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Problem 9
I, Tameez Latib, declare that this work is my own. I did this work honestly and can fully stand behind everything that I have written.
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From the previous malaria model, we had

$$H' = IB\gamma \frac{P-H}{P} - \rho H$$

$$I' = -\mu I + (M-I)B\frac{H}{P}$$

Now, given the fact that $\rho = \mu$ and $P = M$ (in values)
Let's non-dimensionalize this, by defining a few variables:
$H = h\bar{h}$
$I = m\bar{i}$
$t = \tau\bar{t}$
Then we have

$$\frac{\bar{h}}{\bar{t}}h' = m\bar{i}B\gamma\frac{P-h\bar{h}}{P} - \rho h\bar{h}$$

$$h' = m\bar{i}\bar{t}B\gamma\frac{P-h\bar{h}}{P\bar{h}} - \rho h\bar{t}$$

Setting $\bar{t} = \frac{1}{\rho}$, $\bar{h} = P$,

$$h' = m\bar{i}\bar{t}B\gamma\frac{1-h}{P} - h$$

And

$$\frac{\bar{i}}{\bar{t}}m' = -\mu m\bar{i} + (M-m\bar{i})B\frac{h\bar{h}}{P}$$

$$m' = -\mu m\bar{t} + (M-m\bar{i})B\frac{h\bar{h}\bar{t}}{P\bar{i}}$$

Since $\mu = \rho$, $\bar{t} = \frac{1}{\mu}$. Now set $\bar{i} = M$

$$m' = -m + (1-m)Bh\bar{t}$$

Now let $B\bar{t} = \epsilon$, as ϵ is small. Then

$$m' = -m + \epsilon(1-m)h$$

$$h' = m\bar{t}B\gamma\frac{(1-h)M}{P} - h$$

$M/P = 1$, so this simplifies to

$$h' = m\epsilon\gamma(1-h) - h$$

Now, lets make the ansantz that
$h = h_0 + \epsilon h_1 + ...$
$m = m_0 + \epsilon m_1 + ...$
As $\epsilon < 1$ and the terms on the order of ϵ^2 or higher are negligible.
We can now solve the differential equation subject to the constraints of
$h(0) = \zeta$
$m(0) = \eta$
Under these constraints, all order epsilon and higher terms are 0 at $\tau = 0$ and the first order terms are
$h_0(0) = \zeta$
$m_0(0) = \eta$
The ansantz creates the following differential equations:

$$h_0' = -h_0$$
$$m_0' = -m_0$$
$$h_1' = m_0\gamma(1-h_0) - h_1$$
$$m_1' = -m_1 + (1-m_0)h_0$$

The first two are simple,

$$h_0 = \zeta * e^{-\tau}$$
$$m_0 = \eta * e^{-\tau}$$

Then

$$h_1' + h_1 = m_0\gamma(1-h_0)$$
$$(h_1 e^{\tau})' = \eta\gamma(1 - \zeta * e^{-\tau})$$
$$(h_1 e^{\tau}) = \eta\gamma(\tau + \zeta * e^{-\tau}) + C$$
$$h_1 = \eta\gamma(\tau e^{-\tau} + \zeta * e^{-2\tau}) + Ce^{-\tau}$$
$$h_1(0) = \eta\gamma(\zeta) + C = 0$$

So finally:

$$h_1 = \eta\gamma(\tau e^{-\tau} + \zeta * e^{-2\tau} - \zeta * e^{-\tau})$$

Similarly

$$m_1' + m_1 = h_0(1-m_0)$$
$$(m_1 e^{\tau})' = \zeta(1 - \eta * e^{-\tau})$$
$$(m_1 e^{\tau}) = \zeta(\tau + \eta * e^{-\tau}) + C$$

$$m_1 = \zeta(\tau e^{-\tau} + \eta * e^{-2\tau} - \eta * e^{-\tau})$$

So up to order ϵ,

$$h = \zeta * e^{-\tau} + \epsilon\eta\gamma(\tau e^{-\tau} + \zeta * e^{-2\tau} - \zeta * e^{-\tau})$$

$$m = \eta * e^{-\tau} + \epsilon\zeta(\tau e^{-\tau} + \eta * e^{-2\tau} - \eta * e^{-\tau})$$