Thoughts:
We are so used to x and y functions that we forget about functions that are not so nice! For example, what is the equation of a circle? If you said $x^2+y^2=r^2$ where r is the radius and this is centered at 0, you are not wrong! The general form is: $(x-h)^2+(y-k)^2=r^2$ where (h,k) is your center! This is not really the point though. Consider the circle centered at 0 equation. This is not a function that we can solve for y nicely for! This will be our end goal of this section. This is why we parameterize. First question, what does perameterize mean?

Definition 0.1 (Parameterize)**.** Make in terms of a single variable. (Can be multiple equations)

Simple definition, but no so easy to do! Let's first look at a line. This is $y-y_1=m(x-x_1)$. Let t be our goal variable. Think for a second what I can make t such that both x and y are in terms of t. My instinct is telling me t should include x. Let's see what that looks like!

$$t=x-x_1.$$

This gives me $x=t+x_1$ and $y=mt+y_1$! Thus, 2 equations in terms of t! Exactly what we wanted. Now say we do not have an equation but I want a line from point (a,b) to point (c,d). This take more thought! You can do the whole process and fine the equation of the Cartesian line (y and x) and then do what we just did, but there is a shorter approach. Let's say $t=0$ at our starting point and $t=1$ at our end point. This $x(0)=a$ and $y(0)=b$ for $t=0$. Similarly, $x(1)=c$ and $y(1)=d$ for $t=1$. Let's try to use this info to finish this! $x(t)=a(1-t)+ct$ and $y(t)=b(1-t)+dt$. Give it a look! This is a nice form to use!
Now let's go to our circle. Let's look at the following circle: $x^2+y^2=1$. The unit circle! If you look closely at this and remember the unit circle a nice substitution for x and y should stand out to you. Hint: what squared plus what squared is 1? Exactly! $x=\cos(t)$ and $y=\sin(t)$ for $0\leq t<2\pi$. The interval is important here, but look at that! We have what we want. The book goes into graphing these, but it is not that interesting since you just plug in values for t and make (x,y) coordinates and graph. Nothing crazy.
There is a specific parametric equation which someone mentioned in class (might have been a cardiod) and I feel as if some professors may require you to know it so here it is:

Definition 0.2 (Cycloids)**.** $x=at-a\sin(t)$, $y=a-a\cos(t)$

Just a form of a circle-like object. The big thing is being able to go from parametric to Cartesian or vice versa. An example is $x=3t$ and $y=9t^2$ for $-\infty<t<\infty$. Here we see $x^2=y$! This is a funky process, but its more of a practice thing than a making sense thing. Try the problems below!

Problems: Find a Cartesian equation and its domain.

1. $x = 3 - 3t$, $y = 2t$, $0 \leq t \leq 1$

2. $x = 4\sin(t)$, $y = 5\cos(t)$, $0 \leq t \leq 2\pi$

3. $x = \sec^2(t) - 1$, $y = \tan(t)$, $-\pi/2 < t < \pi/2$

4. $x = \dfrac{t}{t-1}$, $y = \dfrac{t-2}{t+1}$, $-1 < t < 1$

5. For the next problems, go from Cartesian to parametric or just find the parametric representation and domain of t.

6. The line segment with endpoints $(-1, -3)$ and $(4, 1)$.

7. The ray starting at $(2, 3)$ passing through $(-1, -1)$.

8. Lower half of the parabola $x - 1 = y^2$

9. Starting at $(a, 0)$, going around $x^2 + y^2 = a^2$ once clockwise. Try with once counter clockwise as well.