MAT 473: Intermediate Real Analysis II

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Problem 41. LEt E be the nonmeasurable set desribed in section 18 of the notes. Prove that if $N \subseteq E$ and N is measurable, then $m(N) = 0$. (Hint: imitate the second part of the proof of Theorem 18.1.)

Problem 42. Let $A \subseteq \mathbb{R}$ be a measurable set with $m(A) > 0$. Prove that there exists a subset $B \subseteq A$ such that B is not measurable. (Hint: if E is the nonmeasurable set described in section 18 of the notes, then $A \subseteq \sqcup_{q \in \mathbb{Q}}(q + E)$.)

Problem 43. Let $\mathcal{E}$ be a collection of Borel sets that generates $\mathcal{B}_{\mathbb{R}}$ (i.e. such that $\mathcal{M}(\mathcal{E}) = \mathcal{B}_{\mathbb{R}}$). Let $f : \mathbb{R} \to \mathbb{R}$. Prove that f is measurable if and only if $f^{-1}(E)$ is measurable for all $E \in \mathcal{E}$. (Hint: show that $\{A \subseteq \mathbb{R} : f^{-1}(A) \text{ is measurable}\}$ is a σ-algebra.)

Problem 44. Let $f_1, f_2, \cdots : \mathbb{R} \to \mathbb{R}$ be measurable function, let $f : \mathbb{R} \to \mathbb{R}$, and suppose that $f_n \to f$ almost everywhere. Prove that f is measurable.