1. A die is thrown 5 times. The experiment results in a sequence $c_1c_2c_3c_4c_5$, where $c_i \in \{1, 2, . . ., 6\}$ is a positive integer obtained in the i-th throw. How many distinct results are there?

   Solution.
   Each positive integer $c_i \in \{1, 2, . . ., 6\}$ of the sequence $c_1c_2c_3c_4c_5$ is a set. By the Product Rule, the Cartesian product of this sequence of sets is $|c|^n$ where $n = 5$, since there are five sets in the sequence.

   $$|c|^n = 6^n = 6^5 = 7776$$

   ■

2. How many four-digit even positive integers are there without a repetition of digits?

   Solution.
   The first digit of a four-digit even positive integer cannot be 0 and the last digit of a four-digit even positive integer cannot be an odd integer. Let the sequence $c_1c_2c_3c_4$, where $c_1 \in \{1, 2, . . ., 9\}$ and $c_2, c_3 \in \{0, 1, . . ., 9\}$ and $c_4 \in \{0, 2, . . ., 8\}$ represent a four-digit even positive integer. The Cartesian product of this sequence of sets gives the answer.

   $$|c_1| \cdot |c_2| \cdot |c_3| \cdot |c_4| = 9 \cdot 10 \cdot 10 \cdot 5 = 4500$$

   ■

3. How many functions $f : \{1, 2, 3, 4\} \to \{a, b, c\}$ are there?

   Solution.

   $$|\{a, b, c\}|^{|\{1,2,3,4\}|} = 3^4 = 36$$

   ■

4. How many onto functions $f : \{1, 2, 3, 4\} \to \{a, b, c\}$ are there?

   Solution.

   $$|\{a, b, c\}|^{|\{1,2,3,4\}|} = 3^4 = 36$$

   ■

5. How many functions $f : \{1, 2, . . ., n\} \to \{1, 2\}$ are there?

   Solution.

   $$|\{1, 2\}|^{|\{1,2,...,n\}|} = 2^n = \{(f(1) \to 1, f(2) \to 1), (f(1) \to 2, f(2) \to 2), ..., (f(1) \to n, f(2) \to n)\}$$

   ■

6. How many ten-digit positive integers are there such that all of the following conditions are satisfied:

   (a) each of the digits 0, 1, . . ., 9 appears exactly once;

   (b) the first digit is odd;

   (c) five even digits appear in five consecutive positions?

   Solution.
   Let the sequence of positive integers $d_1 d_2 \cdots d_9$ satisfy the above requirements. There are $|\{0, 2, ..., 8\}| = 5$ possible positive even digits and are $|\{1, 3, ..., 9\}| = 5$ possible positive odd digits. Five even digits must appear consecutively and form the sequence $d_i d_{i+1}, ..., d_{i+4}$ where $1 < i \leqslant 5$; there are five possible initial points in the sequence to occur. Since no digit can appear more than once, we subtract one element from the set of odds or from the set of evens with each new digit in the sequence.

$$(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot 5 = 72000$$

■