Brief Temporal Equilibrium Logic

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Abstract. Equilibrium Logic is a logical characterisation of Answer Set Programming (ASP) that is very successfully used for the study of ASP foundations and extensions. One of such extensions is Temporal Equilibrium Logic (TEL), that allowed for incorporating temporal modal operators from Linear Temporal Logic (LTL) into ASP. Following the steps of Equilibrium Logic, TEL is defined by selecting models from a temporal extension of the intermediate logic of Here-and-There (HT). These models have the form of traces obtained by some kind of truth minimisation analogous to the one performed in non-temporal ASP. Recently, the variant called *contracted* TEL (cTEL) introduced an additional minimisation that may also shorten the length of the trace. Each contracted trace obtained with cTEL can be seen as a "summarisation" of a family of temporal stable models of a given theory. One irregularity of cTEL is that it is based on a variant of THT which, for non-temporal formulas, does not collapse to HT, but corresponds to a weaker intermediate logic instead. In this paper, we propose another variant of TEL, called *brief TEL* (bTEL), that performs a different trace length minimisation in which traces become "shortest witnesses" of the formula or theory. The new variant bTHT collapses to HT for non-temporal theories and it allows for applying Kamp's translation of temporal logic into Monadic First Order Logic, interpreting the obtained formulas under Quantified HT with dynamic domains. Specifically, we provide the bTEL definition, illustrate its effect on some examples, and prove that Kamp's translation is sound.

1 Introduction

(Linear-time) *Temporal Equilibrium Logic* (TEL) extends Equilibrium Logic [4] to the temporal setting by selecting, among the models of *Temporal Here-and-There* (THT) [1], those traces that are in *equilibrium* (a.k.a. stable traces). Recently, a contraction-based additional selection criterion for equilibrium models has been investigated [2]: the idea is to discard candidate traces when a sequence of states in the "there" trace can be *contracted* into a single state in the "here" trace. The monotonic base studied there coincides, in the atemporal case, with an intermediate logic (Bounded-Depth–2, BD2) strictly weaker than Here-and-There in terms of valid formulas: for example $\neg\neg p \vee \neg p$ is not a valid formula there.

In this work, we introduce a new, intrinsically logical selection criterion for TEL that we call *brief TEL*. The goal is to logically capture a typical feature of incremental temporal reasoning in (temporal, incremental) ASP: intuitively, the length of a trace must be *founded*, in the ASP sense, by what the program justifies. Brief TEL enforces this foundedness by preferring traces that advance only as required by the temporal connectives; informally, one may "hop" ahead unless a *next*-operator forces a step to the immediate successor, internalizing brevity in the trace via a semantic selection principle.

Trace selection by length has been previously addressed in the literature. For instance, the authors in [5] focus on selecting models by identifying the shortest counterexamples for model-checking purposes. In planning contexts, ASP solvers often operate up to a predetermined plan horizon to generate the shortest possible plans. Additional approaches involve minimization criteria applied to weighted atoms; for example, [3] addresses LTL over finite traces, among others.

The rest of the paper proceeds as follows. In Section 2 we introduce brief TEL (bTEL) and its monotonic basis, brief THT (bTHT). In the next Section, we revisit the first order extension of Here-and-There we will use later for translating temporal formulas ($QHT^{+}(<)$), and whose particular feature is that allows a different domain in the worlds "here" and "there". Section 4 contains Kamp's translation from LTL into Monadic First Order Logic, and proves that the translation is sound when done between bTHT and $QHT^{+}(<)$. Finally, Section 5 concludes the paper.

2 Brief THT and Brief TEL

All logics considered in this paper share the same syntax as for LTL. Given a (countable, possibly infinite) set $\mathcal{A}$ of propositional atoms (called *alphabet*), a *temporal formula* φ follows the grammar:

$$\varphi ::= a \mid \bot \mid \varphi_1 \otimes \varphi_2 \mid \circ\varphi \mid \varphi_1 \,\mathbf{U}\, \varphi_2 \mid \varphi_1 \,\mathbf{R}\, \varphi_2$$

where $a \in \mathcal{A}$ is an atom and $\otimes$ is any binary Boolean connective $\otimes \in \{\to, \wedge, \vee\}$. The last three cases correspond to the temporal connectives whose names are listed as follows: $\circ$ for *next*; $\mathbf{U}$ for *until*; and $\mathbf{R}$ for *release*. A formula φ is said to be $\otimes$-*free* if it does not contain any occurrence of some connective $\otimes$. We also define several common derived operators like the Boolean connectives $\neg\varphi \stackrel{df}{=} \varphi \to \bot$, $\top \stackrel{df}{=} \neg\bot$, $\varphi \leftrightarrow \psi \stackrel{df}{=} (\varphi \to \psi) \wedge (\psi \to \varphi)$, and the following temporal operators: *always* as $\Box\varphi \stackrel{df}{=} \bot \,\mathbf{R}\, \varphi$, *eventually* as $\Diamond\varphi \stackrel{df}{=} \top \,\mathbf{U}\, \varphi$, *final* as $\mathbf{F} \stackrel{df}{=} \neg\circ\top$, and *weak next* as $\widehat{\circ}\varphi \stackrel{df}{=} \circ\top \to \circ\varphi$. A *(temporal) theory* is a (possibly infinite) set of temporal formulas.

The semantics of formulas and theories is defined in terms of *traces*. We introduce the notation $x \in [a..b)$ to stand for $a \leq x < b$. An LTL-*trace* $\mathbf{T}$ is a sequence $\mathbf{T} = (T_i)$, $i \in [0..\lambda)$ of sets of atoms (or propositional interpretations) $T_i \subseteq \mathcal{A}$, where $\lambda \in \mathbb{N} \cup \{\omega\}$ is the *length* of the trace. As in contracted THT (cTHT) [2], we have two traces $\mathbf{H}$ and $\mathbf{T}$ of lengths $\lambda_h = |\mathbf{H}|$ and $\lambda_t = |\mathbf{T}|$ respectively, where $\lambda_h \leq \lambda_t \in \mathbb{N} \cup \{\omega\}$.

Definition 1. *An* HT-trace *is a triple* $\langle \mathbf{H}, \mathbf{T}, \rho \rangle$ *satisfying* $\lambda_h \leq \lambda_t$ *and* $\rho : [0..\lambda_h) \to [0..\lambda_t)$ *such that:*

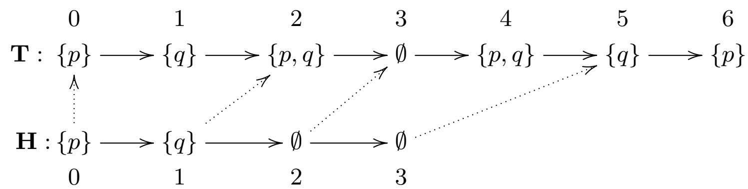

Fig. 1. Example of HT-trace.

- *ρ is strictly monotonic, that is, if $j > i$ then $\rho(j) > \rho(i)$;*
- $H_i \subseteq T_{\rho(i)}$. □

Intuitively, the function ρ selects reference points in the There-trace $\mathbf{T}$ that should be taken into account for evaluating a formula similarly as in an HT-trace for temporal HT-logic (we shall discuss this more in detail below). It can be compactly represented as a vector $[v_0, \ldots, v_{n-1}]$ of length $n = \lambda_h$ so that $\rho(i) = v_i$. An example of HT-trace is shown in Figure 1. In the example, $\lambda_t = 7$, $\lambda_h = 4$ and $\rho = [0, 2, 3, 5]$. An HT-trace of the form $\langle \mathbf{T}, \mathbf{T}, id \rangle$ corresponds to a standard LTL trace $\mathbf{T}$. For this reason, we will sometimes write $\mathbf{T}$ instead of $\langle \mathbf{T}, \mathbf{T}, id \rangle$.

We next define satisfaction of a formula in an HT-trace.

Definition 2 (Satisfaction). *Given* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$

- $\mathbf{M}, i \models \top$, $\mathbf{M}, i \not\models \bot$
- $\mathbf{M}, i \models p$ *iff* $p \in H_i$
- $\mathbf{M}, i \models \varphi \wedge \psi$ *iff* $\mathbf{M}, i \models \varphi$ *and* $\mathbf{M}, i \models \psi$
- $\mathbf{M}, i \models \varphi \vee \psi$ *iff* $\mathbf{M}, i \models \varphi$ *or* $\mathbf{M}, i \models \psi$
- $\mathbf{M}, i \models \varphi \rightarrow \psi$ *iff both:*
  1. $\mathbf{M}, i \not\models \varphi$ *or* $\mathbf{M}, i \models \psi$
  2. $\mathbf{T}, \rho(i) \not\models \varphi$ *or* $\mathbf{T}, \rho(i) \models \psi$
- $\mathbf{M}, i \models \circ\varphi$ *iff* $i + 1 < \lambda_h$ *and* $\rho(i+1) = \rho(i) + 1$ *and* $\mathbf{M}, i+1 \models \varphi$
- $\mathbf{M}, i \models \varphi \,\mathsf{U}\, \psi$ *iff there exists* $j \in \big[i..\lambda_h\big)$ *such that* $\mathbf{M}, j \models \psi$ *and both*
  1. *for all* $k \in \big[i..j\big)$, $\mathbf{M}, k \models \varphi$
  2. *for all* $k \in \big[\rho(i)..\rho(j)\big)$, $\mathbf{T}, k \models \varphi$
- $\mathbf{M}, i \models \varphi \,\mathsf{R}\, \psi$ *iff both:*
  1. *for all* $j \in \big[i..\lambda_h\big)$, $\mathbf{M}, j \models \psi$ *or there exists* $k \in \big[i..j\big)$, $\mathbf{M}, k \models \varphi$
  2. *for all* $j \in \big[\rho(i)..\lambda_t\big)$, $\mathbf{T}, j \models \psi$ *or there exists* $k \in \big[\rho(i)..j\big)$, $\mathbf{T}, k \models \varphi$

An important property in intermediate logics is *persistence*: it guarantees that anything true in a world is maintained true in all accessible worlds. In our context, this means that any formula φ true "here" $\mathbf{H}$ should still be true "there" $\mathbf{T}$. However, in the temporal setting, we must also specify the time point i at which φ is satisfied, and any time point i for $\mathbf{H}$ must be mapped into $\rho(i)$ for $\mathbf{T}$. As a consequence, we get the following formulation of the persistence property.

Theorem 1 (Persistence). *For any formula φ, any* HT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$*:* $\mathbf{M}, i \models \varphi$ *implies* $\mathbf{T}, \rho(i) \models \varphi$ *for every* $i \in \big[0..\lambda_h\big)$.

Proof. We proceed by structural induction on the form of the formula φ:

- If $\varphi = \bot$ both $\mathbf{M}, i \models \bot$ and $\mathbf{T}, \rho(i) \models \bot$ are false and the result trivially follows.
- If $\varphi = p$ is an atom, $\mathbf{M}, i \models p$ implies $p \in H_i \subseteq T_{\rho(i)}$ and so $\mathbf{T}, \rho(i) \models p$.
- If $\varphi = \alpha \wedge \beta$ then $\mathbf{M}, i \models \alpha \wedge \beta$ is equivalent to both $\mathbf{M}, i \models \alpha$ and $\mathbf{M}, i \models \beta$. Applying induction in both subformulas, we conclude $\mathbf{T}, \rho(i) \models \alpha$ and $\mathbf{T}, \rho(i) \models \beta$, and so, $\mathbf{T}, \rho(i) \models \alpha \wedge \beta$.
- If $\varphi = \alpha \vee \beta$ then $\mathbf{M}, i \models \alpha \wedge \beta$ is equivalent to $\mathbf{M}, i \models \alpha$ or $\mathbf{M}, i \models \beta$. Suppose $\mathbf{M}, i \models \alpha$: applying induction we conclude $\mathbf{T}, \rho(i) \models \alpha$ and this implies $\mathbf{T}, \rho(i) \models \alpha \vee \beta$. If $\mathbf{M}, i \models \beta$ instead, the result is analogous.
- If $\varphi = \alpha \rightarrow \beta$ then the definition of $\mathbf{M}, i \models \alpha \rightarrow \beta$ implies (in its second condition) $\mathbf{T}, \rho(i) \not\models \alpha$ or $\mathbf{T}, \rho(i) \models \beta$. But this immediately implies $\mathbf{T}, \rho(i) \models \alpha \rightarrow \beta$.
- If $\varphi = \circ\alpha$ then $\mathbf{M}, i \models \circ\alpha$ is equivalent to $i + 1 < \lambda_h$ and $\rho(i+1) = \rho(i) + 1$ and $\mathbf{M}, i + 1 \models \alpha$. Since $i + 1 < \lambda_h$ we can applying induction to conclude $\mathbf{T}, \rho(i + 1) \models \alpha$. As $\rho(i + 1) = \rho(i) + 1$, we get $\mathbf{T}, \rho(i) + 1 \models \alpha$, and so, $\mathbf{T}, \rho(i) \models \circ\alpha$.
- If $\varphi = \alpha \,\mathbf{U}\, \beta$ then $\mathbf{M}, i \models \alpha \,\mathbf{U}\, \beta$ implies there exists some $j \in \big[i..\lambda_h\big)$ satisfying $\mathbf{M}, j \models \beta$ and (condition 2) for all $k \in \big[\rho(i)..\rho(j)\big)$ we have $\mathbf{T}, k \models \alpha$. By induction, $\mathbf{M}, j \models \beta$ implies $\mathbf{T}, \rho(j) \models \beta$. Also, by the definition of ρ as monotonic function mapping to $\big[0..\lambda_t\big)$, $j \in \big[i..\lambda_h\big)$ implies $\rho(j) \in \big[\rho(i)..\lambda_t\big)$. This means we have obtained some $l = \rho(j)$ such that $\mathbf{T}, l \models \beta$ and for all $k \in \big[\rho(i)..l\big)$, $\mathbf{T}, k \models \alpha$, which is the condition for $\mathbf{T}, \rho(i) \models \alpha \,\mathbf{U}\, \beta$.
- If $\varphi = \alpha \,\mathbf{R}\, \beta$ then the second condition of the definition of $\mathbf{M}, i \models \alpha \,\mathbf{R}\, \beta$ directly corresponds to $\mathbf{T}, \rho(i) \models \alpha \,\mathbf{R}\, \beta$. □

Proposition 1. *Given* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$*, we consequently get the following satisfaction for derived operators:*

(i) $\mathbf{M}, i \models \neg\varphi$ *iff* $\mathbf{T}, \rho(i) \not\models \varphi$
(ii) $\mathbf{M}, i \models \widehat{\circ}\varphi$ *iff both:*
  (a) if $i + 1 < \lambda_h$ *and* $\rho(i + 1) = \rho(i) + 1$ *then* $\mathbf{M}, i + 1 \models \varphi$
  (b) if $\rho(i) + 1 < \lambda_t$ *then* $\mathbf{T}, \rho(i) + 1 \models \varphi$
(iii) $\mathbf{M}, i \models \Diamond\varphi$ *iff there exists* j*,* $i \leq j < \lambda_h$ *such that* $\mathbf{M}, j \models \varphi$
(iv) $\mathbf{M}, i \models \Box\varphi$ *iff both:*
  (a) for all $j \in \big[i..\lambda_h\big)$*,* $\mathbf{M}, j \models \varphi$
  (b) for all $j \in \big[\rho(i)..\lambda_t\big)$ $\mathbf{T}, j \models \varphi$

Proof. (Sketch) Conditions (ii)-(iv) all follow immediately from the respective definition of the derived operator and the satisfaction relation. For (i), note that $\neg\varphi \stackrel{df}{=} \varphi \rightarrow \bot$ and $\mathbf{M}, i \models \varphi \rightarrow \bot$ amounts to the conjunction of $\mathbf{M}, i \not\models \varphi$ and $\mathbf{T}, \rho(i) \not\models \varphi$ but the first condition can be removed since, due to Persistence (Th. 1), it is implied by the second condition. □

Proposition 2. *Fixing* $\rho = id$ *collapses to* THT*-satisfaction. Furthermore, fixing* $\mathbf{H} = \mathbf{T}$ *(with* $\rho = id$*) collapses to LTL.*

Proof. (Sketch) This can be proven by structural induction on the formula φ. Note that $\langle \mathbf{H}, \mathbf{T}, id \rangle$ amounts to $\langle \mathbf{H}, \mathbf{T} \rangle$ in THT-logic, and if $\mathbf{H} = \mathbf{T}$, then $\langle \mathbf{H}, \mathbf{T} \rangle$ amounts to $\mathbf{T}$ in LTL. $\square$

Proposition 3. *For the syntax of non-temporal, propositional formulas,* bTHT *collapses to* HT*, that is, if φ does not contain temporal operators: $\models \varphi$ in* bTHT *iff $\models \varphi$ in* HT.

Proof. We prove both directions by contraposition. For the left to right direction, suppose there is some HT-interpretation $\langle H, T \rangle \not\models \varphi$. Then, we can build the bTHT trace $\langle \mathbf{H}, \mathbf{T}, id \rangle$ where $\mathbf{H} = (H)$, $\mathbf{T} = (T)$, $\rho = id$ where $\lambda_h = \lambda_t = 1$ and we can easily see that $\langle H, T \rangle \not\models \varphi$ amounts to $\langle \mathbf{H}, \mathbf{T}, id \rangle, 0 \not\models \varphi$ by inspection of bTHT satisfaction of non-temporal operators. For the right to left direction, suppose there is some bTHT-trace $\langle \mathbf{H}, \mathbf{T}, id \rangle \not\models \varphi$. Then, we can build the HT-interpretation $\langle H, T \rangle$ with $H = H_0$ and $T = T_{\rho(0)}$. Again, by inspection of bTHT satisfaction of non-temporal operators, we may easily see that $\langle \mathbf{H}, \mathbf{T}, id \rangle \not\models \varphi$ amounts to $\langle H, T \rangle \not\models \varphi$ in HT. $\square$

As well-known, the logic of HT does not satisfy the excluded middle axiom $\varphi \vee \neg\varphi$: for instance, the HT interpretation $\langle H, T \rangle = \langle \emptyset, \{p\} \rangle$ does not satisfy $p \vee \neg p$ since neither $\langle H, T \rangle \models p$, because $p \notin H$, nor $\langle H, T \rangle \models \neg p$ because $p \in T$. Adding the axiom schemata $p \vee \neg p$ for all atoms in the signature forces $H = T$ and makes HT collapse to classical, propositional logic. Analogously, THT does not satisfy excluded middle either, whereas adding the axiom schemata:

$$\square(p \vee \neg p) \qquad \textbf{(EM)}$$

for every $p \in \mathcal{A}$ forces $\mathbf{H} = \mathbf{T}$ and makes THT collapse to LTL. In bTHT, however, axiom (**EM**) does not suffice to guarantee $\mathbf{H} = \mathbf{T}$. Instead, the addition of (**EM**) only guarantees that $H_i = T_{\rho(i)}$, but traces $\mathbf{H}$ and $\mathbf{T}$ may still have different lengths. To put an example, trace $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$ with $\mathbf{T} = \emptyset \cdot \{p\} \cdot \emptyset$, $\mathbf{H} = \{p\}$ and $\rho(0) = 1$ is a model of (**EM**). This effect is somehow due to the fact that requiring (**EM**) on atoms does not suffice to satisfy the excluded for any formula φ. In particular, the formula $\circ\top \vee \neg\circ\top$ (which is a THT-tautology) is not bTHT-valid. As a counterexample, we may use the same example trace introduced above: on the one hand, we may easily see that $\mathbf{M}, 0 \not\models \circ\top$ since the $\mathbf{H}$ trace ends at state $H_0 = \{p\}$. On the other hand, $\mathbf{T}, \rho(0) \not\models \neg\circ\top$ because $\rho(0) = 1$ and, at trace $\mathbf{T}$, state $T_1 = \{p\}$ is followed by a next state $T_2 = \emptyset$. Let us define the axiom:

$$\square(\circ\top \vee \neg\circ\top) \qquad \textbf{(EM}'\textbf{)}$$

Note that $\mathbf{M}, i \models \neg\circ\top \vee \circ\top$ means: (1) either $\rho(i) + 1 = \lambda_t$ (and so $i + 1 = \lambda_h$) so both traces are at the end; or (2) $i + 1 < \lambda_h$ and $\rho(i + 1) = \rho(i) + 1$. In principle, one could expect that the addition of (**EM**$'$) forced $\rho = id$ (or if preferred $\rho(i) = i$), and so, bTHT-traces in that context would amount to THT-traces. However, in general, this does not hold. This is because we may build bTHT-traces where $\rho(0) \neq 0$, so that the $\mathbf{H}$ trace "starts" at a non-initial point $\rho(0) > 0$ in the $\mathbf{T}$ trace. The actual effect of axiom (**EM**$'$) is captured by following result:

Proposition 4. *The models of* (**EM**$'$) *are* bTHT*-traces* $\langle \mathbf{T}, \mathbf{H}, \rho \rangle$ *where* $\mathbf{T} = \gamma \cdot \mathbf{T}'$ *being* γ *an arbitrary prefix of finite length* $|\gamma|$ *and* $\rho(i) = i + |\gamma|$ *for every* $i \in \big[0..\lambda_h\big)$.

Notice that the states in the $\mathbf{T}$-prefix γ are never accessible through bTHT-satisfaction since, in this paper, we do not consider temporal operators that can move backwards in time. For this reason, there is no possible additional axiom that could enforce $\rho = id$. However, although we do not have a one-to-one mapping between THT-models of γ and bTHT-models of $\{\gamma\} \cup$ (**EM**') we can still prove that bTHT plus (**EM**$'$) collapses to THT.

Proposition 5 **(**bTHT$\subseteq$ THT**).** *Any* bTHT*-tautology is also a* THT*-tautology.*

Proof. The result is trivial, since all THT-models are also bTHT-models.

Proposition 6 **(**THT$\subseteq$ bTHT**+ (EM$'$)).** *If* φ *is a* THT*-tautology then any* bTHT*-trace satisfying* (**EM**$'$) *also satisfies* φ.

Proof. Suppose there is a bTHT-trace $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$ satisfying (**EM**$'$) such that $\mathbf{M}, 0 \not\models \varphi$: we will prove that φ is not a THT-tautology. By Proposition 4, trace $\mathbf{M}$ has the form $\mathbf{T} = \gamma \cdot \mathbf{T}'$ and $\rho(i) = i + |\gamma|$. We may then build the bTHT-trace $\mathbf{M}' = \langle \mathbf{T}', \mathbf{H}, id \rangle$ which is also a THT-trace. It is easy to see that $\mathbf{M}, 0 \not\models \varphi$ iff $\mathbf{M}', 0 \not\models \varphi$ in bTHT, but since $\mathbf{M}'$ is also a THT-trace, we obtain that φ is not a THT-tautology. □

Definition 3. *A trace* $\mathbf{M} = \langle \mathbf{T}, \mathbf{T}, id \rangle$ *is a* temporal equilibrium (TEL) model *of a theory* Γ *iff (1)* $\mathbf{M}$ *is a model of* Γ *and (2) there is no other different model of* Γ *of the form* $\langle \mathbf{H}, \mathbf{T}, \rho \rangle$. *When this holds, we also say that* $\mathbf{T}$ *is a* bTHT*-stable model of* Γ.

A first observation is that, as desired, bTHT-stable models are selected from the TEL-models.

Proposition 7. *Any* bTHT*-stable model of* Γ *is also a* THT*-stable model of* Γ.

Proof. It follows from the fact that THT-models are also bTHT-models. Then, by contraposition, if $\mathbf{T}$ is not THT-stable, then there exists some THT-model $\langle \mathbf{H}, \mathbf{T} \rangle$ of Γ (with $\lambda_h = \lambda_t$) but the latter is also a bTHT-model and, thus, $\mathbf{T}$ is not bTHT-stable either. □

Let us next look at a couple of examples.

Example 1. Take the formula $\Diamond p$. Its TEL models have the form $\emptyset^* \cdot \{p\} \cdot \emptyset^{*,\omega}$. Models of the form $\emptyset^+ \cdot \{p\} \cdot \emptyset^{*,\omega}$ or $\emptyset^* \cdot \{p\} \cdot \emptyset^{+,\omega}$ will not be stable because we can form the trace $\mathbf{H} = \{p\}$ and take $\rho(0) = k$ where k is the position of the leftmost state $\{p\}$ in $\mathbf{T}$. It is easy to see that $\langle \mathbf{H}, \mathbf{T}, \rho \rangle, 0 \models \Diamond p$. Thus, the only temporal stable model is the trace $\mathbf{T} = \{p\}$ of length $\lambda_t = 1$.

Example 2. Take the formula $\Box p$. Its TEL models have the form $\{p\}^{+,\omega}$. Models of the form $\{p\} \cdot \{p\}^{+,\omega}$ will not be stable because we can form the trace $\mathbf{H} = \{p\}$ and take $\rho(0) = 0$. Again, it is easy to see that $\langle \mathbf{H}, \mathbf{T}, \rho \rangle, 0 \models \Box p$. Thus, the only temporal stable model is the trace $\mathbf{T} = \{p\}$ of length $\lambda_t = 1$.

Example 3. Take the formula $\Diamond p \wedge \neg \Box p$. Its TEL models are traces containing at least one state $\{p\}$ and at least one $\emptyset$, namely:

$$\emptyset^* \cdot \{p\} \cdot \emptyset^* \cdot \emptyset^{+,\omega} \; + \; \emptyset^+ \cdot \{p\} \cdot \emptyset^{*,\omega}$$

The stable models are be $\emptyset \cdot \{p\}$ and $\{p\} \cdot \emptyset$.

Example 4. Take the formula $\Box(p \vee \neg p) \wedge \Box(\circ\top \vee \neg\circ\top)$. This is a TEL-tautology in the sense that any LTL-trace is a TEL-model $(\emptyset + \{p\})^{+,\omega}$. Yet, take any LTL infinite trace $\mathbf{T}$: we can always form a smaller model with $\mathbf{H}$ by taking any suffix of $\mathbf{T}$, that is, $\mathbf{T} = \gamma \cdot \mathbf{H}$ with $|\gamma| > 0$ and $\rho(i) = i + |\gamma|$, so infinite traces do not form any bTEL-model in this example. For finite traces of length $n > 0$, we can always pick $\mathbf{H} = (T_{n-1})$ of length 1 with $\rho(0) = n - 1$ to form a bTHT-model for this example. This means that we only get two bTEL-models $(\emptyset)$ and $(\{p\})$, both of length 1. This example shows that the abbreviation of traces also takes place even in the presence of excluded middle for atoms and for $\circ\top$. Notice that this effect would be removed if we forced $\rho(0) = 0$ for all the bTHT models.

3 Quantified Here-and-There with dynamic domains

A *first-order signature* is a tuple $\Sigma = \langle \mathcal{C}, \mathcal{F}, \mathcal{P} \rangle$, where $\mathcal{C}$ is the set of constants (or 0-ary functions), $\mathcal{F}$ is the set of (non-0-ary) function symbols and $\mathcal{P}$ is the set of predicate symbols. The function $arity : \mathcal{P} \cup \mathcal{F} \rightarrow \mathbb{N}$ denotes the number of arguments of each predicate in $\mathcal{P}$ and each function in $\mathcal{F}$.

Well-formed formulas follow the syntax of classical predicate calculus with equality $=$. We assume a countably infinite set of variables, connectives '$\bot$', '$\vee$', '$\wedge$', '$\rightarrow$', '$\exists$', '$\forall$' and auxiliary parentheses. Additionally, we define negation by $\neg\varphi \stackrel{df}{=} \varphi \rightarrow \bot$ and double implication by $\varphi \leftrightarrow \psi \stackrel{df}{=} (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$. We denote by $Atoms(\mathcal{C}, \mathcal{F}, \mathcal{P})$, or At for short, the set of ground predicate atoms over the language. We also write $Terms(\mathcal{C}, \mathcal{F})$ to stand for the set of ground terms formed with constants in $\mathcal{C}$ and functions in $\mathcal{F}$. As usual, a *sentence* is a formula where all its variables are bound by some quantifier. A set of sentences is called a *theory*.

Definition 4 **(QHT^+-Interpretation).** *A QHT^+-*interpretation *is a tuple $\langle D_h, D_t, I, \sigma \rangle$ where*

- *$D_h \subseteq D_t \neq \emptyset$ are the domains at the "here" and the "there" worlds, respectively.*
- *I is a function that maps each predicate $p \in \mathcal{P}$ with $arity(p) = n$ at each world $w \in \{h, t\}$ to a set of tuples $I(p, w) \subseteq D_w^n$ formed with elements from the domain of that world. It additionally satisfies $I(p, h) \subseteq I(p, t)$ for every predicate $p \in \mathcal{P}$.*
- *$\sigma : Terms(\mathcal{C} \cup D_t) \rightarrow D_t$ is the interpretation of terms, satisfying the conditions:*
  - *$\sigma(d) = d$ for all $d \in D_t$*
  - *if $\sigma(t) \in D_h$ and t' is a subterm of t then $\sigma(t') \in D_h$.*

Given a tuple of terms $\mathbf{t} = \langle t_1, \ldots, t_n \rangle$, we write $\sigma(\mathbf{t})$ to stand for the tuple $\langle \sigma(t_1), \ldots, \sigma(t_n) \rangle$. For simplicity, a monadic tuple $\langle t \rangle$ is just written as t, omitting the angle brackets $\langle \cdot \rangle$.

The order relation between worlds is such that $h \leq t$ and $w \leq w$ for $w \in \{h, t\}$.

We say that a QHT^+ interpretation M is *total* when M has $M = \langle D_t, D_t, I, \sigma\rangle$ and $I(p, h) = I(p, t)$ for every predicate $p \in \mathcal{P}$. Given a QHT^+ interpretation $M = \langle D_h, D_t, I, \sigma\rangle$, we write M^t to stand for the corresponding total interpretation $\langle D_t, D_t, I', \sigma\rangle$ where $I'(p, h) = I'(p, t) = I(p, t)$ for every predicate $p \in \mathcal{P}$.

Definition 5. *A QHT^+-interpretation $M = \langle D_h, D_t, I, \sigma\rangle$ for signature $\langle \mathcal{C}, \mathcal{F}, \mathcal{P}\rangle$* satisfies *a formula φ at world $w \in \{h, t\}$, written $M, w \models \varphi$, recursively as follows:*

- $M, w \not\models \bot$
- $M, w \models p(\mathbf{t})$ *iff* $\sigma(\mathbf{t}) \in I(p, w)$.
- $M, w \models \varphi \wedge \psi$ *iff* $M, w \models \varphi$ *and* $M, w \models \psi$,
- $M, w \models \varphi \vee \psi$ *iff* $M, w \models \varphi$ *or* $M, w \models \psi$,
- $M, w \models \varphi \rightarrow \psi$ *iff* $M, w' \not\models \varphi$ *or* $M, w' \models \psi$ *for all* $w' \geq w$,
- $M, w \models \forall x\, \varphi(x)$ *iff* $M, w' \models \varphi(d)$ *for all* $w' \geq w$ *and* $d \in D_{w'}$.
- $M, w \models \exists x\, \varphi(x)$ *iff* $M, w \models \varphi(d)$ *for some* $d \in D_w$.

4 Kamp's translation

We will deal with monadic $QHT^+(\leq)$ for signature $\langle \mathcal{C}, \mathcal{F}, \mathcal{P}\rangle$ where $\mathcal{C} = \{0\}$ has the unique constant 0, $\mathcal{F} = \emptyset$ and $\mathcal{P} = \mathcal{A} \cup \{\leq\}$ consists of a set of unary predicates $p \in \mathcal{A}$, $arity(p) = 1$, plus a binary predicate '$\leq$' that will satisfy the properties of a discrete linear order.

We define the following derived predicates:

$$x < y \stackrel{df}{=} x \leq y \wedge \neg(y \leq x)$$
$$x = y \stackrel{df}{=} x \leq y \wedge y \leq x$$
$$succ(x, y) \stackrel{df}{=} x < y \wedge \neg\exists z(x < z \wedge z < y)$$

As said before, $\leq$ satisfies the axioms of a discrete linear order. Besides, we assume there exists a minimum element denoted by constant 0.

$$\forall x(x \leq x) \quad (1)$$
$$\forall x\forall y\forall z(x \leq y \wedge y \leq z \rightarrow x \leq z) \quad (2)$$
$$\forall x\forall y(x \leq y \vee y \leq x) \quad (3)$$
$$\forall x\forall y\forall x(succ(x, z) \wedge succ(y, z) \rightarrow x = y) \quad (4)$$
$$\forall x\forall y\forall x(succ(y, z) \wedge x < z \rightarrow x \leq y) \quad (5)$$
$$\neg\exists x(x < 0) \quad (6)$$

Let $M = \langle D_h, D_t, I, \sigma\rangle$ be a $QHT^+(\leq)$ interpretation and $w \in \{h, t\}$. The expression $a \leq^M_w b$ represents $(a, b) \in I(\leq, w)$. Furthermore, for any world $w \in \{h, t\}$, we define the *level* of every element $d \in D_w$ at w as the function $level_{M,w} : D_w \rightarrow \big[0..|D_w|\big)$:

$$level_{M,w}(d) \stackrel{df}{=} \begin{cases} 0 & \text{if there is no } c \neq d \in D_w \text{ s.t. } c \leq^M_w d \\ level_{M,w}(c) + 1 & \text{otherwise, where} \\ & c = \max\{x \mid x \in D_w, x \neq d, x \leq^M_w d\} \end{cases}$$

Notice that the maximum in the second case above is well-defined: the set is not empty (if empty, the level is 0 due to the first case) and there exists a maximum because $\leq_w^M$ is a discrete linear order relation. A second observation is that function $level_{M,w}(\cdot)$ is bijective, so it has an inverse $level^-_{M,w}(i)$ that, essentially, returns the element in D_w positioned at level i in the order relation.

Example 5. As an example of interpretation $M = \langle D_h, D_t, I, \sigma\rangle$, consider the domains $D_h = \{a, c, d, f\}$ and $D_t = \{a, b, c, d, e, f, g\}$, and predicates p and q interpreted as follows:

$$
\begin{aligned}
I(p, t) &= \{a, c, e, g\}\\
I(q, t) &= \{b, c, e, f\}\\
I(p, h) &= \{a\}\\
I(q, h) &= \{c\}
\end{aligned}
$$

Lel in addition $I(\leq, t)$ correspond to the alphabetical order of the letters we used to represent the elements in D_t, namely $a \leq_t^M b \leq_t^M c \leq_t^M d \leq_t^M e \leq_t^M f \leq_t^M g$ plus the reflexive and transitive closure of those relations. The same relations $\leq_h^M$ apply among elements in D_h. The level of each element in world t corresponds to:

> $level_{M,t}(a) = 0$, $level_{M,t}(b) = 1$, $level_{M,t}(c) = 2$, $level_{M,t}(d) = 3$, $level_{M,t}(e) = 4$, $level_{M,t}(f) = 5$, $level_{M,t}(g) = 6$

whereas in world h we get

> $level_{M,h}(a) = 0$, $level_{M,h}(c) = 1$, $level_{M,h}(d) = 2$, $level_{M,h}(f) = 3$.

Note that a has the same level 0 both in world h and world t while element f, for instance, appears at different levels, 3 and 5, respectively. Finally, note how the inverse of the level relation at some world w can be used to get the element positioned at some given level between 0 and $|D_w|$. For instance $level^-_{M,t}(2) = c$ whereas $level^-_{M,h}(2) = d$.

Given a $QHT^+(\leq)$ interpretation $M = (D_h, D_t, I, \sigma)$, we define its corresponding HT-trace $f(M) = \langle \mathbf{H}, \mathbf{T}, \rho\rangle$ as follows. The lengths of $\mathbf{H}$ and $\mathbf{T}$ respectively correspond to $\lambda_h = |D_h|$ and $\lambda_t = |D_t|$. The traces $\mathbf{H} = (H_i)$ with $i \in [0..\lambda_h)$ and $\mathbf{T} = (T_i)$ with $i \in [0..\lambda_t)$ are defined as follows:

$$
\begin{aligned}
H_i &\stackrel{df}{=} \{p \in \mathcal{A} \mid level^-_{M,h}(i) \in I(p, h)\}\\
T_i &\stackrel{df}{=} \{p \in \mathcal{A} \mid level^-_{M,t}(i) \in I(p, t)\}
\end{aligned}
$$

That is, for each state H_i we collect the names (becoming propositional atoms) of all those monadic predicates $p \in \mathcal{A}$ such that the element in D_h positioned at i belongs to the extension $I(p, h)$ of that predicate. The same applies for T_i with respect to world t. Finally, the mapping ρ is defined in the following way:

$$
\rho(i) \stackrel{df}{=} level_{M,t}(level^-_{M,h}(i))
$$

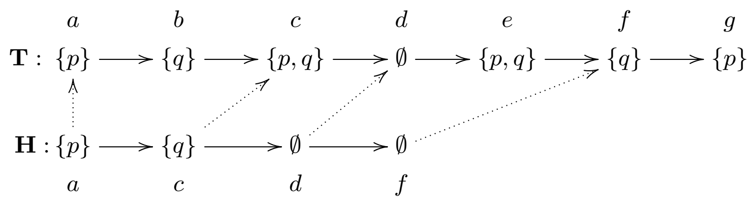

Fig. 2. HT-trace $f(M)$ for the interpretation M in Example 5.

Example 6. For instance, if we take the interpretation M from Example 5, then $f(M)$ corresponds to the HT-trace $\langle \mathbf{H}, \mathbf{T}, \rho \rangle$:

$$\begin{aligned}
H_0 &= \{x \in \mathcal{A} \mid a \in I(x,h)\} = \{p\} \\
H_1 &= \{x \in \mathcal{A} \mid c \in I(x,h)\} = \{q\} \\
H_2 &= \{x \in \mathcal{A} \mid d \in I(x,h)\} = \emptyset \\
H_3 &= \{x \in \mathcal{A} \mid f \in I(x,h)\} = \emptyset
\end{aligned}$$

$$\begin{aligned}
T_0 &= \{x \in \mathcal{A} \mid a \in I(x,h)\} = \{p\} \\
T_1 &= \{x \in \mathcal{A} \mid b \in I(x,h)\} = \{q\} \\
T_2 &= \{x \in \mathcal{A} \mid c \in I(x,h)\} = \{p,q\} \\
T_3 &= \{x \in \mathcal{A} \mid d \in I(x,h)\} = \emptyset \\
T_4 &= \{x \in \mathcal{A} \mid e \in I(x,h)\} = \{p,q\} \\
T_5 &= \{x \in \mathcal{A} \mid f \in I(x,h)\} = \{q\} \\
T_6 &= \{x \in \mathcal{A} \mid g \in I(x,h)\} = \{p\}
\end{aligned}$$

with the mapping:

$$\begin{aligned}
\rho(0) &= level_{M,t}(level^{-}_{M,h}(0)) = level_{M,t}(a) = 0 \\
\rho(1) &= level_{M,t}(level^{-}_{M,h}(1)) = level_{M,t}(c) = 2 \\
\rho(2) &= level_{M,t}(level^{-}_{M,h}(2)) = level_{M,t}(d) = 3 \\
\rho(3) &= level_{M,t}(level^{-}_{M,h}(3)) = level_{M,t}(f) = 5
\end{aligned}$$

To sum up, it can be easily observed that the HT-trace $f(M)$ we have obtained, which is shown in Figure 2 corresponds to the one shown in Figure 1.

The correspondence function $f(\cdot)$ is not injective: different interpretations $M \neq M'$ may produce the same HT-trace $f(M) = f(M')$. As an example of this situation, simply take M from Example 5 and another M' obtained by switching the roles of elements a and b everywhere in M. It is easy to see that the elements themselves are irrelevant, provided that we have the same number of them and keep their same roles in the domains and the order relations. On the other hand, function $f(\cdot)$ is surjective, as stated below.

Proposition 8. *Function $f(\cdot)$ is surjective: given any* HT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$ *there always exist some $QHT^{+}(\leq)$ interpretation M such that $f(M) = \mathbf{M}$.*

Proof. We proceed to build one possible M such that $f(M) = \mathbf{M}$. To this aim, the domains will be subsets of rational numbers $D_h \subseteq D_t \subseteq \mathbb{Q}$ being:

$$\begin{aligned} D_h &:= \big[0..\lambda_h\big) \\ D_t &:= D_h \cup \{\ \rho(i) + (1/k)\ \mid\ i, i+1 \in D_h,\ k \in \big[2..\rho(i+1) - \rho(i)\big]\ \} \end{aligned}$$

and we take $\leq_h^M$ and $\leq_t^M$ to be the standard ordering for rationals $\mathbb{Q}$. As we can see, for D_h we just take the integers in the interval $\big[0..\lambda_h\big)$ with the standard ordering, so $level_{M,h}(i) = i$. For D_t, the idea is to include all the integer numbers in D_h and, between each pair i and $i+1$, add to i as many fractions like $1/2$, $1/3$, $1/4, \dots$ as needed to keep the same distance as from $\rho(i)$ to $\rho(i+1)$. Doing this, we may also observe that $level_{M,t}(i) = \rho(i)$. For instance, if $\rho(3) = 6$ and $\rho(4) = 10$, we have $k \in \big[2..(10-6)\big] = \big[2..4\big]$ and we in D_t the three fractions:

$$3 + 1/2 = 7/2\ \geq\ 3 + 1/3 = 10/3\ \geq\ 3 + 1/4 = 13/4$$

all of them between 3 and 4. Starting with $level_{M,t}(3) = 6$, we then can see that $level_{M,t}(13/4) = 7$, $level_{M,t}(10/3) = 8$, $level_{M,t}(7/2) = 9$ and $level_{M,t}(4) = 10$. Finally, we define the extensions $I(p, w)$ of monadic predicates as follows:

$$\begin{aligned} I(p,h) &\stackrel{df}{=} \{i \mid p \in H_i, i \in \big[0..\lambda_h\big)\} \\ I(p,t) &\stackrel{df}{=} \{level^{-}_{M,t}(i) \mid p \in T_i, i \in \big[0..\lambda_t\big)\} \end{aligned}$$

From here, it is then routine to check that $f(M) = \mathbf{M}$. $\square$

Thus, the correspondence between HT-traces and $QHT^{+}(\leq)$ is actually one-to-many, and we can partition the set of $QHT^{+}(\leq)$ interpretations into equivalence classes, each one consisting of the set of interpretations M that map to the same HT-trace $f(M)$.

We eventually consider Kamp's translation of linear temporal logic into first-order logic.

Definition 6 (Kamp's translation). *Given a temporal formula φ, we define its translation $\tau(\varphi)_x$ with respect to some given domain element $x \in D_t$ into a first-order formula as follows:*

$$\begin{aligned} \tau(\bot)_x &\stackrel{df}{=} \bot \\ \tau(p)_x &\stackrel{df}{=} p(x) \\ \tau(\alpha \otimes \beta)_x &\stackrel{df}{=} \tau(\alpha)_x \otimes \tau(\beta)_x \quad \textit{for all} \otimes \in \{\wedge, \vee, \rightarrow\} \\ \tau(\circ\alpha)_x &\stackrel{df}{=} \exists y(succ(x,y) \wedge \tau(\alpha)_y) \\ \tau(\widehat{\circ}\alpha)_x &\stackrel{df}{=} \forall y(succ(x,y) \rightarrow \tau(\alpha)_y) \\ \tau(\alpha\, \mathbf{U}\, \beta)_x &\stackrel{df}{=} \exists y(x \leq y \wedge \tau(\beta)_y \wedge \forall z(x \leq z \wedge z < y \rightarrow \tau(\alpha)_z)) \\ \tau(\alpha\, \mathbf{R}\, \beta)_x &\stackrel{df}{=} \forall y(x \leq y \rightarrow \tau(\beta)_y \vee \exists z(x \leq z \wedge z < y \wedge \tau(\alpha)_z)) \end{aligned}$$

The main result of this section is then that Kamp's translation works beyond LTL also for bTHT and *a fortiori*, for THT as well.

Theorem 2 (Soundness of Kamp's translation). *For every $QHT^+(\leq)$ interpretation $M = (D_h, D_t, I, \sigma)$, $f(M) = \langle \mathbf{H}, \mathbf{T}, \rho \rangle$ and every $i \in [0..\lambda_h)$:=, it holds that*

$$f(M), i \models \varphi \textit{ in } \mathrm{THT} \textit{ iff } M, h \models \tau(\varphi)_d$$

where $d = level^-_{M,h}(i)$.

Proof. Note that $d = level^-_{M,h}(i)$ iff $i = level_{M,h}(d)$. We proceed by structural induction on φ.

- If $\varphi = \bot$ then trivially $f(M), i \not\models \bot$ and $M, h \not\models \tau(\bot)_d = \bot$
- If $\varphi = p \in \mathcal{A}$ is some atom, then $\mathbf{M}, h \models \tau(p)_d = p(d)$ with $d = level^-_{M,h}(i)$, that is, $d \in I(p, h)$ and $level_{M,h}(d) = i$. By definition of $f(M)$ this is equivalent to $p \in H_i$ and so, to $f(M), i \models p$.
- If $\varphi = \alpha \wedge \beta$ or $\varphi = \alpha \vee \beta$ it directly follows from induction on subformulas
- If $\varphi = \alpha \to \beta$ then:

$$\begin{aligned}
M, h \models \tau(\alpha \to \beta)_d &\Leftrightarrow M, h \models \tau(\alpha)_d \to \tau(\beta)_d \\
&\Leftrightarrow \quad M, h \not\models \tau(\alpha)_d \text{ or } M, h \models \tau(\beta)_d \\
&\quad\ \text{and } M, t \not\models \tau(\alpha)_d \text{ or } M, t \models \tau(\beta)_d \\
&\Leftrightarrow \text{(by induction)} \\
&\quad\quad f(M), i \not\models \alpha \text{ or } f(M), i \models \beta \\
&\quad\ \text{and } f(M^t), \rho(i) \not\models \alpha \text{ or } f(M^t), \rho(i) \models \beta \\
&\Leftrightarrow \quad f(M), i \not\models \alpha \text{ or } f(M), i \models \beta \\
&\quad\ \text{and } \mathbf{T}, \rho(i) \not\models \alpha \text{ or } \mathbf{T}, \rho(i) \models \beta \\
&\Leftrightarrow f(M), i \models \alpha \to \beta
\end{aligned}$$

- If $\varphi = \circ\alpha$ then:

$$\begin{aligned}
M, h \models \tau(\circ\alpha)_d &\Leftrightarrow M, h \models \exists y(succ(d, y) \wedge \tau(\alpha)_y) \\
&\Leftrightarrow \text{(by induction)} \\
&\quad\ \text{there exists } j \in D_h, j = i + 1, f(M), j \models \alpha \text{ with } j = level_{M,h}(y) \\
&\Leftrightarrow i + 1 < \lambda_h, f(M), i + 1 \models \alpha \\
&\Leftrightarrow f(M), i \models \circ\alpha
\end{aligned}$$

- If $\varphi = \alpha \,\mathbf{U}\, \beta$ then:

$$\begin{aligned}
&M, h \models \tau(\alpha \,\mathbf{U}\, \beta)_d \\
&\quad\Leftrightarrow M, h \models \exists y(d \leq y \wedge \tau(\beta)_y \wedge \forall z(d \leq z \wedge z < y \to \tau(\alpha)_z)) \\
&\quad\Leftrightarrow \text{there is } y \in D_h, d \leq^M_h y, M, h \models \tau(\beta)_y \\
&\quad\quad\ \text{and for all } z \in D_h, \text{ s.t. } d \leq^M_h z \text{ and } z <^M_h y \text{ implies } M, h \models \tau(\alpha)_z \\
&\quad\quad\ \text{and for all } z \in D_h, \text{ s.t. } d \leq^M_t z \text{ and } z <^M_t y \text{ implies } M, t \models \tau(\alpha)_z \\
&\quad\quad\ \text{and for all } z \in D_t, \text{ s.t. } d \leq^M_t z \text{ and } z <^M_t y \text{ implies } M, t \models \tau(\alpha)_z \\
&\quad\quad\ \text{(the last `for all' implies the second one, that can be removed)}
\end{aligned}$$

$\Leftrightarrow$ (by induction)
there is $y \in D_h, d \leq_h^M y, f(M), j \models \beta$, with $j = level_{M,h}(y)$
and for all $z \in D_h,$ s.t. $d \leq_h^M z$ and $z <_h^M y$ implies $f(M), k \models \alpha$,
with $k = level_{M,h}(z)$
and for all $z \in D_t,$ s.t. $d \leq_t^M z$ and $z <_t^M y$ implies $f(M^t), k \models \alpha$,
with $k = level_{M,t}(z)$

$\Leftrightarrow$ there is $j \in \big[0..\lambda_h\big), i \leq j, f(M), j \models \beta$
and for all $k \in \big[0..\lambda_h\big),$ s.t. $i \leq k$ and $k < j$ implies $f(M), k \models \alpha$
and for all $k \in \big[0..\lambda_t\big),$ s.t. $\rho(i) \leq k$ and $k < \rho(j)$ implies $f(M^t), k \models \alpha$
$\Leftrightarrow$ there is $j \in \big[0..\lambda_h\big), i \leq j, f(M), j \models \beta$
and for all $k \in \big[0..\lambda_h\big),$ s.t. $i \leq k$ and $k < j$ implies $f(M), k \models \alpha$
and for all $k \in \big[0..\lambda_t\big),$ s.t. $\rho(i) \leq k$ and $k < \rho(j)$ implies $f(M^t), k \models \alpha$

$\Leftrightarrow$ there exists $j \in \big[i..\lambda_h\big)$ such that $f(M), j \models \beta$
and for all $k \in \big[i..j\big), f(M), k \models \alpha$
and for all $k \in \big[\rho(i)..\rho(j)\big), \mathbf{T}, k \models \alpha$

$\Leftrightarrow f(M), i \models \alpha\, \mathbf{U}\, \beta$

– If $\varphi = \alpha\, \mathbf{R}\, \beta$ then:

$M, h \models \tau(\alpha\, \mathbf{R}\, \beta)_d$
$\Leftrightarrow M, h \models \forall y(d \leq y \rightarrow \tau(\beta)_y \vee \exists z(d \leq z \wedge z < y \wedge \tau(\alpha)_z))$
$\Leftrightarrow$ for all $y \in D_h, d \leq_h^M y$ implies :
$M, h \models \tau(\beta)_y$ or there exists $z \in D_h,$ s.t. $d \leq_h^M z$ and $z <_h^M y$ and
$M, h \models \tau(\alpha)_z$and for all $y \in D_t, d \leq_t^M y$ implies :
$M, t \models \tau(\beta)_y$ or there exists $z \in D_t,$ s.t. $d \leq_t^M z$ and $z <_t^M y$ and
$M, t \models \tau(\alpha)_z$

$\Leftrightarrow$ (by induction)
for all $y \in D_h, d \leq_h^M y$ implies :
$f(M), j \models \beta$ or there exists $z \in D_h,$ s.t. $d \leq_h^M z$ and $z <_h^M y$ and
$f(M), k \models \alpha$with $j = level_{M,h}(y)$ and $k = level_{M,h}(z)$
and for all $y \in D_t, d \leq_t^M y$ implies :
$f(M^t), j \models \beta$ or there exists $z \in D_t,$ s.t. $d \leq_t^M z$ and $z <_t^M y$ and
$f(M^t), k \models \alpha$with $j = level_{M,t}(y)$ and $k = level_{M,t}(z)$

$\Leftrightarrow$ for all $j \in \big[0..\lambda_h\big), i \leq j$ implies :
$f(M), j \models \beta$ or there exists $k \in \big[0..\lambda_h\big)$ s.t. $i \leq k$ and $k < j$ and
$f(M), k \models \alpha$ and for all $j \in \big[0..\lambda_t\big), \rho(i) \leq j$ implies :
$f(M^t), j \models \beta$ or there exists $k \in \big[0..\lambda_t\big)$ s.t. $\rho(i) \leq k$ and
$k < j$ and $f(M^t), k \models \alpha$

$\Leftrightarrow$ for all $j \in \big[i..\lambda_h\big)$:
$f(M), j \models \beta$ or there exists $k \in \big[i..j\big)$ and $f(M), k \models \alpha$
and for all $j \in \big[\rho(i)..\lambda_t\big)$:
$\mathbf{T}, j \models \beta$ or there exists $k \in \big[\rho(i)..j\big)$ and $\mathbf{T}, k \models \alpha$
$\Leftrightarrow f(M), i \models \alpha\, \mathbf{R}\, \beta$

5 Conclusion

In this paper, we have introduced brief THT (bTHT) as a new variant of temporal Here-and-There logic that aims at shortening traces by disregarding states and gives rise to bTELas a restriction of temporal equilibrium logic (TEL). Furthermore, we have shown that Kamp's translation of LTL into first-order logic can be applied to bTHT as well, and thus also to THT.

Our ongoing work aims to elucidate the relationship between bTHT and contracted THT (cTHT) in depth, es well es of the ensuing brief TEL and contracted TEL logics. As we have seen, bTHT avoids some irregularities of cTHT, but a more comprehensive picture of its semantics properties ought to be drawn. Likewise, the computational properties and algorithms for efficient evaluation remain to be investigated. Furthermore, the potential use of Kamp's translation in that but also in other contexts reemains to be explored.

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