Multi-Agent Systems

Final Homework Assignment

MSc AI, VU

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IMPORTANT

- This project is an **individual assignment.** So everyone should hand in their own copy. Instructions on how to upload your solution paper will follow.
- This assignment will be **graded.** The max score is 4 and will count towards your final grade.
- Your **final grade (on 10)** will be computed as follows:

  Assignments 1 thu 5 (max 1) + Individual assignment (max 4) + Final exam (max 5)

  During the lectures and lab-sessions in the last lecture week (9-13 Dec), there will be an opportunity to get clarification on any aspect of this homework that seems unclear to you.

1 Multi-Armed Bandits

1.1 Thompson Sampling for Single bandit

Consider a bandit that for each pull of the arm, produces a binary reward: $r = 1$ (with probability p) or $r = 0$ (with probability $1-p$). We model our uncertainty about the actual (but unknown) value p using a beta-distribution (cf. `https://en.wikipedia.org/wiki/Beta_distribution`). This is a probability distribution on the interval $[0, 1]$ which depends on two parameters: $\alpha, \beta \geq 1$. The explicit distribution is given by (for α, β integers!):

$$B(x;\,\alpha,\beta) = \frac{(\alpha+\beta-1)!}{(\alpha-1)!\,(\beta-1)!} x^{\alpha-1}(1-x)^{\beta-1} \qquad (\text{for} 0 \leq x \leq 1).$$

The parameters α and β determine the shape of the distribution:

- If $\alpha = \beta = 1$ then we have the uniform distribution;
- If $\alpha = \beta$ the distribution is symmetric about $x = 1/2$.

- If $\alpha > \beta$ the density is right-leaning (i.e. concentrated in the neighbourhood of 1). In fact, one can compute the mean explicitly:

$$X \sim B(x;\, \alpha, \beta) \quad \Longrightarrow \quad EX = \frac{\alpha}{\alpha + \beta}.$$

- Larger values of α and β produce a more peaked distribution. This follows from the formula for the variance:

$$X \sim B(x;\, \alpha, \beta) \quad \Longrightarrow \quad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

Thompson update rule Assume that we don't know the success probability p for the bandit. The Thompson update rule for a single bandit proceeds as follows:

- Initialise $\alpha = \beta = 1$ (resulting in a uniform distribution, indicating that all possible values for p are equally likely). Now repeat the following loop:
  1. Sample from the bandit and get reward r (either 1 or 0);
  2. Update the values for α and β as follows:

$$\alpha \leftarrow \alpha + r \qquad \beta \leftarrow \beta + (1 - r)$$

Questions

- Make several plots of the Beta-density to illustrate the properties (dependence on the parameters) outlined above;
- Implement the Thompson update rule and show experimentally that the Beta-density increasingly peaks at the correct value for p. Plot both the evolution of the mean and variance over (iteration)time.

Thompson sampling Suppose we have a 2-bandit problem. The first one delivers a reward $r = 1$ with (unknown!) probability p_1, while the second one does so with (unknown!) probability p_2. For each bandit ($k = 1, 2$), the uncertainty about the corresponding p_k is modelled using a Beta-distribution with coefficients α_k, β_k. Thompson sampling now tries to identify the bandit that delivers the maximal output and proceeds as follows:

- Initialise all parameters to 1: $\alpha_k = 1 = \beta_k$;
  Now repeat the following loop:
  1. Sample a value u_k from each of the two Beta-distributions:

$$u_k \sim B(x; \alpha_k, \beta_k)$$

  2. Determine the max: $k_m = \arg\max\{u_1, u_2\}$
  3. Sample the corresponding bandit and get reward r (either 1 or 0);
  4. Update the corresponding parameters: α_{k_m} and β_{k_m} as follows:

$$\alpha_{k_m} \leftarrow \alpha_{k_m} + r \qquad \text{and} \qquad \beta_{k_m} \leftarrow \beta_{k_m} + (1 - r)$$

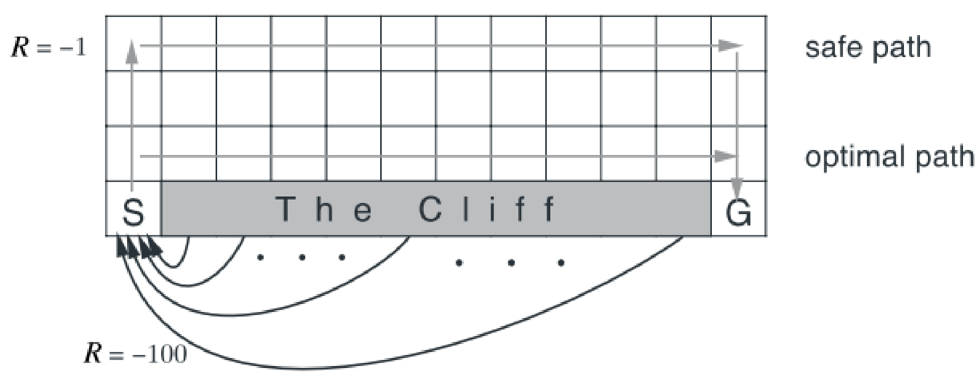
Questions

- Write code to implement Thompson sampling for the above scenario;
- Perform numerical experiments in which you compare Thompson sampling with the UCB and ϵ-greedy approach. Define some performance measures and discuss the performance of each algorithm with respect to these measures.

2 Reinforcement Learning: Cliff Walking

Consider the cliff-walking example (Sutton & Barto, ex. 6.6. p.108). Assume that the grid has 10 columns and 5 rows (above or in addition to the cliff). This is a standard undiscounted, episodic task, with start and goal states, and the usual actions causing movement up, down, right, and left. Reward is -1 on all transitions except:

- the transition to the terminal goal state (G) which has an associated reward of $+20$;
- transitions into the region marked *The Cliff*. Stepping into this region incurs a "reward" of -100 and also terminates the episode.



Questions

1. Use both SARSA and Q-Learning to construct an appropriate policy. Do you observe the difference between the SARSA and Q-learning policies mentioned in the text (safe versus optimal path)? Discuss.
2. Try different values for ϵ (parameter for ϵ-greedy policy). How does the value of ϵ influence the result? Discuss.