ID #33
Problem Set 12
Physics 202
February 18, 2018

1. (a) You would find a beaker of slightly cooler water.
   (b) You would never see an outcome like the large ice cube in a cloud of steam because its multiplicity is incredibly low compared to the multiplicity of the beaker of cool water.

2. The equation relating entropy and energy inputed is

$$dS = \frac{dQ}{T}$$

However, $dS = k_B \log(3)$. Therefore, the change in heat (the heat input) is

$$dQ = Tk_B \log(3)$$

3. (a) There are 52! ways to arrange a deck of cards.
   (b) At first, you have a multiplicity of 1, since there is only one way to arrange the cards in the way they initally were. However, after repeatedly shuffling the cards, the deck has a multiplicity of 52!, since it could be in any possible state. Therefore, the entropy is

$$S = k_B \log(52!) \approx 156.36 k_B = 2.16 \times 10^{-21}\,\text{J/K}$$

   (c) Multiplying the mass of the card by the specific heat and temperature increase gives that the heat added is 2.41 J. Averaging the temperature to be 20.5 °C = 293.65 K, the change in entropy is

$$dS = \frac{dQ}{T} = \frac{2.41\,\text{J}}{293.65\,\text{K}} = 0.008\,21\,\text{J/K}$$

This increase is far, far larger than the change from shuffling the cards.

4. (a) In an isothermal process, $P = \frac{Nk_BT}{V}$, where Nk_BT is constant. Therefore, the heat involved in this process is

$$dQ = \int_{V_i}^{V_f} \frac{Nk_BT}{V} dV = Nk_BT \log(V_f/V_i) \implies dS = \frac{dQ}{T} = Nk_B \log(V_f/V_i)$$

   (b) In the adiabatic compression ($i \to m$), dQ is by definition 0, so $dS = 0$ for that segment. We just showed that the entropy change in an isothermal process is

$$dS_{therm} = Nk_B \log(V_f/V_m)$$

where V_m is the solution to

$$\frac{P_iV_i}{V_m} = \frac{P_fV_f^{(C_P/C_V)}}{V_m^{(C_P/C_V)}} \implies V_m = \frac{P_fV_f^{(C_P/C_V)}}{P_iV_i}^{\frac{1}{(C_P/C_V)-1}}$$

so

$$dS_{therm} = Nk_B \log\left(V_f \left(\frac{P_fV_f^{(C_P/C_V)}}{P_iV_i}\right)^{-\frac{C_V}{C_P-C_V}}\right)$$

For an isochoric process, the difference in heat is given by $dQ = mC_VdT$. Therefore, the entropy is

$$S = \int_{P_i}^{P_f} \frac{mC_VdT}{T} dV = \int_{P_i}^{P_f} \frac{mC_VdT}{PV/(Nk_b)} dP = dS_{therm} = Nk_B \log\left(V_f \left(\frac{P_fV_f^{(C_P/C_V)}}{P_iV_i}\right)^{-\frac{C_V}{C_P-C_V}}\right)$$

5. (a) The entropy of the mixed system is equal to the entropy of the two component systems after they have the chance to mix together. Each of the expansions is isothermal (because the two gasses are the same temperature, the total temperature will not change.) Therefore, the entropy change of the each gas is

$$dS = nR\log(2V/V)$$

The total entropy is then

$$S = 2R\log(2) = 11.5257\,\mathrm{J/K}$$

(b) If one container is twice as big as the other, the total entropy is

$$S = R\log(3) + R\log(3/2) = 12.5049\,\mathrm{J/K}$$