Thoughts:
This one will be short. An alternating series is defined as follows:

Definition 0.1 (Alternating Series)**.**

$$\sum_{n=1}^{\infty}(-1)^n a_n \text{ or } \sum_{n=1}^{\infty}\cos(\pi n)a_n \text{ or any series alternating positive and negative terms.}$$

The test is almost as simple.

Theorem 0.1 (Alternating Series Test). *If*

$$(1)\sum_{n=1}^{\infty}(-1)^n a_n \;\; or \sum_{n=1}^{\infty}\cos(\pi n)a_n,$$

(2) $a_{n+1} \leq a_n$ for all n, (3) $\lim_{n\to\infty} a_n = 0$, and (4) $a_n > 0$ for all n, then the series is convergent.

The last 3 conditions are pretty normal in that we have a decreasing aspect and a limit of 0 aspect all positives. Thus, this is a pretty simple test to use and apply. Seek an alternating piece and apply. The big thing is how strong this is. This completely explains conditional versus absolute convergence. Here is the crown jewel of examples.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

This is called the **Alternating Harmonic Series**. It is convergent by alternating series test, but the absolute value is the harmonic series which we hopefully know very well by now and know it is divergent. Thus, the AHS (not American Horror Story) is conditionally convergent. This test is easy to apply since it is only useful for alternating series which are pretty easy to identify themselves.

Problems: Determine if the series is convergent or divergent. (For added difficulty, determine if conditionally or absolutely convergent if you find the series is convergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \arctan(n)$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{2^n}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

6. $\displaystyle\sum_{n=1}^{\infty} (-1)^n(\sqrt{n+1} - \sqrt{n})$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+p}$ (Determine for what values of p this is convergent if any)

8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln^p(n)}{n}$ (Determine for what values of p this is convergent if any)