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1. $f(x)$ defined on $[-1,1]$ and $f \in C^4[-1,1]$

a. If $h(x)$ is the interpolating polynomial at the points -1, 0, 1, then let

$$L_{-1}(x) = \frac{(x-0)(x-1)}{(-1-0)(-1-1)} = \frac{x(x-1)}{2}$$

$$L_0(x) = \frac{(x+1)(x-1)}{(0+1)(0-1)} = 1 - x^2$$

$$L_1(x) = \frac{(x-0)(x+1)}{(1-0)(1+1)} = \frac{x(x+1)}{2}$$

$$h(x) = L_{-1}(x)f(-1) + L_0(x)f(0) + L_1(x)f(1)$$

b. The error term can be given by

$$E(x) = \frac{(x-1)(x)(x+1)}{6} f'''(\xi(x)), \xi(x) \in [-1,1]$$

By the interpolation formula

c.

$$\int_{-1}^{1} h(x)dx = \int_{-1}^{1} h(x)L_{-1}(x)f(-1) + L_0(x)f(0) + L_1(x)f(1)dx$$

$$= \left[f(-1)(\frac{x^3}{6} - \frac{x^2}{4}) + f(0)(x - \frac{x^3}{3}) + f(1)(\frac{x^2}{4} + \frac{x^3}{6}) \right] \Bigg|_{-1}^{1}$$

$$= \frac{1}{3}(f(-1) + 4f(0) + f(1))$$

d.

Yes, it is true that

$$\int_{-1}^{1} h(x)dx = \int_{-1}^{1} f(x)dx$$

If f is a polynomial of degree 2 or less. Note that in this case, f is the interpolating polynomial of $(-1, f(-1)), (0, f(0)), (1, f(1))$. Since f and h both interpolate these points, they must be the same polynomial since the Lagrange interpolating polynomial is unique. Therefore, their integrals must be the same.

e.

$$\int_{-1}^{1} E(x)dx = \int_{-1}^{1} \frac{(x-1)(x)(x+1)}{6} f'''(\xi(x))dx$$

However, note that if f is of degree 2 or less, $f'''(x) = 0$. Therefore

$$\int_{-1}^{1} E(x)dx = \int_{-1}^{1} \frac{(x-1)(x)(x+1)}{6} f'''(\xi(x))dx = 0$$

2. Given the following $x, f(x)$ pairs: (0,1), (1,2), (2,1), (3,2), (4,1)

a. Use Simpsons rule at nodes 0, 2, 4 to calculate

$$\int_{0}^{4} f(x)dx$$

$$= \frac{2}{3}(1 + 4 * 1 + 1) = 4$$

b. Use Composite Simpsons rule at nodes 0, 1, 2, 3, 4 to calculate

$$\int_{0}^{4} f(x)dx = \int_{0}^{2} f(x)dx + \int_{2}^{4} f(x)dx$$

$$= \frac{1}{3}(1 + 4 * 2 + 1 + 1 + 4 * 2 + 1) = \frac{20}{3}$$