Abstract Algebra Chapter 2

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Important Statements:

Groups:

1. *Associativity:* $(ab)c = a(bc)\ \forall\ a, b, c \in G$
2. *Identity:*
3. *Inverses:*

Uniqueness of the Identity:

In a group G, there is only one identity element.

Cancellation:

In a group G, the right and left cancellation laws hold; that is,

$ba = ca \Rightarrow b = c$ and $ab = ac \Rightarrow b = c$

Uniqueness of Inverses:

For each element a in a group G, there is a unique element b in G such that $ab = ba = e$.

Socks-Shoes Principle:

For group elements a and b, $(ab)^{-1} = b^{-1}a^{-1}$.

End of Chapter Exercises

Question 1.

Give two reasons why the set of odd integers under addition is not a group.

Question 2.

Referring to Example 13, verify the assertion that subtraction is not associative.

Question 3.

Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.