Math 115 Final Review

1 Algebraic Expressions

1. Simplify each expression. Use absolute values if necessary.

   (a) $\sqrt{(-3)^2}$

   3

   (b) $\sqrt{(-x)^2}$

   x

   (c) $(a^4)^{\frac{1}{2}}$

   a^2

   (d) $\left(\frac{d^6}{25}\right)^{-2}$

   $\frac{25^2}{d^{12}}$

(e) $\dfrac{(12-2(-3+5))^3}{5^2-7(5-2)}+7$

$= \dfrac{(12-2(2))^3}{25-7(3)} + 7$

$= \dfrac{8^3}{4} + 7$

$= 2 \cdot 8^2 + 7$

$= 128 + 7 = \boxed{135}$

2. Find each product, combine any like term.

(a) $(2x-2)(5x+7)$

$10x^2 + 4x - 14$

(b) $(x^2+2x+5)(2x-1)$

$2x^3 - x^2 + 4x^2 - 2x + 10x - 5$

$2x^3 + 3x^2 + 8x - 5$

(c) $(2x^2 - 3x + 4)^2$

$= 4x^4 - 6x^3 + 8x^2 - 6x^3 + 9x^2 - 12x + 8x^2 - 12x + 16$

$= 4x^4 - 12x^3 + 25x^2 - 24x + 16$

3. **Factor each polynomial.**

(a) $a^2 - 8a + 7$

$(a-7)(a-1)$

(b) $4t^2 + 5t - 9$

[illegible]

$4t^2 - 4t + 9t - 9$

$4t(t-1) + 9(t-1)$

$(4t+9)(t-1)$

(c) $27w^4 - 8w$

$w(27w^3-8)$

$w(3w-2)(9w^2+6w+4)$

4. Simplify each expression use absolute values if necessary.

(a) $\sqrt{x^2 - 2x + 1}$

$x-1$

(b) $\dfrac{x^2 + x - 6}{x^2 + 2x + 1} \div \dfrac{x^2 - 4}{x^2 + 3x + 2}$

$\dfrac{(x+3)\cancel{(x-2)}}{(x+1)^2} \cdot \dfrac{\cancel{(x+2)}\cancel{(x+1)}}{\cancel{(x-2)}\cancel{(x+2)}}$

$\dfrac{(x+3)}{(x+1)}$

2 Algebraic Equations and Graphing Basics

1. Solve $|2x - 5| = 6$ for x.

$2x-5=6$

$2x=11$

$x=11/2$

$2x-5=-6$

$2x=-1$

$x=-1/2$

2. Solve $K = 5/9(F - 32) + 273$ for F

$\frac{9(k-273)}{5}+32=F$

3. How many gallons of a 60% antifreeze solution must be mixed with 60 gallons of 20% antifreeze to get a mixture that is 50% antifreeze?

$x(.6)+60(.2)=(x+60).5$

$.6x \quad 12 \quad =.5x+30$

$.1x=18$

$x=180$

4. Find the equation of the line in point-slope form and slope-intercept form that passes through the points $(-5, -2)$ and $(5, 12)$.

$\frac{12+2}{5+5} = \frac{14}{10} = \frac{7}{5}$

$y-12 = \frac{7}{5}(x-5)$

$y-12 = \frac{7}{5}x - 7$

$y = \frac{7}{5}x + 5$

5. Find the equation of the line in slope-intercept form that passes through $(7, -3)$ and perpendicular to the line $y = \frac{1}{2}x + 3$.

$m = -2$

$y+3 = -2(x-7)$

$y+3 = -2x+14$

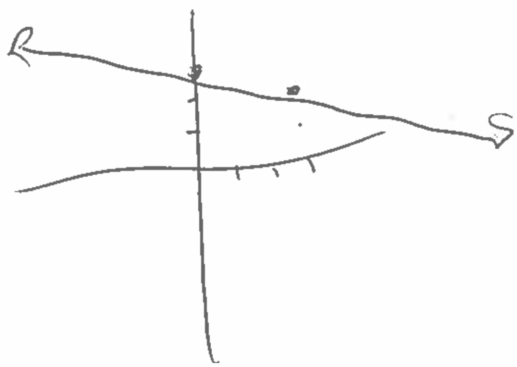
$y = -2x + 11$

6. Find the equation of the line in slope-intercept form that passes through $(1, 2)$ and parallel to the line $y = 2x - 5$.

$y-2 = 2(x-1)$

$y = 2x$

7. Graph the line $y - 2 = \frac{1}{3}(x+3)$. $y = \frac{1}{3}x + 3$



8. Solve the following quadratic equations by factoring if posibble. If not use the quadratic formula to find all real or imaginary solutions.

(a) $x^2 - 7x = 30$

$x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$

$x = 10$ or $x = -3$

(b) $2x^2 - x + 5 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(2)(5)}}{4}$

$x = \frac{1 \pm \sqrt{-39}}{4}$

$= \frac{1 \pm i\sqrt{39}}{4}$

3 Functions

1. Is $f = \{(2,-1),(3,4),(1,0),(2,5)\}$ a function?

   no

2. Is $f = \{(1,2),(2,3),(3,3),(4,2)\}$ a function?

   ✓

3. What test can be used to tell if the graph of a relation is the graph of a function?

   Vertical line test

4. Determine whether the following equations defines y as a function of x.

   (a) $y = -10x + 2$

   ✓

   (b) $x = y^6$

   no $(1,1)$ $(1,-1)$

   (c) $x = y^{\frac{1}{4}}$ $y = x^4$

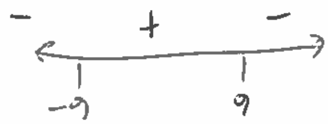
   ~~no~~

   yes

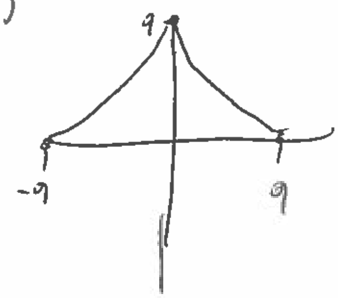
5. Let $f(x) = \sqrt{81 - x^2}$. Sketch the graph and state the domain and range. Identify any intervals on which $f(x)$ is increasing, decreasing, or constant.

D: $81 - x^2 \geq 0$ $\boxed{[-9, 9]}$

$(9-x)(9+x) \geq 0$



R: $[0, \infty)$



6. Let

not a function $$f(x) = \begin{cases} \sqrt{x+6} & \text{for } -6 \leq x \leq 2 \\ x & \text{for } x \geq 2 \end{cases}.$$

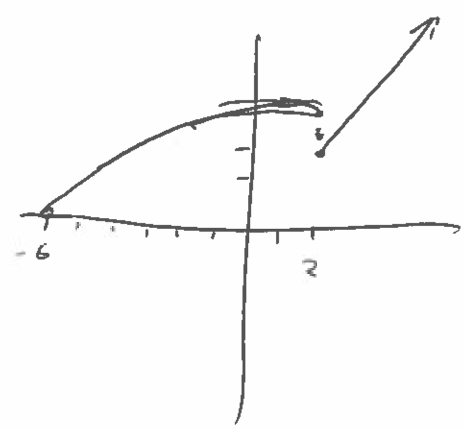
Graph the function and determine the domain and range.

$-6 \leq x \leq 2$

D: $x + 6 \geq 0$ $x \geq -6$

$\boxed{x \geq -6}$

R: $[0, \infty)$



7. For each of the following find and simplify the difference quotient.

(a) $f(x) = 3x^2 - 8x + 7$

$$\frac{3(x+h)^2 - 8(x+h) + 7 - (3x^2 - 8x + 7)}{h}$$

$$= \frac{6xh + h^2 - 8h}{h} = \boxed{6x + h - 8}$$

(b) $f(x) = \sqrt{x+2}$

$$\frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$$

$$\frac{x+h+2-(x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$\boxed{\frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}}$$

(c) $f(x) = \dfrac{1}{x+1}$

$$\frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \frac{\left(\frac{x+1-(x+h+1)}{(x+1)(x+h+1)}\right)}{h}$$

$$= \frac{-h}{h(x+1)(x+h+1)} = \frac{-1}{(x+1)(x+h+1)}$$

8. For $f(x) = 4x + 3$ and $g(x) = \sqrt{x+1}$ find $f \circ g(x)$ and $g \circ f(x)$.

$f \circ g(x) = 4\sqrt{x+1} + 3$

$g \circ f(x) = \sqrt{4x+4} = 2\sqrt{x+1}$

9. Let $f(x) = |x|$, $g(x) = x - 3$, and $h(x) = \sqrt{x}$ Write $N(x) = \sqrt{|x| - 3}$ as a composition of f, g, and h.

$N(x) = h \circ g \circ f(x)$

10. What test, given the graph of a function, can be used to test if that function has an inverse function?

horizontal line test

11. For each function determine if the function is one-to-one.

(a) $f = \{(1,2), (2,3), (3,2), (4,5)\}$

no

(b) $f = \{(1,2), (2,5), (3,11), (4,17)\}$

yes

(c) $f(x) = x^2$

no

(d) $f(x) = x^5$

yes

12. Find the inverse function of each of the following functions

(a) $f = \{(1,2), (2,3), (3,5), (4,7)\}$

$f^{-1} = \{(2,1), (3,2), (5,3), (7,4)\}$

(b) $f(x) = x^3 + 5$.

$x = y^3 + 5$

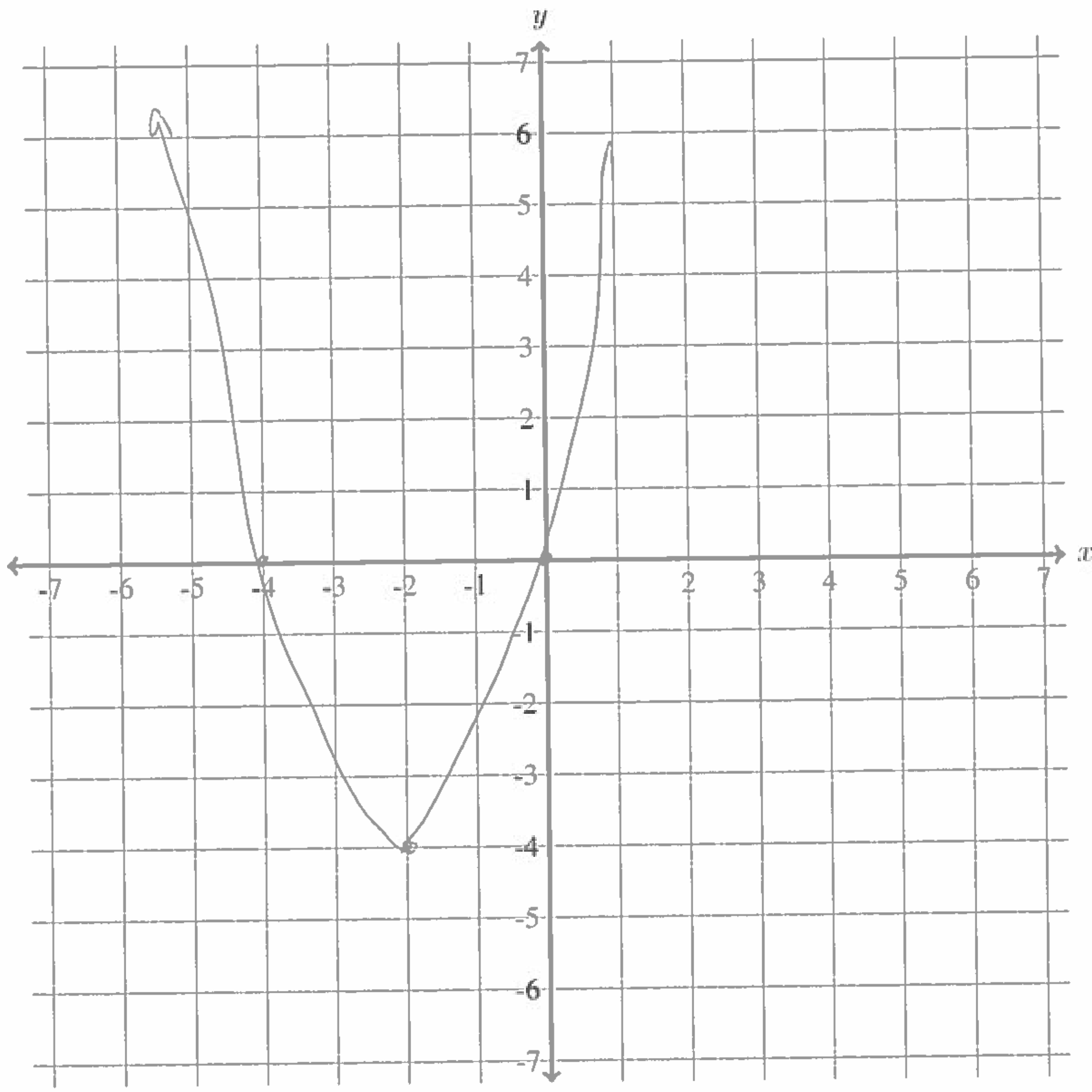
$\sqrt[3]{x-5} = y = f^{-1}(x)$

4 Polynomials

1. Write the quadratic function, $y = x^2 + 4x$, in vertex form($y = a(x-h)^2 + k$) and sketch its graph.(Hint complete the square!)

   Vertex form:

   $y = (x+2)^2 - 4$



2. Let $P(x) = x^4 - 2x^3 - 2x^2 + 2x + 1$.

(a) The possible rational roots of $P(x)$ are: ± 1

(b) Find all roots of $P(x)$.

$\cancel{\pm 1}$

1 | 1 -2 -2 2 1

1 -1 -3 -1

1 -1 -3 -1 0

~~1 | 1 0 -3~~

~~1 0 -3 -4~~

-1 | 1 -1 -3 -1

-1 2 1

1 -2 -1 0

~~-1 | -1 3~~

~~1 -3 2~~

$(x+1)(x-1)(x^2-2x-1)$

$\frac{2 \pm \sqrt{4-4(1)(-1)}}{2}$

$\frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$\cancel{x} = \pm 1,\ 1 \pm \sqrt{2}$

3. How many roots(real or complex) does $x^3 + 29x^2 + 100x + 7$ have?

3

4. How many roots does a degree n polynomial have?

n

5. Let $P(x)$ be a polynomial with real coefficients and with $2 - 3i$ as a root. What is one other root of $P(x)$.

$2+3i$

6. Find a polynomial in general form with real coefficients that has 4 and $5i$ as roots.

$(x-4)(x-5i)(x+5i)$

$(x-4)(x^2+25)$

$x^3-4x^2+25x-100$

7. Use Decrates' Rule of Signs to find the possibilities for the roots of

$$x^7 + 10x^6 - 100x^5 - 50x^4 + 35x^2 + 40x - 5$$

8. Find all real and imaginary solutions to $x^4 + 6x^2 - 40$.(Simplify your answer, but give an exact answer using radicals as needed. Express complex numbers in terms of i.)

5 Exponential and Logarithmic Functions

1. Solve the following equations for x.

   (a) $10^x = 0.0001$

   $\frac{1}{10,000} = 10^{-4}$

   $x = -4$

   (b) $5^x = 125$

   $x = 3$

   (c) $\log_2(x) = 4$

   $x = 16$

(d) $\log_3(81) = x$

$x = 4$

(e) $\log_x(\frac{1}{27}) = 3$

$x^3 = \frac{1}{27}$

$x = \frac{1}{3}$

2. Find the inverse function for each of the following functions.

(a) $f(x) = e^{x+2} - 5$

$x = e^{y+2} - 5$

$x + 5 = e^{y+2}$

$\ln(x+5) = y + 2$

$\ln(x+5) - 2 = y = f^{-1}(x)$

(b) $f(x) = \log_6(3x - 10) + 3$

$x = \log_6(3y-10) + 3$

$6^{x-3} = 3y - 10$

$\frac{6^{x-3}+10}{3} = y = f^{-1}(x)$

3. For each of the following logarithmic expressions use logarithm laws to rewrite each as a single logarithm.

   (a) $2\ln(x) + \frac{1}{2}\ln(y) - 5\ln(z)$

   $\ln(x^2) + \ln(\sqrt{y}) - \ln(z^5)$

   $\ln\left(\frac{x^2\sqrt{y}}{z^5}\right)$

   (b) $5\log_5(x) - \log_5(y) - \frac{1}{3}\log_5(y) + 7\log_5(z)$

   $\log_5\left(\frac{x^5 z^7}{y^{4/3}}\right)$

4. For each of the following rewrite each logarithmic expression as a sum and/or difference of simple logarithms. Simplify any simple logarithms if possible.

   (a) $\ln\left(\frac{x^5\sqrt[3]{y}}{z^2}\right)$

   $5\ln(x) + \frac{1}{3}\ln(y) - 2\ln(z)$

(b) $\log_3\left(\frac{\sqrt{3}(x+y)^5}{z^{\frac{3}{2}}}\right)$

$\frac{1}{2} + 5\log_3(x+y) - \frac{3}{2}\log_3(z)$

5. Solve the following equations for x

(a) $e^{2x-3} = 1$

$2x-3=0$

$x=\frac{3}{2}$

(b) $\frac{1}{27}\cdot 9^{x^2} = 3^{-1}$

$3^{-3}\cdot 3^{2x^2} = 3^{-1}$

$3^{-3+2x^2} = 3^{-1}$

$-3+2x^2=-1$

$2x^2=2$

$x^2=1$

$x=\pm 1$

(c) $5^{x+2} = 7$

$x+2 = \log_5(7)$

$x = \log_5(7) - 2$

(d) $\ln(x-1) = \ln(x+1) + 2$

$\ln\left(\frac{x-1}{x+1}\right) = 2$

$\frac{x-1}{x+1} = e^2$

$x - 1 = e^2 x + e^2$

$x - e^2 x = e^2 + 1$

$x = \frac{e^2+1}{1-e^2}$

(e) $\log_3(x-2) = 1 - \log_3(x+2)$

$\log_3((x-2)(x+2)) = 1$

$x^2 - 4 = 3$

$x^2 = 7$

$x = \pm\sqrt{7}$

$x \neq -\sqrt{7}$ because $\log_3(\cancel{x+2})$

$x - 2 < 0$

but $\log_3(x-2)$ is not possible when $x-2<0$.

6 Trigonometric Functions

1. Determine if the given angles, α and β, are coterminal.

   (a) $\alpha = 1000°, \beta = -440°$

   640 -80

   280 280 ✓

   (b) $\alpha = 117\pi/7, \beta = 5\pi/7$

   14 $\frac{5\pi}{7}$ ✓

   8

   112

2. Find the **exact** value of each: (by exact I mean if you give me a decimal because you found it using a calculator you will recieve no credit)

   (a) $\sin(-\pi/6) = -1/2$

   (b) $\cos(4\pi/3) = -1/2$

   (c) $\tan(1001\pi/4) = 1$

   $\pi/4$

   (d) $\sec(17\pi/3) = \frac{1}{\cos(5\pi/3)} = 2$

   $5\pi/3$

   (e) $\csc(-300°) = -\frac{1}{\sin(300°)} = +\frac{1}{\sqrt{3}/2} = \frac{2\sqrt{3}}{3}$

   (f) $\cot(-1290°) = \frac{1}{-\tan(1290°)} = \frac{-1}{\tan(210)} = \frac{-\cos(210)}{\sin(210)} = \frac{-\sqrt{3}}{\sqrt{3}}$ $-\sqrt{3}$

3. Find the exact value of the other five trigonometric functions, given that $\cos(\alpha) = \frac{8}{17}$ and α is in quadrant I.

(a) $\sin(\alpha) = \frac{15}{17}$

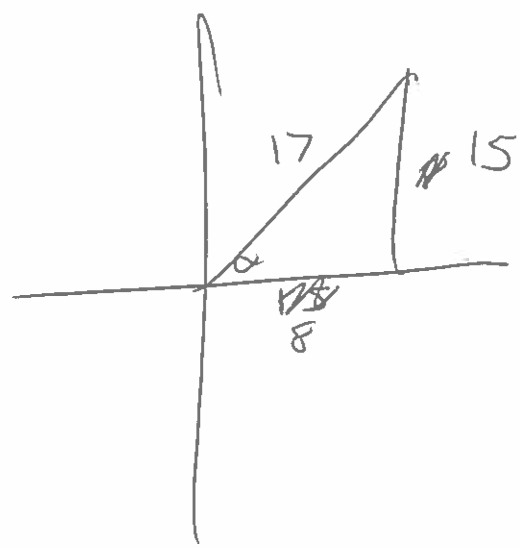
$$\left(\frac{8}{17}\right)^2 + \sin^2\alpha = 1$$
$$\sin^2\alpha = \left(\frac{15}{17}\right)^2$$
$$\sin\alpha = \frac{15}{17}$$

(b) $\tan(\alpha) = \frac{15}{8}$

(c) $\sec(\alpha) = \frac{17}{8}$

(d) $\csc(\alpha) = \frac{17}{15}$

(e) $\cot(\alpha) = \frac{8}{15}$

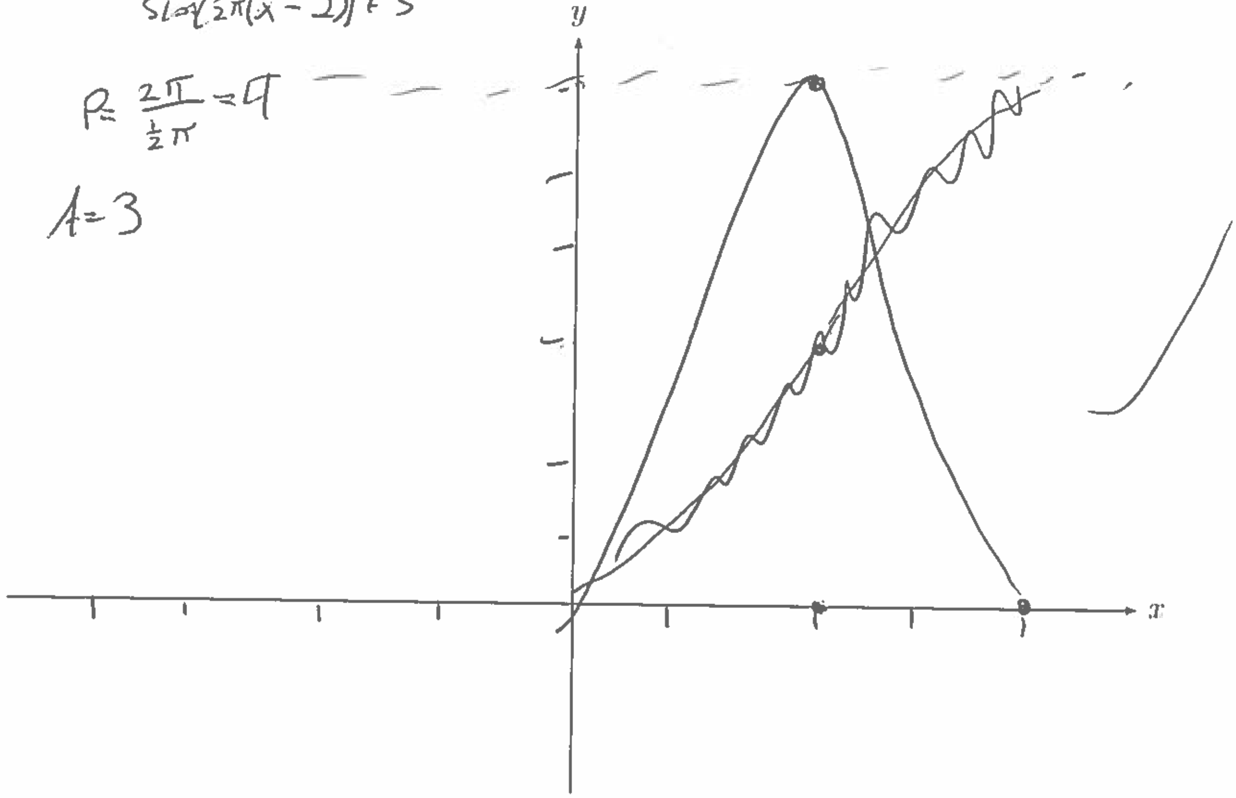


4. Graph $y = 3\cos(\frac{1}{2}\pi x - \pi) + 3$.

$3\cos(\frac{1}{2}\pi(x-2))+3$

$P = \frac{2\pi}{\frac{1}{2}\pi} = 4$

$A = 3$



5. Find the exact value of each in radians, if any value is undefined write "undefined":

(a) $\arcsin(-1) = -\frac{\pi}{2}$

$\sin(y) = -1$

(b) ~~$\sec^{-1}(\sqrt{3}) =$~~ no

(c) $\tan^{-1}(-1) = -\pi/4$

(d) $\cos^{-1}(\cos(\frac{7\pi}{4})) = \pi/4$

$\cos^{-1}(\sqrt{3}/2)$

(e) $\sin(\sin^{-1}(\frac{\sqrt{3}}{2})) =$ $\sqrt{3}/2$

(f) $\tan(\arcsin(-\frac{1}{2})) =$ $-\frac{\sqrt{3}}{3}$

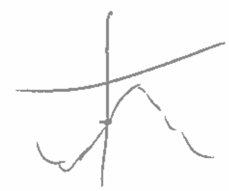
$\tan(-\pi/6)$

(g) $\csc(\tan^{-1}(0)) =$ undefined

0

6. Find the inverse of the function and state its domain.

$$f(x) = \frac{1}{2}\cos(3x) - 1, \quad \text{for } 0 \le x \le \frac{\pi}{3}$$



$x = \frac{1}{2}\cos(3y) - 1$

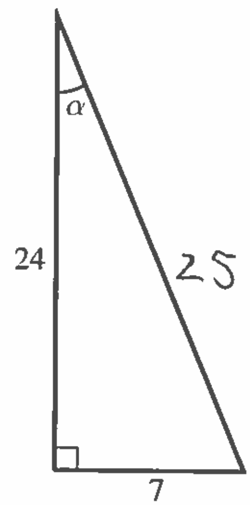
$\frac{\cos^{-1}(2(x+1))}{3} = y = f^{-1}(x)$

~~D: E~~

(a) $f^{-1}(x) =$

(b) Domain of $f^{-1}(x)$:

7. For the given triangle find the inticated trigononetric function values



(a) $\sin(\alpha) = \frac{7}{25}$

(b) $\cos(\alpha) = \frac{24}{25}$

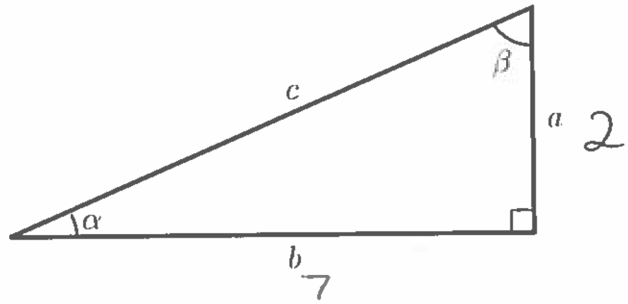
(c) $\tan(\alpha) = \frac{7}{24}$

(d) $\sec(\alpha) = \cancel{\frac{25}{7}}\ \frac{25}{24}$

(e) $\csc(\alpha) = \frac{25}{7}$

(f) $\cot(\alpha) = \frac{24}{7}$

8. Solve the right triangle shown, where $a = 2$ and $b = 7$.



(a) $c = \sqrt{53}$

(b) $\alpha =$ α ~~$\sin^{-1}$~~ $\tan^{-1}\left(\frac{2}{7}\right)$

(c) $\beta = \tan^{-1}\left(\frac{7}{2}\right)$

7 Trigonometric Identities

1. For each of the following express as sines and cosines then use any identities to simplify.

   (a) $\sin^4 x - \cos^4 x$

$(\sin x + \cos x)(\sin x - \cos x)$

   (b) $(1 + \sin x)(1 - \csc x)$

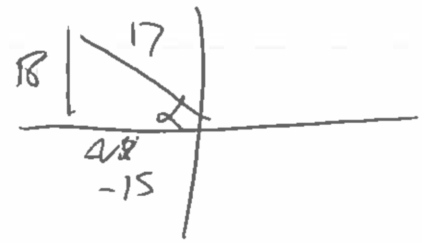
$1 + \sin x - \csc x + 1$

$\cdot \sin x - \frac{1}{\sin x}$

$\frac{\sin^2 x - 1}{\sin x} = -\frac{\cos^2 x}{\sin x}$

2. For the following, use identities to find the exact values for the remaining five trigonometric functions.

$$\tan\alpha = -\frac{8}{15}, \quad \frac{\pi}{2} < \alpha < \pi.$$



(a) $\sin\alpha = \frac{\cancel{1}8}{17}$

(b) $\cos\alpha = \frac{-15}{17}$

(c) $\tan\alpha =$ $\cancel{8/15}$ e

(d) $\sec\alpha = \frac{-17}{15}$

(e) $\csc\alpha = \frac{17}{\cancel{1}8}$

3. Determine if $f(x) = x - \sin x$ is symmetric to the y-axis, the origin, or $f(x)$ has no symmetry.

$$f(-x) = -x - \sin(-x)$$
$$= -x + \sin x$$
$$= -(x - \sin x)$$
$$= -f(x)$$

4. Verify the following identities:

(a) $\ln|\csc x - \cot x| = -\ln|\csc x + \cot x|$

$\csc x - \cot x = \dfrac{1}{\csc x + \cot x}$

RHS: $\dfrac{\csc x - \cot x}{1}$ ✓ by Pythagorean id.

(b) $\dfrac{1 - \tan^2 w + \sin^2 w \tan^2 w}{\sec^2 w} = \cos^4 w$

LHS $\dfrac{1 + \tan^2 w(-1 + \sin^2 w)}{\sec^2 w}$

$\dfrac{1 - \sin^2 w}{\sec^2 w} = \dfrac{\cos^2 w}{\sec^2 w} = \cos^4 w$ ✓

5. For each of the following equations find the solution set using the indicated units.

(a) $\cos x = -0.9135$ (in degrees)

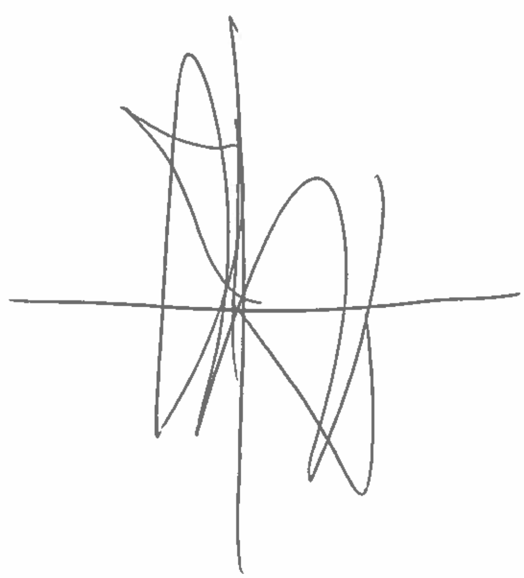
(b) $\tan(2x) = \sqrt{3}$ (in radians)

$2x = \tan^{-1}(\sqrt{3})$

$2x = \pi/3$

$x = \pi/6 + k\pi$

$x = 7\pi/6 + k\pi$



6. For each equation find all solutions in the interval $[0, 2\pi)$ or $[0°, 360°)$ depending on the indicated units.

   (a) $4 \cdot 16^{\cos^2(x)} = 64^{\cos(x)}$ (in radians)

$4 \cdot 4^{2\cos^2 x} = 4^{3\cos x}$

$1 + 2\cos^2 x = 3\cos x$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1)$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = 1$

$x = \pi/3, \frac{5\pi}{3}$ $\quad\quad$ $x = 0$

   (b) $9\sec^2\theta\tan\theta = 12\tan\theta$ (in radians)

$\tan\theta(9\sec^2\theta - 12) = 0$

$3\tan\theta(3\sec^2\theta - 4) = 0$

$3\tan\theta(\sqrt{3}\sec\theta - 2)(\sqrt{3}\sec\theta + 2) = 0$

$\theta = 0, \pi$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \pi/6$ $\quad$ $5\pi/6$

$= 11\pi/6$ $\quad$ $7\pi/6$

(c) $\csc^4\theta - 5\csc^2\theta + 4 = 0$ (in degrees)

$(\csc^2\theta - 4)(\csc^2\theta - 1) = 0$

$(\csc\theta - 2)(\csc\theta + 2)$ $\cot^2\theta = 0$

$\sin\theta = \pm\frac{1}{2}$

$\theta = \pi/2,\ 3\pi/2$

$\theta = \pi/6,\ 5\pi/6,\ 7\pi/6,\ \frac{11\pi}{6}$