When Do Stop-Loss Rules Stop Losses?[☆]

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Abstract

We propose a simple analytical framework to measure the value added or subtracted by stop-loss rules—predetermined policies that reduce a portfolio's exposure after reaching a certain threshold of cumulative losses—on the expected return and volatility of an arbitrary portfolio strategy. Using daily futures price data, we provide an empirical analysis of stop-loss policies applied to a buy-and-hold strategy using index futures contracts. At longer sampling frequencies, certain stop-loss policies can increase expected return while substantially reducing volatility, consistent with their objectives in practical applications.

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JEL Classification: G11, G12

1. Introduction

Thanks to the overwhelming dominance of the mean-variance portfolio optimization framework pioneered by Markowitz (1952), Tobin (1958), Sharpe (1964), and Lintner (1965), much of the investments literature—both in academia and in industry—has focused on constructing well-diversified static portfolios using low-cost index funds. With little use for active trading or frequent rebalancing, this passive perspective comes from the recognition that individual equity returns are difficult to forecast and trading is not costless. The questionable benefits of day-trading are unlikely to outweigh the very real costs of changing one's portfolio weights. It is, therefore, no surprise that a "buy-and-hold" philosophy has permeated the mutual-fund industry and the financial planning profession.[3]

However, this passive approach to investing is often contradicted by human behavior, especially during periods of market turmoil.[4] behavioral biases sometimes lead investors astray, causing them to shift their portfolio weights in response to significant swings in market indexes, often "selling at the low" and "buying at the high." On the other hand, some of the most seasoned investment professionals routinely make use of systematic rules for exiting and re-entering portfolio strategies based on cumulative losses, gains, and other "technical" indicators.

In this paper, we investigate the efficacy of such behavior in the narrow context of stop-

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[3]This philosophy has changed slightly with the recent innovation of a slowly varying asset allocation that changes according to one's age (e.g., a "lifecycle" fund).

[4]For example, psychologists and behavioral economists have documented the following systematic biases in the human decisionmaking process: overconfidence (Fischoff and Slovic, 1980; Barber and Odean, 2001; Gervais and Odean, 2001), overreaction (DeBondt and Thaler, 1986), loss aversion (Kahneman and Tversky, 1979; Shefrin and Statman, 1985; Kahneman and Tversky, 1992; Odean, 1998), herding (Huberman and Regev, 2001), psychological accounting (Kahneman and Tversky, 1981), miscalibration of probabilities (Lichtenstein, Fischoff, and Phillips, 1982), hyperbolic discounting (Laibson, 1997), and regret (Bell, 1982a,b; Clarke, Krase, and Statman, 1994).

loss rules (i.e., rules for exiting an investment after some threshold of loss is reached and re-entered after some level of gains is achieved). We wish to identify the economic motivation for stop-loss policies so as to distinguish between rational and behavioral explanations for these rules. While certain market conditions may encourage irrational investor behavior (e.g., large rapid market declines) stop-loss policies are sufficiently ubiquitous that their use cannot always be irrational.

This raises the question we seek to answer in this paper: When do stop-loss rules stop losses? In particular, because a stop-loss rule can be viewed as an overlay strategy for a specific portfolio, we can derive the impact of that rule on the return characteristics of the portfolio. The question of whether or not a stop-loss rule stops losses can then be answered by comparing the expected return of the portfolio with and without the stop-loss rule. If the expected return of the portfolio is higher with the stop-loss rule than without it, we conclude that the stop-loss rule does, indeed, stop losses.

Using simple properties of conditional expectations, we are able to characterize the marginal impact of stop-loss rules on any given portfolio's expected return, which we define as the "stopping premium." We show that the stopping premium is inextricably linked to the stochastic process driving the underlying portfolio's return. If the portfolio follows a random walk (i.e., independently and identically distributed returns) the stopping premium is always negative. This may explain why the academic and industry literature has looked askance at stop-loss policies to date. If returns are unforecastable, stop-loss rules simply force the portfolio out of higher-yielding assets on occasion, thereby lowering the overall expected return without adding any benefits. In such cases, stop-loss rules never stop losses.

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However, for non-random-walk portfolios, we find that stop-loss rules *can* stop losses. For example, if portfolio returns are characterized by "momentum" or positive serial correlation, we show that the stopping premium can be positive and is directly proportional to the magnitude of return persistence. Not surprisingly, if conditioning on past cumulative returns changes the conditional distribution of a portfolio's return, it should be possible to find a stop-loss policy that yields a positive stopping premium. We provide specific guidelines for finding such policies under several return specifications: mean reversion, momentum, and Markov regime-switching processes. In each case, we are able to derive explicit conditions for stop-loss rules to stop losses.

Of course, focusing on expected returns does not account for risk in any way. It may be the case that a stop-loss rule increases the expected return but also increases the risk of the underlying portfolio, yielding ambiguous implications for the risk-adjusted return of a portfolio with a stop-loss rule. To address this issue, we compare the variance of the portfolio with and without the stop-loss rule and find that, in cases where the stop-loss rule involves switching to a lower-volatility asset when the stop-loss threshold is reached, the unconditional variance of the portfolio return is reduced by the stop-loss rule. A decrease in the variance coupled with the possibility of a positive stopping premium implies that, within the traditional mean-variance framework, stop-loss rules may play an important role under certain market conditions.

To illustrate the empirical relevance of our analysis, we apply a simple stop-loss rule to a standard asset-allocation problem of stocks versus bonds using daily futures data from January 1993 to November 2011. We find that stop-loss rules exhibit positive stopping

premiums over longer sampling frequencies over larger range of threshold values. These policies also provide substantial reduction in volatility creating larger Sharpe ratios as a result. This is a remarkable feat for a buy-high/sell-low strategy. For example in one calibration, using stop loss over monthly intervals in daily data can increase the return by 1.5% and decrease the volatility by 5% causing an increase in the Sharpe Ratio by as much as 20%. These results suggest that stop-loss rules may exploit conditional momentum effects following periods of losses in equities. These results suggest that the random walk model is a particularly poor approximation to U.S. stock returns and may improperly value the use of non-linear policies such as stop-loss rules. This is consistent with Lo and MacKinlay (1999) and others using various methods to examine limitations of the random walk.

2. Literature Review

Before presenting our framework for examining the performance impact of stop-loss rules, we provide a brief review of the relevant portfolio-choice literature, and illustrate some of its limitations to underscore the need for a different approach.

The standard approach to portfolio choice is to solve an optimization problem in a multi-period setting, for which the solution is contingent on two important assumptions: the choice of objective function and the specification of the underlying stochastic process for asset returns. The problem was first posed by Samuelson (1969) in discrete time and Merton (1969) in continuous time, and solved in both cases by stochastic dynamic programming. As the asset-pricing literature has grown, this paradigm has been extended in a number of important directions.[5]

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[5]For a comprehensive summary of portfolio choice see Brandt (2004). Recent extensions include pre-

However, in practice, household investment behavior seems to be at odds with finance theory. In particular, Ameriks and Zeldes (2004) observe that most observed variation in an individuals portfolio is attributed to a small number of significant decisions they make as opposed to marginal adjustments over time. Moreover, other documented empirical characteristics of investor behavior include non-participation (Calvet, Campbell, and Sodini 2006); under-diversification (Calvet, Campbell, and Sodini 2006); limited monitoring frequency and trading (Ameriks and Zeldes 2004); survival-based selling decisions or a "flight to safety" (Agnew 2003); an absence of hedging strategies (Massa and Simonov, 2004); and concentration in simple strategies through mutual-fund investments (Calvet, Campbell, and Sodini 2006). Variations in investment policies due to characteristics such as age, wealth, and profession have been examined as well.[6]

In fact, in contrast to the over-trading phenomenon documented by Odean (1999) and Barber and Odean (2000), Agnew (2003) asserts that individual investors actually trade infrequently. By examining asset-class flows, she finds that investors often shift out of equities after extremely negative asset returns into fixed-income products, and concludes that in retirement accounts, investors are more prone to exhibit a "flight to safety" instead of explicit return chasing. Given that one in three of the workers in the United States participate in 401(k) programs, it is clear that this flight to safety could have a significant

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dictability and autocorrelation in asset returns (Kim and Omberg, 1996; Liu, 1999; Campbell and Viceria, 1999; Brennan and Xia, 2001; Xia, 2001; and Wachter, 2002), model uncertainty (Barberis, 2000), transaction costs (Balduzzi and Lynch, 1999), stochastic opportunity sets (Brennan, Schwartz, and Lagnado, 1997; Campbell, Chan, and Viceria, 2003; and Brandt, Goyal, Santa-Clara, and Stroud, 2005), and behavioral finance (see the references in footnote 4).

[6]For example, lack of age-dependence in allocation, lower wealth and lower education with greater non-participation and under-diversification, and greater sophistication in higher wealth investors have all been considered (see Ameriks and Zeldes, 2004).

impact on market prices as well as demand. Consistent with Agnew's flight-to-safety in the empirical application of stop-loss, we find momentum in long-term bonds as a result of sustained periods of loss in equities. This suggests conditional relationships between stocks and bonds, an implication that is also confirmed by our empirical results.[7]

Although stop-loss rules are widely used, the corresponding academic literature is rather limited. The market microstructure literature contains a number of studies about limit orders and optimal order selection algorithms (Easley and O'Hara, 1991; Biais, Hillion, and Spatt, 1995; Chakravarty and Holden, 1995; Handa and Schwartz, 1996; Harris and Hasbrouck, 1996; Seppi, 1997; and Lo, MacKinlay, and Zhang, 2002). Carr and Jarrow (1990) investigate the properties of a particular trading strategy that employs stop-loss orders, and Shefrin and Statman (1985) and Tschoegl (1988) consider behavioral patterns that may explain the popularity of stop-loss rules. However, to date, there has been no systematic analysis of the impact of a stop-loss rule on an existing investment policy, an oversight that we remedy in this paper.

3. A Framework for Analyzing Stop-Loss Rules

In this section, we outline a framework for measuring the impact of stop-loss policies on investment performance. In Section 3.1, we begin by specifying a simple stop-loss policy and deriving some basic statistics for its effect on an existing portfolio strategy. We describe several generalizations and qualifications of our framework in Section 3.2, and then apply our framework in Section 4 to various return-generating processes including the Random

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[7]Although excess performance in long-term bonds may seem puzzling, from a historical perspective, the deregulation of long-term government fixed-income products in the 1950s could provide motivation for the existence of these effects.

Walk Hypothesis, momentum and mean-reversion models, and regime-switching models.

3.1. Assumptions and definitions

Consider any arbitrary portfolio strategy P with returns $\{r_t\}$ that satisfy the following assumptions:

(**A1**) The returns $\{r_t\}$ for the portfolio strategy P are stationary with finite mean μ and variance σ^2.

(**A2**) The expected return μ of P is greater than the risk-free rate r_f, and let $\pi \equiv \mu - r_f$ denote the risk premium of P.

Our use of the term "portfolio strategy" in Assumption (A1) is meant to underscore the possibility that P is a complex dynamic investment policy, not necessarily a static basket of securities. Assumption (A2) simply rules out perverse cases where stop-loss rules add value because the "safe" asset has a higher expected return than the original strategy itself.

Now suppose an investor seeks to impose a stop-loss policy on a portfolio strategy. This typically involves tracking the cumulative return $R_t(J)$ of the portfolio over a window of J periods, where:[8]

$$R_t(J) \equiv \sum_{j=1}^{J} r_{t-j+1} \tag{1}$$

and when the cumulative return crosses some lower boundary, reducing the investment in P by switching into cash or some other safer asset. This heuristic approach motivates the

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[8]For simplicity, we ignore compounding effects and define cumulative returns by summing simple returns r_t instead of multiplying $(1+r_t)$. For purposes of defining the trigger of our stop-loss policy, this approximation does not have significant impact. However, we do take compounding into account when simulating the investment returns of a portfolio with and without a stop-loss policy.

following definition:

Definition 1. *A **simple stop-loss policy** $\mathcal{S}(\gamma, \delta, J)$ for a portfolio strategy P with returns $\{r_t\}$ is a dynamic binary asset-allocation rule $\{s_t\}$ between P and a risk-free asset F with return r_f, where s_t is the proportion of assets allocated to P, and:*

$$s_t \equiv \begin{cases} 0 & \text{if} & R_{t-1}(J) & < & -\gamma & \text{and} & s_{t-1} = 1 & \text{(exit)} \\ 1 & \text{if} & r_{t-1} & \geq & \delta & \text{and} & s_{t-1} = 0 & \text{(re-enter)} \\ 1 & \text{if} & R_{t-1}(J) & \geq & -\gamma & \text{and} & s_{t-1} = 1 & \text{(stay in)} \\ 0 & \text{if} & r_{t-1} & < & \delta & \text{and} & s_{t-1} = 0 & \text{(stay out)} \end{cases} \tag{2}$$

for $\gamma \geq 0$. Denote by r_{st} the return of portfolio strategy S, which is the combinaton of portfolio strategy P and the stop-loss policy $\mathcal{S}$, hence:

$$r_{st} \equiv s_t r_t + (1 - s_t) r_f . \tag{3}$$

Definition 1 describes a 0/1 asset-allocation rule between P and the risk-free asset F, where 100% of the assets are withdrawn from P and invested in F as soon as the J-period cumulative return $R_{t_1}(J)$ reaches some loss threshold γ at t_1. The stop-loss rule stays in place until some future date $t_2 - 1 > t_1$ when P realizes a return r_{t_2-1} greater than δ, at which point 100% of the assets are transferred from F back to P at date t_2. Therefore, the stop-loss policy $\mathcal{S}(\gamma, \delta, J)$ is a function of three parameters: the loss threshold γ, the re-entry threshold δ, and the cumulative-return window J. Of course, the performance of the stop-loss policy also depends on the characteristics of F—lower risk-free rates imply a more significant drag on performance during periods when the stop-loss policy is in effect.

Note that the specification of the loss and re-entry mechanisms are different; the exit decision is a function of the cumulative return $R_{t-1}(J)$, whereas the re-entry decision involves

only the one-period Return, r_{t-1}. This is intentional, and motivated by two behavioral biases. The first is loss aversion and the disposition effect, in which an individual becomes less risk-averse when facing mounting losses. The second is the "snake-bite" effect, in which an individual is more reluctant to re-enter a portfolio after experiencing losses from that strategy. The simple stop-loss policy in Definition 1 is meant to address both of these behavioral biases in a systematic fashion.

To gauge the impact of the stop-loss policy $\mathcal{S}$ on performance, we define the following metric:

Definition 2. *The **stopping premium** $\Delta_\mu(\mathcal{S})$ of a stop-loss policy $\mathcal{S}$ is the expected return difference between the stop-loss policy $\mathcal{S}$ and the portfolio strategy P:*

$$\Delta_\mu \;\equiv\; \mathrm{E}[r_{st}] - \mathrm{E}[r_t] \;\;=\;\; p_o\left(r_f - \mathrm{E}[r_t|s_t=0]\right), \tag{4}$$

$$\text{where} \;\; p_o \;\equiv\; \mathrm{Prob}(s_t=0) \tag{5}$$

*and the **stopping ratio** is the ratio of the stopping premium to the probability of stopping out:*

$$\frac{\Delta_\mu}{p_o} \;\;=\;\; r_f \;-\; \mathrm{E}[r_t|s_t=0] \;. \tag{6}$$

Note that the difference of the expected returns of r_{st} and r_t reduces to the product of the probability of a stop-loss p_o and the conditional expectation of the difference between r_f and r_t, conditioned on being stopped out. The intuition for this expression is straightforward: the only times r_{st} and r_t differ are during periods when the stop-loss policy has been trig-

gered. Therefore, the difference in expected return should be given by the difference in the conditional expectation of the portfolio with and without the stop-loss policy—conditioned on being stopped out—weighted by the probability of being stopped out.

The stopping premium (4) measures the expected-return difference per unit time between the stop-loss policy $\mathcal{S}$ and the portfolio strategy P, but this metric may yield misleading comparisons between two stop-loss policies that have very different parameter values. For example, for a given portfolio strategy P, suppose $\mathcal{S}_1$ has a stopping premium of 1% and $\mathcal{S}_2$ has a stopping premium of 2%; this suggests that $\mathcal{S}_2$ is superior to $\mathcal{S}_1$. But suppose the parameters of $\mathcal{S}_2$ implies that $\mathcal{S}_2$ is active only 10% of the time (i.e., one month out of every 10 on average), whereas the parameters of $\mathcal{S}_1$ implies that it is active 25% of the time. On a total-return basis, $\mathcal{S}_1$ is superior, even though it yields a lower expected-return difference per-unit-time. The stopping ratio Δ_μ/p_o given in (6) addresses this scale issue directly by dividing the stopping premium by the probability p_o. The reciprocal of p_o is the expected number of periods that $s_t=0$ or the expected duration of the stop-loss period. Multiplying the per-unit-time expected-return difference Δ_μ by this expected duration $1/p_o$ then yields the total expected-return difference Δ_μ/p_o between r_f and r_t.

The probability p_o of a stop-loss is of interest in its own right because more frequent stop-loss events imply more trading and, consequently, more transactions costs. Although we have not incorporated transactions costs explicitly into our analysis, this can be done easily by imposing a return penalty in (3):

$$r_{st} \quad \equiv \quad s_t r_t \; + \; (1-s_t)r_f \; - \; \kappa \, |s_t - s_{t-1}|, \tag{7}$$

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where $\kappa > 0$ is the one-way transactions cost of a stop-loss event. For expositional simplicity, we shall assume $\kappa = 0$ for the remainder of this paper.

Using the metrics proposed in Definition 2, we now have a simple way to answer the question posed in our title: stop-loss policies can be said to stop losses when the corresponding stopping premium is positive. In other words, a stop-loss policy adds value if and only if its implementation leads to an improvement in the overall expected return of a portfolio strategy.

Of course, this simple interpretation of a stop-loss policy's efficacy is based purely on expected return, and ignores risk. Risk matters because it is conceivable that a stop-loss policy with a positive stopping premium generates so much additional risk that the risk-adjusted expected return is less attractive with the policy in place than without it. This may seem unlikely because by construction, a stop-loss policy involves switching out of P into a risk-free asset, implying that P spends more time in higher-risk assets than the combination of P and $\mathcal{S}$. However, it is important to acknowledge that P and $\mathcal{S}$ are dynamic strategies and static measures of risk such as standard deviation are not sufficient statistics for the intertemporal risk/reward trade-offs that characterize a dynamic rational expectations equilibrium (e.g., Merton, 1973; Lucas, 1978). Nevertheless, it is still useful to gauge the impact of a stop-loss policy on volatility of a portfolio strategy P, as only one of possibly many risk characteristics of the combined strategy. To that end, we have:

Definition 3. *Let the variance difference Δ_{σ^2} of a stopping strategy be given by:*

$$\Delta_{\sigma^2} \;\equiv\; \text{Var}[r_{st}] \;-\; \text{Var}[r_t] \tag{8}$$

$$= \; \text{E}\big[\text{Var}[r_{st}|s_t]\big] \;+\; \text{Var}\big[\text{E}[r_{st}|s_t]\big] \;-\; \text{E}\big[\text{Var}[r_t|s_t]\big] - \text{Var}\big[\text{E}[r_t|s_t]\big] \tag{9}$$

$$= \; -p_o \text{Var}[r_t|s_t = 0] \; + $$
$$p_o(1 - p_o)\left[\big(r_f - \text{E}[r_t|s_t = 0]\big)^2 \;-\; \left(\frac{\mu - \text{E}[r_t|s_t = 0]}{1 - p_o}\right)^2 \right] \tag{10}$$

From an empirical perspective, standard deviations are often easier to interpret, hence we also define the quantity $\Delta_\sigma \equiv \sqrt{\text{Var}[r_{st}]} - \sigma$.

Given that a stop-loss policy can affect both the mean and standard deviation of the portfolio strategy P, we can also define the difference between the Sharpe ratios of P with and without $\mathcal{S}$:

$$\Delta_{\text{SR}} \;\equiv\; \frac{\text{E}[r_{st}] - r_f}{\sigma_s} \;-\; \frac{\mu - r_f}{\sigma} \;. \tag{11}$$

However, given the potentially misleading interpretations of the Sharpe ratio for dynamic strategies such as P and $\mathcal{S}$, we refrain from using this metric for evaluating the efficacy of stop-loss policies.[9]

3.2. Generalizations and Qualifications

The basic framework outlined in Section 3.1 can be generalized in many ways. For example, instead of switching out of P and into a completely risk-free asset, we can allow F to be a lower-risk asset but with some non-negligible volatility. More generally, instead

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[9]See Sharpe (1994), Spurgin (2001), and Lo (2002) for details.

of focusing on binary asset-allocation policies, we can consider a continuous function $\omega(\cdot) \in [0, 1]$ of cumulative returns that declines with losses and rises with gains. Also, instead of a single "safe" asset, we might consider switching into multiple assets when losses are realized, or incorporate the stop-loss policy directly into the portfolio strategy P itself so that the original strategy is affected in some systematic way by cumulative losses and gains. Finally, there is nothing to suggest that stop-loss policies must be applied at the portfolio level—such rules can be implemented security-by-security or asset-class by asset-class.

Of course, with each generalization, the gains in flexibility must be traded off against the corresponding costs of complexity and analytic intractability. These trade-offs can only be decided on a case-by-case basis, and we leave it to the reader to make such trade-offs individually. Our more modest objective in this paper is to provide a complete solution for the leading case of the simple stop-loss policy in Definition (1). From our analysis of this simple case, a number of generalizations should follow naturally, some of which are explored in Kaminski (2006).

However, an important qualification regarding our approach is the fact that we do not derive the simple stop-loss policy from any optimization problem—it is only a heuristic, albeit a fairly popular one among many institutional and retail investors. This is a distinct departure from much of the asset-pricing literature in which investment behavior is modelled as the outcome of an optimizing individual seeking to maximize his expected lifetime utility by investing in a finite set of securities subject to a budget constraint (e.g., Merton, 1971). While such a formal approach is certainly preferable if the consumption/investment problem is well posed. For example, if preferences are given and the investment opportunity set is

completely specified, the simple stop-loss policy can still be studied in the absence of such structure.

Moreover, from a purely behavioral perspective, it is useful to consider the impact of a stop-loss heuristic even if it is not derived from optimizing behavior, precisely because we seek to understand the basis of such behavior. Of course, we can ask the more challenging question of whether the stop-loss heuristic can be derived as the optimal portfolio rule for a specific set of preferences, but such inverse-optimal problems become intractable very quickly (e.g., Chang, 1988). Instead, we have a narrower set of objectives in this paper: to investigate the basic properties of simple stop-loss heuristics without reference to any optimization problem, and with as few restrictions as possible on the portfolio strategy P to which the stop-loss policy is applied. The benefits of our narrower focus are the explicit analytical results described in Section 4, and the intuition that they provide for how stop-loss mechanisms add or subtract value from an existing portfolio strategy.

Although this approach may be more limited in the insights it can provide to the investment process, the siren call of stop-loss rules seems so universal that we hope to derive some useful implications for optimal consumption and portfolio rules from our analysis. Moreover, the idea of overlaying one set of heuristics on top of an existing portfolio strategy has a certain operational appeal that many institutional investors have found so compelling recently (e.g., so-called "portable alpha" strategies). Overlay products can be considered a general class of "superposition strategies," which is explored in more detail in Kaminski (2006).

4. Analytical Results

Having defined the basic framework in Section 3 for evaluating the performance of simple stop-loss rules, we now apply them to several specific return-generating processes for $\{r_t\}$, including the Random Walk Hypothesis in Section 4.1, mean-reversion and momentum processes in Section 4.2, and a statistical regime-switching model in Section 4.3. The simplicity of our stop-loss heuristic will allow us to derive explicit conditions under which stop-loss policies can stop losses in each of these cases.

4.1. The Random Walk Hypothesis

Since the Random Walk Hypothesis is one of the most widely used return-generating processes in the finance literature, any analysis of stop-loss policies must consider this leading case first. Given the framework proposed in Section 3, we are able to derive a surprisingly strong conclusion about the efficacy of stop-loss rules:

Proposition 1. *If $\{r_t\}$ satisfies the Random Walk Hypothesis so that:*

$$r_t \ = \ \mu \ + \ \epsilon_t \ , \ \ \epsilon_t \ \overset{\text{IID}}{\sim} \ \text{White Noise}(0, \sigma_\epsilon^2), \tag{12}$$

then the stop-loss policy has the following properties:

$$\Delta_\mu \ = \ p_o(r_f - \mu) \ = \ -p_o\pi. \tag{13a}$$

$$\frac{\Delta_\mu}{p_o} \ = \ -\pi. \tag{13b}$$

$$\Delta_{\sigma^2} \ = \ -p_o\sigma^2 \ + \ p_o(1 - p_o)\pi^2. \tag{13c}$$

$$\Delta_{\text{SR}} \ = \ -\frac{\pi}{\sigma} \ + \ \frac{\Delta_\mu + \pi}{\sqrt{\Delta_{\sigma^2} + \sigma^2}}. \tag{13d}$$

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Proof: See Appendix A.1. ∎

Proposition 1 shows that, for any portfolio strategy with an expected return greater than the risk-free rate r_f, the Random Walk Hypothesis implies that the stop-loss policy will always reduce the portfolio's expected return since $\Delta_\mu \leq 0$. In the absence of any predictability in $\{r_t\}$, whether or not the stop-loss is activated has no informational content for the portfolio's returns; hence, the only effect of a stop-loss policy is to replace the portfolio strategy P with the risk-free asset when the strategy is stopped out, thereby reducing the expected return by the risk premium of the original portfolio strategy P. If the stop-loss probability p_o is large enough and the risk premium is small enough, (13) shows that the stop-loss policy can also reduce the volatility of the portfolio.

The fact that there are no conditions under which the simple stop-loss policy can add value to a portfolio with IID returns may explain why stop-loss rules have been given so little attention in the academic finance literature. The fact that the Random Walk Hypothesis was widely accepted in the 1960s and 1970s, and considered to be synonymous with market efficiency and rationality, eliminated the motivation for stop-loss rules altogether. In fact, our simple stop-loss policy may be viewed as a more sophisticated version of the "filter rule" that was tested extensively by Alexander (1961) and Fama and Blume (1966). Their conclusion that such strategies did not produce any excess profits was typical of the outcomes of many similar studies during this period.

However, despite the lack of interest in stop-loss rules in academic studies, investment professionals have been using such rules for many years, and part of the reason for this dichotomy may be the fact that the theoretical motivation for the Random Walk Hypothesis is

stronger than the empirical reality. In particular, Lo and MacKinlay (1988) presented compelling evidence against the Random Walk Hypothesis for weekly U.S. stock-index returns from 1962 to 1985, which has subsequently been confirmed and extended to other markets and countries by a number of other authors. In the next section, we shall see that, if asset-returns do not follow random walks, there are several situations in which stop-loss policies can add significant value to an existing portfolio strategy.

4.2. Mean Reversion and Momentum

In the 1980s and 1990s, several authors documented important departures from the Random Walk Hypothesis for U.S. equity returns (e.g., Fama and French,1988; Lo and MacKinlay, 1988, 1990, 1999; Poterba and Summers, 1988; Jegadeesh, 1990; Lo, 1991; and Jegadeesh and Titman, 1993) and, in such cases, the implications for the stop-loss policy can be quite different than in Proposition 1. To see how, consider the simplest case of a non-random-walk return-generating process, the AR(1):

$$r_t \;=\; \mu \;+\; \rho(r_{t-1} - \mu) \;+\; \epsilon_t \;\;,\;\; \epsilon_t \stackrel{\text{IID}}{\sim} \text{White Noise}(0, \sigma_\epsilon^2) \;\;,\;\; \rho \in (-1, 1) \qquad (14)$$

where the restriction that ρ lies in the open interval $(-1, 1)$ is to ensure that r_t is a stationary process (see Hamilton, 1994).

This simple process captures a surprisingly broad range of behavior depending on the single parameter ρ, including the Random Walk Hypothesis ($\rho = 0$), mean reversion ($\rho \in (-1, 0)$), and momentum ($\rho = (0, 1)$). However, the implications of this return-generating process for our stop-loss rule are not trivial to derive because the conditional distribution of r_t, conditioned on $R_{t-1}(J)$, is quite complex. For example, according to Definition (4), the

expression for the stopping premium Δ_μ is given by:

$$\Delta_\mu \quad = \quad p_o(r_f - \mathrm{E}[r_t|s_t = 0]) \tag{15}$$

but the conditional expectation $\mathrm{E}[r_t|s_t = 0]$ is not easy to evaluate in closed-form for an AR(1). For $\rho \neq 0$, the conditional expectation is likely to differ from the unconditional mean μ since past returns do contain information about the future, but the exact expression is not easily computable. Fortunately, we are able to obtain a good first-order approximation under certain conditions, yielding the following result:

Proposition 2. *If $\{r_t\}$ satisfies an AR(1) (14), then the stop-loss policy (2) has the following properties:*

$$\frac{\Delta_\mu}{p_o} \quad = \quad -\pi \; + \; \rho\sigma \; + \; \eta(\gamma, \delta, J) \tag{16}$$

and for $\rho > 0$ and reasonable stop-loss parameters, it can be shown that $\eta(\gamma, \delta, J) \geq 0$, which yields the following lower bound:

$$\frac{\Delta_\mu}{p_o} \quad \geq \quad -\pi \; + \; \rho\sigma, \tag{17}$$

Proof: See Appendix A.2. ∎

Proposition 2 shows that the impact of the stop-loss rule on expected returns is the sum of three terms: the negative of the risk premium, a linear function of the autoregressive parameter ρ, and a remainder term. For a mean-reverting portfolio strategy, $\rho < 0$; hence, the stop-loss policy hurts expected returns to a first-order approximation. This is consistent

with the intuition that mean-reversion strategies benefit from reversals, thus a stop-loss policy that switches out of the portfolio after certain cumulative losses will miss the reversal and lower the expected return of the portfolio. On the other hand, for a momentum strategy, $\rho > 0$, in which case there is a possibility that the second term dominates the first, yielding a positive stopping premium. This is also consistent with the intuition that in the presence of momentum, losses are likely to persist, therefore, switching to the risk-free asset after certain cumulative losses can be more profitable than staying fully invested.

In fact, (17) implies that a sufficient condition for a stop-loss policy with reasonable parameters to add value for a momentum-AR(1) return-generating process is:

$$\rho \; \geq \; \frac{\pi}{\sigma} \; \equiv \; \text{SR}, \tag{18}$$

where SR is the usual Sharpe ratio of the portfolio strategy. In other words, if the return-generating process exhibits enough momentum, then the stop-loss rule will indeed stop losses. This may seem like a rather high hurdle, especially for hedge-fund strategies that have Sharpe ratios in excess of 1.00. However, note that (18) assumes that the Sharpe ratio is calibrated at the same sampling frequency as ρ. Therefore, if we are using monthly returns in (14), the Sharpe ratio in (18) must also be monthly. A portfolio strategy with an annual Sharpe ratio of 1.00, annualized in the standard way by multiplying the monthly Sharpe ratio by $\sqrt{12}$, implies a monthly Sharpe ratio of 0.29, which is still a significant hurdle for ρ but not quite as imposing as 1.00.[10]

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[10]Of course, the assumption that returns follow an AR(1) makes the usual annualization factor of $\sqrt{12}$ incorrect, which is why we use the phrase "annualized in the standard way." See Lo (2002) for the proper method of annualizing Sharpe ratios in the presence of serial correlation.

4.3. Regime-Switching Models

Statistical models of changes in regime, such as the Hamilton (1989) model, are parsimonious ways to capture apparent nonstationarities in the data, such as sudden shifts in means and variances. Although such models are, in fact, stationary, they do exhibit time-varying conditional means and variances, conditioned on the particular state that prevails. Moreover, by assuming that transitions from one state to another follow a time-homogenous Markov process, regime-switching models exhibit rich time-series properties that are surprisingly difficult to replicate with traditional linear processes. Regime-switching models are particularly relevant for stop-loss policies because one of the most common reasons investors put forward for using a stop-loss rule is to deal with a significant change in market conditions, such as October 1987 or August 1998. To the extent that this motivation is genuine and appropriate, we should see significant advantages to using stop-loss policies when the portfolio return $\{r_t\}$ follows a regime-switching process.

More formally, let r_t be given by the following stochastic process:

$$r_t \;=\; I_t r_{1t} \;+\; (1 - I_t) r_{2t} \quad , \quad r_{it} \overset{\text{IID}}{\sim} \mathcal{N}(\mu_i, \sigma_i^2) \quad , \quad i = 1, 2 \tag{19a}$$

$$A \;\equiv\; \begin{array}{c} \\ I_t = 1 \\ \\ I_t = 0 \end{array} \begin{array}{cc} I_{t+1} = 1 & I_{t+1} = 0 \\ \left(\begin{array}{cc} p_{11} & p_{12} \\ \\ p_{21} & p_{22} \end{array} \right) \end{array} \tag{19b}$$

where I_t is an indicator function that takes on the value 1 when state 1 prevails and 0 when state 2 prevails, and A is the Markov transition probabilities matrix that governs the transitions between the two states. The parameters of (19) are the means and variances of the two states, $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$, and the transition probabilities (p_{11}, p_{22}). Without any loss

in generality, we adopt the convention that state 1 is the higher-mean state so that $\mu_1 > \mu_2$. Given assumption (A2), this convention implies that $\mu_1 > r_f$, which is an inequality we will use below. The six parameters of (19) may be estimated numerically via maximum likelihood (Hamilton, 1994).

Despite the many studies in the economics and finance literatures that have implemented the regime-switching model (19), the implications of regime-switching returns for the investment process has only recently been considered (see Ang and Bekaert, 2004). This is due, in part, to the analytical intractability of (19)—while the specification may seem simple, it poses significant challenges for even the simplest portfolio optimization process. However, numerical results can easily be obtained via Monte Carlo simulation, and we provide such results in Sections 5.

In this section, we investigate the performance of our simple stop-loss policy for this return-generating process. Because of the relatively simple time-series structure of returns within each regime, we are able to characterize the stopping premium explicitly:

Proposition 3. *If $\{r_t\}$ satisfies the two-state Markov regime-switching process (19), then the stop-loss policy (2) has the following properties:*

$$\Delta_\mu \;=\; p_{o,1}(r_f - \mu_1) \;+\; p_{o,2}(r_f - \mu_2) \tag{20}$$

$$\frac{\Delta_\mu}{p_o} \;=\; (1 - \tilde{p}_{o,2})(r_f - \mu_1) \;+\; \tilde{p}_{o,2}(r_f - \mu_2) \tag{21}$$

where:

$$p_{o,1} \equiv \text{Prob}\,(\,s_t{=}0, I_t{=}1\,). \tag{22a}$$

$$p_{o,2} \equiv \text{Prob}\,(\,s_t{=}0, I_t{=}0\,). \tag{22b}$$

$$\tilde{p}_{o,2} \equiv \frac{p_{o,2}}{p_o} = \text{Prob}\,(I_t{=}0\,|\,s_t{=}0)\,. \tag{22c}$$

If the risk-free rate r_f follows the same two-state Markov regime-switching process (19), with expected returns r_{f1} and r_{f2} in states 1 and 2, respectively, then the stop-loss policy has the following properties:

$$\Delta_\mu = p_{o,1}(r_{f1} - \mu_1) + p_{o,2}(r_{f2} - \mu_2). \tag{23}$$

$$\frac{\Delta_\mu}{p_o} = (1 - \tilde{p}_{o,2})(r_{f1} - \mu_1) + \tilde{p}_{o,2}(r_{f2} - \mu_2)\,. \tag{24}$$

The conditional probability $\tilde{p}_{o,2}$ can be interpreted as the accuracy of the stop-loss policy in anticipating the low-mean regime. The higher is this probability, the more likely it is that the stop-loss policy triggers during low-mean regimes (regime 2), which should add value to the expected return of the portfolio as long as the risk-free asset-return r_f is sufficiently high relative to the low-mean expected return μ_2.

In particular, we can use our expression for the stopping ratio Δ_μ/p_o to provide a bound on the level of accuracy required to have a non-negative stopping premium. Consider first the case where the risk-free asset r_f is the same across both regimes. For levels of $\tilde{p}_{o,2}$ satisfying the inequality:

$$\tilde{p}_{o,2} \geq \frac{\mu_1 - r_f}{\mu_1 - \mu_2} \tag{25}$$

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the corresponding stopping premium Δ_μ will be non-negative. By convention, $\mu_1 > \mu_2$, and by assumption (A2), $\mu_1 > r_f$, therefore the sign of the right side of (25) is positive. If r_f is less than μ_2, then the right side of (25) is greater than 1, and no value of $\tilde{p}_{o,2}$ can satisfy (25). If the expected return of equities in both regimes dominates the risk-free asset, then the simple stop-loss policy will always decrease the portfolio's expected return, regardless of how accurate it is. To see why, recall that returns are independently and identically distributed within each regime, and we know from Section 4.1 that our stop-loss policy never adds value under the Random Walk Hypothesis. Therefore, the only source of potential value-added for the stop-loss policy under a regime-switching process is if the equity investment in the low-mean regime has a lower expected return than the risk-free rate (i.e., $\mu_2 < r_f$). In this case, the right side of (25) is positive and less than 1, implying that sufficiently accurate stop-loss policies will yield positive stopping premia.

Note that the threshold for positive stopping premia in (25) is decreasing in the spread $\mu_1 - \mu_2$. As the difference between expected equity returns in the high-mean and low-mean states widens, less accuracy is needed to ensure that the stop-loss policy adds value. This may be an important psychological justification for the ubiquity of stop-loss rules in practice. If an investor possesses a particularly pessimistic view of the low-mean state, implying a large spread between μ_1 and μ_2, then our simple stop-loss policy may appeal to him even if its accuracy is not very high.

5. Empirical Analysis

To illustrate the potential relevance of our framework for analyzing stop-loss rules, we consider the performance of the simple stop-loss rule when applied to equity portfolios.

Given that most financial hedging is done in the futures markets, we apply stop-loss rules on equities using daily futures prices from January 5, 1993 until November 7, 2011. Similar to how a futures position would be held, the futures prices represent a weighted basket of prices over various maturities shorter in the curve, which are rolled over to avoid jumps in prices near maturity. The IMM S&P futures contract is used for a position in U.S. Equities and the 10-year CBT Treasury note futures contract. In this given sample period, the two portfolios have a negative correlation of -17.29%. In Table 1, the basic statistical properties of the two return series is detailed. In Table 2, the parameter estimates for a two-state regime-switching model are also detailed.

Despite the net positive serial autocorrelation in the IMM S&P contract, over smaller time intervals the serial autocorrelation seems to be time varying. In Figures 1 and 2, rolling point estimates of serial autocorrelation are plotted using both a 150-day and a 75-day window. These graphs suggests that there are periods when stocks can be either momentum driven or mean reverting. Given the theoretical analysis in this paper, during periods of sufficient momentum stop-loss policies might provide a stopping premium. In the same vein, during periods of mean reversion stop-loss policies may produce negative stopping premiums. When a simple regime-switching model is applied to the IMM S&P contract basket, the estimates also suggest that there are two regimes: one positive low volatility regime and one negative higher volatility regime, which occurs less often. Given the analytic results in Section 4, these parameter estimates indicate stopping rules, which can accurately determine low performance regimes in stocks may add stopping premium.

5.1. Basic Results

To examine the performance of stop-loss rules, the approach can be applied from short-term to longer time where the strategy can be applied daily (1 day), weekly (5 days), monthly (20 days), and quarterly (60 days). For each frequency, the stopping windows will be multiples of 3, 5, and 10 times the length of the data frequency (daily, weekly, monthly, and quarterly). The different strategy combinations include daily (3,1), (5,1), (10,1), weekly (15,5), (25,5), (50,5), monthly (60,20), (100,20), (200,20), and quarterly (180,60), (300,60), (600,60). Consistent with our theoretical framework, (i,j) represent the size of the stopping window and the re-entry window is one period for each time frequency. The stopping thresholds, (γ), will vary from -1.5 to -0.5 standard deviations from the mean at the relevant frequency. For example, if the stopping window is three months long the stops will be set relative to deviations from -1.5 to -0.5 standard deviations. To avoid data selection bias, we review a large range of stops to demonstrate how the performance depends on threshold choices. The re-entry threshold, (δ), will also be modulated to the data frequency and will simply vary between -0.5 to 1 standard deviation from the mean. This approach is used to allow for comparison across different frequencies of time. For example, a one standard deviation stop-loss in weekly versus a one standard deviation stop-loss quarterly can be compared to see how the time frequency impacts the results.

Given the large set of parameters we analyze in this experiment, it is not surprising that the performance of these strategies varies. There are a few key trends in the results. First, shorter term, lower frequency stop-loss policies have negative stopping premiums over large ranges of parameters. Longer term stop-loss at frequencies above one month perform

better and can achieve positive stopping premiums. In Figure 3, the stopping premium for all frequencies and combinations of threshold parameters is plotted as a function of the stopping threshold with delta the re-entry threshold at a 0 standard deviation threshold. In Figures 4 and 5, the empirical results for the change in Sharpe Ratio and change in standard deviation demonstrate how for longer term stop-loss strategies the Sharpe ratio can improve as standard deviation decreases. Second, the decision to exit and the stopping threshold seems to have a larger impact on the variation in results than the re-entry threshold. Putting these results together, the empirical results suggest that the use of longer term stop-loss strategies might have improved performance consistent with anecdotal discussion of the strategy in practice.

6. Conclusion

In this paper, we provide a concrete answer to the question of when do stop-loss rules stop losses? The answer depends, of course, on the return-generating process of the underlying investment for which the stop-loss policy is implemented, as well as the particular dynamics of the stop-loss policy itself. If "stopping losses" is interpreted as having a higher expected return with the stop-loss policy than without it, then for a specific binary stop-loss policy, we derive various conditions under which the expected-return difference, which we call the stopping premium, is positive. We show that under the most common return-generating process, the Random Walk Hypothesis, the stopping premium is always negative. The widespread cultural affinity for the Random Walk Hypothesis, despite empirical evidence to the contrary, may explain the general indifference to stop-loss policies in the academic finance literature.

However, under more empirically plausible return-generating processes such as momentum or regime-switching models, we show that stop-loss policies can generate positive stopping premia. When applied to a standard buy-and-hold strategy using daily U.S. futures contracts from January 1993 to November 2011 where the stop-loss asset is U.S. long-term bonds futures, we find that at longer sampling frequencies, certain stop-loss policies add value over a buy-and-hold portfolio while substantially reducing risk by reducing strategy volatility, consistent with their objectives in practical applications. These empirical results suggest important nonlinearities in aggregate stock and bond returns that have not been fully explored in the empirical finance literature.

Our analytical and empirical results contain several points of intersection with the behavioral finance literature. First, the flight-to-safety phenomena, which is best illustrated by events surrounding the default of Russian government debt in August 1998, may create momentum in equity returns and increase demand for long-term bonds, creating positive stopping premia as a result. Second, systematic stop-loss policies may profit from the disposition effect and loss aversion, the tendency to sell winners too soon and hold on to losers too long. Third, if investors are ambiguity-averse, large negative returns may cause them to view equities as more ambiguous which, in relative terms, will make long-term bonds seem less ambiguous. This may cause investors to switch to bonds to avoid uncertainty about asset returns.

More generally, there is now substantial evidence from the cognitive sciences literature that losses and gains are processed by different components of the brain. These different components provide a partial explanation for some of the asymmetries observed in exper-

imental and actual markets. In particular, in the event of a significant drop in aggregate stock prices, investors who are generally passive will become motivated to trade because mounting losses will cause them to pay attention when they ordinarily would not. This influx of uninformed traders, who have less market experience and are more likely to make irrational trading decisions, can have a significant impact on equilibrium prices and their dynamics. Therefore, even if markets are usually efficient, on occasions where a significant number of investors experience losses simultaneously, markets may be dominated temporarily by irrational forces. The mechanism for this coordinated irrationality is cumulative loss.

Of course, our findings shed little light on the controversy between market efficiency and behavioral finance. The success of our simple stop-loss policy may be due to certain nonlinear aspects of stock and bond returns from which our strategy happens to benefit (e.g., avoiding momentum on the downside and exploiting asymmetries in asset returns following periods of negative cumulative returns). And from the behavioral perspective, our stop-loss policy is just one mechanism for avoiding or anticipating the usual pitfalls of human judgment (e.g., the disposition effect, loss aversion, ambiguity aversion, and flight-to-safety).

In summary, both behavioral finance and rational asset-pricing models may be used to motivate the apparent effectiveness of stop-loss policies, in addition to the widespread use of such policies in practice. This underscores the importance of learning how to deal with loss as an investor, of which a stop-loss rule is only one dimension. As difficult as it may be to accept, for the many investors who lamented, after the subprime mortgage meltdown of 2007–2008, that "if only I had gotten out sooner, I wouldn't have lost so much," they may have been correct.
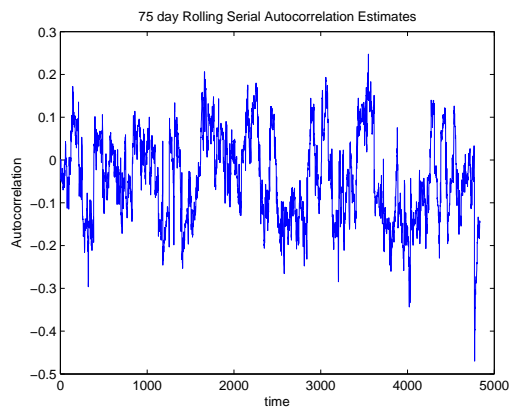
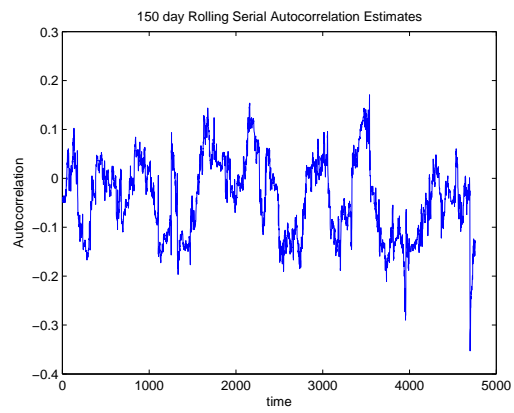**Fig. 1.** Rolling serial autocorrelation coefficient estimates for the IMM S&P contact with a 75-day Window

Fig. 2. Rolling serial autocorrelation coefficient estimates for the IMM S&P contact with a 150-day Window
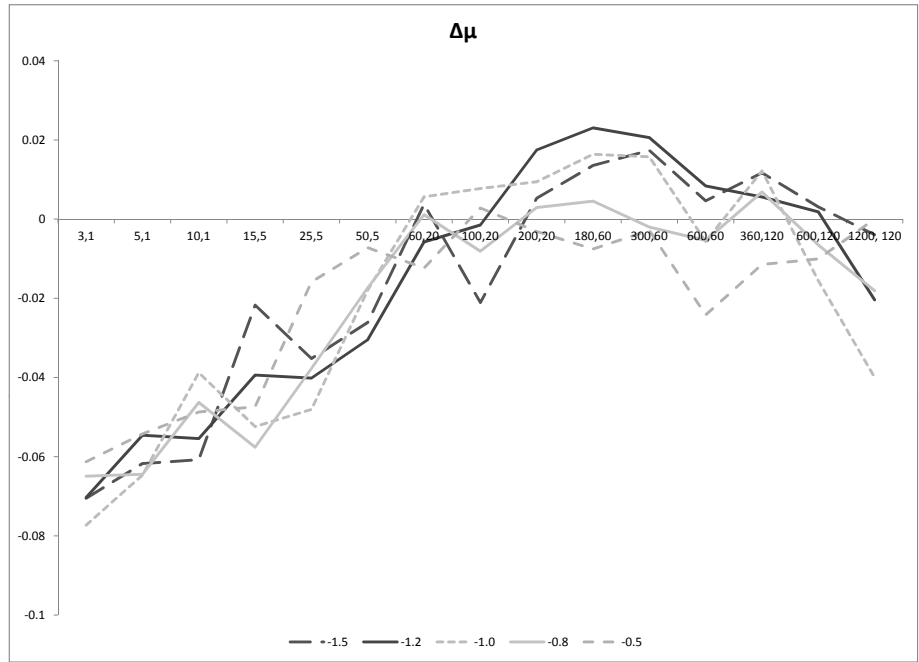
Fig. 3. Stopping premium, (Δ_μ), from short-term $(3,1)$ stop-loss strategies to longer-term $(1200,120)$ stop-loss strategies with exit thresholds, (γ), of 1.5, 1.2, 1, 0.8, and 0.5 standard deviations from the mean with a 0 standard deviation re-entry threshold, (δ)
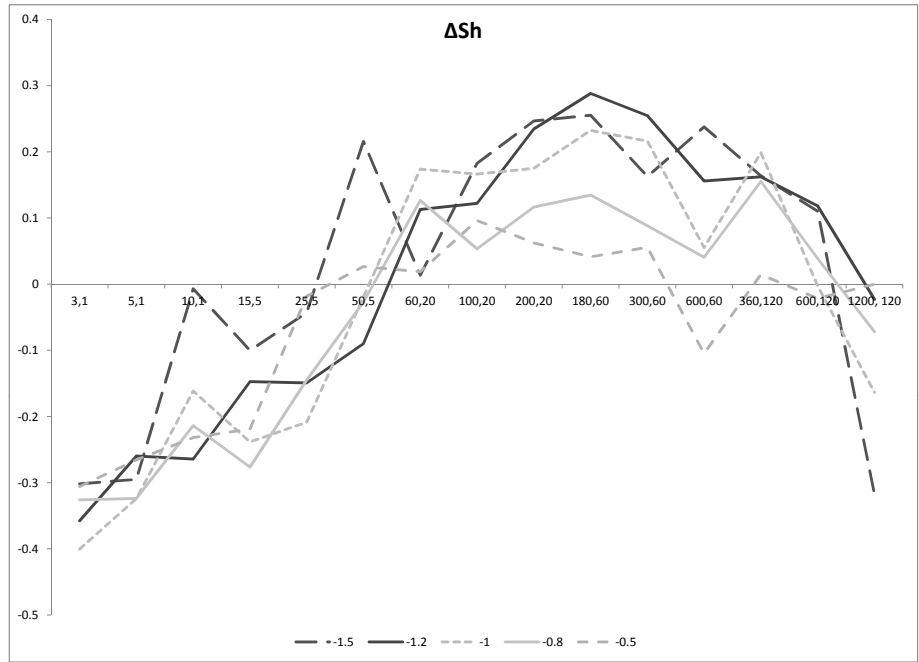
Fig. 4. Change in Sharpe Ratio, (Δ_{SR}), from short-term (3,1) stop-loss strategies to longer-term (1200,120) stop-loss strategies with exit thresholds, (γ), of 1.5, 1.2, 1, 0.8, and 0.5 standard deviations from the mean with a 0 standard deviation re-entry threshold, (δ)
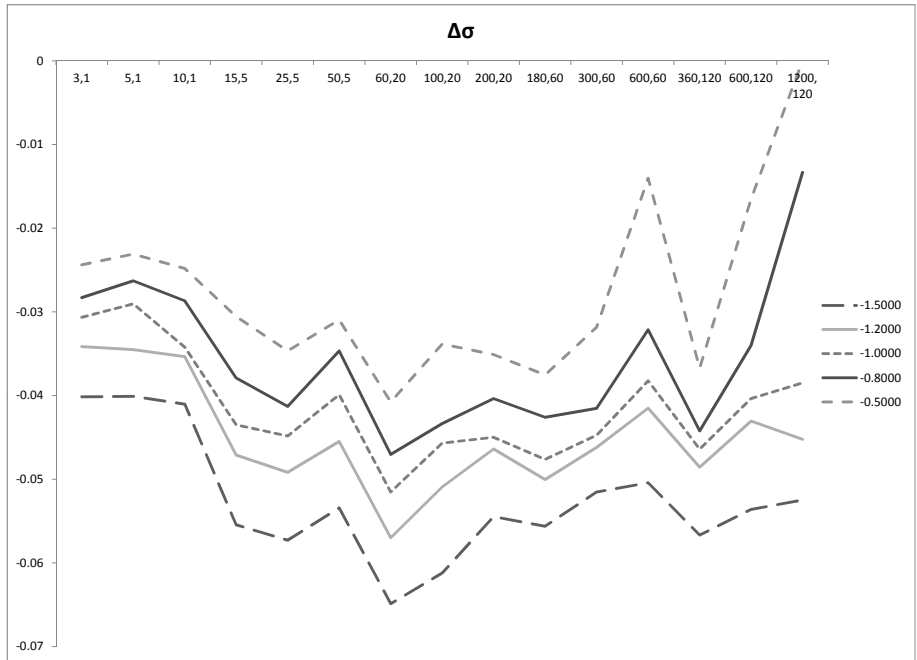
Fig. 5. Change in standard deviation, (Δ_σ), from short-term (3,1) stop-loss strategies to longer-term (1200,120) stop-loss strategies with exit thresholds, (γ), set at 1.5, 1.2, 1, 0.8, and 0.5 standard deviations from the mean with a 0 standard deviation re-entry threshold, (δ)

	Ann. Mean (%)	Ann. SD (%)	ρ	Skew	Kurt	Min (%)	Max (%)	Ann. Sharpe	MDD (%)
IMM S&P	7.295	19.499	-0.06	0.12	14.24	-9.88	14.12	0.37	26.62
CBT 10YR TN	1.181	6.331	0.02	-0.09	6.11	-2.43	3.61	0.19	5.25

Table 1

Summary statistics for U.S. Equities (IMM S&P) and long-term U.S. Government Bonds (CBT 10 YR) from January 5, 1993 to November 7,2011. Statistics are annualized assuming 250 days per year.

	μ_0	μ_1	σ_0	σ_1	p
IMM S&P	22.3%	-39.0%	10.3%	33.0%	0.72

Table 2

Annualized parameter estimates for a two-state Hamilton model estimated via the EM Algorithm

on the IMM S&P Futures Contracts from January 5, 1993 to November 7, 2011.

Appendix

In this Appendix, we provide proofs of Propositions 1 and 2 in Sections A.1 and A.2.

A.1. Proof of Proposition 1

The conclusion follows almost immediately from the observation that the conditional expectations in (4) and (6) are equal to the unconditional expectations because of the Random Walk Hypothesis (conditioning on past returns provides no incremental information), hence:

$$\Delta_\mu \;=\; -\,p_o\pi \;\leq\; 0 \tag{.1}$$

$$\frac{\Delta_\mu}{p_o} \;=\; -\,\pi \;\leq\; 0 \tag{.2}$$

and the other relations follow in a similar manner. ∎

A.2. Proof of Proposition 2

Let r_t be a stationary AR(1) process:

$$r_t \;=\; \mu \;+\; \rho(r_{t-1} - \mu) \;+\; \epsilon_t, \; \epsilon_t \stackrel{\text{IID}}{\sim} \text{White Noise}(0, \sigma_\epsilon^2), \; \rho \in (-1, 1) \tag{.3}$$

We seek the conditional expectation of r_t given that the process is stopped out. If we let J be sufficiently large and $\delta = -\infty$, $s_t = 0$ is equivalent to $R_{t-1}(J) < -\gamma$ and $s_{t-1} = 1$ with $R_{t-2}(J) \geq -\gamma$. Using log returns, we have:

$$\mathrm{E}[r_t | s_t = 0] \;=\; \mathrm{E}[r_t | R_{t-1}(J) < -\gamma, R_{t-2}(J) \geq -\gamma] \tag{.4}$$

$$=\; \mu(1 - \rho) \;+\; \rho\,\mathrm{E}[r_{t-1} + \epsilon_t | R_{t-1}(J) < -\gamma, R_{t-2}(J) \geq -\gamma] \tag{.5}$$

$$=\; \mu(1 - \rho) \;+\; \rho\,\mathrm{E}[r_{t-1} | R_{t-1}(J) < -\gamma, R_{t-2}(J) \geq -\gamma]. \tag{.6}$$

By definition $R_{t-1}(J) \equiv r_{t-1} + \cdots + r_{t-J}$ and $R_{t-2}(J) = r_{t-2} + \cdots + r_{t-J-1}$. Setting $y \equiv r_{t-2} + \cdots + r_{t-J}$ then yields:

$$E[r_t|s_t = 0] \;=\; \mu(1-\rho) \;+\; \rho E[r_{t-1}|R_{t-1}(J) < -\gamma, R_{t-2}(J) \geq -\gamma] \tag{.7}$$

$$=\; \mu(1-\rho) \;+\; \rho E_y\big[E[r_{t-1}|r_{t-1} < -\gamma - y, r_{t-J-1} \geq -\gamma - y]\big]. \tag{.8}$$

For J large enough, the dependence between r_{t-J-1} and r_{t-1} is of order $\mathbf{o}(\rho^J) \approx 0$, hence:

$$E_y\big[E[r_{t-1}|r_{t-1} < -\gamma - y]\big] \;\leq\; E_{r_{t-J-1}}\big[E[r_{t-1}|r_{t-1} < r_{t-J-1}]\big] \tag{.9}$$

$$\leq\; \mu \;-\; \sigma, \tag{.10}$$

which implies:

$$E[r_t|s_t = 0] \;\leq\; \mu(1-\rho) \;+\; \rho(\mu - \sigma). \tag{.11}$$

$$\leq\; \mu \;-\; \rho\sigma. \tag{.12}$$

∎

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References

Agnew, J., 2003. An analysis of how individuals react to market returns in one 401(K) plan. Working paper, The College of William and Mary.

Alexander, S., 1961. Price movements in speculative markets: Trends or random walks. Industrial Management Review 2, 7–26.

Ameriks, J., Zeldes, S., 2004. How do household portfolio shares vary with age?. Working paper, Columbia University.

Ang, A., Bekaert, G., 2004. How regimes affect asset allocation. Financial Analysts Journal 60, 86–99.

Balduzzi, P., Lynch, A., 1999. Transaction costs and predictability: Some utility cost calculations. Journal of Financial Economics 52, 47–78.

Barber, B., Odean, T., 2000. Trading is hazardous to your wealth: The common stock investment performance of individual investors. Journal of Finance 55, 773–806.

Barber, B., Odean, T., 2001. Boys will be boys: Gender, overconfidence, and common stock investment. Quarterly Journal of Economics 116, 261–292.

Barberis, N., 2000. Investing in the long run when returns are predictable. Journal of Finance 55, 225–264.

Bell, D., 1982a. Regret in decision making under uncertainty. Operations Research 30, 961–981.

Bell, D., 1982b. Risk premiums for decision regret. Management Science 29, 1156–1166.

Biais, B., Hillion, P., Spatt, C., 1995. An empirical analysis of the limit order book and the order flow in the Paris Bourse. Journal of Finance 50, 1655–1689.

Brandt, M., 2010. Portfolio Choice Problems. In: Ait-Sahalia, Y., Hansen, L. (Eds.), Handbook of Financial Econometrics, Volume 1: Tools and Techniques. Elsevier, North Holland, pp. 269–336.

Brandt, M., Goyal, A., Santa-Clara, P., Stroud, J., 2005. A Simulation approach to dynamic portfolio choice with an application to learning about return predictability. Review of Financial Studies 18, 831–873.

Brennan, M., Xia, Y., 2001. Persistence, predictability, and portfolio planning. Working paper.

Brennan, M., Schwartz, E., Lagnado, R., 1997. Strategic Asset Allocation. Journal of Economic Dynamics and Control 21, 1377–1403.

Calvet, L., Campbell, J., Sodini, P., 2006. Down or out: Assesing the welfare costs of household investment mistakes. Working Paper No. 12030, NBER.

Campbell, J., Chan, Y., Viceira, L., 2003. A multivariate model of strategic asset allocation. Journal of Financial Economics 67, 41–80.

Campbell, J., Lo, A., MacKinlay, C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton.

Campbell, J., Viceira, L., 1999. Consumption and portfolio decisions when expected returns are time-varying. Quarterly Journal of Economics 114, 433–495.

Carr, P., Jarrow, R., 1990. The stop-loss start-gain paradox and option valuation: A new decomposition into intrinsic and time value. Review of Financial Studies 3, 469–492.

Chakravarty, S., Holden, C., 1995. An integrated model of market and limit orders. Journal of Financial Intermediation 4, 213–241.

Chang, F., 1988. The inverse optimal problem: A dynamic programming approach. Econometrica 56, 147–172.

Clarke, R., Krase, S., Statman, M., 1994. Tracking errors, regret, and tactical asset allocation. Journal of Portfolio Management 20, 16–24.

DeBondt, W., Thaler, R., 1986. Does the stock market overreact?. Journal of Finance 40, 793–807.

Easley, D., O'Hara, M., 1991. Order form and information in securities markets. Journal of Finance 46, 905–927.

Fama, E., Blume, M., 1966. Filter rules and stock market trading profits. Journal of Business 39, 226–241.

Fama, E., French, K., 1988. Permanent and temporary components of stock prices. Journal of Political Economy 96, 246–273.

Fischoff, B., Slovic, P., 1980. A little learning...: Confidence in multicue judgment tasks. In: Nickerson, R. (Eds.), Attention and Performance, VIII. Erlbaum, Hillsdale, pp. 779–800.

Gervais, S., Odean, T., 2001. Learning to be overconfident. Review of Financial Studies 14, 1–27.

Hamilton, J., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357–384.

Hamilton, J., 1994. Time Series Analysis. Princeton University Press, Princeton.

Handa, P., Schwartz, R., 1996. Limit order trading. Journal of Finance 51, 1835–1861.

Harris, L., Hasbrouck, J.,1996. Market vs. limit-orders: The SuperDot evidence on order submission strategy. Journal of Financial and Quantitative Analysis 31, 213–231.

Huberman, G., Regev, T., 2001. Contagious speculation and a cure for cancer: A nonevent that made stock prices soar. Journal of Finance 56, 387–396.

Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. Journal of Finance 45, 881–898.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. Journal of Finance 48, 65–91.

Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk. Econometrica 47, 263–291.

Kahneman, D., Tversky, A., 1981. The framing of decisions and the psychology of choice. Science 211, 453—458.

Kahneman, D., Tversky, A., 1992. Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and Uncertainty 5, 297–323.

Kaminski, K., 2006. General Superposition Strategies. unpublished manuscript.

Kim, T., Omberg, E., 1996. Dynamic non-myopic portfolio behavior. Review of Financial Studies 9, 141–161.

Laibson, D., 1997. Golden eggs and hyperbolic discounting. Quarterly Journal of Economics 62, 443–477.

Lichtenstein, S., Fischoff, B., Phillips, L., 1982. Calibration of Probabilities: The State of the Art to 1980. In: Kahneman, D., Slovic, P., Tversky, A. (Eds.), Judgment Under Uncertainty: Heuristics and Biases. Cambridge University Press, New York, pp. 306–334.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47, 13–37.

Liu, J., 1999. Portfolio selection in stochastic environments. unpublished working paper, University of California, Los Angeles.

Lo, A., 1991. Long-term memory in stock market prices. Econometrica 59, 1279–1313.

Lo, A., 2002. The statistics of Sharpe ratios. Financial Analysts Journal 58, 36–50.

Lo, A. and C. MacKinlay, 1988. Stock market prices do not follow random walks: Evidence from a simple specification test. Review of Financial Studies 1, 41–66.

Lo, A., MacKinlay, C., 1990. An econometric analysis of nonsynchronous trading. Journal of Econometrics 45, 181–212.

Lo, A., MacKinlay, C., 1999. A Non-Random Walk Down Wall Street. Princeton University Press, Princeton.

Lo, A., MacKinlay, C., Zhang, J., 2002. Econometric models of limit-order executions. Journal of Financial Economics 65, 31–71.

Lucas, R., 1978. Asset pricing in an exchange economy. Econometrica 46, 1429–1446.

Markowitz, H., 1952. Portfolio selection. Journal of Finance 7, 77–91.

Massa, M., Simonov, A., 2004. Hedging, familiarity and portfolio choice. CEPR Discussion Paper No. 4789.

Merton, R. 1969. Lifetime portfolio selection under uncertainty; The continuous-time case. Review of Economics & Statistics 51, 247–257.

Merton, R., 1971. Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory 3, 373–413.

Merton, R., 1973. An intertemporal capital asset pricing model. Econometrica 41, 867–887.

Odean, T., 1998. Are investors reluctant to realize their losses?. Journal of Finance 53, 1775–1798.

Odean, T., 1999. Do investors trade too much. American Economic Review 89, 1279–1298.

Poterba, J., Summers, L., 1988. Mean reversion in stock returns: Evidence and implications. Journal of Financial Economics 22, 27–60.

Samuelson, P. 1969. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics 51, 239–246.

Seppi, D., 1997. Liquidity provision with limit orders and a strategic specialist. Review of Financial Studies 10, 103–150.

Sharpe, W., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19, 425–442.

Sharpe, W., 1994. The Sharpe ratio. Journal of Portfolio Management Fall, 49–58.

Shefrin, M., Statman, M., 1985. The disposition to sell winners too early and ride losers too long: Theory and evidence. Journal of Finance 40, 777–790.

Spurgin, R., 2001. How to game your Sharpe ratio. The Journal of Alternative Investments 4, 38–46.

Tobin, J., 1958. Liquidity preference as behavior towards risk. Review of Economic Studies 25, 65–86.

Tschoegl, A., 1988. The source and consequences of stop orders: A conjecture. Managerial and Decision Economics 9, 83–85.

Wachter, J., 2002. Portfolio and consumption decisions under mean-reverting returns: An exact solution for complete markets. Journal of Financial and Quantitative Analysis 37, 63–91.

Xia, Y., 2001. Learning about predictability: The effects of parameter uncertainty on dynamic asset allocation. Journal of Finance 56, 205–246.