Thoughts:

The antiderivative is intuitive verbally. Yes, it is the opposite of a derivative so what does that mean? Understanding the history will actually help here. So a derivative came first. The natural question to follow was can we go the other way? Thus, the antiderivative was born and defined in terms of derivatives! Here is how we define it.

Definition 0.1 (Antiderivative)**.** Let F be a function such that on an interval, say I, $F'(x) = f(x)$ for all x in I.

Seems pretty obvious but there are subtlies here that are in disguise. For example, why the interval? We will address this later. The thing now is how do we find them. Let's take an example. Find the antiderivative of $f(x) = x^3$. In other words, find the function that has the derivative x^3. Well the power rule tells us that the antiderivative, $F(x)$, has power 4 for x. So, lets take the derivative and see what we are missing.

$$(x^4)' = 4x^3.$$

Thus, we need a (1/4) in the front to take care of that 4 we didnt have in $f(x)$. Therefore, $F(x) = (1/4)x^4$. Now I have a very important question: Is this the only one? Reall think about this before you move on. What would have to be true for their to be another antiderivative?
The real question is 'what is 0 when i take the derivative?' The answer is any constant! Therefore, $F(x) = (1/4)x^4 + C$ where C is any constant is an antiderivative! Is this true for any antiderivative? If I have one example of an antiderivative will I always be able to attach a constant and say that is also an antiderivative? Test it out with the antiderivative of x^2. The answer is yes!

Theorem 0.1 (All Antiderivatives). *If F is an antiderivative of f on interval I, then*

$$F(x) + C$$

for any C is also an antiderivative.

Now I do not want to do these examples, but I want to encourage you to try to find the antiderivatives of the following functions: $\sin(x)$, $\cos(x)$, $1/x$, and x^n for $n \neq -1$.

Function	Antiderivatives
x^n $n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\|x\| + C$
e^x	$e^x + C$
$\sin(x)$	$-\cos(x) + C$
$\cos(x)$	$\sin(x) + C$
$\sec^2(x)$	$\tan(x) + C$
$\sec(x)\tan(x)$	$\sec(x) + C$

With this we can find solutions to questions that are posed like: Find all functions f given f' or find the antiderivatives of f or find f given f' and a point $f(a) = b$ or continuing this last one, find f given f'', $f'(a) = b$, and $f(c) = d$ (sometimes both just points on f). These last 2 are ones I want to address since they take slightly more thinking. However, I am going to write the example I am doing which you should try to do first before looking at my solution!
Find f given $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$.
Notice it doesn't ask for all f! This means we need the exact function with specific constants. The first thing to do is find a general form for f'.

$$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x + C = 4x^3 + 3x^2 - 4x + C.$$

Notice none of the information is in terms of f' so we musr continue.

$$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + D = x^4 + x^3 - 2x^2 + Cx + D.$$

Now our information applies since it is in terms of f. 0 will get rid of the Cx term and tell us D, so let's start with that.

$$4 = f(0) = 0^4 + 0^3 - 2(0)^2 = C(0) + D \Rightarrow D = 4.$$

Now we use this and our last piece of information to find C.

$$1 = f(1) = 1^4 + 1^3 - 2(1)^2 + C(1) + 4 \Rightarrow C = -3.$$

Thus, $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$. Done!
Now I want to address a common application which you have seen before! This is position/height, velocity, and acceleration. The derivative works down this chain (ex: $s'(t) = v(t)$). Now we work the other way of antiderivatives! I will use capital letters to represent antiderivative.

$$A(t) = v(t) + C, V(t) = s(t) + C, AA(t) = s(t) + Cx + D$$

The last one is due to two antiderivatives with resulting in a constant being 'lifted' to x like the previous example.

I also showed how to find a sketch of f from f' in the past. This is the essence of the antiderivative! Use this same process to get a sketch (where f' is 0 is when f has a max, min, or critical point. Positive f' is increasing interval in f and so on.)

Problems: Find f.

1. $f'(x) = 8x^3 + 12x + 3,\ f(1) = 6$
2. $f^{(121)}(x) = e^x,\ f^{(n)}(0) = 0$ for all $n \leq 120$
3. $f'(x) = x^{-1/3},\ f(1) = 1,\ f(-1) = -1$
4. $f''(x) = \sin(x) + \cos(x),\ f(0) = 3,\ f'(0) = 4$
5. $f''(x) = \dfrac{3}{\sqrt{x}},\ f(4) = 20,\ f'(4) = 7$
6. $f''(x) = 20x^3 + 12x^2 + 4,\ f(0) = 8,\ f(1) = 5$