Abstract Algebra Chapter 2

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Important Statements:

Groups:

1. *Associativity:* $(ab)c = a(bc) \; \forall \; a, b, c \in G$
2. *Identity:*
3. *Inverses:*

Uniqueness of the Identity:

In a group G, there is only one identity element.

Cancellation:

In a group G, the right and left cancellation laws hold; that is,

$ba = ca \Rightarrow b = c$ and $ab = ac \Rightarrow b = c$

Uniqueness of Inverses:

For each element a in a group G, there is a unique element b in G such that $ab = ba = e$.

Socks-Shoes Principle:

For group elements a and b, $(ab)^{-1} = b^{-1}a^{-1}$.

End of Chapter Exercises

Question 1.

Give two reasons why the set of odd integers under addition is not a group.

Question 2.

Referring to Example 13, verify the assertion that subtraction is not associative.

Question 3.

Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.

Question 4.

Show that the group $GL(2, \mathbb{R})$ of Example 9 is non-Abelian by exhibiting a pair of matrices A and B in $GL(2, \mathbb{R})$ such that $AB \neq BA$.

Question 5.

Find the inverse of the element a in $GL(2, \mathbb{Z}_{11})$.

Question 6.

Give an example of group elements a and b with the property that $a^{-1}ba \neq b$.

Question 7.

Translate each of the following multiplicative expressions into its additive counterpart. Assume that the operation is commutative.

(a) a^2b^3

(b) $a^{-2}(b^{-1}c)^2$

(c) $(ab^2)^{-3}c^2 = e$