ID #33
Problem Set 18
Physics 202
March 7, 2018

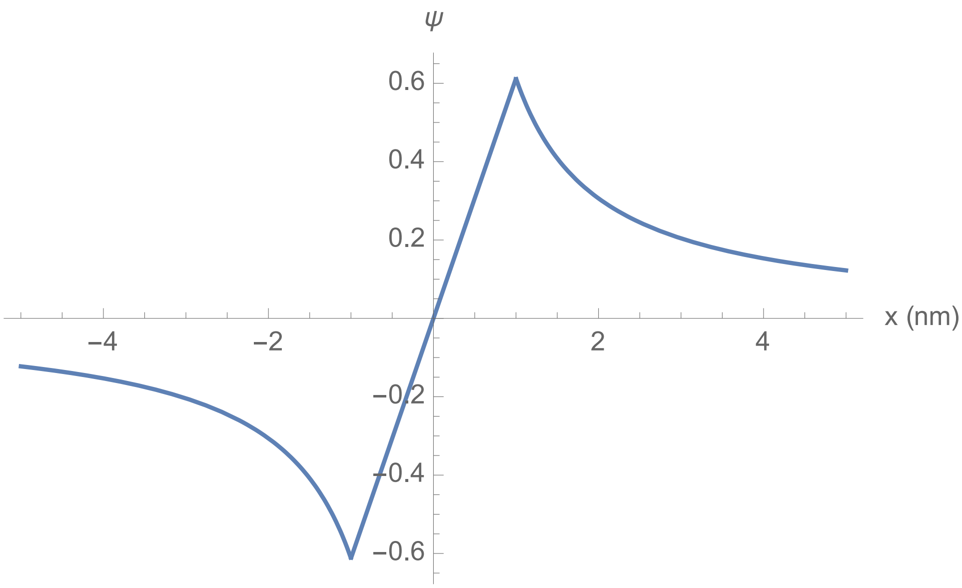
1. (a) The PDF for this function is

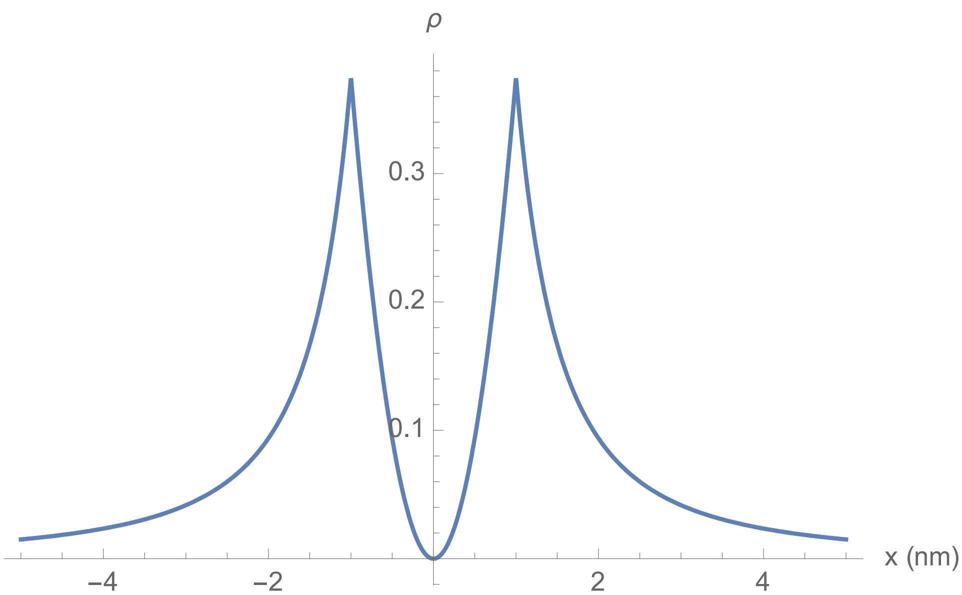
$$\psi(x) = \begin{cases} (xc)^2 & |x| \leq 1\,\text{nm} \\ (c/x)^2 & |x| \geq 1\,\text{nm} \end{cases}$$

Since the PDF must be normalized, we want c^2 s.t.

$$1 = 2\int_{x=1}^{\infty} c^2/x^2 dx + \int_{x=-1}^{1} c^2x^2 dx = 2c^2 + \frac{2c^2}{3} \implies c^2 = \frac{3}{8} \implies c = \frac{\sqrt{3}}{2\sqrt{2}}$$

(b) The graph of the wavefunction between $\pm 5\,\text{nm}$ is





(c)

(d) Using our known value for c^2,

$$\rho = \int_{-1}^{1} |\psi(x)|^2 dx = \int_{-1}^{1} \frac{3}{8}x^2 = \frac{1}{4}$$

Therefore, 2.5×10^5 electrons will be found between $\pm 1\,\text{nm}$.

2. (a) Assuming $\alpha > 0$,

$$1 = 2b\int_{k_0}^{\infty} C^2 e^{-2\alpha(k-k_0)}dk = \frac{bC^2}{\alpha} \implies b = \frac{\alpha}{C^2}$$

Since the Gaussian is symmetric, the total integral is twice the right half, as calculated above. Therefore,

$$\psi(k) = \frac{\sqrt{\alpha}}{C}e^{-\alpha(k-k_0)^2}$$

(b) The Fourier transform of $\psi(k)$ is given by

$$\psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(k)e^{ikt}dt = \left(\frac{\alpha}{2C}\right)^{\frac{1}{2}} e^{ik_0x}e^{-\frac{x^2}{4a}}$$

I know the constant here isn't right, sorry.

3. (a) Conservation of energy is violated by $\Delta E = 135\,\mathrm{MeV}/{c_0}^2$. Because the pi meson has this mass, and appears out of nowhere, this is the amount of energy that must be spontaneously created.

(b)

$$\Delta E\Delta t = \frac{\hbar}{2} \implies \Delta t = \frac{\hbar}{2\Delta E} = \frac{6.583\times10^{-16}\,\mathrm{J\,s}}{2(1.35\times10^{8}\,\mathrm{eV})} = 2.44\times10^{-24}\,\mathrm{s}$$

(c) If the pi meson travels near the speed of light, then the distance it can travel is about

$$c(\Delta t) = (2.44\times10^{-24}\,\mathrm{s})(3\times10^{8}\,\mathrm{m/s}) = 7.31\times10^{-16}\,\mathrm{m}$$

4. Letting the angle of the incident proton be 0, the angle of the two scattered photons must be the same. Call this angle θ. Then the angle between the photons, which we're trying to calculate, is given by 2θ. The momentum of each of the protons is also give by

$$E^2 = (pc)^2 + (mc^2)^2 \implies p = \frac{\sqrt{E^2-(mc^2)^2}}{c}$$

We have that

$$2\frac{\sqrt{(E/2)^2-(mc^2)^2}}{c}\sin\theta = \frac{\sqrt{E^2-(mc^2)^2}}{c}$$

because the x-direction momentum must be conserved. This gives that

$$\theta = \arcsin\left(\sqrt{\frac{E^2-(mc^2)^2}{E^2-(2mc^2)^2}}\right)$$

and so the angle between the two protons is

$$2\theta = 2\arcsin\left(\sqrt{\frac{E^2-(mc^2)^2}{E^2-(2mc^2)^2}}\right)$$