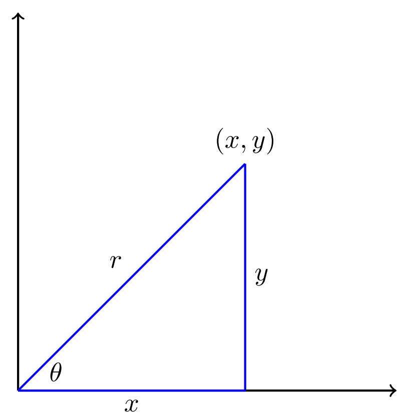
Thoughts:
This is the last topic for you guys! I know I know this is great news. Let's finish strong. First we have to explain a polar coordinate. Let's consider the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. This point is on the unit circle with a an angle of $\pi/4$ with respect to the x-axis. Thus, its polar coordinates are (I know you do not know what that means yet, but this shoulld help you take a guess) $(1, \pi/4)$. Any guesses? The idea behind polar coordinates is the following:

Definition 0.1 (Polar Coordinate)**.** The point (x, y) (a Cartesian point) has the polar form (r, θ) where r is the distance of the point (x, y) from the origin and θ is the angle with respect to the x-axis.

Let's do a general example: (x, y). The best way to illustrate the conversion to polar coordinates is by a picture.



This image should work wonders! How do we find r? That's right! Pythagorean Theorem! Thus, $r^2 = x^2 + y^2$ is quite useful. How do we find θ? Soh Cah Toa! Any of them! For description sake I will write all of them: $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$. In the case of going from Cartesian to polar, you will see all of these work as long as you find r first (tangent you do not need r). Going the other direction (polar to Cartesian) is a different story! Knowing r and θ to find x and y we can do some manipulation and the useful equations are: $x = r\cos\theta$ and $y = r\sin\theta$. That's all you need to know to convert between polar and Cartesian coordinates!
With new coordinates comes new equations. Let's see the general form: $r = f(\theta)$ or $f(r, \theta) = 0$. An example!

Example 0.1. $r = 2\cos(\theta)$. Try sketching this curve! I will show how to convert to Cartesian equations!

$$r = 2\left(\frac{x}{r}\right) \Rightarrow r^2 = 2x \Rightarrow x^2 + y^2 = 2x \Rightarrow (x-1)^2 + y^2 = 1$$

The last part is by completing the square. Hopefully you see that this is the equation of a circle centered at (1,0) with radius 1. Another similar example is the equation $r = 1 + \sin\theta$ which is the equation of a cardioid.

Now let's do some calculus!

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d(r\sin\theta)}{d\theta}}{\frac{d(r\cos\theta)}{d\theta}} = \frac{\sin\theta \cdot \frac{dr}{d\theta} + \cos\theta \cdot r}{\cos\theta \cdot \frac{dr}{d\theta} - \sin\theta \cdot r}$$

These are all the different ways you can find $\frac{dy}{dx}$ with polar equations! Thus, now applications like tangent lines are accessible.

We can also use integration to tell us things. First, we can determine the area of a sector of the function when we have polar equations. What is the sector of a polar equation you ask? Well, the sector is the area of the region made my drawing a straight line from each point $(a, f(a))$ and $(b, f(b))$ where a and b are the end points to the origin. Then use the function $r = f(\theta)$ to be the last side of the solid. This shape has the following formula for area:

$$A = \int_a^b (1/2)(f(\theta))^2 d\theta = \int_a^b (1/2) r^2 d\theta.$$

Now we have the idea of arc length again. I do not want to go into why the formula is the way it is. I will instead leave it up to you to ask me if you want to know where it comes from. Here is the formula.

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

These last two concepts are more plug and chug than anything else. That concludes everything for the class!

Problems:

1. Convert the following from Cartesian to Polar for $r > 0$ and $0 \leq \theta < 2\pi$.

   a. $(-1, \sqrt{3})$

   b. $(3\sqrt{3}, 3)$

   c. $(1, -2)$

2. Convert from Polar to Cartesian.

   a. $(1, \pi)$

   b. $(-2, 3\pi/4)$

   c. $(2, 7\pi/6)$

3. Convert the Cartesian equation to polar.

   a. $y = 2$

   b. $xy = 4$

   c. $x^2 + y^2 = 2x$

4. Convert the polar equations to Cartesian.

   a. $r = 4\sin(3\theta)$

   b. $r^2 = 9\sin(2\theta)$

   c. $r = 1 - 2\sin\theta$

   d. $r = 3 + 4\cos\theta$

5. Find the slope of the tangent lines for the following polar equations.

   a. $r = 1/\theta$

   b. $r = 1 + 2\cos\theta$

   c. $r = e^{\theta}$

6. Find the area inside $r = \sqrt{3}\cos\theta$ and inside $r = \sin\theta$.

7. Find the length of $r = 2(1 + \cos\theta$

8. Find the area enclosed by the curve $r = 1 + 5\sin\theta$.