ID #33
Problem Set 10
Physics 202
February 14, 2018

1. (a) Since each atom of Helium has the same temperature (and therefore thermal energy) as each atom of Argon, and $K = (1/2)mv^2$, the Helium atoms must have a higher v_{rms}, since their mass is lower.

   (b) The argon has more thermal energy. On average, each atom has the same amount of energy, but there are twice as many atom of argon as there are of helium.

2. The thermal energy of a monatomic gas is given by

$$E_{th} = \frac{3}{2}Nk_BT$$

Here, the amount of helium, N, is given by

$$N = \frac{PV}{k_BT}$$

Then the thermal energy is

$$E_{th} = \frac{3}{2}\frac{PV}{k_BT}k_BT = \frac{3}{2}PV = \frac{3}{2}(1 \times 10^5\,\text{Pa})(0.001\,\text{m}^3) = 150\,\text{J}$$

3. (a) The Knight textbook walks through calculating the number of particles hitting the container's wall, giving the equation

$$N_{coll} = \frac{NAv_x\Delta t}{2V} = \frac{\left(\frac{PV}{k_BT}\right)Av_x\Delta t}{2V} = \frac{PAv_x\Delta t}{2k_BT} = \frac{PA\Delta t}{2m\overline{v_x}}$$

   (b) The total kinetic energy of an ideal gas particle is given by

$$v_{rms}^2 = \frac{3k_BT}{m} = \overline{v_x}^2 + \overline{v_y}^2 + \overline{v_z}^2 \implies \overline{v_z}^2 = \frac{3k_BT}{m} \implies \sqrt{\overline{v_x^2}} = \sqrt{\frac{k_BT}{m}}$$

   (c) Dividing what we had earlier by Δt gives the rate at which the gas is escaping

$$\frac{PA}{2m\overline{v_x}} = \frac{Nk_BTA}{2vm\sqrt{\overline{v_x^2}}} = \frac{A}{2V}\frac{k_BT}{m\sqrt{\overline{v_x^2}}}N = \frac{A}{2V\sqrt{k_BT/m}}N$$

   Therefore

$$N(t) = N_0e^{-t(2V)/A\sqrt{kBT/m}}$$

   where

$$\tau = \frac{2V}{A\sqrt{k_BT/m}}$$

   (d) The characteristic time is

$$\tau_{N_2} = \frac{2V}{A\sqrt{k_BT/m}} = \frac{2(0.001\,\text{m}^3)}{(1 \times 10^{-6}\,\text{m}^2)\sqrt{(1.380 \times 10^{-23}\,\text{J/K})(293\,\text{K})/(4.65 \times 10^{-23}\,\text{kg})}} = 89.1\,\text{s}$$

4.

$$E_{th} = \frac{3}{2}Nk_BT \implies 1\,\text{J} = \frac{3}{2}(1 \times 10^{20})(1.380 \times 10^{-23}\,\text{J/K})T \implies T = 483.092\,\text{K}$$