A physicists personal thoughts on writing on a tablet

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1 Introduction

I happen to have worked a long time with measurements, measurement errors and mathematical models for physical and measurement processes. If I look at the "writing on a tablet" issue, all the points I considered in completely different context (radar tracking of planes, robot navigation, star formation in galaxies, hydrodynamical simulations of star formation) pop up in my mind. For someone not familiar with the measurement process in physical experiments and not familiar with stochastic processes (think of clouds on the sky), some statements seem to be highly erratic and confusing, I guess. Here I like to elaborate on some of my lines of reasoning.

Without any further ado:

2 Goal:

Writing on a tablet should have the same look and feel as writing on a piece of paper.

Expressing this wish in a machine-understandable way is actually quite difficult (I get reminded ot this: https://en.wikipedia.org/wiki/I_know_it_when_I_see_it)

How to tackle this?

First, observe,

Second, write down the obvious and not so obvious.

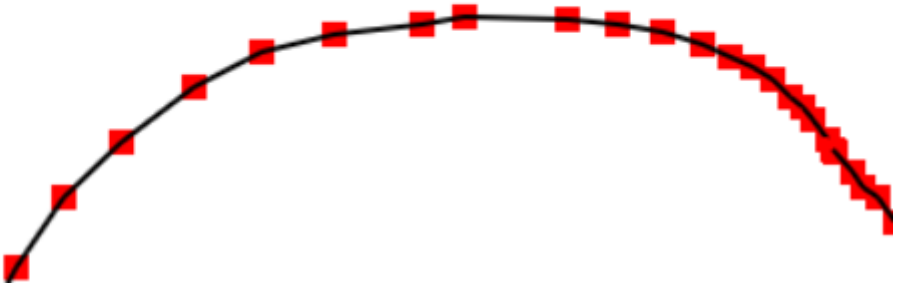
Third, build a mathematical model taking into account all the observed

3 Observations:

(Black lines in the images are a linear interpolation, red are the actual measurement. In some images the red boxes are moved by some pixels to the side.)

In random order:

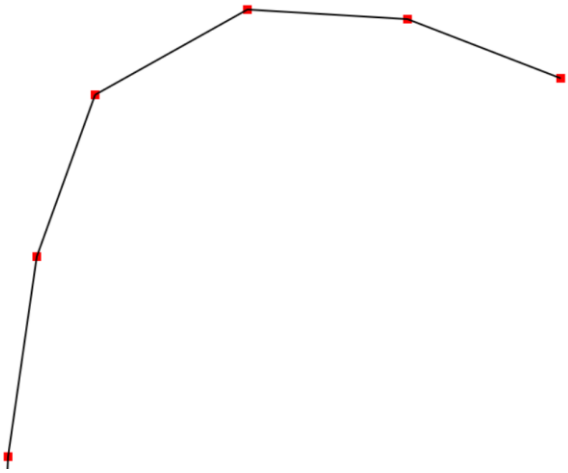
3.1 Writing curved lines in just the right speed



There is a "sweet" spot when you do not write too slow and not too fast. At this speed a linear interpolation between the points is sufficient.

But who does write like this?

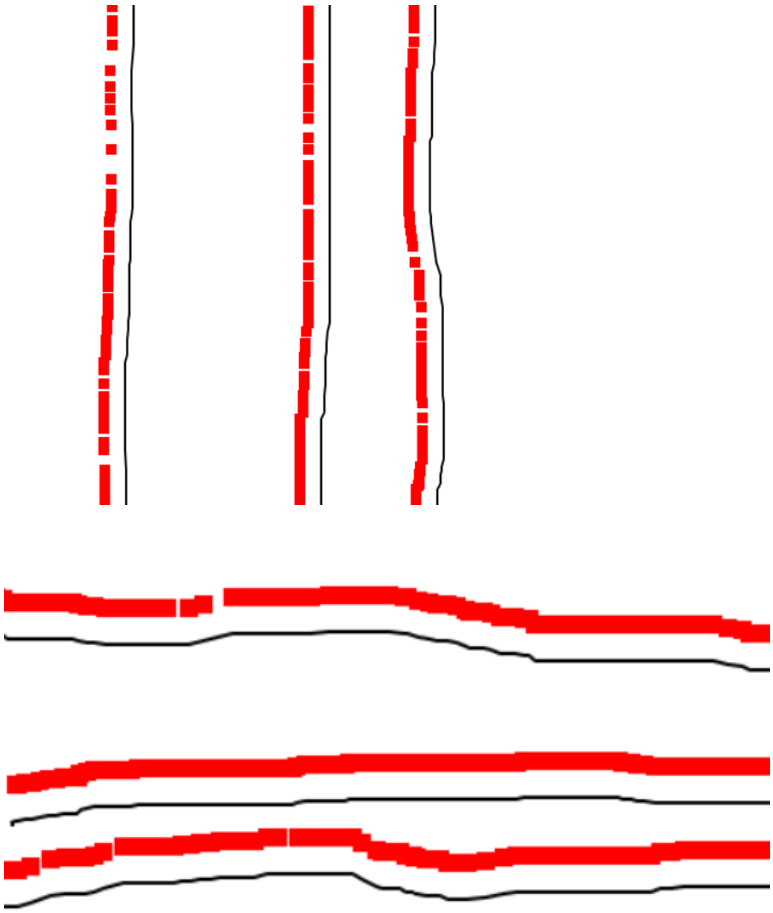
3.2 Writing curved lines quickly



If you move the pen quickly over the tablet, the frequency of measurements is too low. The pen has moved too far between one pen (mouse) event and the next one.

Therefore it is necessary to interpolate between two measurements for drawing the "real" motion of the pen

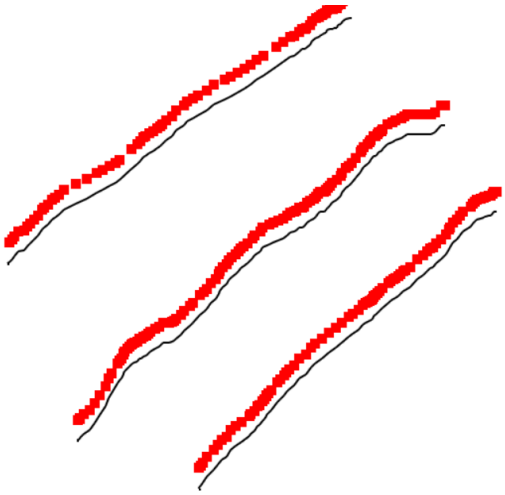
3.3 Writing horizontal or vertical lines rather slowly



There are "steps" in the lines. This is due to the fact, that the sensor is discretised in x and y (pixels).

If you write more quickly, this discretisation vanishes as you are not measuring it, because the measurement frequency is too low.

3.4 Writing diagonal lines slowly



Diagonal lines appear to be wobbly if you write slowly. But why? I think, two reasons are there for this behaviour:

1) Discretisation. That we already discussed. This in in my opinion, hovever, not the main reason

2) Different friction resistance of a plastic pen tip on a plastic display surface compared to a pencil or ballpoint pen on a piece of paper. Paper is much more coarse and rough than a tablet display. A stylus on a tablet is quite slippery. With this slipperyness all the tremble and shiver of the human hand is measured. This is not really wanted.

3.5 Writing zig-zag lines quickly

Ok, this one is hard.

If you write zig-zag very quickly, then I think the following happens:

The measurement frequency is too low on the tablet to actually provide measurements at the tip of the "zig". So in the picture the measurements (red squares) marked yellow would be missing. If they miss, then any algorithm not accounting for this would make a mistake and draw the red line as connection (excuse my bad mouse drawing). As far as I understand, this is what Oleg complained about.

3.6 Summary of the observations

The following problems ("errors" in my lingo) arise:

- the (low) measurement frequency requires for fast movement of the stylus an interpolation between the measurements for drawing (insufficent measurement rate, "frequency" error)
- the discretised pixels do not account for fractional values (discretisation error)
- the smoothness of stylus and tablet lead to the measurement of the human tremble ("tremble" error)
- missing data due to low measurement frequency ("missing data" error)

4 Deep dive

The following sections are the rather dry writeup of what I think of if I look at "writing on a tablet" as a physics process. Sorry if it reads too much like a paper as this.

4.1 Experimental setup:

A human has some pen with a plastic tip (aka stylus) in his hand and moves it over the smooth surface of the display of a tablet.

The tablet has built-in sensors that detect the presence of the stylus and measure position (pressure, angle) at more or less frequent points in time.

Time is typically discretised and not given in physical units like seconds.

The sensor in the tablet is discretised in position via pixels.

Thus, we measure position in two dimensions at points in time. In maths: *x(t), y(t)* and *t*,

4.2 Data analysis (aka "the algorithm")

Data analysis means that you take the measured data (positions), do something with them and show the results on the screen. If possible in real-time. Performance issues, though, are considered later, below.

4.2.1 Linear interpolation

4.2.1.1 Method

Just take a straight line through all points.

4.2.1.2 Result

a)Not nice. If writing quickly lines are kinky (pardon the joke. should read: have kinks).

b) Many other issues

4.2.1.3 Reason for problems

a) Method does not predict values between two measurements.

b) I am too lazy to write it up. Linear interpolation is just wrong.

4.2.1.4 Conclusion

Linear interpolation is a nice try, but too simple.

4.2.2 Splines

4.2.2.1 Method

Just take the d3 library (https://d3js.org/) and put a spline (https://en.wikipedia.org/wiki/Spline_(mathematics)) through the points.

Try different splines.

4.2.2.2 Result

Quite ok, but some problems:

a) Writing slowly may result in wobbly lines

b) Writing quickly letter like an "l" (Ell) misses the edgy top bits

4.2.2.3 Reason for problems

a) Splines are an interpolation algorithm, they do not account for measurement errors (discretisation error, tremble error)

b) Splines do not into account missing data or a dynamical model.

4.2.2.4 Conclusion

Imho there is no way to mend splines so that they account for measurement errors, missing data or a dynamical model. People have tried, solutions become very complicated (random citation: https://arxiv.org/pdf/1804.00793.pdf) Just fries my brain.

Maybe, start with a mathematical model that takes everything into account instead of trying to make a shirt out of a pair of trousers.

5 Confluence (Darmstadt)

- Bereiche
- Personen
- Glossare
- Kalender
- **ErstellenErstellen**
- Hilfe
- 2

Konfigurieren

- Bearbeiten
- Favorit
- Beobachtung
- Teilen

1. Seiten
2. ...
3. tm sandbox

Zum Ende des Banners springen

Zum Anfang des Banners springen

6 A physicists personal thoughts on writing on a tablet

Zum Ende der Metadaten springen

- Angelegt von Maschberger, Thomas, zuletzt geändert vor 11 Minuten

Zum Anfang der Metadaten

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10.2.3 Bezier Curves

10.2.3.1 Method

A Bezier curve is mathematically more or less the same as a spline (see https://en.wikipedia.org/wiki/B%C3%A9zier_curve). I am not sure, but I think I read somewhere that a Bezier curve can be expressed mathematically the same way as a certain class of splines. Would not bet on it, though.

10.2.3.2 Result

probably the same as for splines

10.2.3.3 Reason for the problems

same as for splines

10.2.3.4 Conclusion

same as for splines

10.3 "Right" way of data analysis

Just a sketch

We measure position as a function of time. This calls for time series analysis (see https://en.wikipedia.org/wiki/Time_series)

The time series we measure has some random elements, the discretisation error and the tremble error. Thus the measured signal is

$Position_{measured} = Position_{real}\sim + Noise_{discretisation} + Noise_{tremble}$,

For practical reasons, it is probably best to combine the discretion error and the tremble error to an effective error. Thus:

$Position_{measured} = Preal + Noise_{effective}$.

Furthermore, we know that the human hand does not do discontinuous movements. So we should somehow have a dynamic model:

$x_{t+1} = f(x_t)$

This should take care of the frequency error (as it allows interpolation) and the missing data error.

In total, we can write something like

$Observation_{t+1} = f(x_t) + Noise_{effective}$

The way to deal with such problems is the Kalman filter (https://en.wikipedia.org/wiki/Kalman_filter)

The Kalman filter is some sort of generalised least squares (https://en.wikipedia.org/wiki/Least_squares) for arbitrary curves.

A nice side-effect is the rather low computational effort (if done correctly, without re-calculating things that do not need to be calculated again between measurements).

I think such an approach to the "Writing on tablet" problem could be really fruitful, as it accounts for all problems resp. errors that I can think of.

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