Linear Algebra Chapter 1

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End of Chapter Exercises

Section 1.1 Introduction

Question 1.

Determine whether the vectors emanating from the origin and terminating at the following pairs of points are parallel.

(a) (3,1,2) and (6,4,2)

(b) (-3,1,7) and (9,-3,-21)

(c) (5,-6,7) and (-5,6,-7)

(d) (2,0,-5) and (5,0,-2)

Question 2.

Find the equations of the lines through the following pairs of points in space.

(a) (3,-2,4) and (-5,7,1)

(b) (2,4,0) and (-3,-6,0)

(c) (3,7,2) and (3,7,-8)

(d) (-2,-1,5) and (3,9,7)

Question 3.

Find the equations of the planes containing the following points in space.

(a) (2,-5,-1), (0,4,6) and (-3,7,1)

(b) (3,-6,7), (-2,0,-4) and (5,-9,-2)

(c) (-8,2,0), (1,3,0) and (6,-5,0)

(d) (1,1,1), (5,5,5) and (-6,4,2)

Question 4.

What are the coordinates of the vector 0 in the Euclidean plane that satisfies Property 3? (There exists a vector denoted 0 such that $x + 0 = x$ for each vector x.)

Question 5.

Prove that if the vector x emanates from the origin of the Euclidean plane and terminates at the point with coordinates (a_1, a_2), then the vector tx that emanates from the origin terminates at the point with coordinates (ta_1, ta_2).

Question 6.

Show that the midpoint of the line segment joining the points (a, b) and (c, d) is $((a + c)/2, (b + d)/2)$.

Question 7.

Prove that the diagonals of a parallelogram bisect each other.

Section 1.2 Vector Spaces

Question 1.

Label the following statements as True or False.

(a) Every vector space contains a zero vector.

(b) A vector space may have more than one zero vectos.

(c) In any vector space, $ax = bx$ implies that $a = b$.

(d) In any vector space, $ax = ay$ implies that $x = y$.

(e) A vector in $\mathbb{F}^n$ may be regarded as a matrix in $M_{n\times 1}(F)$.

(f) An $m \times n$ matrix has m columns and n rows.

(g) In $\mathbf{P}(F)$, only polynomials of the same degree may be added.

(h) If f and g are polynomicals of degree n, then $f + g$ is a polynomial of degree n.

(i) If f is a polynomial of degree n and c is a nonzero scalar, then cf is a polynomial of degree n.

(j) A nonzero scalar of F may be considered to be a polynomial in $\mathbf{P}(F)$ having degree zero.

(k) Two functions in $\mathcal{F}(S, F)$ are equal if and only if they have the same value at each element of S.

Question 2.

Write the zero vector of $M_{3\times 4}(F)$.

Question 3.

If

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

what are $M_{1,3}$, $M_{2,1}$, and $M_{2,2}$?

Question 4.

Perform the indicated operations.

(a) $\begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 5 \\ -5 & 3 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -6 & 4 \\ 3 & -2 \\ 1 & 8 \end{pmatrix} + \begin{pmatrix} 7 & -5 \\ 0 & -3 \\ 2 & 0 \end{pmatrix}$

(c) $4 \begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix}$

(d) $-5\begin{pmatrix} -6 & 4 \\ 3 & -2 \\ 1 & 8 \end{pmatrix}$

Question 5.

Section 1.3 Subspaces

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Section 1.4 Linear Combinations and Systems of Linear Equations

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Section 1.5 Linear Dependence and Linear Independence

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Section 1.6 Bases and Dimension

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Section 1.7 Maximal Linearly Independent Subsets

Question 1.

Question 2.

Question 3.

Question 4.

Question 5.

Question 6.

Question 7.

Question 8.

Question 9.

Question 10.