Box Num. 33
Problem Set 30
April 13, 2018

1. (a) There are 3 possible macrostates:

0	1	2	3
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*	*	*	
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   (b) The number of microstates for each of the macrostates above is 3, 6, and 1, respectively.

   (c) The probability of finding one of the particles with 2 units of energy is 0.6. The probability of finding one with an energy of 0 is 0.9.

2. The possible macrostates are:

									# of microstates		
0	1	2	3	4	5	6	7	8	classical	integral spin	half-int spin
***								*	4	1	0
**	*						*		12	1	1
*	**					*			12	1	1
**	*	*				*			12	1	1
**			*		*				12	1	1
*	*	*			*				24	1	1
	***				*				4	1	0
**				**					6	1	1
*	*		*	*					24	1	1
	**	*		*					12	1	1
*		**		*					12	1	1
*		*	**						12	1	1
	**		**						6	1	1
	*	**	*						12	1	1
		****							1	1	0

With classical distinguishable particles, the probability of finding a particle with energy 2 is $(85/165) = 0.515$. With integer-spin particles, the probability is $(7/15) = 0.467$. With half-spin particles, the probability is $(6/12) = 0.5$.

3. The expression

$$\frac{1}{2}m_1\langle v_1^2\rangle = \langle K_1\rangle,$$

since kinetic energy is given by $\frac{1}{2}mv^2$, and the value $\langle v^2\rangle$ is the expectation value of velicty squared. Since the relation $\langle \mathbf{w}\cdot\mathbf{v}_{cm}\rangle = 0$ when the system is in thermal equilibrium, the kinetic energy of all molecules of gas must, on average, be equal. Therefore,

$$K_1 = K_2 \implies \frac{1}{2}m_2\langle v_2^2\rangle = \frac{1}{2}m_2\langle v_2^2\rangle$$

4. (a) For standing waves to occur, the total length of the standing wave must be an integer multiple of $\lambda/2$. Since this condition is satisfied here, a standing wave can form.

   (b) The Bohr model states that the angular momentum must be an integer multiple of $\hbar$—that is, that $mvr = n\hbar$. The de Broglie wavelength is defined to be $\lambda = h/p$, giving that $p = h/\lambda$. Then $L = r \times p = rh/\lambda$ and, since $C = 2\pi r = n\lambda$, we get that

$$L = \frac{nh}{2\pi} = n\hbar$$

   (c) We have that $mvr = n\hbar$, and that $r = n\lambda/2\pi$ Combining the two gives

$$\frac{mvn\lambda}{2\pi} = n\hbar \implies v = \frac{h}{m\lambda}$$