

AMMM Final Project

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Topological Sort (Kahn's algorithm)

Cyclic graphs don't have valid topological orderings.

ILP Model

Decision Variables

$Priority_{ij}$: Binary $\forall i, j \in Members$

$Order_n$: Integer $\forall n \in Members$

Objective Function

maximize $\sum_{i,j \in Members} Priority_{ij} \cdot m_{ij}$

Each node appears before all the nodes it points to

$Priority_{ij} = 1 \rightarrow Order_i < Order_j$

$Priority_{ij} = 0 \rightarrow Order_j < Order_i$

Constraints

$Priority_{ij} + Priority_{ji} = 1$

$\forall i, j \in Members \text{ and } m_{ij} + m_{ji} \neq 0 \quad (1)$

$Order_i + 1 \leq Order_j + (1 - Priority_{ij}) \cdot N$

$\forall i, j \in Members \text{ and } m_{ij} + m_{ji} \neq 0 \quad (2)$

Pseudo Code & Greedy Cost Function

Greedy Cost Function

$$q(<i,j>, S) = \begin{cases} -1 & \text{if } G(S, <i,j>) \text{ is not a directed acyclic graph} \\ Bids_{ij} & \text{if } G(S, <i,j>) \text{ is a directed acyclic graph} \end{cases}$$

Algorithm Greedy Method

```

Solution  $\leftarrow \phi$ 
Candidates  $\leftarrow \{<i,j> | i,j \in Members \text{ and } Bids_{ij} + Bids_{ji} \neq 0\}$ 
while Candidates  $\neq \phi$  do
    Evaluate  $q(<i',j'>, Solution)$   $\forall <i',j'> \in Candidates$ 
     $< i, j > \leftarrow argmax\{q(<i',j'>, Solution) | <i',j'> \in Candidates\}$ 
    Candidates  $\leftarrow Candidates \setminus \{<i,j>, <j,i>\}$ 
    Solution  $\leftarrow Solution \cup \{<i,j>\}$ 
return Solution

```

Greedy Procedure (data=project.4.dat)

Local Search Method

Target Edges

$\{<i,j> | <j,i> \in Solution \text{ and } Bids_{ij} > Bids_{ji}\}$

Neighborhood

Solutions obtained by swapping target edges not included in the solution with their flipped counterparts that are included in the solution.

Knockon Flipped Edges Cost Function

$q(<i',j'>, edges^{flip}) =$

$$\begin{cases} -1 & \text{if } Bids_{i'j'} - Bids_{j'i'} + \sum_{<i'',j''> \in edges^{flip}} Bids_{i''j''} - Bids_{j''i''} \geq Bids_{ij} - Bids_{ji} \\ & \text{or } edges^{residual} | edges^{flip} = edges^{residual} | edges^{flip} \cup \{<i',j'>\} \\ \sum_{k' \in \{i',j'\}} indegree(k') + outdegree(k') & \text{if } Bids_{i'j'} - Bids_{j'i'} + \sum_{<i'',j''> \in edges^{flip}} Bids_{i''j''} - Bids_{j''i''} < Bids_{ij} - Bids_{ji} \\ & \text{and } edges^{residual} | edges^{flip} > edges^{residual} | edges^{flip} \cup \{<i',j'>\} \end{cases}$$

Knockon Flipped Edges Selection Process

Pseudo Code

Algorithm Local Search Method

```

 $solution^{current} \leftarrow doGreedyConstructionPhase()$ 
 $improved \leftarrow \text{True}$ 
while  $improved = \text{True}$  do
     $improved \leftarrow \text{False}$ 
    for each  $< i, j > \in \{< i, j > | < j, i > \in solution^{current} \text{ and } bids_{ij} > bids_{ji}\}$  do
         $solution^{flip} \leftarrow solution^{current}$ 
        Swap  $< j, i >$  with  $< i, j >$  in  $solution^{flip}$ 
         $edges^{residual} \leftarrow doTopologicalSort(solution^{flip})$ 
         $edges^{candidate}, edges^{flip} \leftarrow edges^{residual}, \phi$ 
        while  $edges^{residual} \neq \phi$  do
            Evaluate  $q(< i', j' >, edges^{flip}) \quad \forall < i', j' > \in edges^{candidate}$ 
             $< i'', j'' > \leftarrow argmax\{q(< i', j' >, edges^{flip}) | < i', j' > \in edges^{candidate}\}$ 
            Swap  $< i'', j'' >$  with  $< j'', i'' >$  in  $edges^{residual}$ 
             $edges^{residual} \leftarrow doTopologicalSort(edges^{residual})$ 
             $edges^{candidate} \leftarrow edges^{residual}$ 
             $edges^{flip} \leftarrow edges^{flip} \cup \{< i'', j'' >\}$ 
        Swap  $< j', i' >$  with  $< i', j' >$  in  $solution^{flip} \quad \forall < i', j' > \in edges^{flip}$ 
        if  $G(solution^{flip})$  is a DAG then
             $improved \leftarrow \text{True}$ 
             $solution^{current} \leftarrow solution^{flip}$ 
    return  $solution^{current}$ 

```

Local Search Procedure (data=project.4.dat)

Pseudo Code & RCL

Algorithm GRASP Constructive Phase

```

solution  $\leftarrow \phi$ 
edgescandidate  $\leftarrow \{<i, j> | i, j \in Members \text{ and } Bids_{ij} + Bids_{ji} \neq 0\}$ 
while edgescandidate  $\neq \phi$  do
    Evaluate  $q(<i', j'>, solution) \forall <i', j'> \in edges^{candidate}$ 
    edgescandidate  $\leftarrow \{<i', j'> \in edges^{candidate} | q(<i', j'>, solution) > 0\}$ 
     $q^{min} \leftarrow \min\{q(<i', j'>, solution) | <i', j'> \in edges^{candidate}\}$ 
     $q^{max} \leftarrow \max\{q(<i', j'>, solution) | <i', j'> \in edges^{candidate}\}$  ▷ RCL Procedure
    RCLmax  $\leftarrow \{<i', j'> \in edges^{candidate} | q(<i', j'>, solution) \geq q^{max} - \alpha(q^{max} - q^{min})\}$ 
     $< i, j > \leftarrow \text{select } < i', j'> \in \text{RCL at random}$ 
    edgescandidate  $\leftarrow edges^{candidate} \setminus \{<i, j>, <j, i>\}$ 
    solution  $\leftarrow solution \cup \{<i, j>\}$ 
return solution

```

Algorithm GRASP Procedure

```

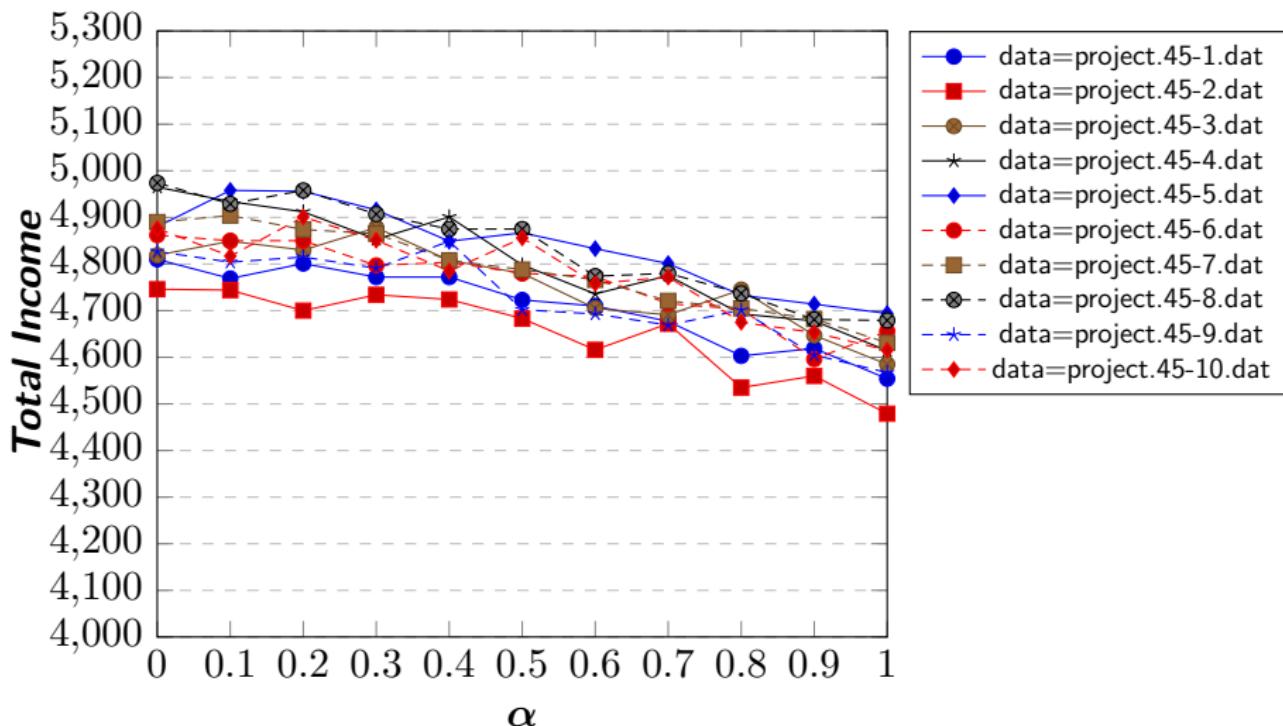
objectbest, solutionbest  $\leftarrow 0, \phi$ 
for retry = 1 to MaxIterations do
    obejective, solution  $\leftarrow 0, \phi$ 
    solution  $\leftarrow doConstructivePhase()$ 
    solution  $\leftarrow doLocalSearchPhase(solution)$ 
    if obejective  $>$  objectbest then
        objectbest  $\leftarrow obejective$ 
        solutionbest  $\leftarrow solution$ 
return solutionbest

```

GRASP Constructive Phase (data=project.4.dat, $\alpha = 0.2$)

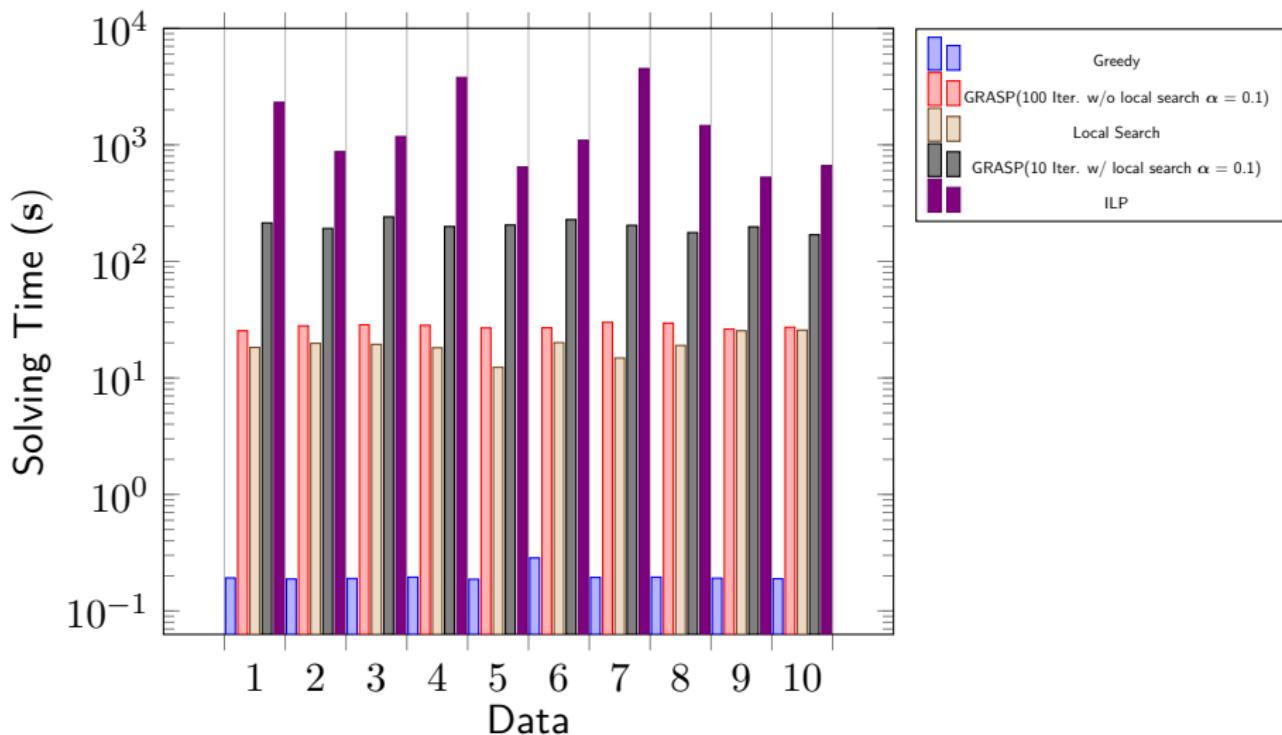
α Tuning for GRASP

Quality of Solutions (100 iter. w/o local search phase)

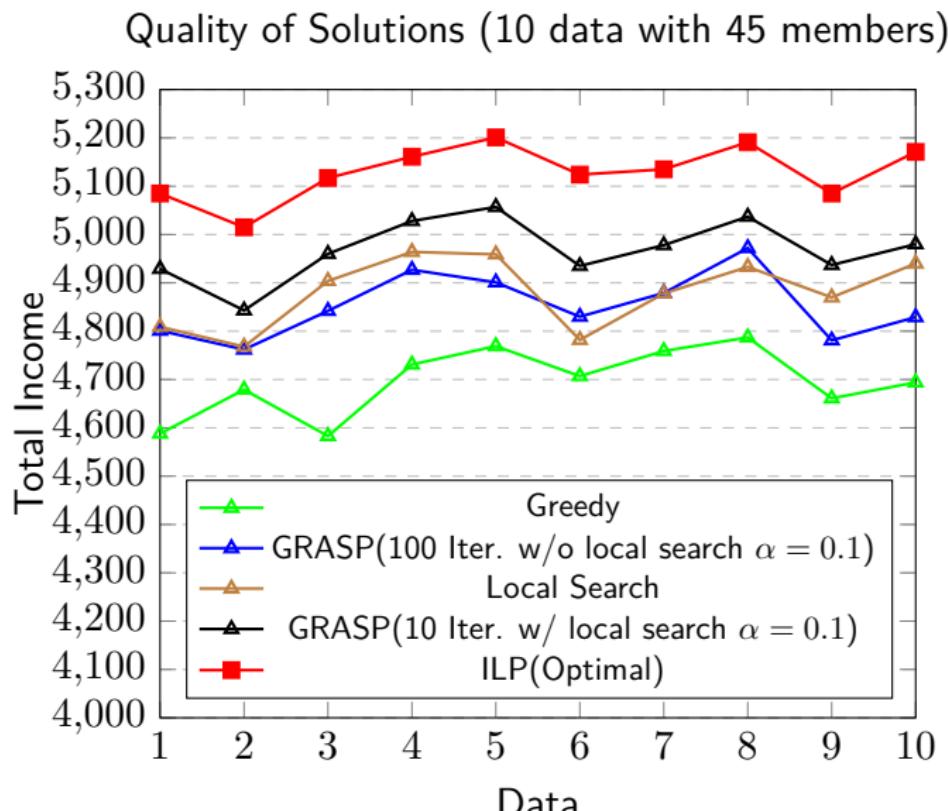


Solving Time

Solving Time (10 data with 45 members)



Solution Quality



Thanks