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1. Let

$$f(x) = x^2 - 0.7x$$

a. Note
$f(-1) = 1.7$
$f(0.5) = -0.1$
$f(1) = 0.3$
Since $f$ is continuous, the intermediate value theorem implies there exists a zero on the interval (-1, 0.5) and there exists a zero on the interval (0.5, 1). Furthermore, by the fundamental theorem of algebra, $f$ only has 2 zeros. Therefore, there is exactly one zero on the interval (0.5, 1).

b. With the bisection method, the error of the nth point $p_n$, $E_n$ is given by

$$E_n = |p_n - p| \leq \frac{b-a}{2^n}$$

So for $a = 0.5, b = 1$, we have

$$E_n \leq \frac{1}{2^{n+1}} \leq 10^{-5}$$

Which gives us a minimum value of n of $n = 16$

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2. if $f(x)$ is continuous on $I = [a, b]$ and $f(x) \in \forall x \in I$, then there exists $c$ such that $f(c) = c \in I$

Proof: Consider the function $g(x) = f(x) - x$. $f$ has a fixed point at $c$ iff g has a root $c$.

Now consider the values at the endpoints,
$g(a) = f(a) - a \geq a - a = 0$
$g(b) = f(b) - b \leq b - b = 0$
We can assume $g(a) \neq 0 \neq g(b)$ otherwise take the endpoint(s) as the fixed point. Therefore $g(b) < 0 < g(a)$

So by the intermediate value theorem value theorem, $g$ has a zero in the interval $I$. Therefore $f$ has a fixed point in $I$

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3.a. Given $p_0 = 3$,
$p_{n+1} = \frac{p_n^2+3}{2p_n}$
Then we find
$p_1 = \frac{3^2+3}{2*3} = 2$
$p_2 = \frac{2^2+3}{2*2} = \frac{7}{4}$
b. Find all possible limits of $p_n$:
If $p$ is a limit of the sequence, then we must have

$$\lim_{n \to \inf} p_{n+1} = \lim_{n \to \inf} \frac{p_n^2 + 3}{2p_n}$$

$$p = \frac{p^2+3}{2p}$$

$$p^2 = 3$$

$$p = \pm\sqrt{3}$$

So the only limits of the sequence are $\sqrt{3}, -\sqrt{3}$

c. If we apply Newtons method to $f(x) = x^2 - 3$, the iterative step to produce the next point in the sequence is given by:

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$$p_{n+1} = p_n - \frac{p_n^2 - 3}{2p_n}$$

$$p_{n+1} = \frac{2p_n^2 - p_n^2 + 3}{2p_n}$$

$$p_{n+1} = \frac{p_n^2 + 3}{2p_n}$$

Which is the given sequence. Therefore the given sequence is precisely the sequence generated by Newton's method to find the zeros of $f(x)$

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4. Let

$$f(x) := x^2 - 3$$

We want to find the zeroes of $f$ on $I = [0, 4]$

a. With the secant method,

$$p_{n+2} = p_{n+1} - \frac{f(p_{n+1}) * (p_{n+1} - p_n)}{f(p_{n+1}) - f(p_n)}$$

So for starting points: $p_0 = 0.5, p_1 = 3$ we have that:

$$p_2 = p_1 - \frac{f(p_1) * (p_1 - p_0)}{f(p_1) - f(p_0)}$$

$$p_2 = 3 - \frac{6 * (3 - 0.5)}{6 - -2.75} = 1.286$$

$$p_3 = 1.286 - \frac{-1.346 * (1.286 - 3)}{-1.346 - 6} = 1.6$$

b. With the method of false position,

We compute

$$c = b - \frac{f(b) * (b-a)}{f(b) - f(a)}$$

Then update $a = c$ if $f(c)f(b) \leq 0$ else $b = c$
so with $a = p_0 = 0.5, b = p_1 = 3$

$$c = b - \frac{f(b) * (b-a)}{f(b) - f(a)} = 1.286 = p_2$$

$f(c) * f(b) \leq 0$ So this means we update $a = c$, and find our new c:

$$c = 3 - \frac{6 * (3 - 1.286)}{6 - -1.346} = 1.6 = p_3$$

This gives us the same $p_2, p_3$ as a.
5. Refer to PDF of code + outputs