ID #33
Problem Set 20
Physics 202
March 19, 2018

1. (a) From Krane, we know that the ground state energy of this system is

$$E_0 = \frac{h^2}{8mL^2} = \frac{(1.626 \times 10^{-34}\,\mathrm{J\,s})^2}{8(9.109 \times 10^{-31}\,\mathrm{kg})(5\,\mathrm{nm})^2} = 1.451 \times 10^{-22}\,\mathrm{J}$$

Then the energy of the n-th energy state is $E_n = n^2 E_0$, so

$$E_4 = 16E_0 = 2.322 \times 10^{-21}\,\mathrm{J}$$

and the energy releassed in the phase transition is

$$E_4 - E_0 = 2.177 \times 10^{-21}\,\mathrm{J}$$

The wavelength of the released photon is then

$$\lambda = \frac{hc}{E} = 91.3\,\mathrm{\mu m}$$

(b) If we approximate a nucleon as an infinite potential energy well, its base energy is given by

$$E_0 = \frac{h^2}{8mL^2}$$

Decreasing the size of the well increases the base energy, and therefore the difference between energy levels. Therefore, decreasing the size of the potential well (relative to the scenario in part a.) will increase the difference between energy levels and therefore the energy of the photons released.

2. First, looking at the case where $m = n$, this integral becomes

$$\int_{x=-\infty}^{\infty} \psi_n^* \psi_n dx = \int_{x=-\infty}^{\infty} (|\psi_n|)^2 dx = 1$$

When $m \neq n$,

$$\int_{x=-\infty}^{\infty} \psi_n^* \psi_m dx$$

However, the wavefunctions are complex exponentials, so this integral is equal to

$$\int_{x=-\infty}^{\infty} (Ae^{-im\pi x/L})(Be^{in\pi x/L}) \implies \int_{x=-\infty} \psi_n^* \psi_m dx = -\int_{x=0} \infty \psi_n^* \psi_m dx \implies \int_{x=-\infty}^{\infty} \psi_n^* \psi_m dx = 0$$

Therefore,

$$\int_{x=-\infty}^{\infty} \psi_n^* \psi_m dx = \delta_{nm}$$

3. (a) From Krane, we know that

$$k = \sqrt{\frac{2mE}{\hbar^2}} \text{ and } k' = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \implies k = k'\sqrt{\frac{E}{U - E}} \implies E = \frac{k^2 U}{k^2 + k'^2}$$

(b) When $U_0 \to \infty$, $k' \to k$, so the base energy is $E = \frac{\pi^2 \hbar^2}{2mL^2}$.

4. (a) The diagram is on the reverse side of the page.

(b) The work done by the gas is

$$W_{12} = Nk_B T_2 \log \frac{V_2}{V_1}$$

$$W_{34} = Nk_B T_1 \log \frac{V_1}{V_2}$$

The change in heat over the isochores is

$$Q_{23} = C_V(T_1 - T_2), Q_{41} = C_V(T_2 - T_1)$$

Then, since

$$\eta = \frac{\Delta W}{\Delta Q}$$

the efficiency of the Stirling engine is

$$\eta = \frac{Nk_B T_2 \log \frac{V_2}{V_1} + Nk_B T_1 \log \frac{V_1}{V_2}}{C_V(T_2 - T_1) + C_V(T_1 - T_2)} = \frac{T_2 - T_1}{T_2 + \frac{C_V(T_2 - T_1)}{Nk \log V_2/V_1}}$$

(c) This efficiency is lower than the Carnot efficiency, because no cycle is more efficient, and only the Carnot cycle is equally efficient.