Box Num. 33
Problem Set 35
April 25, 2018

1. (a) $^{27}\mathrm{Si} \rightarrow {}^{27}\mathrm{Al} + \mathrm{e}^{+} + \mathrm{v}_e$
   (b) $^{74}\mathrm{As} \rightarrow {}^{74}\mathrm{Se} + \mathrm{e}^{-} + \overline{\mathrm{v}_e}$
   (c) $^{228}\mathrm{U} \rightarrow \alpha + {}^{224}\mathrm{Th}$
   (d) $^{93}\mathrm{Mo} + \mathrm{e}^{-} \rightarrow {}^{93}\mathrm{Nb}$
   (e) $^{131}\mathrm{I} \rightarrow {}^{131}\mathrm{Xe} + \mathrm{e}^{+} + \mathrm{v}_e$

2. (a) Radioactive decay for nuclide A is described by the differential equation

$$dN_A = -N_A \lambda_A dt$$

A similar differential equation describes the decay of nuclide B, but the supply of nuclide B is also being replenished over time:

$$dN_B = (\lambda_A N_A - \lambda_B N_B)dt$$

   (b) When B is stable, the differential equation is

$$dNB_B = \lambda_A N_A dt$$

When $\lambda = \lambda_A = \lambda_B$, the differential equation is

$$dN_B = (N_A - N_B)\lambda dt$$

   (c) The activity of nuclide A is

$$A_A(t) = \lambda_A N_{A,0} e^{-\lambda_A t}$$

Solving the ODE above for $N_B(t)$ gives

$$N_B(t) = e^{-\lambda_B t}\left(\frac{e^{(\lambda_B - \lambda_A)t} N_{0,A}\lambda_A}{\lambda_B - \lambda_A}\right)$$

and the activity is

$$A_B(t) = e^{-\lambda_B t}\lambda_B\left(\frac{e^{(\lambda_B - \lambda_A)t} N_{0,A}\lambda_A}{\lambda_B - \lambda_A}\right)$$

When $\lambda = \lambda_A = \lambda_B$,

$$N_B(t) = N_{0,A} t\lambda e^{-\lambda t}$$

and

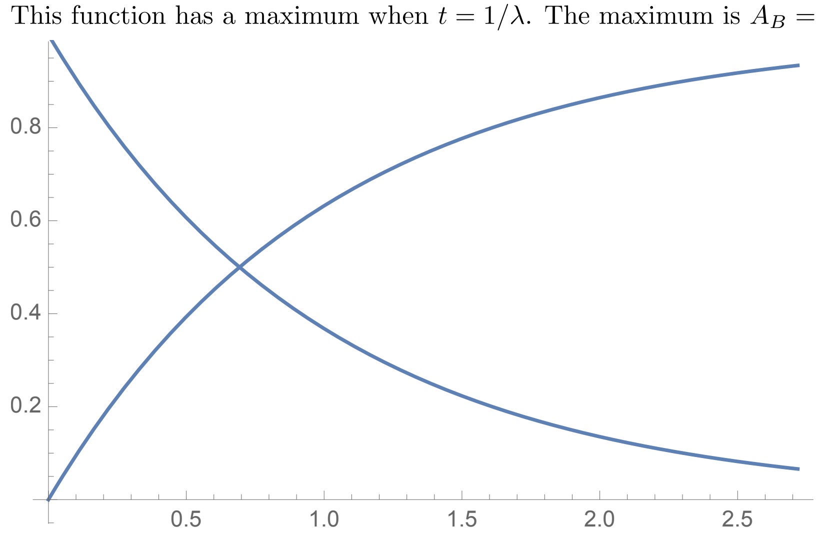
$$A_B(t) = N_{0,A} t\lambda^2 e^{-\lambda t}$$

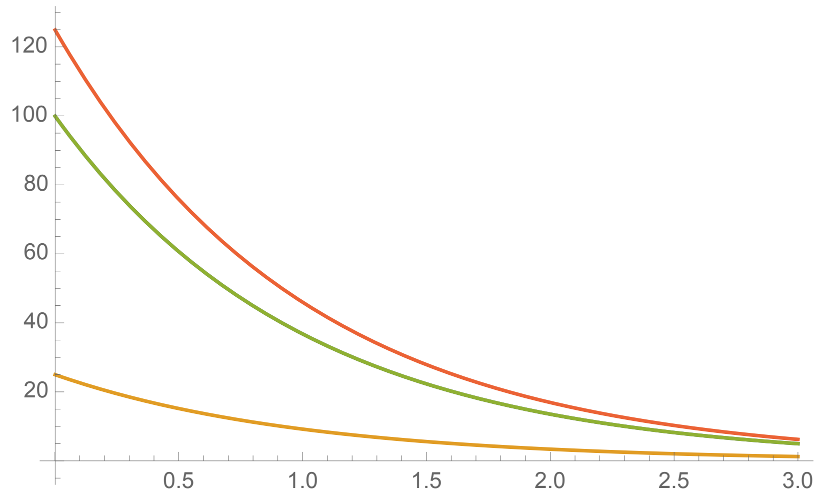
   (d) In the $\lambda = \lambda_A = \lambda_B$ case, we have $A_B(t) = N_{0,A} t\lambda^2 e^{-\lambda t}$. Taking the derivative,

$$\frac{dA_B(t)}{dt} = -N_{0,A}\lambda^2 e^{-\lambda t}(\lambda t - 1)$$

This function has a maximum when $t = 1/\lambda$. The maximum is $A_B = N_{0,A}\lambda/e$.

   (e)





(f)

3. (a) The atomic mass of ^{149}Eu is $148.917931238u$, which is smaller than the mass of ^{149}Gd, so β^- decay is not energetically favorable. An alpha decay would result in a total product mass of 148.915352277, so this is (barely) energetically favorable. A β^+/neutron capture reaction would result in a product mass of 148.917184735u, so this reaction is also favorable.
   (b) ^{151}Eu is stable. All of the potential decay products ($^{147}\text{Pm} + \alpha$, ^{151}Gd, and ^{151}Sm) have a higher mass.
   (c) ^{152}Eu decays via β^+ and β^- decays. The products of these reactions, ^{152}Gd and ^{152}Sm, both have lower masses than ^{152}Eu, making this reaction energetically favorable.
   (d) The only decay mode this isotope can undergo is β^-. Alpha decay results in ^{155}Pm and an alpha particle, which together have a higher mass than the original. Only β^- decay results in a decrease in energy.

4. (a) This reaction is endothermic, with a threshold energy of $0.0045267uc^2 = 4.2\,\text{MeV}$.
   (b) This reaction is exothermic.
   (c) This reaction is exothermic.
   (d) This reaction is endothermic, with a threshold energy of $0.005368007uc^2 = 5\,\text{MeV}$.