ID #33
Problem Set 17
Physics 202
March 5, 2018

1. (a) These neutrons' kinetic energy is

$$K = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\,\mathrm{J/K})(293\,\mathrm{K}) = 6.06 \times 10^{-21}\,\mathrm{J} = 0.038\,\mathrm{eV}$$

(b) To calculate the momentum of one of these neutrons, we need its velocity. This is given by

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.06 \times 10^{-21}\,\mathrm{J})}{1.67 \times 10^{-27}\,\mathrm{kg}}} = 2695\,\mathrm{m/s}$$

Then the momentum is

$$p = mv = (1.67 \times 10^{-27}\,\mathrm{kg})(2695\,\mathrm{m/s}) = 7.20 \times 10^{-24}\,\mathrm{kgm/s} \tag{1}$$

and the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\,\mathrm{J\,s}}{7.20 \times 10^{-24}\,\mathrm{kgm/s}} = 9.21 \times 10^{-11}\,\mathrm{m} = 92.08\,\mathrm{pm}$$

2. (a) Since $K = \frac{1}{2}mv^2 = \frac{5}{2}kT$, and $p = mv$, the momentum

$$p = m\sqrt{\frac{5kT}{m}} = \sqrt{5mkT}$$

and the de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{5mkT}} = \frac{6.626 \times 10^{-34}\,\mathrm{J\,s}}{\sqrt{5(2.3 \times 10^{-24}\,\mathrm{kg})(1.38 \times 10^{-23}\,\mathrm{J/K})(293\,\mathrm{K})}} = 1.14 \times 10^{-10}\,\mathrm{m} = 3.073\,\mathrm{pm}$$

(b) In a cubic meter of air, there are 1.292 kg of gas. Assuming all of this gas is N_2, which has a molar mass of 28.03 g/mol. Therefore, there are $1.292/0.02803 = 46.09\,\mathrm{mol/m^3}$. Each molecule therefore occupies a volume of about

$$V_{\mathrm{molecule}} \approx \frac{1\,\mathrm{m^3}}{(46.09\,\mathrm{mol/m^3})(6.022 \times 10^{23}\,/\mathrm{mol})} = 3.6 \times 10^{-26}\,\mathrm{m^3}$$

and so the distance between molecules is about $\sqrt[3]{3.6 \times 10^{-26}\,\mathrm{m^3}} = 3.3 \times 10^{-9}\,\mathrm{m} = 3.303\,\mathrm{nm}$. Since this is about 3 orders of magnitude larger than the de Broglie wavelength, quantum effects are not very important at room temperature.

(c) Since the general rule of thumb is that quantum effects become important when the distance between molecules is less than the de Broglie wavelength, we need to find T s.t.

$$\lambda = \frac{h}{p} \approx 3.303\,\mathrm{nm}$$

which can be rewritten as

$$3.303\,\mathrm{nm} = \frac{6.602 \times 10^{-34}\,\mathrm{J\,s}}{\sqrt{5(2.3 \times 10^{-24}\,\mathrm{kg})(1.38 \times 10^{-23}\,\mathrm{J/K})T}} \implies T = 0.000\,25\,\mathrm{K}$$

3. (a) According to the email, we didn't have to do part (a.)

(b) The rest mass energy of a proton is

$$E = mc^2 = (1.672 \times 10^{-27}\,\mathrm{kg})(3 \times 10^8\,\mathrm{m/s})^2 = 1.505 \times 10^{-10}\,\mathrm{J} = 939.35\,\mathrm{MeV}$$

Since the rest mass energy is so much larger than the kinetic energy, we can use classical mechanics. The velocity of the proton in the nucleus is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(10\,\mathrm{MeV})}{1.672 \times 10^{-27}\,\mathrm{kg}}} = 4.38 \times 10^7\,\mathrm{m/s}$$

Therefore, the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s})}{(4.38 \times 10^7\,\mathrm{m/s})(1.672 \times 10^{-27}\,\mathrm{kg})} = 9.05 \times 10^{-15}\,\mathrm{m}$$

and

$$\frac{\lambda}{r} = \frac{9.05 \times 10^{-15}\,\mathrm{m}}{1 \times 10^{-12}\,\mathrm{m}} = 0.00905$$

so wave mechanics are not needed for this problem.

(c) The rest mass energy of an electron is 510 keV, so we can use classical mechanics for this problem. The electrons have a velocity of

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.602 \times 10^{-15}\,\mathrm{J})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 5.93 \times 10^7\,\mathrm{m/s}$$

and a de Broglie wavelength of

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\,\mathrm{J\,s}}{(5.93 \times 10^7\,\mathrm{m/s})(9.11 \times 10^{-31}\,\mathrm{kg})} = 1.22 \times 10^{-11}\,\mathrm{m}$$

Even if we wanted the resolution of our screen to be 1 μm, we would not have to worry about wave effects. Since real resolutions are far less fine than this, these designers do not have to account for quantum physics.