Theorems: Group Exercises

CSCI 246

January 21, 2026

Problem 1. Translate each of the following statements into the if-then form.

A. Every integer is even or odd.

Ans. If x is an Integer, then x is *even* or x is *odd*.

B. The sum of two even integers is even.

Ans. If integers x and y are *even*, then $x + y$ is *even*.

C. Every blue car looks cool.

Ans. If a car is blue, then the car looks cool.

D. Every non-negative power of an odd number is odd.

Ans. If $0 \leq x$ is a non-negative integer and $y \in \mathbb{Z}$ is *odd*, then y^x is *odd*.

E. Every positive power of an even number is even.

Ans. If $0 < x$ is a positive integer and $y \in \mathbb{Z}$ is *even*, then y^x is *even*.

Problem 2. For every pair of statements A and B, determine if the following statements:

1. if A, then B,
2. if B, then A,
3. A if and only if B,

are *always* true, *always* false, or *sometimes* true and *sometimes* false.

A. For $x, y \in \mathbb{Z}$. A: $x < y$. B: $x + 1 \leq y$.

1. *always* true 2. *always* true 3. *always* true

B. A: Polygon $PQRS$ is a rectangle. B: Polygon $PQRS$ is a square.

1. *sometimes* true 2. *always* true 3. *sometimes* true

C. For $x, y \in \mathbb{R}$. A: $x < y$. B: $x < \frac{x+y}{2} < y$.

1. *always* true 2. *always* true 3. *always* true

D. For $x \in \mathbb{R}$. A: $x > 0$. B: $x^2 > 0$.

1. *always* true 2. *sometimes* true 3. *sometimes* true

E. A: James lives in Bozeman. B: James lives in Montana.

1. *always* true 2. *sometimes* true 3. *sometimes* true

F. A: Ellen earned at least 93% in this class. B: Ellen earned an A in this class.

1. *always* true 2. *always* true 3. *always* true

G. A: Lines l_1 and l_2 are parallel. B: Lines l_1 and l_2 are orthogonal.

1. *always* false 2. *always* false 3. *always* false

Problem 3. Show that the following pair of statements are equivalent via truth tables.

A. A: If P, then Q
B: If (not Q), then (not P)

P	Q	A	B
false	false	true	true
false	true	true	true
true	false	false	false
true	true	true	true

B. A: P iff Q
B: (not P) iff (not Q)

P	Q	A	B
false	false	true	true
false	true	false	false
true	false	false	false
true	true	true	true

C. A: if P, then (if Q, then R) B: If (P and Q), then R

P	Q	R	A	B
false	false	false	true	true
false	false	true	true	true
false	true	false	true	true
false	true	true	true	true
true	false	false	true	true
true	false	true	true	true
true	true	false	false	false
true	true	true	true	true

Problem 4. Come up with a conjecture for the following examples:

A. What can you say about the sum of the sum of consecutive odd numbers starting from 1?
That is 1, $1+3$, $1+3+5$, $1+3+5+7$, ...

Ans. If you let n be the number of consecutive odd numbers added, then the sum is equal to n^2.

B. Consider a sequence of n doors. Doors can be open or closed. There are n people numbered 1 to n.

- The first person will close every door.
- The second person will flip every even numbered door (i.e., close opened doors and open closed doors). Now, every even numbered door will be open.
- The third will flip every door whose number is divisible by 3.
- The process continues until person n flips door n (the only door whose number is divisible by n).

What can you say about the sequence of numbers corresponding to the doors that are left open?

The sequence of numbers on the doors that are open corresponds exactly to the sequence of prime numbers less than or equal to n.