Abstract Algebra Chapter 1

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December 27, 2022

End of Chapter Exercises

Question 1.

WIth words, describe each symmetry in D_3 (the set of symmetries of an equilateral triangle).

Question 2.

Write out a complete Cayley table for D_3.

Question 3.

Is D_3 Abelian?

Question 4.

Describe in words the elements of D_5 (symmetries of a regular pentagon).

Question 5.

For $n \geq 3$, describe the elements of D_n. *(Hint: Consider two cases - n even and n odd.)* How many elements does D_n have?

Question 6.

In D_n, explain geometrically why a reflection followed by a reflection must be a rotation.

Question 7.

In D_n, explain geometrically why a rotation followed by a rotation must be a rotation.

Question 8.

In D_n, explain geometrically why a rotation and a reflection taken together in either order must be a reflection.

Question 9.

Associate the number $+1$ with a rotation and the number -1 with a reflection. Describe an analogy between multiplying these two numbers and multiplying elements of D_n.

Question 10.

If r_1, r_2, and r_3 represent rotations from D_n and f_1, f_2, and f_3 represent reflections from D_n, determine whether $r_1 r_2 f_1 r_3 f_2 f_3 r_3$ is a rotation or a reflection.

Question 11.

Find elements A, B, and C in D_4 such that $AB = BC$ but $A \neq C$. (Thus, "cross cancellation" is not valid.)

Question 12.

Explain what the following diagram proves about the group D_n.

Question 13.

Describe the symmetries of a nonsquare rectangle. Construct the corresponding Cayley table.

Question 14.

Describe the symmetries of a parallelogram that is neither a rectangle nor a rhombus. Describe the symmetries of a rhombus that is not a rectangle.

Question 15.

Describe the symmetries of a noncircular ellipse. Do the same for a hyperbola.

Question 16.

Consider an infinitely long strip of equally spaced H's:

... H H H H ...

Describe the symmetries of this strip. Is the group of symmetries of the strip Abelian?

Question 17.

For each of the snowflakes in the figure, find the symmetry group and locate the axes of reflective symmetry.

Question 18.

Determine the symmetry group of the outer shell of the cross section of the human immunodeficiency virus (HIV).

Question 19.

Does an airplane propeller have a cyclic symmetry group or a dihedral symmetry group?

Question 20.

Bottle caps that are pried off sypically have 22 ridges around the rim. Fine the symmetry group of such a cap.

Question 21.

What group theoretic property do upper-case letters F, G, J, K, L, P, Q, R have that is not shared by the remaining upper-case letters in the alphabet?

Question 22.

For each design below, determine the symmetry group.

Question 23.

What would the effect be if a six-bladed ceiling fan were designed so that

the centerlines of two of the blades were at 70^o angle and all the other blades were set at 58^o angle?