Box Num. 33
Problem Set 34
April 23, 2018

1. (a) The binding energy is given by

$$\mathrm{BE} = a_{\mathrm{vol}}A - a_{\mathrm{surf}}A^{2/3} - a_{\mathrm{coul}}\frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}}\frac{(N-Z)^2}{A} + \epsilon\frac{a_{\mathrm{pair}}}{A^{1/2}}$$

Assuming A is odd, $\epsilon = 0$, so the simplified binding energy is

$$\mathrm{BE} = a_{\mathrm{vol}}A - a_{\mathrm{surf}}A^{2/3} - a_{\mathrm{coul}}\frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}}\frac{(N-Z)^2}{A}$$

and the semiempirical mass is

$$m_{\mathrm{nuc}} = Zm_p + Nm_n - \frac{a_{\mathrm{vol}}A - a_{\mathrm{surf}}A^{2/3} - a_{\mathrm{coul}}\frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}}\frac{(N-Z)^2}{A}}{c^2}$$

(b) If we take the derivative of the SEMF wrt. Z, we get that

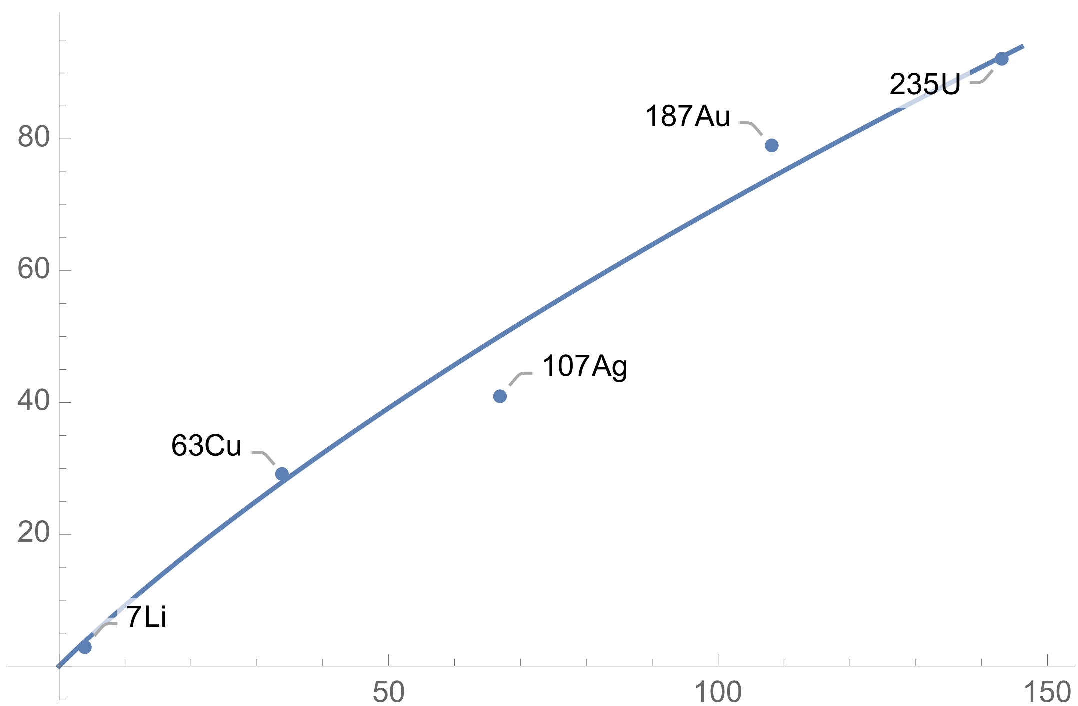
$$\frac{dm_{\mathrm{nuc}}}{dZ} = m_p - m_n + \frac{-4Aa_{\mathrm{sym}} + 2A^{2/3}a_{\mathrm{coul}Z} + 8a_{\mathrm{sym}}Z}{Ac^2},$$

which has a zero at

$$Z = \frac{A(4a_{\mathrm{sym}} + c^2m_n - c^2m_p)}{2(A^{2/3}a_{\mathrm{coul}} + 4a_{\mathrm{sym}})} = \frac{A}{2}\frac{1+\alpha}{1+\beta A^{2/3}},$$

with α and β defined as in the problem.

(c) For $A = 37$, the most stable Z is $Z = 17.25 \approx 17$. In reality, the most stable isotope with $A = 37$ is ^{37}Cl, with $Z = 17$ as expected.
For $A = 115$, $Z = 49.32 \approx 49$. The most stable isotope for $A = 115$ is ^{115}Sn, with $Z = 50$. This is close to the predicted value.
For $A = 185$, $Z = 75.11 \approx 75$. The most stable isotope here is ^{185}Re, with $Z = 75$, right as expected.



2.

3. (a) In all of these cases, there is an even number of neutrons, so they do not contribute to j_{tot}. For $^{39}_{19}\mathrm{K}_{20}$, there is a lone nucleon in the $1d_{3/2}$ state, so it has a total spin of 3/2. For $^{40}_{20}\mathrm{Ca}_{20}$, the even number of protons means there is no net spin. For $^{41}_{21}\mathrm{Sc}_{20}$, there is again an unpaired nucleon in the $1d_{3/2}$ state, for a total spin of 3/2.

(b) See attached page.

(c) ^{12}C has a j_{tot} of 1/2, since there is a single unpaired neutron in the $1p_{1/2}$ state, and no unpaired protons.

(d) ^{13}N has an even number of neutrons, so they do not contribute to j_{tot}. In the ground state, there is a single unpaired proton in the $1p_{1/2}$ state, for a total spin of 1/2. When excited, this unpaired proton moves to the $1d_{5/2}$ level, for a total spin of 5/2.

4. (a) See attached page.

(b) Process ii. takes in heat, and process iv. expels it. Processes i and iii do not change the heat of the system.

(c) Let P_1 be the pressure at points b and c, and let P_2 be the pressure at a and d. If we are given the difference in pressure, ΔP, and the distance between a and d, the volume at b and c can be found. We have that

$$P_1 V_b^\gamma = P_2 V_a^\gamma \implies V_b = V_a \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$P_1 V_c^\gamma = P_2 V_d^\gamma \implies V_c = V_d \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

Then the total work of the cycle is

$$W = \oint P dV = \int_{V_a}^{V_b} P_2 \left(\frac{V_a}{V}\right)^\gamma dV + \int_{V_b}^{V_c} P_1 dV + \int_{V_c}^{V_d} P_1 \left(\frac{V_d}{V}\right)^\gamma dV + \int_{V_d}^{V_a} P_2 dV$$