ID #33
Problem Set 4
Physics 202
January 30, 2018

1. (a)

$$\Delta t = \Delta t_0 \frac{1}{\sqrt{1-(u/c)^2}} = 100\,\text{ns}\frac{1}{\sqrt{1-(0.96c)^2}} = 357\,\text{ns}$$

(b)

$$d = v\Delta t = (0.96c)357\,\text{ns} = 102.82\,\text{m}$$

(c)

$$d_0 = v\Delta t_0 = (0.96c)100\,\text{ns} = 28.8\,\text{m}$$

2. (a) Let the period of the light given off by the source be Δt, and A and B be successive points where the wave reaches a peak. Assuming these paths are near-identical, the difference in path lengths is

$$v\Delta t\cos(\theta)$$

since $v\Delta t$ is the distance traveled by the source in that time, and $\cos(\theta)$ is the component of that velocity going towards Q. Since the time between peaks from a stationary source is $c\Delta t$, the distance between peaks is

$$\Delta t_0' = (1-\beta\cos(\theta))\Delta tc$$

However, we also need to factor in the length dilation due to relativistic speeds. This gives that

$$\frac{f_{obs}}{f_{sce}} = \frac{1}{(1-\beta\cos(\theta))\Delta tc}\frac{\Delta tc}{\gamma} \implies f_{obs} = \frac{f_{sce}}{(1-\beta\cos(\theta))\gamma}$$

(b) When the source is approaching head-on, $\theta = 0$. Using the equation from above,

$$f_{obs} = \frac{f_{sce}}{(1-\beta\cos(0))\gamma} = \frac{f_{sce}}{(1-\beta)\gamma}$$

which matches what we expect for an approaching source. When the source is receding directly away, $\theta = \pi$, and

$$f_{obs} = \frac{f_{sce}}{(1-\beta\cos(\pi))\gamma} = f_{obs} = \frac{f_{sce}}{(1+\beta)\gamma}$$

which is again what we expected.

(c) The frequency observed on the detector is

$$f_{obs} = \frac{f_{sce}}{(1-\beta\cos(\pi/2\pm\pi/4))\gamma} = \frac{250\times10^{18}\,\text{Hz}}{(1\mp0.3/\sqrt{2})(1.048)} = 302\times10^{18}\,\text{Hz}, 196\times10^{18}\,\text{Hz}$$

With detectors at 135°, the readings would be identical, since θ would simply be negative, and $\cos(-\theta) = \cos(\theta)$. With detectors at 90°,

$$f_{obs} = \frac{f_{sce}}{(1\pm\beta)\gamma} = 341\times10^{18}\,\text{Hz}, 183\times10^{18}\,\text{Hz}$$

3. (a) This is calculated using the Doppler shift equation:

$$(1\,\mathrm{yr}^{-1})\sqrt{\frac{1-0.6}{1+0.6}} = 0.5\,\mathrm{yr}^{-1}$$

(b)

$$(1\,\mathrm{yr}^{-1})\sqrt{\frac{1+0.6}{1-0.6}} = 2\,\mathrm{yr}^{-1}$$

The reason these results are the same as found for Casper's signals from Amelia is because, from Amelia's perspective, Casper is going away from her at $0.6c$, and then returning—same as her travels appear to Casper.

(c) Amelia measured the 20-year journey to take

$$20\,\mathrm{yr} = \Delta t \frac{1}{\sqrt{1-(0.6c)^2}} \implies \Delta t = 16\,\mathrm{year}$$

Therefore, Casper will receive 16 signals, since Amelia measured the passing of 16 years.

4.

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