Box Num. 33
Problem Set 267
April 6, 2018

1. (a) The magnitude of the spin for this particle is

$$|\hat{\mathbf{S}}| = \sqrt{s(s+1)}\hbar = \sqrt{\frac{15}{4}}\hbar$$

with z-component $S_z = m_s\hbar = \pm(1/2)\hbar$. The angle with the $z$-axis is then given by

$$\theta = \arccos\left(\frac{1}{2\sqrt{15/4}}\right) \approx 0.417\pi$$

(b) Because no electric field is applied, the degeneracy with spin is the same as without spin, so this state has a degeneracy of 25.

2. (a) The energy levels would be larger in a muonic hydrogen atom, compared to a normal hydrogen atom. This happens because the energy levels of electrons in hydrogen are given by

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2 n^2}.$$

Therefore, replacing an electron with a higher-mass particle with the same charge would increase the energy levels.

(b) The magnetic moment would be smaller. It is given by

$$\mu_L = -(e/2m)\mathbf{L}$$

so increasing the mass would decrease the magnetic moment.

3. (a)
   i. This decreases the count rate, since high-angle scatters have a lower probability of occurring.
   ii. This decreases the count rate, since the scattering angle off each atom is (on average) decreased.
   iii. The count rate increases, since the higher-charge nuclei exert a greater force on the incident particle.
   iv. The count rate decreases, since the charge-mass ratio of the incident particle decreases.
   v. The count rate decreases, since there are on average less interactions per particle.

(b) Let the foil be $n$ atoms thick, with a distance $D$ between atoms. Assume the charge-to-mass ratio of both the target and projectile is the same (true within an order of magnitude, since both are made of nucleons). Then

$$F_E = \frac{q_t q_p}{4\pi\epsilon_0} \propto \frac{R_t^3 R_p^3}{r^2} \implies a_E \propto \frac{R_t^3}{r^2}$$

Note that the acceleration only depends on the target radius—this does not contradict part iv. from above, because we are assuming a constant charge-to-mass ratio here. Assume that the particle will be scattered $n$ times as it passes through the foil, each time passing a random distance $d$ (where $0 < d < D$, uniformly distributed) from the nearest nucleus.