Box Num. 33
Problem Set 31
April 16, 2018

1. The energy of any given wave must be

$$E = pc = c\sqrt{p_x^2 + p_y^2}.$$

Since every momentum component must be consistent with the boundary conditions for that wavelength,

$$E = c\sqrt{\left(\frac{\hbar\pi n_x}{L}\right)^2 + \left(\frac{\hbar\pi n_y}{L}\right)^2} = \frac{\hbar c\pi}{L}\sqrt{n_x^2 + n_y^2} = \frac{\hbar c n\pi}{L}$$

Since there are only two dimensions in this problem (instead of three as shown in class), we need to imagine a disk instead of a sphere. The number of points corresponding to a value of $E$ is then $\frac{1}{2}\pi n$, where $n = \sqrt{n_x^2 + n_y^2}$. Since there are two possible spins for photons, the number of states per unit volume is

$$g(n)dn = \frac{1}{2}\frac{2}{A}\pi n dn = \frac{\pi n}{L^2}dn = g(E)dE$$

Since $E = \frac{\hbar c n\pi}{L}$, we have $dE = \frac{\hbar c\pi}{L}$, so $g(E) = \frac{n}{cL\hbar}$.

2. The Maxwell-Boltzmann energy distribution for this gas of electrons is

$$g(E) = \frac{2}{\sqrt{\pi}(kT)^{3/2}}E^{1/2}e^{-E/kT} = \frac{2}{\sqrt{\pi}(0.0252\,\mathrm{eV})^{3/2}}E^{1/2}e^{-E/0.0252\,\mathrm{eV}}$$

The interval of 1% around the most probable energy is the range from 0.024 95 eV to 0.2545 eV. According to the Stefan-Boltzmann distribution, the

4. (a) The partition function is defined as:

$$Z = \sum_i e^{-E_i/k_BT} = e^{-\epsilon/k_BT} + 2e^0 + e^{\epsilon/k_BT} = 2 + e^{\epsilon/k_BT} + e^{-\epsilon/k_BT}$$

(b) The probability of the system having energy 0 is

$$P_0 = \frac{2}{2 + e^{\epsilon/k_BT} + e^{-\epsilon/k_BT}}$$

And the probability that it has energy $\epsilon$ or $-\epsilon$, respectively, is

$$P_\epsilon = \frac{E^{-\epsilon/k_BT}}{2 + e^{\epsilon/k_BT} + e^{-\epsilon/k_BT}}, P_{-\epsilon} = \frac{E^{\epsilon/k_BT}}{2 + e^{\epsilon/k_BT} + e^{-\epsilon/k_BT}}$$

(c) These figures show the probabilities of $E = 0$, $E = \epsilon$, and $E = -\epsilon$, respectively.