ID #33
Problem Set 1
Physics 202
January 23, 2018

1. (a) The law of cosines gives that

$$v'^2 = v^2 + u^2 - 2uv\cos\theta' \implies v' = \sqrt{v^2 + u^2 - 2uv\cos\theta'}$$

   (b) The lowest value of v' is when $\theta' = 0$, when the flatcar is moving towards the receiver at an infinite distance. The highest value is when $\theta' = pi$, when the flatcar is moving away from the receiver.

   (c) The lowest velocity is given by

$$v' = \sqrt{(330\,\mathrm{m/s})^2 + (15\,\mathrm{m/s})^2 - 2(330\,\mathrm{m/s})(15\,\mathrm{m/s})\cos(0)} = 315\,\mathrm{m/s}$$

   and the highest velocity is given by

$$v' = \sqrt{(330\,\mathrm{m/s})^2 + (15\,\mathrm{m/s})^2 - 2(330\,\mathrm{m/s})(15\,\mathrm{m/s})\cos(\pi)} = 345\,\mathrm{m/s}.$$

   This difference should be easily detectable.

2. (a) Using the equations derived in class,

$$\begin{aligned}\Delta t = t_2 - t_1 &= \frac{2L}{c}\left(\frac{1}{1-(v/c)^2} - \frac{1}{\sqrt{1-(v/c)^2}}\right)\\ &\approx \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{1-\frac{v^2}{2c^2}}\right)\\ &= \frac{2cL}{c^2-v^2} - \frac{2cL}{c^2-\frac{v^2}{2}}\end{aligned}$$

   (b) The period of the wave is given by $T = \lambda/c$. Then the phase shift is given by

$$N = \frac{\frac{2cL}{c^2-v^2} - \frac{2cL}{c^2-\frac{v^2}{2}}}{\frac{\lambda}{c}} = \frac{2c^2L}{\lambda\left(c^2-v^2\right)} - \frac{4c^2L}{\lambda\left(2c^2-v^2\right)}$$

   (c) If the second leg has length $L + \Delta L$, the new difference in time is given by

$$\Delta t = \frac{2(L+\Delta L)}{c-\frac{v^2}{c}} - \frac{2L}{c-\frac{v^2}{2c}}$$

   and the subsequent phase shift is given by

$$N = \frac{2(L+\Delta L)\lambda}{c^2-v^2} - \frac{2L\lambda}{c^2-\frac{v^2}{2}}$$

   The difference between this phase shift and the one calculated earlier is

$$\frac{2\Delta L\lambda}{c^2-v^2}.$$

   However, rotating the apparatus by 90° doubles this difference, so

$$\Delta N = \frac{4\Delta L\lambda}{c^2-v^2}$$

   (d) Using the equation from 2b, the expected fringe shift is

$$N = \frac{2c^2L}{\lambda\left(c^2-v^2\right)} - \frac{4c^2L}{\lambda\left(2c^2-v^2\right)} = 0.186$$