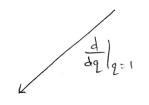
Homological Toolkit For the Quantum Mechanic
Part I

Recap: Pp a multipardite density State; we motivated:



Mutual INF.

=) is dim(H_T) & rank (P_T) the dimension of some vector Space?

Thm:

A State 4 & H, & HB is Factor 1 Zable (=> 3 (For X = A, B)

- · Left BAx modules Mx
- · Distinguished Points Mx E Mx
- · Equiverient maps Mx: Mx > HA & HB

S.t.

- · BHx · mx = Mx (Cyclic)
- · Mx (mx) = 7
- · O C (Ker do = Spene (ma, mB)).

Thus, given 4 ∈ H, Q HB we want to Find (Mx, Mx, Tx) w/ Smallest Cohomology.

The GNS-module (Gelfond-Neuman - Segal)

Quesi - Defs:

· A Cu-elgebra is a V.S. A low/

- Norm 11-11

- M: A x A - A rassociative mult.

- +: A -> A an anti-Iner involution

S.t.

- A is complete

- 4 plays nicely w/ M

- 11x x 11 = 11x112 (C* - cond2)

Exis BH, C°(X) & Commutative, The Mnil & All F.d. C*-algs

= to this

· A W - algebra 15 à Ct-algebra w/ a produal. A.

C: (Ax) A
"Normal linear Functionals / density States)

Ex: (B,H) = BX

· (Lop(X, M)) = Lop(X, M).

· Every F.d. Cx - 21gebra is Wx.

DeF: A State on a C - algebra A is a positive linear Foretimal

P: A -> C

[No normalization in this talk: i.e. p(1) not hec. 1).

· A normal State on a W* - algebra is an element of (A+)+

Why? $(A_{+})^{\vee} = A = A \Rightarrow A_{+} \longrightarrow (A_{+}^{\vee})^{\vee} \longrightarrow A^{\vee}$ $E_{\times}: \circ D_{ens}(\mathcal{H}) \subseteq (B_{1}\mathcal{H})_{+} \qquad A = B\mathcal{H}$ $p \longrightarrow p(-) = Tr[\hat{p}(-)]: A \longrightarrow C$ $p \longrightarrow p(-) = Tr[\hat{p}(-)]: A \longrightarrow C$ $p \longrightarrow p(-) = Tr[\hat{p}(-)]: A \longrightarrow C$

The GNS-module

A & Cu -arg.; p: A -> C & State. DeFine:

$$\mathcal{I}_{p} = \left\{ a \in A : p(a^*a) = 0 \right\}$$

$$\frac{\text{Couch'l}}{\text{Summer } 2} \left\{ a \in A : p(x^*a) = 0 \right\} \forall x \in A$$

$$\text{GNS}(p) = A/\mathcal{I}_{p}, \quad g_{p} = \mathcal{I}_{A} + \mathcal{I}_{p}$$

Cloim: Zp is a left ideal => GNS(p) is a left A-module.

raez, Vreh

Interpretation:

GNS(p) = Right Essential equivalence (12:35=5 OF operators)

$$a \sim b <=> p(x*a) = p(x*b) \forall x \in A$$
.

Non-Commutative analog OF a.e. equiv Functions (take a, be La (X)).

Submodules and Support Projections

Claim:
$$\mathcal{Z}_{p} = A(1-S_{p})$$

$$AS_{p} \xrightarrow{\sim} GNS(p)$$

$$a \mapsto a + \mathcal{Z}_{p}$$

Cor:
$$\hat{\rho} \in Dens(\mathcal{H})$$
 -v. $GNS(\hat{p}) \cong B\mathcal{H} S_{\hat{p}}$
 $= \mathcal{H}om b(Image(\hat{p}), \mathcal{H})$
 $\cong \mathcal{H} \otimes Image(\hat{p})^{\vee}$
 $= \mathcal{H} \otimes Image(\hat{p})^{\vee}$
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PF. that GNS-module is the Smallest pointed module:

· Show that every improphismmer pointed modules

Fectors through the "cyclification" of (N, n) unquery $\exists ! \quad \neg (N/Ann_A(n), n) \cong (A \cdot n, n)$ $(M, m) \quad f \quad (N, n)$

- · Show ! map is Sprjective
- · Take N ~ HB

 A ~ BXX

Constructing Multipartite Complexes

Multipostite State over Set OF 8- Factors P:

=> H°(M) = H°(G) + H°(K)

$$\underline{P} = ((A_s)_{s \in P}, P: \bigotimes_{s \in P} A_s \longrightarrow \mathbb{C})$$

Claim: PP M Presheat OF Orivector Cech

Spaces over P (B)

(A) Define
$$G = G(p_p) : Open(p) \longrightarrow Vector \qquad V_T : \bigotimes_{t \in T} A_t \longrightarrow \bigotimes_{t \in T} A$$