

Solutions of Question on Linear Equations

$$\underline{1.} \quad \frac{112x}{5} \geq \frac{9-3x}{8} + \frac{3}{4}, \quad x \in \mathbb{N}$$

$$\Rightarrow \frac{112x}{5} \geq \frac{9-3x+6}{8}$$

$$\frac{112x}{5} \geq \frac{15-3x}{8}$$

$$\Rightarrow 896x \geq 75 - 15x$$

$$\Rightarrow 911x \geq 75$$

$$\Rightarrow x \geq \frac{75}{911}$$

$$\therefore x = 1, 2, 3, 4, \dots \quad (\because x \in \mathbb{N})$$



2. $-\frac{1}{3} \leq \frac{3x-8}{6} < 1\frac{1}{6}, \quad x \in \mathbb{R}$

$$\Rightarrow -\frac{1}{3} \leq \frac{3x-8}{6} < \frac{7}{6}$$

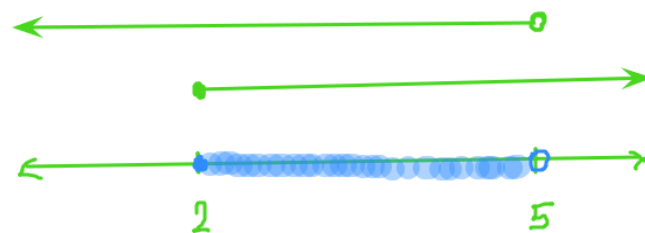
$$\Rightarrow -2 \leq 3x-8 < 7 \quad \left\{ \text{multiplying by 6} \right\}$$

$$\Rightarrow -2+8 \leq 3x-8+8 < 7+8$$

$$\Rightarrow 6 \leq 3x < 15$$

$$\Rightarrow \frac{6}{3} \leq \frac{3x}{3} < \frac{15}{3} \quad \left\{ \text{dividing by 3} \right\}$$

$$\Rightarrow \boxed{2 \leq x < 5}$$



3. Let no. of rows = x

and no. of seats in each row = y , then $\{x = y, \text{ initially}\}$

no. of total seats in auditorium = xy

Now, later no. of rows = $2x$

and no. of seats in each row = $(y - 5)$

No. of total seats in auditorium = $2x(y - 5)$

According to question,

$$2x(y - 5) = xy + 375$$

$$\Rightarrow 2xy - 10x = xy + 375$$

$$\Rightarrow 2y - 10x - 375 = 0$$

$$\Rightarrow x^2 - 10x - 375 = 0 \quad \left\{ \because x = y \right\}$$

$$\Rightarrow x^2 - 25x + 15x - 375 = 0$$

$$\Rightarrow (x - 25)(x + 15) = 0$$

$$\Rightarrow x = 25 \text{ or } x = -15$$

(not possible)

$$\therefore \boxed{x = 25}$$

Thus, 25 rows were there in the beginning.

$$\underline{4.} \quad -\frac{1}{3} \leq \frac{x}{2} - \frac{1}{3} < \frac{1}{6}, \quad x \in \mathbb{R}$$

$$\Rightarrow -\frac{1}{3} \leq \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

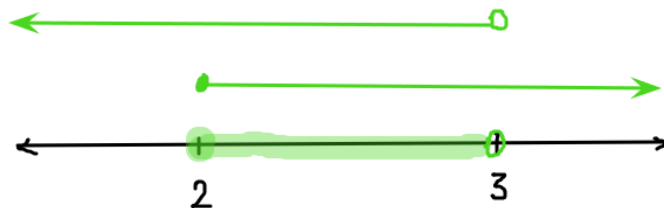
$$\Rightarrow -\frac{1}{3} + \frac{4}{3} \leq \frac{x}{2} - \frac{4}{3} + \frac{4}{3} < \frac{1}{6} + \frac{4}{3}$$

$$\Rightarrow \frac{3}{3} \leq \frac{x}{2} < \frac{9}{6}$$

$$\Rightarrow 1 \leq \frac{x}{2} < \frac{3}{2}$$

$$\Rightarrow 2 \leq 2 \times \frac{x}{2} < 2 \times \frac{3}{2}$$

$$\Rightarrow \boxed{2 \leq x < 3}$$



5. let shopkeeper buys 'x' no. of books and price per book be 'y' ₹, then

$$xy = 80 \quad \text{--- (1)}$$

later,

$$(x+4)(y-1) = 80 \quad \text{--- (2)}$$

$$\Rightarrow xy - x + 4y - 4 = 80$$

$$\Rightarrow \cancel{80} - x + 4y - 4 = \cancel{80}$$

$$\Rightarrow x - 4y + 4 = 0$$

$$\Rightarrow x - 4\left(\frac{80}{x}\right) + 4 = 0$$

$$\Rightarrow x^2 - 320 + 4x = 0$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow (x+20)(x-16) = 0$$

$$\Rightarrow x = -20, \quad x = 16$$

(Not possible)

$$\therefore \boxed{x = 16}$$

Thus, he buys 16 books.

6. $2x - 3 < x + 2 \leq 3x + 5$, $x \in \mathbb{W}$

$$\Rightarrow 2x - 3 < x + 2 \quad \text{and} \quad x + 2 \leq 3x + 5$$

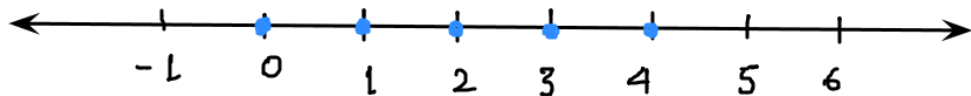
$$\Rightarrow 2x - x < 2 + 3 \quad \text{and} \quad 2 - 5 \leq 3x - x$$

$$\Rightarrow \boxed{x < 5} \quad \text{and} \quad -1 \leq 2x$$

$$2x \geq -1$$

$$\boxed{x \geq -\frac{1}{2}}$$

$\therefore x \in \mathbb{W}$ (Whole Numbers)



7. $13x - 6 < 15x + 4 < 7x + 12, x \in \mathbb{I}$

$$\Rightarrow 13x - 6 < 15x + 4 \quad \text{and} \quad 15x + 4 < 7x + 12$$

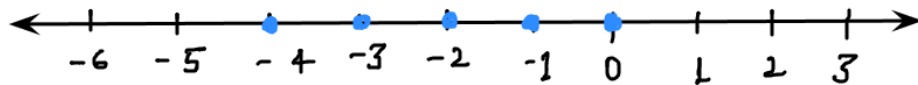
$$\Rightarrow -6 - 4 < 15x - 13x \quad \text{and} \quad 15x - 7x < 12 - 4$$

$$\Rightarrow -10 < 2x \quad \text{and} \quad 8x < 8$$

$$\Rightarrow 2x > -10 \quad \text{and} \quad \boxed{x < 1}$$

$$\Rightarrow \boxed{x > -5}$$

$$\boxed{-5 < x < 1}$$



8.

$$M = \{x: 5 < 2x-1 \leq 11, x \in \mathbb{I}\}$$

$$\Rightarrow 5 < 2x-1 \leq 11$$

$$\Rightarrow 5+1 < 2x-1+1 \leq 11+1$$

$$\Rightarrow 6 < 2x < 12$$

$$\Rightarrow \frac{6}{2} < \frac{2x}{2} < \frac{12}{2}$$

$$\Rightarrow \boxed{3 < x \leq 6}$$

$$\Rightarrow x = 4, 5, 6$$

$$\therefore M = \{4, 5, 6\}$$

$$N = \{x: -1 \leq 3+4x < 23, x \in \mathbb{I}\}$$

$$\Rightarrow -1 \leq 3+4x < 23$$

$$\Rightarrow -1-3 \leq 3+4x-3 < 23-3$$

$$\Rightarrow -4 \leq 4x < 20$$

$$\Rightarrow \frac{-4}{4} \leq \frac{4x}{4} < \frac{20}{4}$$

$$\Rightarrow \boxed{-1 \leq x < 5}$$

$$\Rightarrow x = -1, 0, 1, 2, 3, 4$$

$$\therefore N = \{-1, 0, 1, 2, 3, 4\}$$

$$\therefore M \cap N = \{4\}$$



9.

Initially, no. of children = x
Amount to be divided = ₹ 480

$$\text{Share of each child} = \left(\frac{480}{x} \right) \text{ ₹}$$

Then, no. of children = $(x+20)$
Share of each child = $\left(\frac{480}{x+20} \right) \text{ ₹}$

According to question,

$$\frac{480}{x+20} = \frac{480}{x} - 12$$

$$\Rightarrow \frac{480}{x+20} = \frac{480 - 12x}{x}$$

$$\Rightarrow 480x = (x+20)(480 - 12x)$$

$$\Rightarrow \cancel{480x} = \cancel{480x} - 12x^2 + 9600 - 240x$$

$$\Rightarrow 12x^2 + 240x - 9600 = 0$$

$$\Rightarrow x^2 + 20x - 800 = 0$$

$$\Rightarrow x^2 + 40x - 20x - 800 = 0$$

$$\Rightarrow (x+40)(x-20) = 0$$

$$\Rightarrow x = -40, x = 20$$

(Not possible) $\therefore x = 20$

10.

Given eq. is

$$x^2 - px - 4 = 0$$

$\therefore x = -4$ is a root of above eq.

$$\therefore (-4)^2 - p(-4) - 4 = 0$$

$$\Rightarrow 16 + 4p - 4 = 0$$

$$\Rightarrow 4p + 12 = 0$$

$$\Rightarrow 4p = -12$$

$$\Rightarrow \boxed{p = -3}$$

Another eq. is

$$x^2 + px - k = 0$$

$$\Rightarrow x^2 - 3x - k = 0$$

for equal roots,

$$D = 0 \quad (\text{Discriminant} = 0)$$

$$\Rightarrow (-3)^2 - 4 \times 1 \times (-k) = 0$$

$$\Rightarrow 9 + 4k = 0$$

$$\Rightarrow 4k = -9$$

$$\Rightarrow \boxed{k = -\frac{9}{4}}$$

In question change x to k in third term.