Math Review

CFA L1 Standard

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To Begin with

ChatGPT is a good coach if and only if you have a good understanding of what you are asking.

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The Time Value of Money

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- Rates, Periods, and Size of Annuity Payments

Introduction

- In short, the calculation of the time value of money involves finding equivalence between cash flows occurring on different dates.
- The real risk-free rate reflects the time preferences of individuals for current versus future real consumption.

$$\begin{aligned} &interest\ rate \\ &= real\ rate + inflation\ premium + default\ risk\ premium \\ &+ liquidity\ premium + maturity\ premium \end{aligned}$$

FV of a Single Cash Flow

- A single CF or lump-sum investment.
- Principle.
- Interest.
- (Frequency of) Compounding.

$$FV_N = PV\left(1 + \frac{r_a}{m}\right)^{mN}$$

Effective Annual Rate (EAR)

- A stated annual interest rate will result in different Effective Annual Rates (EAR) depending on the compounding frequency.

$$EAR_{DT} = \left(1 + \frac{r_a}{m}\right)^m - 1$$

- For continuous-time case:

$$EAR_{CT} = e^{r_a} - 1 > EAR_{DT}$$

FV of a Series of Cash Flows

- Annuity and Perpetuity.
- Equal CFs case:

$$FV_N = A \sum_{t=0}^{N-1} [(1+r)^t]$$

$$FV_N = A[\frac{(1+r)^N - 1}{r}]$$

FV of a Series of Cash Flows

- Unequal CFs case:

$$FV_N = \sum_{t=1}^{T} CF_t(1+r)^{T-t}$$

From FV to PV

- Do the opposite.

$$PV_N = A\sum_{t=1}^{N}[\frac{1}{(1+r)^t}]$$

$$PV_N = A[\frac{1-\frac{1}{(1+r)^N}}{r}]$$

From FV to PV

- Unequal CFs case:

$$PV_N = \sum_{t=1}^{T} CF_t(1+r)^{-t}$$

$$FV_N = PV(1+r)^N$$

The equation above will yield the same value as the one you calculated two slides earlier.

From FV to PV

- Infinite case (when interest rates are positive):

$$PV = A\sum_{t=1}^{\infty}[\frac{1}{(1+r)^t}]$$

$$PV = \frac{A}{r}$$

Consol Bond

- There used to be such bond issued by British government that promised to pay a level CF forever. Say the bond paid £100 per year in perpetuity, how would you price the bond if the required rate of return were 5%?

Consol Bond

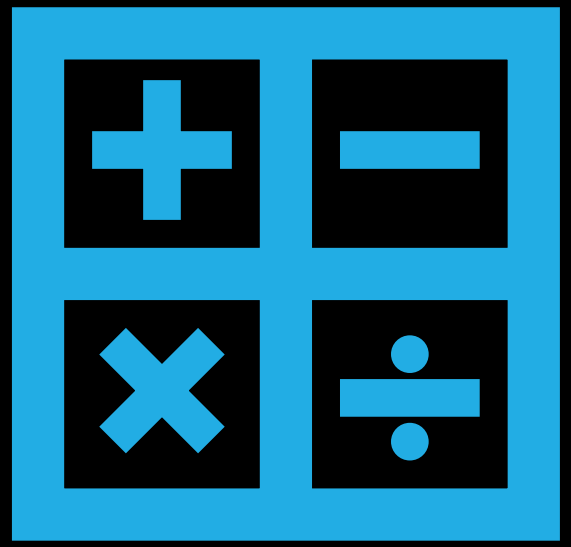
- What if the first payment starts at t=5?

$$PV_4 = 2000$$

$$PV_0 = 2000/(1.05)^4$$

Application

- Now you know all the essential equations in the field of time value of money.
- By now, you should know how to use CF and r to get PV/FV.
- So automatically, you know how to use PV and FV to get r.
- If you know PV, FV, and r, you know N.
- If you know PV, r, and N, you know A.
- ...



Mathematically, this is all you need for the course.