ENUM GENERATOR

BD
AMAR
INV
AR

BIV.
CAP
INT

SW. (GEN) CREATE OBJ

.GEN
.GEN
.GEN
.GEN

RIV

AMAKE CALL

1. $J \neq K$ $\quad W_t^J, W_t^K \quad \forall t$ indep

2. $\mathbb{E}[W_t^J W_t^K] = 0$

$\mathbb{E}[W_s W_t] = \min\{s, t\}$

3. $\int_0^T W_t \, dW_t = \dfrac{W_T^2 - T}{2}$

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t$$

$$\underbrace{d \log S_t = \left(\mu - \sigma^2/2\right) dt + \sigma \, dW_t}_{X_t \;\sim\; N\left(\left(\mu - \sigma^2/2\right)T,\; \sigma^2 T\right)}$$

$$\left(e^{X_T}\right) = S_T \sim LN\left(\left(\mu - \sigma^2/2\right)T,\; \frac{\sigma^2 T}{\sigma}\right)$$

$\rightarrow$ WIKIPEDIA

$\cdot\ e^{2\mu+\sigma^2/2}$

$L(\mu, \sigma^2)$

$\mu - \sigma^2/2$

$\cdot\ e^{2\mu+\sigma^2}(e - 1)$

$\Rightarrow \mathbb{E}[S_t] = e^{2\mu}$

$Var[S_t] = e^{2\mu+\sigma^2} - e^{2\mu}$

$$d\,\mathbb{E}[S_T] = \mathbb{E}[\mu S_t]\,dt + \underbrace{\cancel{\mathbb{E}[\sigma S_t\, dW_t]}}_{=0}$$

$$\mathbb{E}[S_t]' = \mu\, \mathbb{E}[S_t]$$

$$\underline{\underline{S_0}} \quad S_0\, e^{\mu T}$$

$$X_m^\Delta = (1 + r\Delta_t) X_{m-1}^\Delta + \sigma X_{m-1}^\Delta \cancel{N(0, \Delta)}$$

$$\mathbb{E}[X_m^\Delta] = (1 + r\Delta_t)\,\mathbb{E}[X_{m-1}^\Delta] +$$

$$\alpha_m = \alpha_{m-1}(1 + r\Delta_t) \qquad \alpha_0 = X_0$$

$$\alpha_m = X_0 (1 + \mu\Delta_t)^m = X_0\left(1 + \frac{\mu T}{m}\right)^m \longrightarrow X_0 \exp(\mu T)$$

$$= T/m$$

$$\mathrm{Var}[X_t^2] = \mathbb{E}[X_t^2] - \mathbb{E}[X_t]^2$$

$$= X_0^2 \exp(2\mu t)$$

$$X_n = (1+\mu\Delta_t)X_{n-1} + \sigma X_{n-1} Y \qquad Y \sim N(0, \Delta_t)$$

$$= X_{n-1}(1+\mu\Delta_t + Y)$$

$$X_n^2 = X_{n-1}^2 (1+\mu\Delta_t + Y)^2$$

$$E[X_n^2] = E[X_{n-1}^2] \cdot E\left((1 + r\Delta_t + \sigma Y)^2\right)$$

$$= E[X_{n-1}^2] \cdot E\left[(1 + r\Delta_t)^2 + 2\sigma Y r\Delta_t + \sigma^2 Y^2\right]$$

$$= E[X_{n-1}^2]\left((1 + r\Delta_t)^2 + \sigma^2 \underbrace{E[Y^2]}_{=\Delta_t}\right) \qquad \alpha_0 = x^2$$

$$\alpha_n = \alpha_{n-1}\left((1 + r\Delta_t)^2 + \sigma^2 \Delta_t\right)$$

$$X_n = X_0\left((1+\mu\Delta_t)^2 + \sigma^2\Delta_t\right)^n$$

$$X_n = X_0\left(1 + 2\mu\Delta_t + \sigma^2\Delta_t + \mu^2\Delta_t^2\right)^n$$

$$= X_0\left(1 + (2\mu+\sigma^2)\underbrace{\Delta_t}_{T/n} + \mu^2\underbrace{\Delta_t^2}_{T/n}\right)^n$$

$$E(X_n^2) = e^{(2\mu+\sigma^2)T}$$

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$$\mathrm{Var}(X_n) = e^{(2\sigma^2+\mu)T} - e^{2\mu T}$$