Solutions of Question On Linear Equations

1. $\frac{112x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}$, $x \in N$

$\Rightarrow \frac{112x}{5} \geq \frac{9-3x+6}{8}$

$\frac{112x}{5} \geq \frac{15-3x}{8}$

$\Rightarrow 896x \geq 75 - 15x$

$\Rightarrow 911x \geq 75$

$\Rightarrow x \geq \frac{75}{911}$

$\therefore x = 1, 2, 3, 4, \ldots$ $(\because x \in N)$

2. $-\frac{1}{3} \leq \frac{3x-8}{6} < 1\frac{1}{6}$, $x \in R$

$\Rightarrow -\frac{1}{3} \leq \frac{3x-8}{6} < \frac{7}{6}$

$\Rightarrow -2 \leq 3x-8 < 7$ {Multiplying by 7}

$\Rightarrow -2+8 \leq 3x-8+8 < 7+8$

$\Rightarrow 6 \leq 3x < 15$

$\Rightarrow \frac{6}{3} \leq \frac{3x}{3} < \frac{15}{3}$ {dividing by 3}

$\Rightarrow \boxed{2 \leq x < 5}$

3. Let no. of rows = x

and no. of seats in each row = y, then $\{x = y$, initially$\}$

no. of total seats in auditorium = xy

Now, Later no. of rows = $2x$

and no. of seats in each row = $(y-5)$

No. of total seats in auditorium = $2x(y-5)$

According to question,

$$2x(y-5) = xy + 375$$

$$\Rightarrow \quad 2xy - 10x = xy + 375$$

$$\Rightarrow \quad xy - 10x - 375 = 0$$

$$\Rightarrow \quad x^2 - 10x - 375 = 0 \quad \{\because x = y\}$$

$$\Rightarrow \quad x^2 - 25x + 15x - 375 = 0$$

$$\Rightarrow \quad (x-25)(x+15) = 0$$

$$\Rightarrow \quad x = 25 \text{ or } x = -15$$

(not possible)

$\therefore$ $\boxed{x = 25}$

Thus, 25 rows were there in the beginning.

4. $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$, $x \in R$

$\Rightarrow -\frac{1}{3} \leq \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$

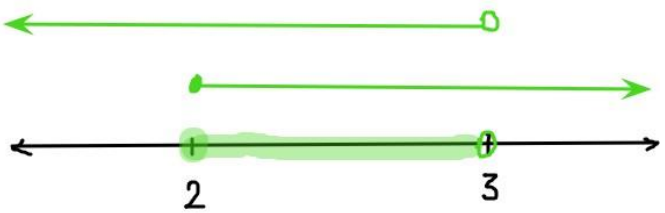
$\Rightarrow -\frac{1}{3} + \frac{4}{3} \leq \frac{x}{2} - \frac{4}{3} + \frac{4}{3} < \frac{1}{6} + \frac{4}{3}$

$\Rightarrow \frac{3}{3} \leq \frac{x}{2} < \frac{9}{6}$

$\Rightarrow 1 \leq \frac{x}{2} < \frac{3}{2}$

$\Rightarrow 2 \leq 2 \times \frac{x}{2} < 2 \times \frac{3}{2}$

$\Rightarrow \boxed{2 \leq x < 3}$



5. Let shopkeeper buys 'x' no. of books and price per book be 'y' ₹, then

$$xy = 80 \quad \text{—(1)}$$

Later,

$$(x+4)(y-1) = 80 \quad \text{—(2)}$$

$$\Rightarrow xy - x + 4y - 4 = 80$$

$$\Rightarrow \cancel{80} - x + 4y - 4 = \cancel{80}$$

$$\Rightarrow x - 4y + 4 = 0$$

$$\Rightarrow x - 4\left(\frac{80}{x}\right) + 4 = 0$$

$$\Rightarrow x^2 - 320 + 4x = 0$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow (x+20)(x-16) = 0$$

$$\Rightarrow x = -20, \quad x = 16$$

(Not possible)

$\therefore \boxed{x = 16}$

Thus, he buys 16 books.

6. $2x - 3 < x + 2 \leq 3x + 5$, $x \in W$

$\Rightarrow$ $2x - 3 < x + 2$ and $x + 2 \leq 3x + 5$

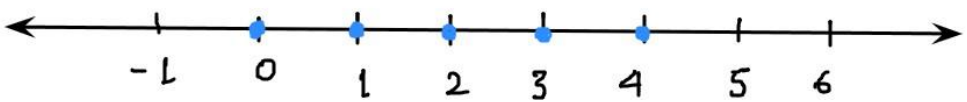
$\Rightarrow$ $2x - x < 2 + 3$ and $2 - 5 \leq 3x - x$

$\Rightarrow$ $\boxed{x < 5}$ and $-1 \leq 2x$

$2x \geq -1$

$\boxed{x \geq \frac{-1}{2}}$

$\therefore x \in W$ (Whole Numbers)



7. $13x - 6 < 15x + 4 < 7x + 12$, $x \in I$

$\Rightarrow$ $13x - 6 < 15x + 4$ and $15x + 4 < 7x + 12$

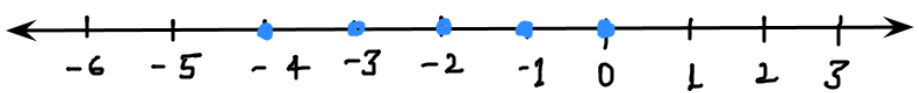
$\Rightarrow$ $-6 - 4 < 15x - 13x$ and $15x - 7x < 12 - 4$

$\Rightarrow$ $-10 < 2x$ and $8x < 8$

$\Rightarrow$ $2x > -10$ and $\boxed{x < 1}$

$\Rightarrow$ $\boxed{x > -5}$

$\boxed{-5 < x < 1}$



8. $M = \{x : 5 < 2x-1 \leq 11,\ x \in I\}$

$\Rightarrow \quad 5 < 2x-1 \leq 11$

$\Rightarrow \quad 5+1 < 2x-1+1 \leq 11+1$

$\Rightarrow \quad 6 < 2x < 12$

$\Rightarrow \quad \frac{6}{2} < \frac{2x}{2} < \frac{12}{2}$

$\Rightarrow \quad \boxed{3 < x \leq 6}$

$\Rightarrow \quad x = 4, 5, 6$

$\therefore M = \{4, 5, 6\}$

$N = \{x : -1 \leq 3 + 4x < 23,\ x \in I\}$

$\Rightarrow \quad -1 \leq 3+4x < 23$

$\Rightarrow \quad -1-3 \leq 3+4x-3 < 23-3$

$\Rightarrow \quad -4 \leq 4x < 20$

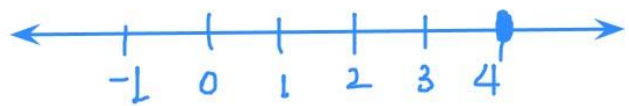
$\Rightarrow \quad \frac{-4}{4} \leq \frac{4x}{4} < \frac{20}{4}$

$\Rightarrow \quad \boxed{-1 \leq x < 5}$

$\Rightarrow \quad x = -1, 0, 1, 2, 3, 4$

$\therefore N = \{-1, 0, 1, 2, 3, 4\}$

$\therefore M \cap N = \{4\}$



9.

Initially, no. of children $= x$

Amount to be divided $= ₹\ 480$

Share of each children $= \left(\frac{480}{x}\right)$ ₹

Then,

no. of children $= (x+20)$

Share of each children $= \left(\frac{480}{x+20}\right)$ ₹

According to question,

$$\frac{480}{x+20} = \frac{480}{x} - 12$$

$$\Rightarrow \frac{480}{x+20} = \frac{480-12x}{x}$$

$$\Rightarrow 480x = (x+20)(480-12x)$$

$$\Rightarrow 480x = 480x - 12x^2 + 9600 - 240x$$

$$\Rightarrow 12x^2 + 240x - 9600 = 0$$

$$\Rightarrow x^2 + 20x - 800 = 0$$

$$\Rightarrow x^2 + 40x - 20x - 800 = 0$$

$$\Rightarrow (x+40)(x-20) = 0$$

$$\Rightarrow x = -40, \quad x = 20$$

(Not possible) $\therefore \boxed{x = 20}$

10. Given eq. is

$$x^2 - px - 4 = 0 \quad ,$$

$\because$ $x = -4$ is a root of above eq.

$\therefore$ $(-4)^2 - p(-4) - 4 = 0$

$\Rightarrow$ $16 + 4p - 4 = 0$

$\Rightarrow$ $4p + 12 = 0$

$\Rightarrow$ $4p = -12$

$\Rightarrow$ $\boxed{p = -3}$

Another eq. is

$$x^2 + px - k = 0$$

$\Rightarrow$ $x^2 - 3x - k = 0$

{ In question change x to k in third term. }

For equal roots,

$D = 0$ (Discriminant $= 0$)

$\Rightarrow$ $(-3)^2 - 4 \times 1 \times (-k) = 0$

$\Rightarrow$ $9 + 4k = 0$

$\Rightarrow$ $4k = -9$

$\Rightarrow$ $\boxed{k = \frac{-9}{4}}$