Box Num. 33
Problem Set 26
April 4, 2018

1. (a) The 16 possible sets of quantum numbers for $n = 4$ are

n	l	m_l	n	l	m_l
4	0	0	4	2	-2
4	1	0	4	3	0
4	1	1	4	3	1
4	1	-1	4	3	-1
4	2	0	4	3	2
4	2	1	4	3	-2
4	2	-1	4	3	3
4	2	2	4	3	-2

   (b)
   - The possible values of l when $n = 6$ are $0, 1, \ldots, 5$.
   - The possible values of m_l when $l = 6$ are $0, 1, \ldots 6$.
   - The smallest n for which l can be 4 is $n = 5$.
   - The smallest possible l that has a z-component of $4\hbar$ is $l = 4$.

2. The radial component of the wavefunction for this set of qunatum numbers is

$$R(r) = \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

so the radial probability is given by

$$|R(r)|^2 = \frac{1}{(2a_0)^3} \left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}$$

Differentiating and solving for roots gives that there are extrema at $r = 2a_0$ and $r = 4a_0$. Plugging these into the original function gives that $R(2a_0) = 0$ and $R(4a_0) = -1/\sqrt{2}a_0^{3/2}e^2$, so the maximum is at $r = 4a_0$.

3. For the 1s state of hydrogen,

$$r_{\text{av}} = \int_0^\infty rP(r)dr = \int_0^\infty r^3|R(r)|^2 dr = \int_0^\infty r^3 \left(\frac{2}{a_0^{3/2}} e^{-r/a_0}\right)^2 dr = \int_0^\infty r^3 \frac{4}{a_0^3} e^{-2r/a_0} dr = \frac{3}{2} a_0$$

This is larger than the Bohr radius because the 'long tail' of the decaying exponential term means there is a small probability of the electron being found at large radii, which then drag the overall mean upwards.

4. The average potential energy in this situation is

$$U_{\text{av}} = \int_0^\infty U(r)P(r) = \int_0^\infty -\frac{e^2}{\pi\epsilon_0 a_0^3} r \exp(-2r/a_0) dr = -\frac{e^2}{4a_0\pi\epsilon_0} = -27.213\,\text{eV}$$

This is larger than the Bohr model prediction for $n = 1$, which predicted an average potential of $-13.6\,\text{eV}$.