

Math Review

CFA L1 Standard

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To Begin with

ChatGPT is a good coach if and only if you have a good understanding about what you are asking.

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The Time Value of Money



Statistical Concepts and Market Returns



Probability Concepts

The Time Value of Money

- Introduction
- FV of a Single Cash Flow
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- PV of a Single CF
- PV of a Series of CFs
- Rates, Periods, and Size of Annuity Payments

Introduction

- In short, the calculation of time value of money is about finding equivalence between cash flows occurring on different dates.
- The real risk-free rate reflects the time preferences of individuals for current versus future real consumption.

interest rate

- $= real \ rate + inflation \ premium + default \ risk \ premium$
- + liquidity premium + maturity premium

FV of a Single Cash Flow

- A single CF or lump-sum investment.
- Principle.
- Interest.
- (Frequency of) Compounding.

$$FV_N = PV\left(1 + \frac{r_a}{m}\right)^{mN}$$

Effective Annual Rate (EAR)

 A stated annual rate will have different levels of EAR w.r.t. different levels of compounding frequency.

$$EAR_{DT} = \left(1 + \frac{r_a}{m}\right)^m - 1$$

For continuous-time case:

$$EAR_{CT} = e^{r_a} - 1 > EAR_{DT}$$

FV of a Series of Cash Flows

- Annuity and Perpetuity.
- Equal CFs case:

$$FV_N = A \sum_{t=0}^{N-1} [(1+r)^t]$$

$$FV_N = A\left[\frac{(1+r)^N - 1}{r}\right]$$

FV of a Series of Cash Flows

Unequal CFs case:

$$FV_N = \sum_{t=1}^{T} CF_t (1+r)^{T-t}$$

From FV to PV

• Do the opposite.

$$PV_N = A \sum_{t=1}^{N} \left[\frac{1}{(1+r)^t} \right]$$

$$PV_N = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right]$$

From FV to PV

Unequal CFs case:

$$PV_N = \sum_{t=1}^{T} CF_t (1+r)^{-t}$$

$$FV_N = PV(1+r)^N$$

The above equation will give you the same value you calculated 2 slides before.

From FV to PV

• Infinite case (when interest rates are positive):

$$PV = A \sum_{t=1}^{\infty} \left[\frac{1}{(1+r)^t} \right]$$

$$PV = \frac{A}{r}$$

Consol Bond

• There used to be such bond issued by British government that promised to pay a level CF forever. Say the bond paid £100 per year in perpetuity, how would you price the bond if the required rate of return were 5%?

Consol Bond

What if the first payment starts at t=5?

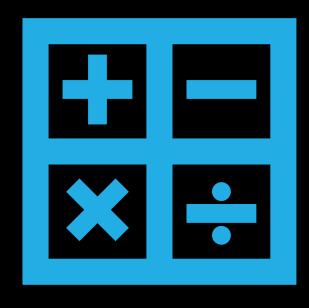
$$PV_4 = 2000$$

$$PV_0 = 2000/(1.05)^4$$

Application

- Now you know all the essential equations in the field of time value of money.
- By now, you should know how to use CF and r to get PV/FV.
- So automatically, you know how to use PV and FV to get r.
- If you know PV, FV, and r, you know N.
- If you know PV, r, and N, you know A.

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Mathematically, this is all you need for the course.