A Cognitively Plausible Model of Human Multigoal Navigation and its Application to the Traveling Salesman Problem

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Abstract

Although problems involving navigating near-optimal routes through multiple goals, such as the traveling salesman problem, are NP-hard, human brains can typically solve these tasks near-optimally in linear time. This paper describes a biologically inspired model of hierarchical spatial maps inspired by hippocampal place cells in the brain, and proposes route planning heuristics based on these representations. The model is shown to accurately reproduce human accuracies and decision times in multigoal navigation tasks.

Introduction

The traveling salesman problem (TSP), a problem requiring the planning of the shortest route reaching a number of goals - for example, the shortest way to visit a number of cities for a salesman - is a combinatorial optimization problem that is extremely hard to solve optimally. It is NP-hard, which means that the number of operations required to solve it increases rapidly with the number of goals - exactly solving it is currently not possible within polynomial time (Applegate et al. 2011).

Despite the difficulty of this problem, the human brain is surprisingly adept at solving it – even for huge problems with more than 100 goals, average human solutions are only about 10% worse than the mathematical optimum (Dry et al. 2006). Also, human solution times increase only linearly with the number of cities, as opposed to greater than polynomial increases in exact solution algorithms. Because of this surprising speed and accuracy, in the 60's and even early 70's, humans were still used in combination with computer algorithms to solve huge TSP problems (Krolak et al. 1970).

This paper presents a simple biologically inspired model of how human brains can solve the traveling salesman problem. As opposed to previously published models (see (Dry et al. 2006) for a good review), this model aims to be neuroscientifically plausible, and hypothesizes that the neuronal correlates of spatial cognition play a major role in the solution of spatial traveling salesman problems. The paper first presents neuroscientific evidence about spatial representations in the brain. A computational model is introduced and

described that is based on this evidence. The model is applied to solving the spatial traveling salesman problem, and the error rates and planning times of simulated problems are compared to human performance in order to substantiate the models claims. Finally, the visually presented traveling salesman problem is discussed, and the model is extended to account for the significantly higher performance of human subjects in visually presented problems compared to spatial TSP problems.

Spatial Memory in the Brain

There is a large amount of accumulated evidence (O'Keefe & Dostrovsky 1971)(McNaughton et al. 2006) in neuroscience and psychology that the brain contains a "cognitive map", an allocentric, two-dimensional, map-like representation of locations in the environment and objects / memories associated with them. Three types of cells in different brain ares play an especially important role for allocentric representations in mammalian brains (Burgess 2008) and humans (Ekstrom et al. 2003)(Doeller et al. 2011):
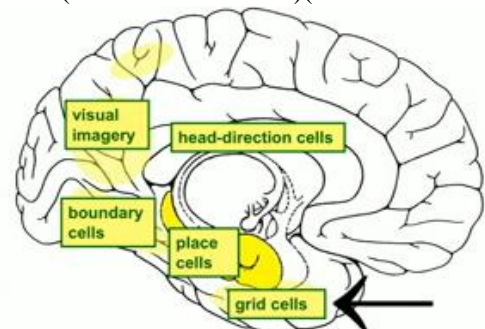

Figure 1. Brain areas playing a role in the neural representation of the "cognitive map"

1) Place cells are pyramidal cells in the hippocampus which exhibit strongly increased firing in specific spatial locations, largely independent from orientation in open environments (O'Keefe & Dostrovsky 1971), thus providing a representation of the animal's (or human's (Ekstrom et al. 2003)) position in the environment. This property of place cells can be explained using the Boundary Vector Cell (BVC) model (Burgess 2008)(Barry et al. 2006), which assumes the existence of BVCs providing inputs

to place cells depending on the presence of boundaries at a given distance and direction (relative to head-direction cells). Thus, place cell firing would depend on the shape, configuration and distance to environmental boundaries (or large, salient and static landmarks, if there are no natural boundaries). *'Border cells'* have been found in the medial entorhinal cortex and subiculum exhibiting these firing properties of BVCs (Lever et al. 2009)(Solstad et al. 2008). Another hypothesis has been suggested as well, implying that the place cells emerge from a combination of grid cell outputs on different scales (Solstad et al. 2006)(Moser et al. 2008); however, this hypothesis contradicts the finding that place cells mature earlier than grid cells during postnatal development (Langston et al. 2010)(Wills et al. 2010).

2) Head-direction cells fire whenever the animal's head is pointing to a certain direction. The primary circuit responsible for head direction signals projects from the dorsal tegmental nucleus to the lateral mammillary nucleus, anterior thalamus and postsubiculum and terminates in the entorhinal cortex (Taube 2007). There is some evidence that head direction cells exist in the human brain within the medial parietal cortex (Baumann & Mattingley 2010).

3) Grid cells in the medial entorhinal cortex show increased firing at multiple locations regularly positioned in a regular grid across the environment consisting of equilateral triangles (Hafting et al. 2005). Grids from neighboring cells share orientation, but have different and randomly distributed offsets, meaning that a small number of them can cover an environment. Grid spacing increases from dorsal to ventral positions from about 25cm to several meters (Doeller et al. 2011)(Hafting et al. 2005) , implying that they are implementing a hierarchy of sub-maps (Derdikman et al. 2009). Grid cells have also been suggested to aid path integration, their activation being updated depending on the animal's movement speed and direction (Burgess 2008)(Hafting et al. 2005)(O'Keefe & Burgess 2005). Grid cells seem to exist in the human entorhinal cortex (Doeller et al. 2011)(Doeller et al. 2010). Metric distance information between grid locations is claimed to be available, either through 'distance cells' in the dentate gyrus (Huhn et al. 2009b), or, indirectly, through a metric representation derived from a modular coding of locations arising from the combination of multiple grid distances and phases (Fiete et al. 2008).
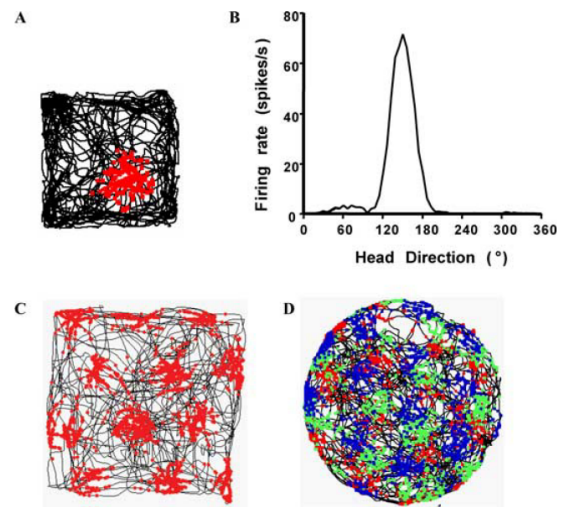

Figure 2. Firing patterns of place cells, head direction cells, and grid cells. A) Spatially constrained firing of a place cell (the black line denotes the rat's path, red dots denoting locations where the cell fired). B) Firing rate of a head direction cell as a function of head direction. C) A grid cell firing in a triangular grid pattern. D) Superimposed firing patterns of three neighboring grid cells with different offsets, tiling most of the environment. (From (Burgess 2008)).

A Computational Representation of Spatial Memory

Based on neuroscientific knowledge of the spatial representation of the environment in the brain, it is possible to design such a representation computationally. A number of such representations have been suggested in literature already (e.g. Byrne et al. 2007; Barrera & Weitzenfeld 2008). While our model bears some similarity to these models, which is to be expected since the same neural substrate is being modeled, there are two major differences. First, most of the existing models of spatial neural correlates are based on spiking neural networks or attractor networks for greater biological plausibility. Our model, on the other hand, approximates spike rates with a single activation value – similarly to traditional neural networks - which makes its computations faster by several orders of magnitude (Maass 1995). The other major difference is that whereas the mentioned models focus on the representation itself and on simple single-goal navigation problems, our focus is on solving multigoal navigation problems.

The "cognitive map" is represented by a two-dimensional grid of simulated place cells in our model. The place fields (areas in the real world which correspond to the place cell) of these place cells are the same size for each cell for simplification, and

partially overlap. Place cells are connected recurrently, and the weights of the connections between them decrease exponentially with the distance of their corresponding place fields (as observed in hippocampal place cells in the brain - Csizmadia & Muller 2008; Samsonovich & McNaughton 1997; Moser et al. 2008).
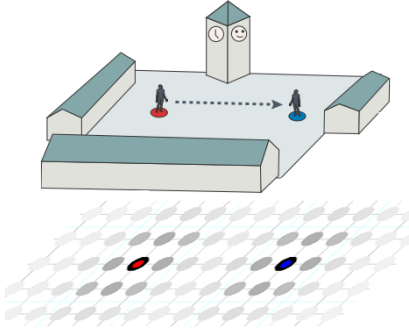

Figure 3. The spatial memory model. Top panel: two locations in an environment. Bottom panel: representations of these locations in the two-dimensional place cell grid. Although the current location (red and blue circles) always has the highest activation, cells with place fields around them also have non-zero activation since their fields overlap.

It should be mentioned that although the illustrations in this paper show the place cells to reside on a topologically accurate two-dimensional grid, we do not claim that this corresponds to the actual neural substrate of the hippocampus. However, the exact locations of these cells in the hippocampus do not make any difference to the models performance in terms of representation or planning (as long as the connection weights are accurate).

There are different theories pertaining to how distances are represented in the cognitive map in the brain, and there does not yet seem to be a consensual solution (Huhn et al. 2009a; Csizmadia & Muller 2008; Moser et al. 2008; McNaughton et al. 2006), some of them emphasizing the importance of entorhinal grid cells in distance representations and others trying to directly decode distances from hippocampal activity. We have decided to adopt the latter view and to encode distances into the connection weights between our modeled place cells, based on (Csizmadia & Muller 2008). The idea is that since the synaptic strength between two hippocampal place cells will depend on how far away the place fields of these cells are, this distance can be decoded from the synaptic strength (the biological equivalent of a connection weight). This dependence follows from the fact that pairs of cells with coincident or overlapping fields will tend to fire together in time, causing an

increase in synaptic strength. This follows from Hebb's rule (Sejnowski & Tesauro 1989) that connections between neurons that are frequently active at the same time tend to become stronger: "neurons that fire together wire together".
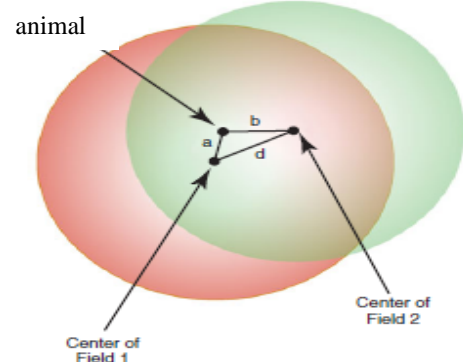

Figure 3. The firing fields of two place cells. Place cell activity (firing rate) depends on how far away the animal is from the center of the cells place field. In this figure, both place cells would be active, place cell 1 having a higher firing rate than cell 2 because the distance from its field center a is smaller. Hebb's rule implies that the synaptic connection between these two cells will be strengthened when the animal is in the indicated location, because both cells will fire at the same time, and that the amount it is strengthened will depend on the distance d.

On the other hand, cells with place fields located very far away will never fire together and there will be no change in synaptic resistance.

Based on a simple mathematical formulation of Hebb's rule, and on the key insight that the firing rate of place cells decreases exponentially with increasing distance from the place field center, Csizmadia & Muller have suggested a way of deriving distances from synaptic strengths (connection weights). If S is a numerical representation of the synaptic strength between two place cells, a the distance from the center of the place field of place cell 1 and b the distance from the center of place field 2 (see Figure 3), the change in synaptic strength ΔS can be calculated from:

$$\Delta S = \begin{cases} 0 & \text{if } e^{-ka}e^{-kb} < S \\ \alpha(e^{-ka}e^{-kb} - S) & \text{otherwise.} \end{cases} \quad (1)$$

From this relation, assuming that the rat knows the environment well (i.e. a very large number of locations have influenced synaptic strengths between place cells), it is trivial to show that the synaptic strength between any two place cells with overlapping fields will converge to

$$S = e^{-kd} \quad (2)$$

with S being the numerical representation of the synaptic strength, d being the metric distance between place field centers, and k being a constant factor that can be determined from published place cell recordings in animal experiments.

Using this relation (2) and numerical activation values instead of firing rates it is straightforward to implement a spatial model capable of representing locations, and metric distances, in our model.

Although we do not explicitly include models of object representations in this simple model, since they are not required for modeling multigoal navigation and the traveling salesman problem, object and object-place representations do play a key role in cognitive maps. Following the neuroscientific evidence presented in (Byrne et al. 2007; Moser et al. 2008) and others, we hypothesize that object representations are built mainly in the Lateral Entorhinal Cortex and the Perirhinal Cortex, both of which have activation patterns that correspond to object identities and both of which are closely connected to place cells in the hippocampus (Jo & Lee 2010).

Paths and Path Planning

A recurrently interconnected network of cells representing locations and metric distances facilitates a very simple and straightforward path planning mechanism. Assuming that every goal location G passes activation through the network, the distance to the goal can be decreased at any starting point S by moving to the adjacent neighbor cell with the highest activation. If the cells representing the locations of possible obstacles are connected with zero or near-zero weights, this mechanism can implement obstacle avoidance as well as path planning.
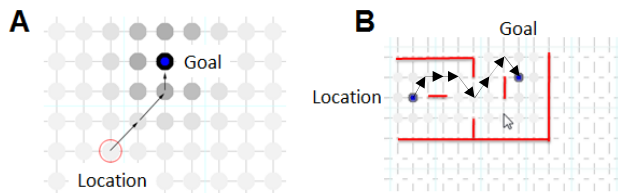

Figure 4. Path planning by climbing activation gradients. A) Path resulting from moving to the neighboring cell with the highest activation. B) The same strategy applied to a more complex setting. The red lines are there for illustration purposes only and show the locations of obstacles, which in the model are represented by cells connected with zero or near-zero weights (and thus do not transfer activation).

This approach works well for single-goal navigation as illustrated in Figure 4. However, since it always results in

the shortest path, it can be suboptimal for multigoal navigation problems such as the one shown in Figure 5 A.

However, this problem can be alleviated by performing hierarchical clustering on the goal locations. It has long been suspected that human cognitive maps are built in a hierarchical way, for more efficient representation and planning, and indeed there is a range of experiments substantiating this hypothesis (Hirtle & Jonides 1985; Shapiro et al. 1997). We have implemented such hierarchical representations by creating multiple levels of grids for each environment, with one grid at the bottom containing every single location in the environment, and subsequent grids on higher levels consisting of "place cells" clustering over (representing) multiple places on lower levels as illustrated in Figure 5.
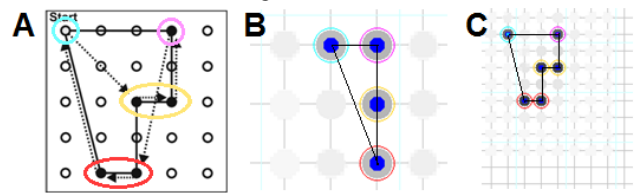

Figure 5. Multigoal routes and hierarchical cognitive map representations. A) An example multigoal navigation problem where the described path planning heuristic yields a suboptimal path (dotted arrows) as opposed to the shortest path (solid lines). B) A higher-level cognitive map representation. Each place cell (gray circles) represents a cluster of goal locations in the lower level representation. Colored ellipses illustrate cluster memberships. C) The lowest-level cognitive map representation (equal to the schematic in panel A). The (optimal) route shown as a black line was derived by first solving the easier multigoal problem on a higher level (B) and then refining the solution by solving the problems represented by the clusters and connecting them according to the higher level solution.

This hierarchical representation significantly shortens path lengths in multigoal planning problems. Instead of just following the activation gradient in a low-level representation of the problem, multiple hierarchical representations are built by clustering goal locations that are close together (Figure 5). Paths are planned on the highest representation level (with the fewest goals) first. Then refined paths are planned in the clusters on the level below, which are interconnected based on the higher level path. This process is iterated until a path in the lowest level representation is found. Despite path planning on all levels being done using the simple activation gradient climbing heuristic described above, this hierarchical planning schema identifies near-optimal routes (see Figure 5 and Figure 8).

Clustering is performed automatically by an interconnected hierarchical neural network: multiple cells on low levels are connected to each cell on high levels, and thus feed activation into them. Furthermore, place cells on lower levels can be connected to more than one higher level cell – it could be said that cells on higher level grids also have "place fields" on the lower level grids which overlap.

This hierarchical interconnection and activation passing, together with a thresholding mechanism (only cells receiving enough activation on high levels are interpreted as goal locations), leads to an automatic clustering mechanism that identifies all clusters and maps on all representation levels immediately after the lowest level map has been initialized and the activation passing has finished.
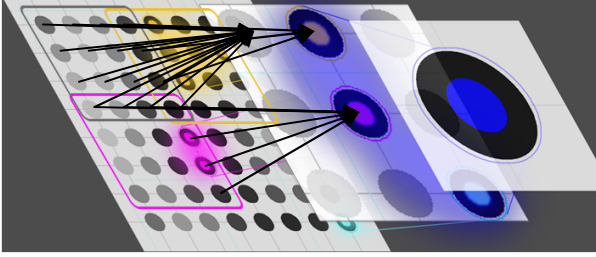

Figure 6. Clustering on hierarchical interconnected cell grids. *Six goals on the lowest representation level on the left - a group of three goals (yellow), two goals (pink) and a single goal (lightblue) - are clustered into three goals on a higher level (middle) and subsequently into one goal (blue) at the highest level. Levels are interconnected, and cells on lower levels are connected to multiple cells on higher levels. Whenever activation is passed by the goal locations in the lowest representation level, this structure automatically leads to clusters receiving high activation on higher levels.*

Results

(Wiener et al. 2009) have performed a spatial multigoal navigation (traveling salesman) experiment on human subjects in a 6.0 x 8.4m room, with 25 different locations marked by boxes with symbols on them (Figure 7).

Figure 7. Photo of the experiment setup (Wiener et al. 2009)

Subjects were given a "shopping list" containing a number of different symbols, each of which denoted a location that they had to visit, and they subsequently had to plan the shortest route visiting all of these locations. The experiment was carried out in different settings – not requiring memory (subjects saw the "shopping list" as well as the symbols on the boxes) as well as requiring working memory (the "shopping list" was taken away during planning and subjects had to remember the items) and long-term memory (the symbols on the boxes were taken away after an initial learning period). Figure 8 shows subjects' performance at this task, and compares it with the simulated performance of an agent using the spatial memory

representation, and path planning heuristic, described above. The models performance closely accounts for human data, as can be seen from this figure, which substantiates the models cognitive plausibility. Since goals are traversed sequentially and never re-visited, the models computational complexity is $O(n)$ and thus solution times also increase linearly, in accordance with human data (Dry et al. 2006).
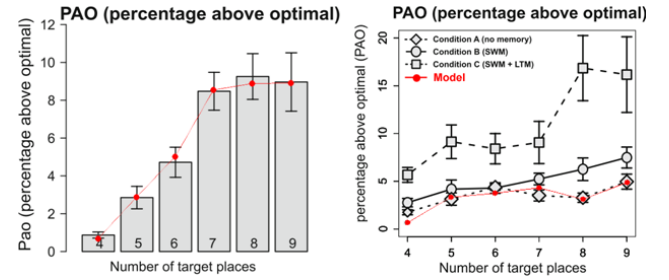

Figure 8. Subjects performance on the spatial TSP task (gray boxes / black lines) (Wiener et al. 2009), compared to the simulated performance of the described model (red lines)

Although the model accounts for human data in spatial TSP settings quite well, as seen in Figure 8, this simple heuristic seems to be insufficient to model huge visually presented TSP problems – performance on visual TSPs is much better than on spatial TSP problems. Humans seem to resort to additional heuristic strategies, performed by different brain areas, if the problem is presented visually. Based on empirical evidence, we suggest the existence of three heuristics explaining the improved performance:

1. Humans quickly identify and avoid path intersections in visual TSPs using the visual cortices (Vickers et al. 2003)
2. Humans use "mental lookahead", they plan ahead for multiple future goals and optimize decisions of where to go next (MacGregor et al. 2004)
3. The human perceptual system easily connects boundary points (MacGregor et al. 2004)

Implementing these three additional heuristics in our model, we were able to model the significantly better visual TSP performances accurately using the same described spatial representation. It should be mentioned that this implementation had $O(n^2)$ complexity because of the simplified intersection detection (in brains, intersections are detected visually by edge detectors in V1 instead of an iterative algorithm).
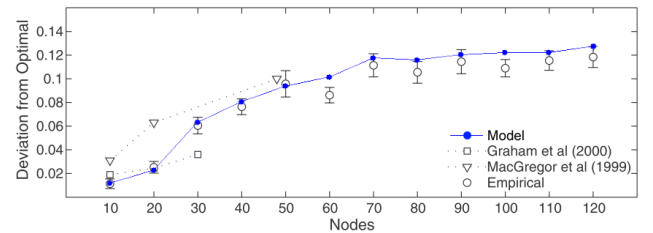

Figure 9. Subject performance at visually presented TSP problems (black circles) compared to the model (blue line).

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