1. A simple function we can work with is the damped harmonic oscillator:

$$m\ddot{x}(t) = -kx(t) - 2mb\dot{x}(t)$$

2. Using the Taylor expansion from the writeup above, we have that

$$x_{j+1} = 2x_j - x_{j-1} + \Delta t^2 \frac{F_j}{m}$$

Then

$$v_j = \frac{x_{j+1} - x_j}{\Delta t} = \frac{x_j - x_{j-1} + \Delta t^2 \frac{F_j}{m}}{\Delta t} = \frac{x_j - x_{j-1}}{\Delta t} + \Delta t \frac{F_j}{m}$$

3. The angular frequency for an oscillator like this is

$$\omega = \sqrt{k/m}$$

so the regular frequency is

$$f = \frac{\sqrt{k/m}}{2\pi} \implies T = \frac{2\pi}{\sqrt{k/m}}$$

The potential energy of the mass at maximum extension is

$$U = \frac{1}{2}ka^2$$

so the maximum speed is

$$v_{\max} = a\sqrt{k/m}$$

Therefore, for

$$v_{\max} = c \implies a = c\sqrt{m/k}$$