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Problem 1

I, Tameez Latib, declare that this work is my own. I did this work honestly and can fully stand behind everything that I have written.

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We will try to find the relative difference between two powers, P_C - the highest input power at which the earth will freeze over, and P_H - the lowest input power at which the earth will be completely covered in water.

To do this, we must first make a few assumptions:

-The earth consists only of ice or water.

-At temperature $T = 0^o C$ (or below), the earth is completely covered in ice.

-At temperature $T = 20^o C$ (or above), the earth is completely filled in water.

-Ice reflects 70% of all radiation it is exposed to.

-Water reflects 10% of all radiation it is exposed to.

-Due to a greenhouse effect, 50% of all radiation reflected is re-absorbed.

-Let P denote the total power coming into the earth.

-The earth's radius is $40,000 km$

Now, let's define $P_a(T)$ - the Power Absorbed by the earth (as a function of temperature, since temperature affects the composition of earth). We know P is the total power coming, and that- depending on the composition of the earth- a certain part is reflected outward, let's call this $P_r(T)$. Furthermore, 50% of the amount reflected is re-absorbed.

So $P_a(T) = P - P_r(T) + .5 * P_r(T) = P - .5 * P_r(T)$

We know $P_r(T) = .7 * P$ for $T \leq 0$, as the earth will be entirely composed of ice at these temperatures. And for $T \geq 20$, $P_r(T) = .1 * P$, as the earth will be entirely composed of water at these temperatures.

Let's assume that $P_r(T)$ in between these regions is linear. This makes physical sense as the earth will be a mix of ice and water. Furthermore, $P_r(T)$ should be decreasing in this region. So for $0 < T < 20$, let

$P_r(T) = \frac{.7*P*(20-T)}{20} + \frac{.1*P*T}{20} = .7 * P - \frac{.6*T}{20}$

Now, by the Stefan-Boltzmann Law, the earth emits $P_e(T) = \sigma * \epsilon * (T + 273)^4 * 4 * \pi * r^2$

Note that we multiple the amount by $4\pi r^2$ as that is the surface area of the earth. Furthermore, T is in Kelvin in this equation, so we convert from Celsius to Kelvin. σ is simply the Stefan-Boltzmann constant. The emissivity of water and ice are .98 and .96 respectively, so for simplicity sake, let's take $\epsilon = .97$. Temperature is directly related to the energy of a system, and power is the rate of change of energy, so we can let $T'(T) = P_a(T) - P_e(T)$

We know that for the input power P_c, we should have earth approach a state of pure ice (from any starting T in the range $0 < T < 20$)

Hence, $T'(T) \leq 0$ for all $0 < T < 20$

In physical terms, no matter what starting Temperature (in the given range), for an input power P_c, the earth should get colder (T decreases, i.e. $T' < 0$) at least until $T = 0$. We will assume (but not prove) that if $T'(T) < 0$ at $T = 20$ for a input power P_c, then $T'(T) < 0$ for all $0 < T < 20$

To solve for P_c, we let $T'(20) = 0$ (choose the greatest possible value as this maximizes P_a, similar to why one would choose a worst case scenario. If it works in the worst case, it should work in all better cases), and find that

$P_c = 8.5 * 10^{18} W$

The graphs (attached) confirm that $T'(T) < 0$ for all $0 < T < 20$. Furthermore, we know this must be the greatest possible value (to 2 significant figures) of P_c, as for any greater value, $T'(T)$ would be positive at values around $T = 20$.

P_h can be found in a similar way, except that we set $T'(T) > 0$ for all $0 < T < 20$. The same reasoning applies as before, except that now the input power P_h is the smallest such number. Solving $T'(0) = 0$, we find that

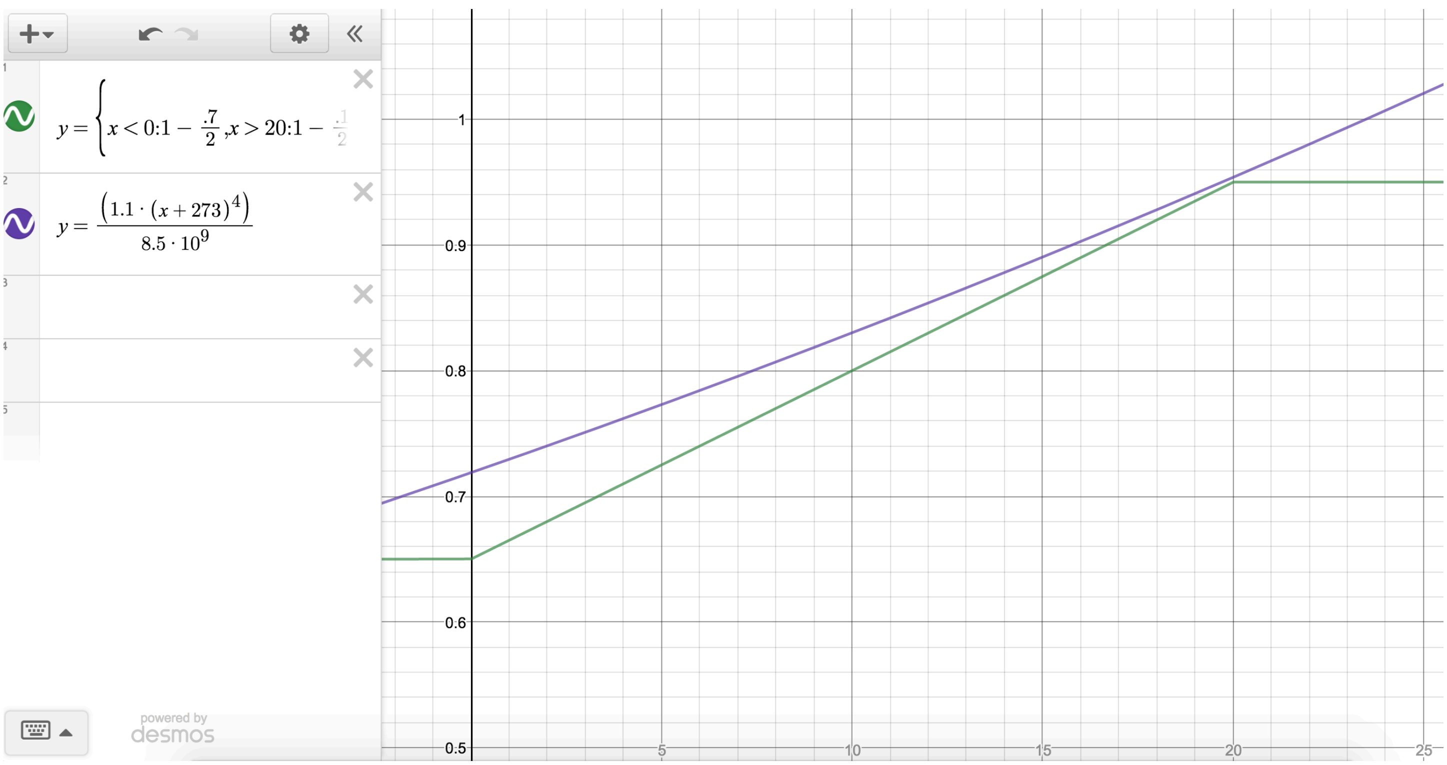
$P_h = 9.4 * 10^{18} W$

The relative difference is therefore $\frac{P_h - P_c}{P_{average}} = 0.1$

Note about graphs: The green is P_a, and the blue is P_e However, I divided both sides by P, the input power (to make it easier to view).

The first page is with $P = P_c$. As seen in the graph, the blue line is always above the green line (for $0 < T < 20$). This indicates that $P_a - P_e = T'(T) < 0$, as it emits more than it absorbs. Therefore, the Temperature will decrease to the point at which $T'(T) = 0$ which occurs after $T < 0$

The second page is with $P = P_h$. As seen in the graph, the green line is always above the blue line (for $0 < T < 20$). This indicates that $P_a - P_e = T'(T) > 0$, as it absorbs more than it emits. Therefore, the Temperature will increase to the point at which $T'(T) = 0$ which occurs after $T > 20$



$y = \{x < 0 : 1 - \frac{.7}{2}, x > 20 : 1 - \frac{.1}{2}$
$y = \frac{(1.1 \cdot (x + 273)^4)}{8.5 \cdot 10^9}$
1
0.9
0.8
0.7
0.6
0.5
5
10
15
20
25
powered by desmos

$y = \begin{cases} x < 0 : 1 - \frac{.7}{2}, x > 20 : 1 - \frac{.1}{2} \end{cases}$
$y = \frac{(1.1 \cdot (x + 273)^4)}{9.4 \cdot 10^9}$
powered by desmos
1
0.9
0.8
0.7
0.6
0.5
5
10
15
20
25