green solid). The latter sub set is further split up between stars that formed *in situ* (blue solid), and those that were accreted (red solid). Shaded regions indicate the 1σ interval. The MW (M31) GCS is shown in purple (magenta). The middle (bottom) panel shows the ratio of the simulated mass to the mass in the MW (M31) GCS.

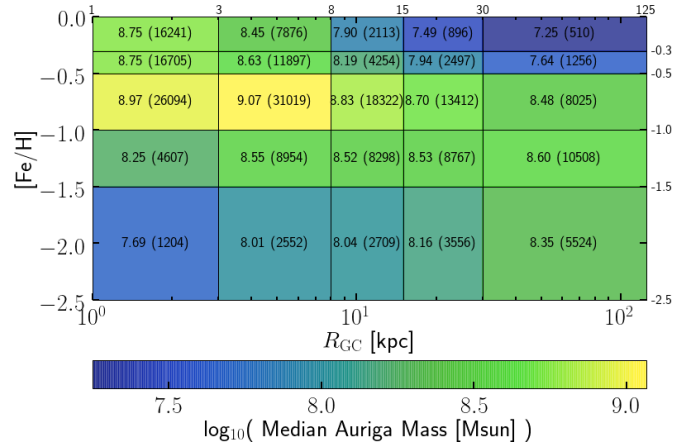


Figure 8. Mass-weighted $[\text{Fe}/\text{H}]$ - R_{GC} distribution of all thirty Auriga L4 haloes. Here we consider only the GC candidates (age > 10 Gyr) stars in and color-code by the **median** (values also shown in each bin). The numbers in parenthesis show how many star particles fall within the bin. Note that the range of the colourmap again differs for improved contrast within the plot.

that the stellar disc is no longer present when applying the latter selection criterion. Furthermore, we find that the dominant contribution to the total mass in GC candidates changes from those formed *in situ* to the accreted population around 10 kpc. Again we show the mass excess of the simulations compared to the Milky Way and Andromeda GCS. We find a decreasing mass excess with increasing radius in the range 0.2 to ~ 5 kpc, followed by an increase attributed to the accreted subpopulation. For the Milky Way we notice that significantly fewer GCs are found beyond 40 kpc than for M31, and that accreted GC candidates contribute mostly to the Auriga GCS at these radii. We further investigate a breakdown of the total mass in Auriga GC candidates into bins of both metallicity and radius in the following section.

4.3 Total mass in metallicity-radial space

We investigate whether the Auriga simulations still produce sufficient mass when the GC candidates are two-dimensionally binned in $[\text{Fe}/\text{H}]$ and R_{GC} . First we sum the total simulated mass in each bin (for an individual Auriga simulation), then we calculate the median over all thirty Auriga L4 haloes (in each bin, see Figure 8 which can be compared to Figure 2). Finally, we divide these value by the total mass in the MW (M31) GCS to obtain the median mass excess produced by the star formation model implemented in the Auriga simulations. See the top (bottom) panel of Figure 9 for mass excess with respect to the MW (M31) GCS.

We notice in Figure 8 that the Auriga simulations, on average, produce fewer GCs in the lowest metallicity bins (-2.5 to -1.5) than at higher metallicities in the three innermost radial bins. In addition, Auriga produces fewer high-metallicity GCs at large radii, similar to the MW and M31 GC systems. However, the three bins in the upper right corner are populated in the Auriga simulations, in contrast to the observations. The top panel of Figure 9 shows that the mass excess for Auriga GCs within the Solar radius decreases with decreasing metallicity. This means that either the formation efficiency should increase with decreasing metallicity, or that the lowest-metallicity GCs in the Auriga simulations are less strongly affected by disruptive processes, if the star formation histories in the

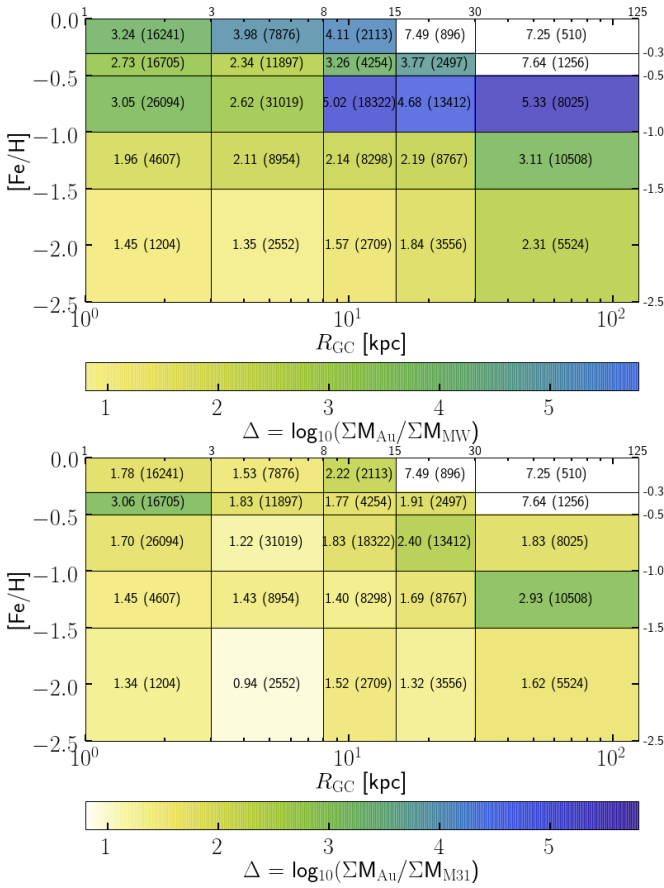


Figure 9. The top (bottom) panel shows the logarithm of the ratio of simulated mass to mass in the MW (M31) GCS, i.e. the logarithm of the mass excess. The color-coded values are also shown in each bin. Note that the three bins in the upper right corner are left blank because the observations have zero mass there, and that the numbers indicate how much (median) mass is produced by the simulations in those bins.

Auriga simulations are to give rise to a GC system that can be similar to the GCS that is observed in the Milky Way. Moreover, either a lower formation efficiency is required at large radii, or stronger disruption. Disruption of star clusters is expected to be more efficient at smaller radii due to higher tidal forces in the innermost region for GCs on orbits that pass through the disk, or with pericenters close to the bar or bulge. Compared to Andromeda, the mass excess (bottom panel of Figure 9) shows a less clear trend than for the Milky Way, which suggests that both galaxies have diverse GC formation histories. **Most bins show mass surplus factors of few tens, meaning that high formation efficiencies would be required in addition to little disruption for the Auriga simulations to yield a GC system consistent with that of the Andromeda galaxy.**

5 DISCUSSION

5.1 Metallicity distribution

Globular clusters are ~ 0.5 dex more metal-poor than spheroid stars observed at the same radius for almost all galaxies (Harris 1991). Our selection function (age cut) does lower the mean metallicity by 0.5 dex. We still find that the metallicity distributions of GC candidates in the Auriga simulations are more metal-rich than the

MW (M31) GCS. Although the (old) star particles represent single stellar populations with a mass (resolution) consistent with that of GCs, they are in fact statistical tracers of the stellar population of the galaxy as a whole. Therefore only a (small) fraction of the star particles may represent plausible formation sites of GCs, whereas the majority represents (halo) field stars - the disk component effectively falls outside our selection of star particles due to the age cut. It is implausible that all halo stars come from disrupted GCs because GC systems that survive down to redshift zero are generally few orders of magnitude less massive than galactic stellar haloes.

In general, the colour (metallicity) distribution of most GC systems is bimodal with typical separating value $[\text{Fe}/\text{H}] \sim -1$ (Zinn 1985; Gebhardt & Kissler-Patig 1999; Larsen et al. 2001; Peng et al. 2006). Indeed, Ashman & Zepf (1998, p. 234) and Harris (2001, p. 38) find that the MW GCS has a bimodality $[\text{Fe}/\text{H}]$ distribution: the latter fit a double Gaussian which peaks at $[\text{Fe}/\text{H}] = -1.59$ (metal-poor) and -0.51 (metal-rich) and dispersions of 0.34 and 0.23. Observations of GCs in M31, however, may be best split into three distinct metallicity groups, one found at small radii with $[\text{Fe}/\text{H}] > -0.4$, one intermediate $[\text{Fe}/\text{H}]$ group, and a dominant (metal-poor) group with $[\text{Fe}/\text{H}] < -1.5$ (Caldwell & Romanowsky 2016). The numerical simulation of Renaud et al. (2017) does yield a bimodal metallicity distribution where the metal-poor population is dominated by accreted star particles and the metal-rich population by *in situ* stars.

We find that none of the Auriga simulations produces a bimodal metallicity distribution for age-selected GC candidates. Interestingly, the cross-over point above (below) which the mass-weighted metallicity distribution of GC candidates is dominated by those that have formed *in situ* (were accreted) does coincide with the separation between the metal-rich and metal-poor populations of GC systems at $[\text{Fe}/\text{H}] = -1$. Moreover, we do find that the mean metallicity shifts when we split the GC candidates up according to birth location (accreted or *in situ*). In particular, the mean values of the old *in situ* GC candidates are roughly consistent with the metal-rich MW GCs. However, we also notice that a substantial number of simulations has similar mean values for all GC candidates, and the simulated *in situ* GC candidates have larger dispersions than the metal-rich GCs. Moreover, the offset between the mean of the metal-rich and metal-poor populations in the MW is 1 dex, a factor 2 – 3 larger than the offset that we find between the *in situ* and accreted populations. The Auriga simulations therefore do not support the view that the metal-poor GCs could have formed in satellites that have later been accreted whereas the metal-rich GC subpopulation has formed *in situ*. With this statement, however, we do caution that Kruijssen et al. (2019) suggests to reserve the ‘*ex situ*’ classification for accretion after $z = 2$, when the central galaxy has formed and accretion unambiguously contributes to the radially extended halo GC population. In fact, our classification appears flawed in Au4-1.

Finally, we note that Brodie & Strader (2006) compares the number of metal-poor GCs to the stellar halo mass and find⁷ $T_{\text{blue}}^n \sim 100$, while the number of metal-rich GCs compared to the bulge mass yields $T_{\text{red}}^n \sim 5$, and therefore conclude that the formation efficiency of metal-poor GCs is twenty times higher than the metal-rich GCs with respect to field stars. We find that gradual linear increase in formation efficiency of GCs with respect to field stars would be required with decreasing metallicity for the GC candidates in the Auriga simulations to yield sufficient total mass to be consistent with the Milky Way GCS.

⁷ The quantity T is the number of GCs per $10^9 M_{\odot}$ of galaxy stellar mass.

5.2 Radial distribution

The M31 GCS has a factor 2-3 more GCs at large radii compared to the MW GCS, which may indicate that M31 has a richer merger history than the MW (Caldwell & Romanowsky 2016). In the Auriga simulations we find that the GC candidates at radii larger than ~ 20 kpc are indeed dominated by accreted star particles. However, we select star particles that are bound to the main halo and main subhalo, which means that we include particles up to the virial radius R_{200} . The Auriga simulations have no problem populating the stellar halo up to the virial radius, even with our additional age cut. On the other hand, the MW and M31 have fewer GCs at large radii. We expect tidal disruption to be less efficient at larger radii, thus, the formation efficiencies of the accreted GC candidates would have to be lower. Our classification as GC candidates could, for example, be improved by selecting a number of globular clusters based on the $M_{\text{vir}} - N_{\text{GC}}$ correlation found by Burkert & Forbes (2019), and taking a virial mass below which none of the star particles is treated as GC candidate.

6 SUMMARY AND CONCLUSIONS

We investigate GC candidates in the Auriga simulations and draw the following conclusions.

- The star formation model implemented in the Auriga simulations produces metallicity distributions that are more metal-rich than the Milky Way and M31 globular cluster systems.
- We reject the null hypothesis H_0 for any one of the Auriga simulations that the GC candidates and the MW GCs are drawn from the same underlying distribution. This statement holds true when we select all old star particles (GC candidates), but also when we select the old accreted or the old *in situ* star particles. In addition, this is also the case when we test against the M31 GCS instead of the MW GCS. Moreover, the difference between the MW and M31 is big compared to the scatter in GC candidate properties in the Auriga simulations.
- GC candidates in the Auriga simulations may be found out to R_{200} , given that our selection function of star particles selects all stars bound to the main subhalo in the main halo. The stellar mass is dominated by accreted star particles at radii beyond 20 kpc. The GCs in the MW and M31, on the other hand, have a much smaller radial extent.
- The cluster formation efficiency would have to increase with decreasing metallicity for GC candidates in the Auriga simulations to be consistent with the Milky Way GC system, given that we expect dynamical evolution to more strongly affect GC candidates at smaller radii. This trend of over-production of old star particles that are metal-rich and at large radii compared to observed GCs is less clear for the M31 GCS.

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Check Auriga boilerplate that we need to acknowledge

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The analysis in this work was performed using the Python (van Rossum & de Boer 1991) programming language, the IPython (Perez & Granger 2007) environment, the NumPy (van der Walt et al. 2011), SciPy (Jones et al. 2001), and Astropy (Astropy Collaboration et al. 2013) packages. Plots were created with Matplotlib (Hunter 2007).

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