SPST502 Homework 7

Thomas Mangan (ID 5712142)

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Question 1.

The Space Shuttle Main Engines (RS-25) have a rated power level of 470,800 lbs of thrust and have an effective exhaust velocity of 4,447 m/s each. What is the mass flow rate of the fuel mixture for one of these engines? (Note: 1 lb of thrust = 4.4482 N)

First, we convert the power level of the RS-25 from lbs of thrust to Newtons:

$$470,800lbs \times \frac{4.4482N}{1lb} = 2.094 \times 10^6 N$$

Then, using the equation:

$$F_{thrust} = \dot{m}C \tag{14-2}$$

where

$$\begin{aligned} F_{thrust} &= \text{Rocket's total thrust } (N) \\ \dot{m} &= \text{Mass flow rate } (kg/s) \\ C &= \text{Effective exhaust velocity } (m/s) \end{aligned}$$

We can rearrange (14-2) to get:

$$\dot{m} = \frac{F_{thrust}}{C}$$

with $F_{thrust} = 2.094 \times 10^6 N$ and $C = 4,447 m/s$.

Finally,

$$\dot{m} = \frac{2.094 \times 10^6 N}{4,447 m/s} = 470.87 kg/s.$$

Question 2.

Based on the information in Question 1, what is the jet power produced by one of the RS-25 engines?

We use the equation:

$$P_J = \frac{1}{2}\dot{m}C^2 \tag{14-4}$$

where

$$\begin{aligned} P_J &= \text{Jet power } (J/s = W) \\ \dot{m} &= \text{Mass flow rate } (kg/s) \\ C &= \text{Effective exhaust velocity } (m/s) \end{aligned}$$

We simply get:

$$\begin{aligned} P_J &= \frac{1}{2}\dot{m}C^2 \\ &= \frac{1}{2}(470.87kg/s)(4,447m/s)^2 \\ &= 4.656 \times 10^9 J/s. \end{aligned}$$

Question 3.

Using the information in Question 1, determine the Specific Impulse of one of the RS-25 engines.

We use the equation:

$$I_{sp} = \frac{C}{g_0} \tag{14-8}$$

where

$$\begin{aligned} I_{sp} &= \text{Specific Impulse } (s) \\ C &= \text{Effective exhaust velocity } (m/s) \\ g_0 &= \text{Gravitational acceleration constant} = 9.81m/s^2 \end{aligned}$$

The Specific Impulse in one RS-25 engine is

$$\begin{aligned} I_{sp} &= \frac{C}{g_0} \\ &= \frac{4,447m/s}{9.81m/s^2} \\ &= 453.3s. \end{aligned}$$

Question 4.

The Space Shuttle main tanks hold approximately 719,000 kg of liquid propellant (combined weight of liquid hydrogen and liquid oxygen). Assume 718,500 kg is consumed by the 3 main engines during launch. Using this data and the specific impulse from Question 3, determine the total impulse of one of the RS-25 engines. (Note: Use 9.81 m/s^2 as the gravitational constant)

We will use the definition of Specific Impulse to determine the Total Impulse of the rocket.

By definition:

$$I_{sp} \equiv \frac{I}{\Delta m_{propellant} \cdot g_0} \tag{14-6}$$

where

$$\begin{aligned} I_{sp} &= \text{Specific Impulse } (s) \\ I &= \text{Total Impulse } (N \cdot s) \\ g_0 &= \text{Gravitational acceleration constant} = 9.81m/s^2 \\ \Delta m_{propellant} &= \text{Change in propellant's mass } (kg) \end{aligned}$$

We may rearrange the definition above to get:

$$I = I_{sp} \cdot \Delta m_{propellant} \cdot g_0$$

Thus,

$$\begin{aligned} I &= I_{sp} \cdot \Delta m_{propellant} \cdot g_0 \\ &= 453.3s \cdot 718,500kg \cdot 9.81m/s^2 \\ &= 3.19 \times 10^9 N \cdot s. \end{aligned}$$

Question 5.

The Space Shuttle Orbital Maneuvering System (OMS) uses 2 AJ10-190 engines (normally used one at a time for maneuvering). Each engine can produce approximately 6,000 lb of thrust at a mass flow rate of $\dot{m}$ = 8.61 kg/s. Using this information

(a) Determine the specific impulse of the AJ10-190 engine.

First we will convert the thrust into Newtons.

$$6000lbs \times \frac{4.4482N}{1lb} = 26,689N$$

Using equation

$$I_{sp} = \frac{F_{thrust}}{\dot{m} \cdot g_0} \tag{14-7}$$

where

$$\begin{aligned} I_{sp} &= \text{Specific Impulse } (s) \\ F_{thrust} &= \text{Force of thrust } (N) \\ g_0 &= \text{Gravitational acceleration constant} = 9.81m/s^2 \\ \dot{m} &= \text{Propellant's mass flow rate } (kg/s) \end{aligned}$$

Then we simply have

$$\begin{aligned} I_{sp} &= \frac{26,689N}{8.61kg/s \cdot 9.81m/s^2} \\ &= 315.98s \end{aligned}$$

(b) What is the effective exhaust velocity of the engine?

We will use the equation:

$$F_{thrust} = \dot{m}C \tag{14-2}$$

where

$$\begin{aligned} F_{thrust} &= \text{Force of thrust } (N) \\ \dot{m} &= \text{Propellant's mass flow rate } (kg/s) \\ C &= \text{Effective exhaust velocity } (m/s) \end{aligned}$$

After a rearrangement, we get:

$$C = \frac{F_{thrust}}{\dot{m}}$$

Finally,

$$\begin{aligned} C &= \frac{26,689N}{8.61kg/s} \\ &= 3099.7m/s \end{aligned}$$

Alternatively, we could have calculated:

$$C = I_{sp} \cdot g_0 = 315.98s \cdot 9.81m/s^2$$

and generated the same result.

Question 6.

The typical weight of the Space Shuttle, once reaching space and dropping the external tank and booster rockets was approximately 125,900 kg. This included 11,800 kg of fuel in the OMS pods for orbital maneuvering. Using this data and the specific impulse or effective exhaust velocity of the AJ10-190 engine from Question 5, what is the total ΔV that can be produced by the OMS?

First, we will calculate the final mass of the Space Shuttle using the initial (wet) mass and the expended fuel mass to determine the final (dry) mass.

$$\begin{aligned} m_{final} &= m_{initial} - m_{fuel} \\ &= 125,900kg - 11,800kg \\ &= 114,100kg \end{aligned}$$

Next, we use the Rocket Equation:

$$\Delta V = I_{sp} g_0 \ln\left(\frac{m_{initial}}{m_{final}}\right) \tag{14-11}$$

where

$$\begin{aligned} \Delta V &= \text{Vehicle's change in velocity } (m/s) \\ I_{sp} &= \text{Specific Impulse of rockst } (s) \\ g_0 &= \text{gravitational acceleration at sea level } (m/s^2) \\ m_{initial} &= \text{vehicle's initial mass } (kg) \\ m_{final} &= \text{vehicle's final mass } (kg) \end{aligned}$$

Then,

$$\begin{aligned} \Delta V &= I_{sp} g_0 \ln\left(\frac{m_{initial}}{m_{final}}\right) \\ &= 315.98s \cdot 9.81m/s^2 \cdot \ln\left(\frac{125,900kg}{114,100kg}\right) \\ &= 304.75m/s \end{aligned}$$

Question 7.

NASA engineers are developing New Horizons II space probe that will be used to scout asteroids for potential resources. Unlike its predecessor, this time, engineers want to use cold propellant thrusters instead of hydrazine. The new probe's mass, with the thruster propellant tank empty, is expected to be 92 kg. The propellant will be stored in a high-pressure spherical tank with a diameter of 0.36 m. The propellant will be stored at 225 bars at a temperature at 273 K. The options have been narrowed to Ammonia (17 kg/k·mol), which can provide a specific impulse of 61 s, and Carbon Dioxide (44 kg/k·mol), which can provide specific impulse of 96 s. Based on this information: (Note: 1 bar = 1.0×10^5Pa)

For both parts (a) and (b), we will use the Ideal Gas Law

$$P = \rho R T \tag{14-12}$$

where

$$\begin{aligned} P &= \text{Pressure } (N/m^2 = 1.0 Pa) \\ \rho &= \text{Density } (kg/m^3) \\ T &= \text{Termperature } K \\ R &= \text{Specific gas constant } (J/kg \cdot K) \end{aligned}$$

Also,

$$225 bar \times \frac{1.0 \times 10^5 Pa}{1 bar} = 2.25 \times 10^7 Pa$$

(a) Determine the density of Ammonia.

We must determine the specific gas constant for Ammonia. It is given by the following formula:

$$R_{Ammonia} = \frac{Ru}{M}$$

where

$$\begin{aligned} Ru &= \text{Universal gas constant } (8314.41 J/kmole \cdot K) \\ M &= \text{Molecular mass } (kg/kmole) \end{aligned}$$

Therefore,

$$\begin{aligned} R_{Ammonia} &= \frac{Ru}{M} \\ &= \frac{8314.41 J/kmole \cdot K}{17 kg/kmole} \\ &= 489.08 J/(kg \cdot K) \end{aligned}$$

Then, a rearrangement of the Ideal Gas Law gives us

$$\rho = \frac{P}{RT}$$

We conclude

$$\begin{aligned} \rho_{Ammonia} &= \frac{2.25 \times 10^7 Pa}{489.08 J/(kg \cdot K) \cdot 273 K} \\ &= 168.52 kg/m^3 \end{aligned}$$

(b) Determine the density of Carbon Dioxide.

Similarly, we must determine the specific gas constant for Carbon Dioxide.

$$R_{CarbonDioxide} = \frac{Ru}{M}$$

where

$$\begin{aligned} Ru &= \text{Universal gas constant } (8314.41 J/kmole \cdot K) \\ M &= \text{Molecular mass } (kg/kmole) \end{aligned}$$

Therefore,

$$\begin{aligned} R_{CarbonDioxide} &= \frac{Ru}{M} \\ &= \frac{8314.41 J/kmole \cdot K}{44 kg/kmole} \\ &= 188.96 J/(kg \cdot K) \end{aligned}$$

Again, we rearrange the Ideal Gas Law to get

$$\rho = \frac{P}{RT}$$

So,

$$\begin{aligned} \rho_{CarbonDioxide} &= \frac{2.25 \times 10^7 Pa}{188.96 J/(kg \cdot K) \cdot 273K} \\ &= 436.16 kg/m^3 \end{aligned}$$

Question 8.

Using the information in Question 7:

(a) What is the volume of the propellant tank?

Based on the above information that the propellant tanks are spherical and have a diameter of $0.36m$ we use the formula for the volume of a sphere:

$$\begin{aligned} Volume_{tank} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \left(\frac{1}{2} 0.36m\right)^3 \\ &= 0.02443m^3 \end{aligned}$$

(b) Calculate the mass of the Ammonia.

We will use the density of Ammonia and the volume of the spherical tank to determine the mass of the Ammonia in kg.

$$\begin{aligned} Mass_{Ammonia} &= \rho_{Ammonia} \times V_{tank} \\ &= 168.52 kg/m^3 \times 0.02443m^3 \\ &= 4.117kg \end{aligned}$$

(c) Calculate the mass of the Carbon Dioxide when the storage tank is full.

Similarly we will use the density of Carbon Dioxide and the volume of the spherical tank to determine the mass of the Ammonia in kg.

$$\begin{aligned} Mass_{CarbonDioxide} &= \rho_{CarbonDioxide} \times V_{tank} \\ &= 188.96kg/m^3 \times 0.02443m^3 \\ &= 4.6163kg \end{aligned}$$

Question 9.

Using the information in Questions 7 and 8:

(a) Determine the ΔV values for Ammonia.

First we will calculate the mass of the probe when it has a full tank of fuel:

$$\begin{aligned} m_{fulltank} &= m_{emptytank} + m_{Ammonia} \\ &= 93kg + 4.117kg \\ &= 97.117kg \end{aligned}$$

Then, using the rocket equation we get:

$$\begin{aligned} \Delta V &= I_{sp} g_0 \ln\left(\frac{m_{initial}}{m_{final}}\right) \\ &= (61s)(9.81\frac{m}{s^2})\ln\left(\frac{97.117kg}{93kg}\right) \\ &= 25.92\frac{m}{s}. \end{aligned}$$

(b) Determine the ΔV values for Carbon Dioxide.

Similarly we will calculate the mass of the probe when it has a full tank of fuel:

$$\begin{aligned} m_{fulltank} &= m_{emptytank} + m_{CarbonDioxide} \\ &= 93kg + 4.6163kg \\ &= 97.6163kg \end{aligned}$$

Then, using the rocket equation we get:

$$\begin{aligned} \Delta V &= I_{sp} g_0 \ln\left(\frac{m_{initial}}{m_{final}}\right) \\ &= (96s)(9.81\frac{m}{s^2})\ln\left(\frac{97.6163kg}{93kg}\right) \\ &= 45.62\frac{m}{s}. \end{aligned}$$

(c) Discuss which would be best for the mission.

It would be better to use Carbon Dioxide as the fuel for the probe because it will give more ΔV to the probe, and thus be able to perform more corrective maneuvering than if the Ammonia were used. This is mostly due to the higher Specific Impulse and negligible difference in mass.

(d) Extra Credit: After working problems 7-9, examine the details of Example 14-1 in the text. Do you notice a flaw in the way the problem was worked by the textbook authors? If you do detect a flaw in their calculations, how might this impact the mission? Your answer can add as much as 3 Points to your score for this assignment.

Question 10.

The Space Shuttle Main Engine Nozzles have an exit diameter of 90.3 in and an expansion ratio of 69:1. What is the nozzle throat diameter?

We will use the equation:

$$\epsilon = \frac{A_e}{A_t} \tag{14-22}$$

Where

$$\begin{aligned}
\varepsilon &= \text{Nozzle's expantion ratio} \\
A_e &= \text{Nozzle's exit area } (in^2) \\
A_t &= \text{Nozzle's throat area } (in^2)
\end{aligned}$$

We calculate the Area of the nozzle's exit,

$$\begin{aligned}
A_e &= \pi r^2 \\
&= \pi \left(\frac{90.3in}{2}\right)^2 \\
&= 6404.21in^2
\end{aligned}$$

Then, applying equation (14-22) we see

$$\begin{aligned}
\varepsilon &= \frac{A_e}{A_t} \\
69 &= \frac{6404.21in^2}{\pi r_t^2} \\
&\Downarrow \\
r_t &= 5.4354in \\
\text{and, } d_t &= 10.871in
\end{aligned}$$

Question 11.

A new micro-satellite launch system is using a hybrid rocket motor with a throat diameter of 1.25 cm and uses HTPB/LOX for propulsion. The fuel and Oxidizer have a combined specific heat ratio of 1.231. Computer modeling calculated the theoretical exhaust velocity at 1320 m/s. During initial experimental testing, the total mass flow rate was calculated to be $\dot{m} = 0.225$ kg/s, the chamber pressure was 22.5 bar, and the motor produced 118 lbs of thrust. From this information:

(a) Determine the measured characteristic exhaust velocity.

First, we calculate the nozzle's throat area.

$$\begin{aligned} A_t &= \pi r_t^2 \\ &= \pi \left(\frac{0.0125m}{2}\right)^2 \\ &= 1.227 \times 10^{-4} m^2 \end{aligned}$$

Then, using the equation,

$$C^* = \frac{P_c A_t}{\dot{m}} \tag{14-23}$$

where

$$\begin{aligned} C^* &= \text{Characteristic exhaust velocity} \\ P_c &= \text{Chamber pressure} \\ A_t &= \text{Throat area} \\ \dot{m} &= \text{Mass flow rate} \end{aligned}$$

Then,

$$\begin{aligned} C^* &= \frac{22.5 \times 10^5 Pa \cdot 1.227 \times 10^{-4} m^2}{0.225 kg/s} \\ &= 1227 m/s \end{aligned}$$

(b) Compare this to the theoretical value from computer modeling.

The experimental characteristic exhaust velocity is lower than the theoretical exhaust velocity. This slight of roughly 93% difference is caused by the rocket not being ideal and not producing the optimal exhaust velocity that an ideal theoretical rocket would. However, we can consider the rocket to have slightly below "good performance."

Question 12.

Based on the information in Question 11:

(a) Compute the measured specific impulse.

First, we convert 118 lbs of thrust into Newtons

$$118 lbs \times \frac{4.44822N}{1lb} = 524.88N$$

Then, we will use the equation

$$I_{sp_{measured}} = \frac{F_{measured}}{g_0 \, \dot{m}_{measured}} \tag{14-25}$$

where

$$\begin{aligned} I_{sp_{measured}} &= \text{Measured specific impulse} \\ F_{measured} &= \text{Measured thrust} \end{aligned}$$

We get

$$\begin{aligned} I_{sp_{measured}} &= \frac{F_{measured}}{g_0\ \dot{m}_{measured}} \\ &= \frac{524.88N}{9.81m/s^2\ 0.225kg/s} \\ &= 237.79s \end{aligned}$$

(b) Compute the theoretical specific impulse.

We will use the equation

$$I_{sp_{Ideal}} = \frac{C^*}{g_0}\gamma\left[\left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}} \tag{14-26}$$

where

$$\begin{aligned} I_{sp_{Ideal}} &= \text{Theoretical ideal specific impulse (s)} \\ C^* &= \text{Characteristic exhaust velocity (m/s)} \\ \gamma &= \text{Ratio of specific heats} \end{aligned}$$

Therefore,

$$\begin{aligned} I_{sp_{Ideal}} &= \frac{C^*}{g_0}\gamma\left[\left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}} \\ &= \frac{1320m/s}{9.81m/s^2}(1.231)\left[\left(\frac{2}{1.231-1}\right)\left(\frac{2}{1.231+1}\right)^{\frac{1.231+1}{1.231-1}}\right]^{\frac{1}{2}} \\ &= 165.639\left[(8.658)(0.8965)^{9.658}\right]^{\frac{1}{2}} \\ &= 287.56s \end{aligned}$$

(c) Compare these figures.

The difference between the Theoretical Specific Impulse and the Measured Specific Impulse is approximately 82%, which makes it less than what would be considered "good performance." The engineers should look into improving the efficiency and performance of the rocket.

Question 13.

An asteroid estimated, with a mass of 4.2×10^8 kg, is in the same orbital plane (1 AU from the Sun) and heading toward Earth. Engineers want to use a solar sail to push the asteroid directly away from the sun and out of Earth's orbital path (sail directly facing the sun). If the asteroid needs to be moved 175,000 km in 5 years, determine the following:

(a) What acceleration rate is needed?

(b) How much force will be needed to achieve that rate of acceleration?

Here we will use Newtons Second Law:

$$F = m \cdot a$$

where

$$\begin{aligned} F &= \text{Force } (N) \\ m &= \text{Mass } (kg) \\ a &= \text{Acceleration } m/s^2 \end{aligned}$$

So the force required becomes

$$\begin{aligned} F &= (4.2 \times 10^8 kg)(acceleration) \\ &= \end{aligned}$$

(c) What size sail will be needed to produce the force needed (assume a reflectivity of $\rho = 0.92$)?

We will use the equation

$$F = \frac{F_s}{c} A_s (1 + \rho) \cos I \tag{14-38}$$

where

$$\begin{aligned} F &= \text{Force on the sail } (N) \\ F_s &= \text{Solar constant } (1358 W/m^2) \\ c &= \text{Speed of light } (3 \times 10^8 m/s) \\ A_s &= \text{Surface area } (m^2) \\ \rho &= \text{Surface reflactance} \\ I &= \text{Incident angle to the Sun } (deg) \end{aligned}$$

We get

$$\begin{aligned} F &= \frac{1358 W/m^2}{3 \times 10^8 m/s} A_s (1 + 0.92) \cos(0) \\ &\Downarrow \\ A_s &= \end{aligned}$$

Question 14.

The Atlantean Warship, Tria (as seen in Stargate Atlantis), traveled at 99.9% the speed of light for 25 years (relative to the crew). How much time passed on Earth while the ship was traveling at this speed?

We will use the Lorentz transformation

$$\tau = \frac{t_{starship}}{t_{Earth}} = \sqrt{1 - \frac{V^2}{c^2}} \tag{14-39}$$

where

$$\begin{aligned} t_{starship} &= \text{Time measure on a starship} \\ t_{Earth} &= \text{Time measure on a Earth} \\ V &= \text{Starship's velocity } (km/s) \\ c &= \text{Speed of light } (300,000 km/s) \end{aligned}$$

We get

$$\begin{aligned}
\frac{t_{starship}}{t_{Earth}} &= \sqrt{1 - \frac{V^2}{c^2}} \\
\frac{25yrs}{t_{Earth}} &= \sqrt{1 - \left(\frac{0.99}{1}\right)^2} \\
&\Downarrow \\
t_{Earth} &= 177.22yrs
\end{aligned}$$

Question 15.

An experimental long-range rocket booster is being designed. Determine the optimal design altitude for the rocket nozzle given the following information:

(a) The burnout altitude is 108 km:

(b) The burnout altitude is 126 km:

Question 16.

A repair satellite in a circular orbit at an altitude of 440 km carries a launchable fuel probe with a 22 km tether.

(a) What is the new apogee and perigee altitude if the probe is released while extended upward?

If the tether is extended upward and released the current location of the probe will be the new perigee of the now elliptical orbit. So,

$$R_{perigee} = 440km + 22km + 6378km = 6842km$$

But because it was attached to the repair satellite, it will be going faster than if it were alone. We can calculate the specific mechanical energy of the original orbit based on the equation

$$\begin{aligned}
\varepsilon &= -\frac{\mu}{2a} \\
&= -\frac{3.986 \times 10^5 km^3/s^2}{2 \cdot (6378km + 440km)} \\
&= -29.2314km^2/s^2
\end{aligned}$$

Thus the original velocity was

$$\begin{aligned}
V &= \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)} \\
&= \sqrt{2\left(\frac{3.986 \times 10^5 km^3/s^2}{6378km + 440km} + -29.2314km^2/s^2\right)} \\
&= 7.6461km/s
\end{aligned}$$

We can now calculate the new specific mechanical energy after the probe is detached

$$
\begin{aligned}
\varepsilon &= \frac{V^2}{2} - \frac{\mu}{R} \\
&= \frac{(7.6461km/s)^2}{2} - \frac{3.986 \times 10^5 km^3/s^2}{6842km} \\
&= -29.0264km^2/s^2
\end{aligned}
$$

Then

$$
\begin{aligned}
\varepsilon &= -\frac{\mu}{2a} \\
&\Downarrow \\
2a &= -\frac{\mu}{\varepsilon} \\
2a &= \frac{3.986 \times 10^5 km^3/s^2}{29.0264km^2/s^2} \\
2a &= 13732km
\end{aligned}
$$

And finally

$$
\begin{aligned}
2a &= R_{perigee} + R_{apogee} \\
13732km &= 6842km + R_{apogee} \\
&\Downarrow \\
R_{apogee} &= 6890km
\end{aligned}
$$

The altitude of the apogee after release is

$$6890km - 6378km = 512km$$

The altitude of the perigee after release is

$$6842km - 6378km = 464km$$

(b) What is the new apogee and perigee altitudes if the probe is released while extended downward?

In a similar fashion, if the tether is extended downward and released the current location of the probe will be the new apogee of the now elliptical orbit. So,

$$R_{apogee} = 440km - 22km + 6378km = 6796km$$

But because it was attached to the repair satellite, it will be going slower than if it were alone. We will use the same numbers as above because we are calculating the specific mechanical energy while the probe and satellite are still tethered.

$$
\begin{aligned}
\varepsilon &= -\frac{\mu}{2a} \\
&= -\frac{3.986 \times 10^5 km^3/s^2}{2 \cdot (6378km + 440km)} \\
&= -29.2314km^2/s^2
\end{aligned}
$$

Thus the original velocity was

$$
\begin{aligned}
V &= \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)} \\
&= \sqrt{2\left(\frac{3.986 \times 10^5 km^3/s^2}{6378km + 440km} + -29.2314km^2/s^2\right)} \\
&= 7.6461km/s \quad \text{just as above.}
\end{aligned}
$$

We can now calculate the new specific mechanical energy after the probe is detached

$$
\begin{aligned}
\varepsilon &= \frac{V^2}{2} - \frac{\mu}{R} \\
&= \frac{(7.6461km/s)^2}{2} - \frac{3.986 \times 10^5 km^3/s^2}{6796km} \\
&= -29.4207km^2/s^2
\end{aligned}
$$

Then

$$
\begin{aligned}
\varepsilon &= -\frac{\mu}{2a} \\
&\Downarrow \\
2a &= -\frac{\mu}{\varepsilon} \\
2a &= \frac{3.986 \times 10^5 km^3/s^2}{29.4207km^2/s^2} \\
2a &= 13548km
\end{aligned}
$$

And finally

$$
\begin{aligned}
2a &= R_{perigee} + R_{apogee} \\
13548km &= 6796km + R_{perigee} \\
&\Downarrow \\
R_{perigee} &= 6752km
\end{aligned}
$$

The altitude of the perigee after release is

$$6752km - 6378km = 374km$$

The altitude of the apogee after release is

$$6796km - 6378km = 418km$$