ID #33
Problem Set 5
Physics 202
February 2, 2018

1. (a)

	First Ball(ρ_1)	Second Ball(ρ_2)	Total
Before:	$\gamma_1 m(a,b)$	$\gamma_2 m(-a,-b)$	$(0,0)$
After:	$\gamma_1 m(a,-b)$	$\gamma_2 m(-a,b)$	$(0,0)$

The γ is the same for both balls because the absolute value of both balls' velocities is the same.

(b)

	First Ball(ρ_1)	Second Ball(ρ_2)	Total
Before:	$m\left(0, \frac{b}{\gamma(1-\beta^2)}\right)$	$m\left(\frac{-2a}{1+\beta^2}, \frac{-b}{\gamma(1-\beta^2)}\right)$	$m\left(-\frac{2a}{b^2+1}, 0\right)$
After:	$m\left(0, \frac{-b}{\gamma(1-\beta^2)}\right)$	$m\left(\frac{-2a}{1+\beta^2}, \frac{b}{\gamma(1-\beta^2)}\right)$	$m\left(-\frac{2a}{b^2+1}, 0\right)$

2. (a)

$$E^2 = \left(\frac{1}{\sqrt{1-(v/c)^2}} mvc\right)^2 + (mc^2)^2 = m^2c^4\left(\frac{v^2}{c^2\sqrt{1-(v/c)^2}} + 1\right) = m^2c^4\left(\frac{dt}{dt_0}\right)^2 \implies E = mc^2\frac{dt}{dt_0}$$

(b) The transformations we will need are

$$dt' = \gamma dt, \qquad d\vec{r}' = d\vec{r}\frac{\hat{x}}{\gamma}$$

Then, in S',

$$\vec{p}' = m\frac{d\vec{r}'}{dt_0} = m\frac{d\vec{r}\frac{\hat{x}}{\gamma}}{dt_0} = m\hat{x}\sqrt{1-(u/c)^2}\frac{d\vec{r}}{dt_0}$$

and

$$E = mc^2\frac{dt}{dt_0} = mc^2\frac{\gamma dt}{dt_0} = \frac{mc^2}{\sqrt{1-(u/c)^2}}\frac{dt}{dt_0}$$

3. (a)

$$\vec{p}_1 = \frac{m_e v_i}{\sqrt{1-(v_i/c)^2}}\hat{x}$$

$$\vec{p}_{1,f} = \frac{m_e \vec{v}_{1,f}}{\sqrt{1-(v_{1,f}/c)^2}} = \frac{m_e v_{1,f}}{\sqrt{1-(v_{1,f}/c)^2}}(\cos\theta_1, \sin\theta_1)$$

(b)

$$\frac{m_e v_i}{\sqrt{1-(v_i/c)^2}}\hat{x} = \frac{m_e v_{1,f}}{\sqrt{1-(v_{1,f}/c)^2}}(\cos\theta_1, \sin\theta_1) + \frac{m_e v_{2,f}}{\sqrt{1-(v_{2,f}/c)^2}}(\cos\theta_2, \sin\theta_2)$$

Since the total momentum only has an x-component, the y-components must cancel out, so

$$p_{1,f}\sin\theta_1 = -p_{2,f}\sin\theta_2 \implies p_{2,f} = -\frac{\sin\theta_1}{\sin\theta_2}p_{1,f}$$

Plugging this back into the earlier equation gives that

$$p_{1,f} - \frac{\sin\theta_1}{\sin\theta_2}p_{1,f} = p_i \implies \frac{m_e\vec{v}_{1,f}}{\sqrt{1-(v_{1,f}/c)^2}}\left(1 - \frac{\sin\theta_1}{\sin\theta_2}\right) = \frac{m_e v_i}{\sqrt{1-(v_i/c)^2}}$$

(c)

$$p_{2,f} = \frac{m_e\vec{v}_{2,f}}{\sqrt{1-(v_{2,f}/c)^2}} \implies v_{2,f}(p_{2,f}) = \frac{cp_{2,f}}{\sqrt{c^2m_e^2 + p_{2,f}^2}}$$