Internal structure of (globular) star clusters

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1 Timescales

- Crossing Time
- Relaxation Time
- Equipartition Time

1.1 Crossing Time

Copy-Pasta from Aarseth (2010)

"The crossing time is undoubtedly the most intuitive time-scale relating to self-gravitational systems. For a system in approximate dynamical equilibrium it is defined by

$$t_{\mathrm{cr}} = 2R_V/\sigma \quad , \tag{1}$$

where R_V is the virial radius, obtained from the potential energy by $R_V = GN^2\bar{m}^2/2|U|$, and σ is the rms velocity dispersion. In a state of approximate equilibrium, $\sigma^2 \approx GN\bar{m}/2R_V$, which gives

$$t_{\mathrm{cr}} \approx 2\sqrt{2}(R_V^3/GN\bar{m})^{1/2} \quad , \tag{2}$$

with $\bar{m}$ the mean mass, or alternatively $t_{\mathrm{cr}} = G(N\bar{m})^{5/2}/(2|E|)^{3/2}$ from $E = \frac{1}{2}U$. Unless the total energy is positive, any significant deviation frmo overall equilibrium causes a stellar system to adjust globally on this timescale which is also comparable to the free-fall time.

1.2 Relaxation Time

"The subject of relaxation time is fundamental and was mainly formulated by Rosseland [1928], Ambartsumian [1938, 1985], Spitzer [1940] and Chandrasekhar [1942]. The classical expression is given by

$$T_E = \frac{1}{16}\left(\frac{3\pi}{2}\right)^{1/2}\left(\frac{NR^3}{Gm}\right)^{1/2}\frac{1}{\ln 0.4N} \tag{3}$$

where R is the size of the homogeneous system [Chandrasekhar, 1942]. For the purposes of star cluster dynamics, the half-mass relaxation time is perhaps more useful since it is not sensitive to the density profile.

Following Spitzer [1987], it is defined by[1]

$$t_{\rm rh} = 0.138\left(\frac{Nr_h^3}{Gm}\right)^{1/2}\frac{1}{\ln(\gamma N)}, \tag{4}$$

where r_h is the half-mass radius and $\Lambda = \gamma N$ is the argument of the Coulomb logarithm. Formally this factor is obtainer by integrating over all impact parameters in two-body encounters, with a historical value of $\gamma = 0.4$."

1.3 Equipartition Time

Analysis of a two-component system dominated by light particles gave rise to the equipartition time for kinetic energy [Spitzer, 1969]

$$t_{\rm eq} = \frac{(\bar{v_1^2} + \bar{v_2^2})^{3/2}}{8(6\pi)^{1/2}G^2\rho_{01}m_2\ln N_1} \tag{5}$$

[1]Also see Spitzer & Hart [1971a] for an alternative derivation.

2 Radii

- Virial Radius
- Close Encounter

2.1 Virial Radius

$$R_V = GN^2\bar{m}^2/2|U| \tag{6}$$

2.2 Close Encounter

$$R_{\rm cl} = 2G\bar{m}/sigma^2 \tag{7}$$

which takes the simple form $R_{\rm cl} \approx 4R_V/N$ at equilibrium.

References

Aarseth S. J., 2010, Gravitational N-Body Simulations