Fluid flow around a shell

Noah

Stokes can be written as

$$-\nabla \cdot (\nabla u + p) = f \tag{1}$$
$$\nabla \cdot u = 0. \tag{2}$$

The naïve finite element approach to solving the Stokes equations is as follows. Find $(u, p) \in W$ such that

$$a((u, p), (v, p)) = L((v, q)) \tag{3}$$

for all $(v, q) \in W$, where

$$a((u, p), (v, q)) = \int_\Omega \nabla u \cdot \nabla v - \nabla \cdot vp + \nabla \cdot uq \, \mathrm{d}x, \tag{4}$$
$$L((v, q)) = \int_\Omega f \cdot v \, \mathrm{d}x. \tag{5}$$

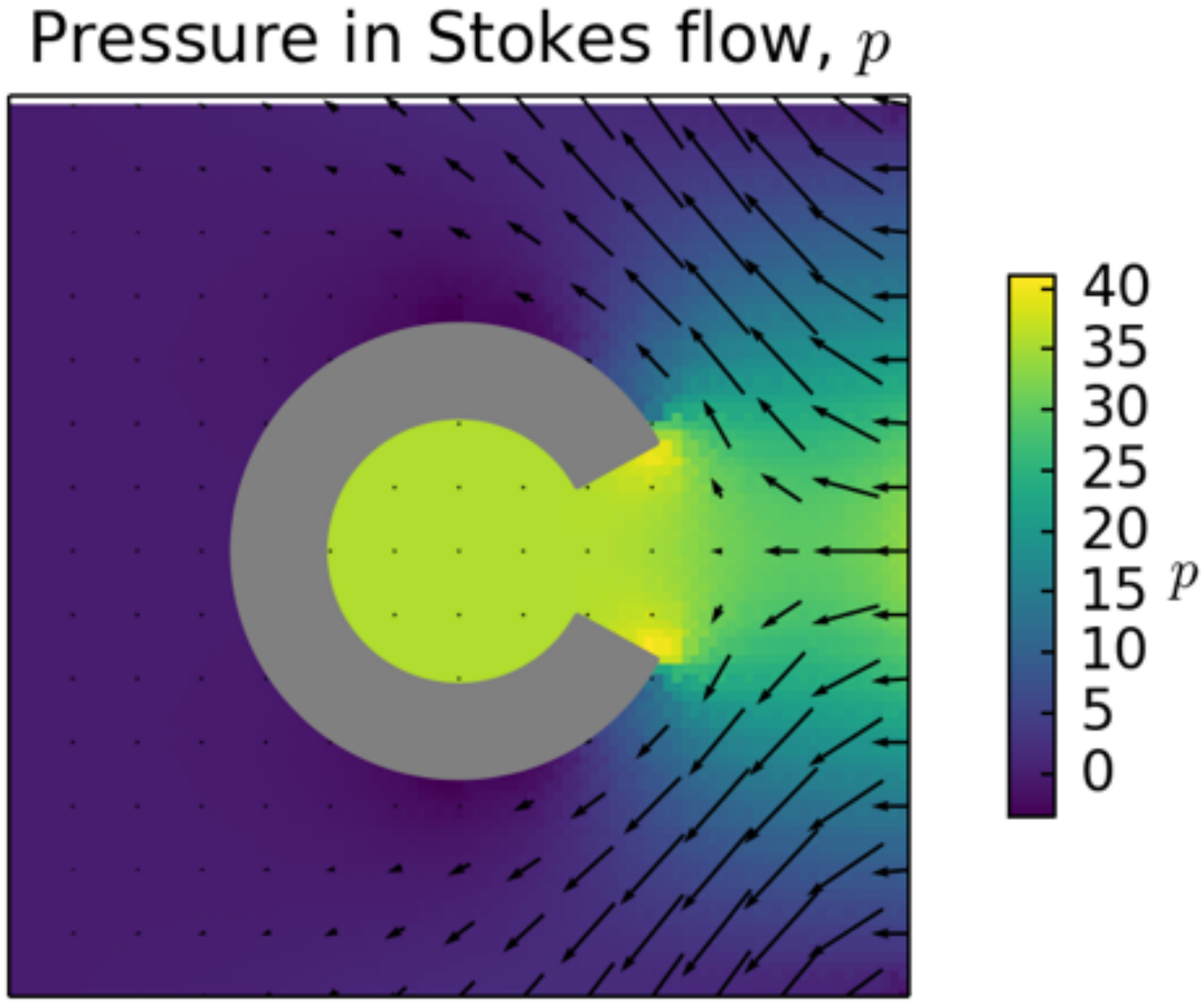
Using first order elements in both velocity and pressure leads to stability problems for reasons that are unclear to me but clear to some in the FEA community. Replacing equations 6 and 7 with

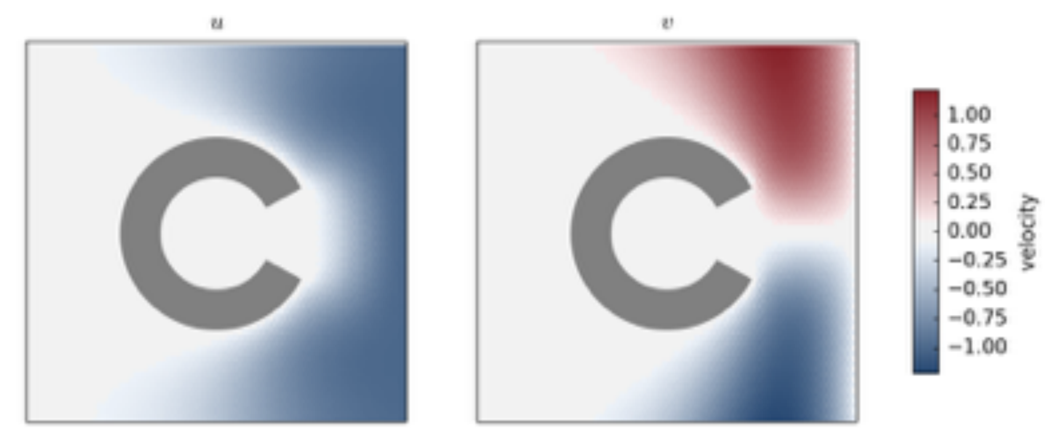
$$a((u, p), (v, q)) = \int_\Omega \nabla u \cdot \nabla v - \nabla \cdot vp + \nabla \cdot uq + \delta \nabla q \cdot \nabla p \, \mathrm{d}x, \tag{6}$$
$$L((v, q)) = \int_\Omega f \cdot v \, \mathrm{d}x + \int_\Omega \nabla q \cdot f \, \mathrm{d}x. \tag{7}$$

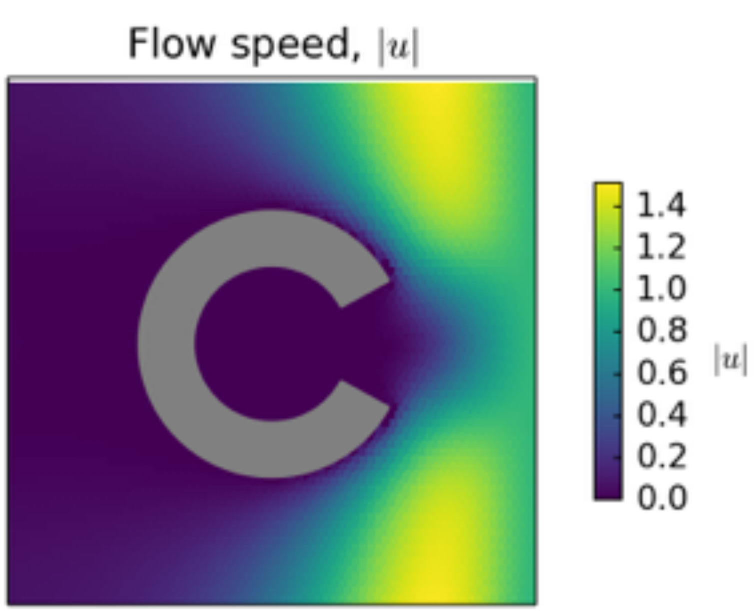
We then implement this on FEniCS with a mixed function space method.

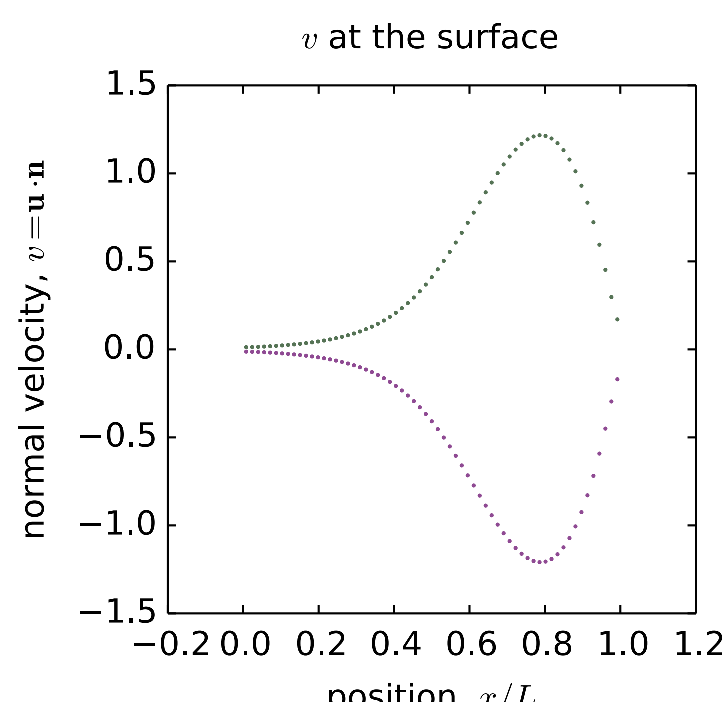
Note that the sign of p has been flipped from its definition as pressure in my notes so that the equations are symmetric.

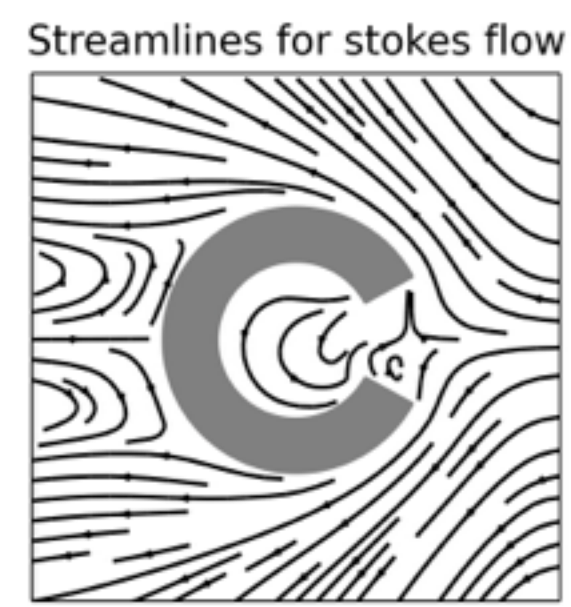
Inward flow from right boundary

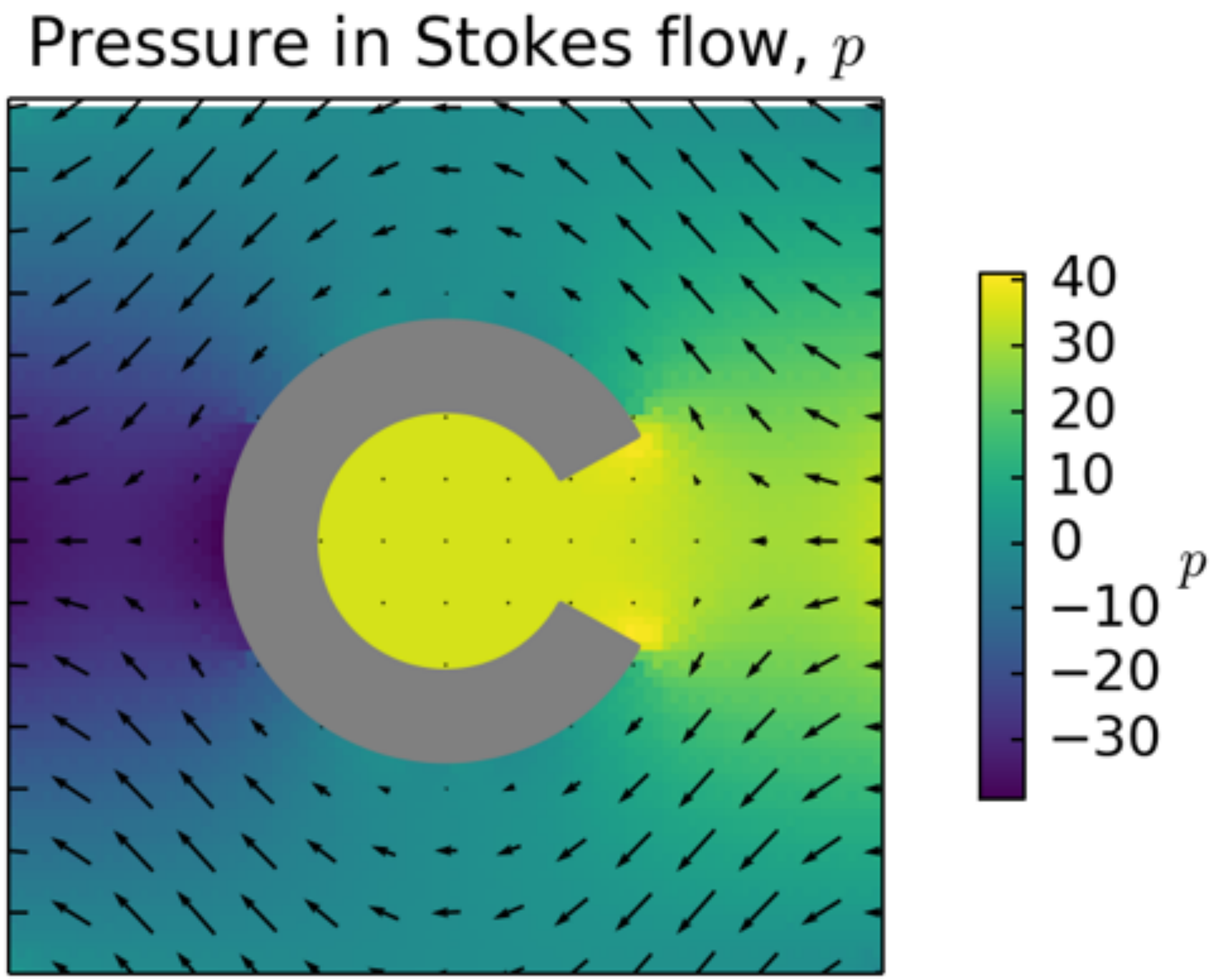




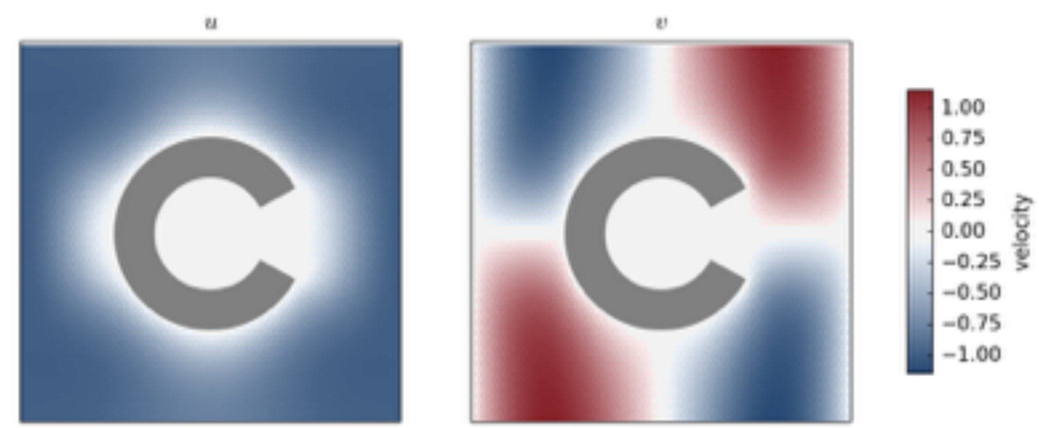


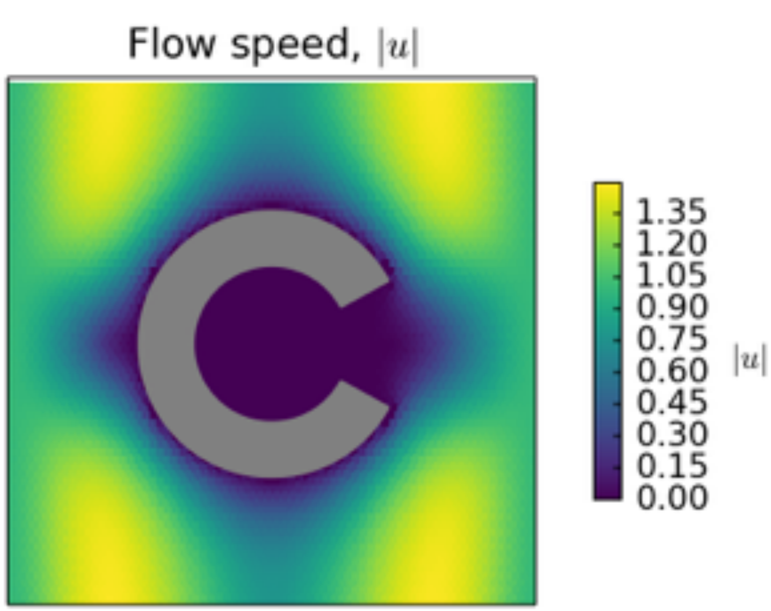


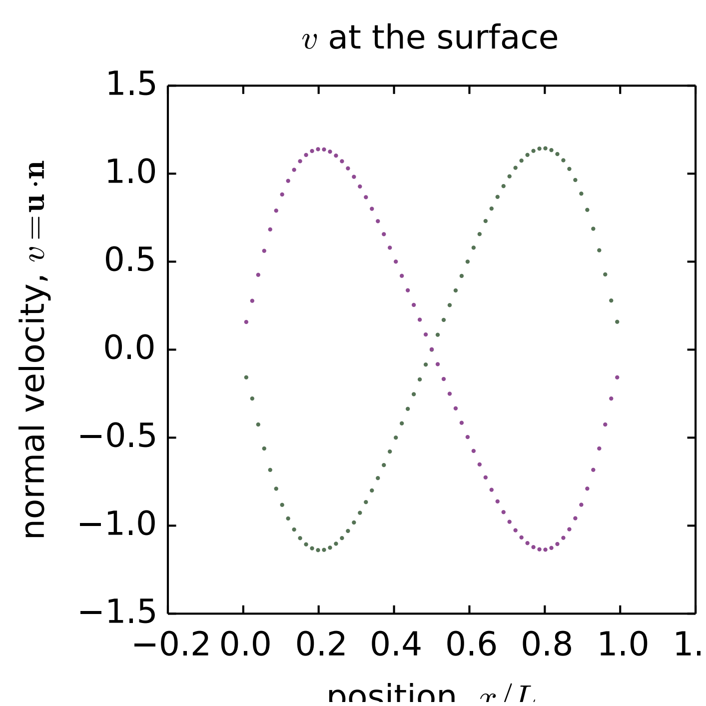


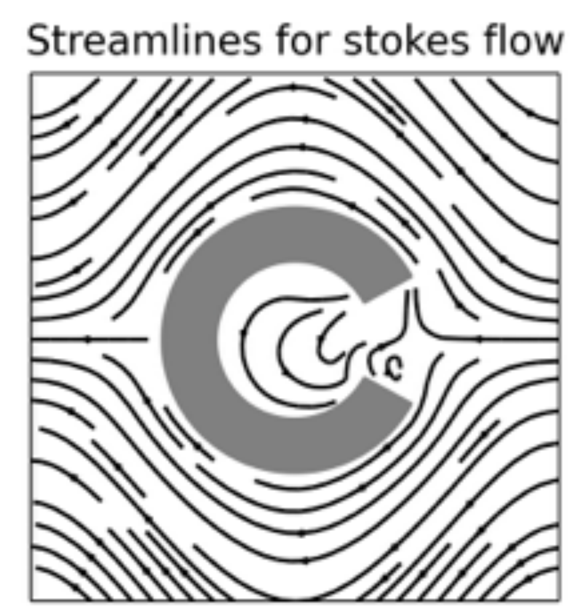


Equal flows on right and left boundary

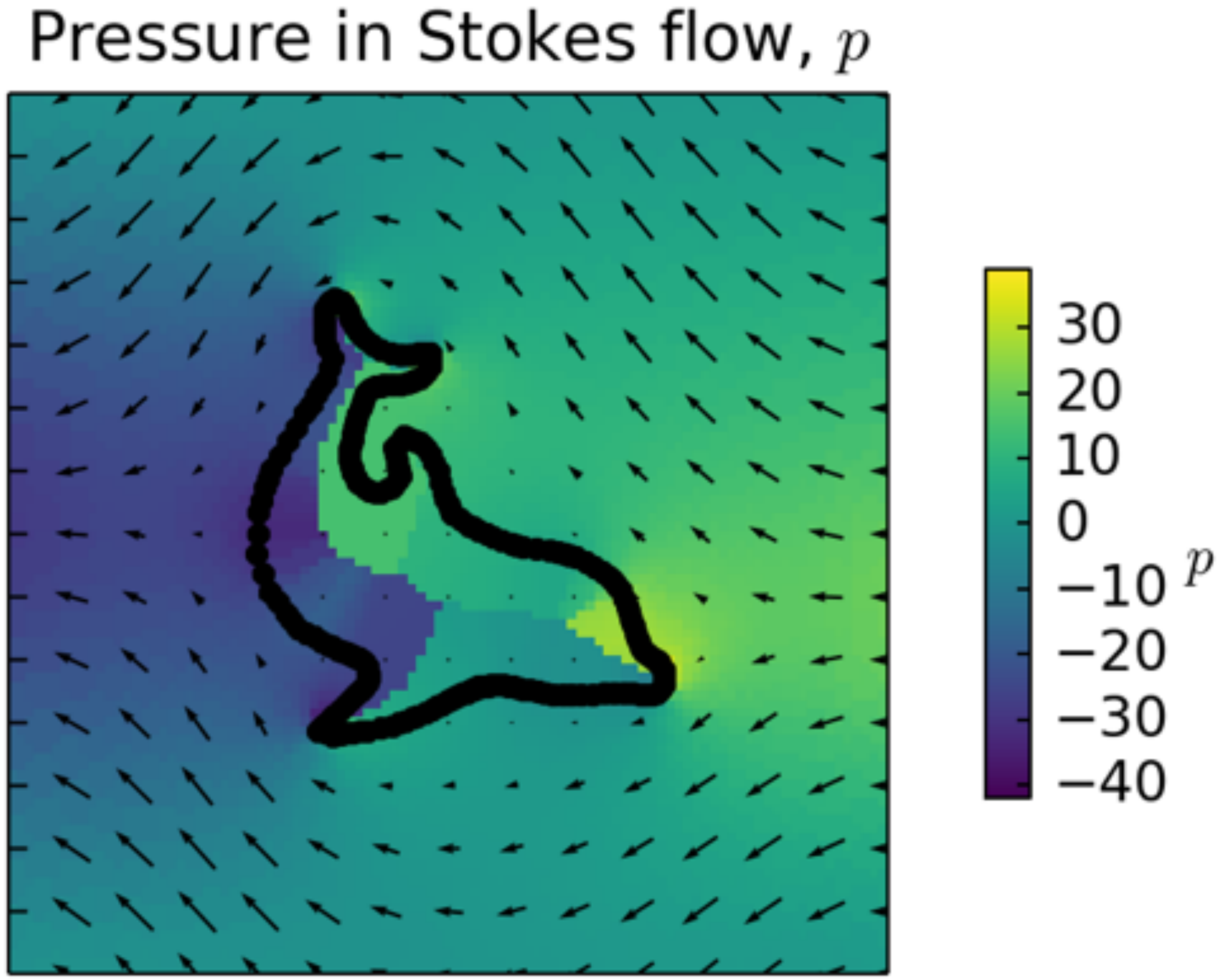


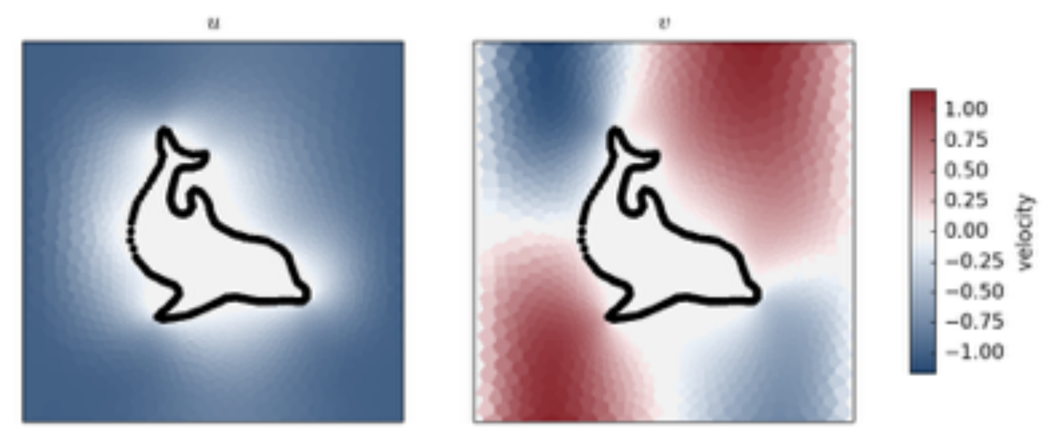


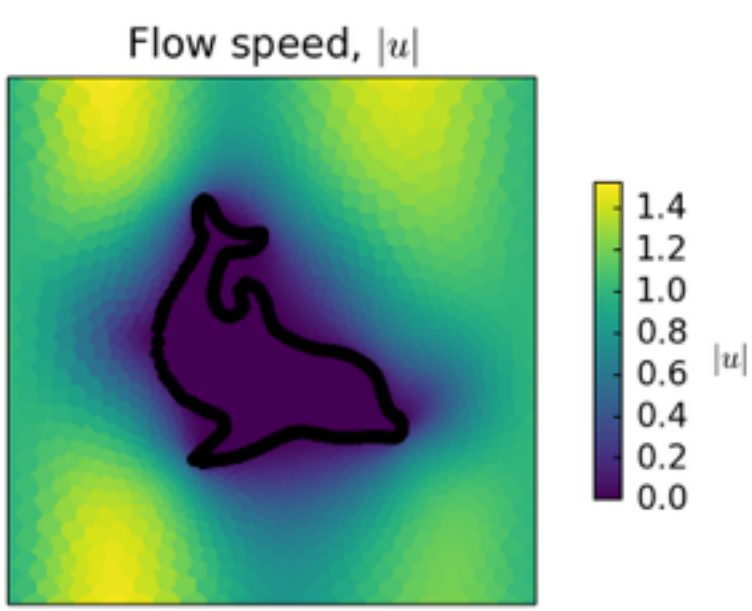


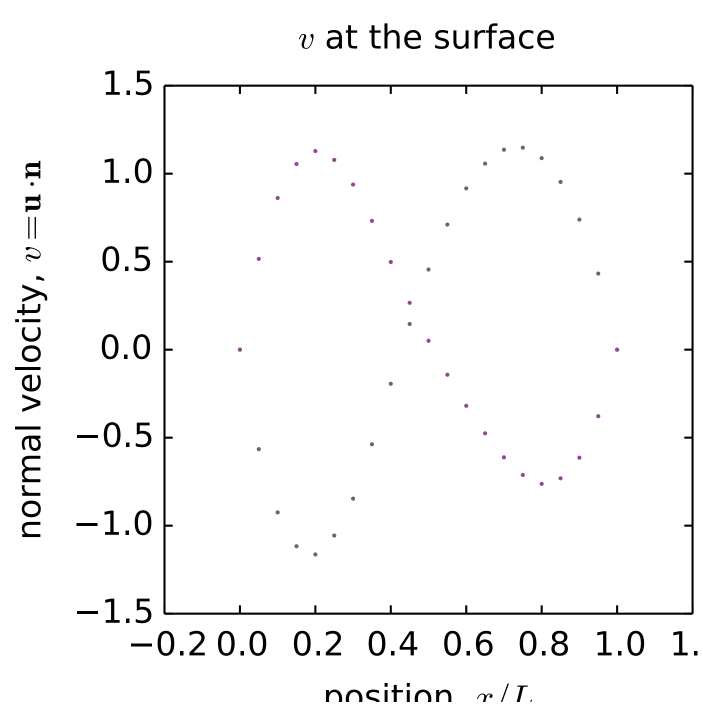


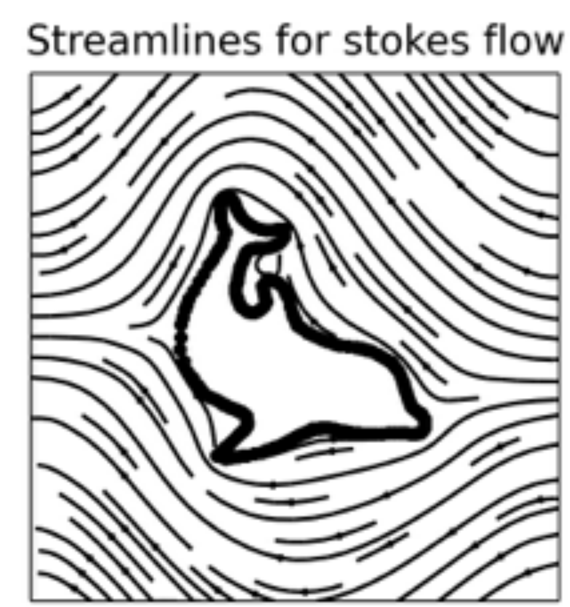
Generalizable to any geometry

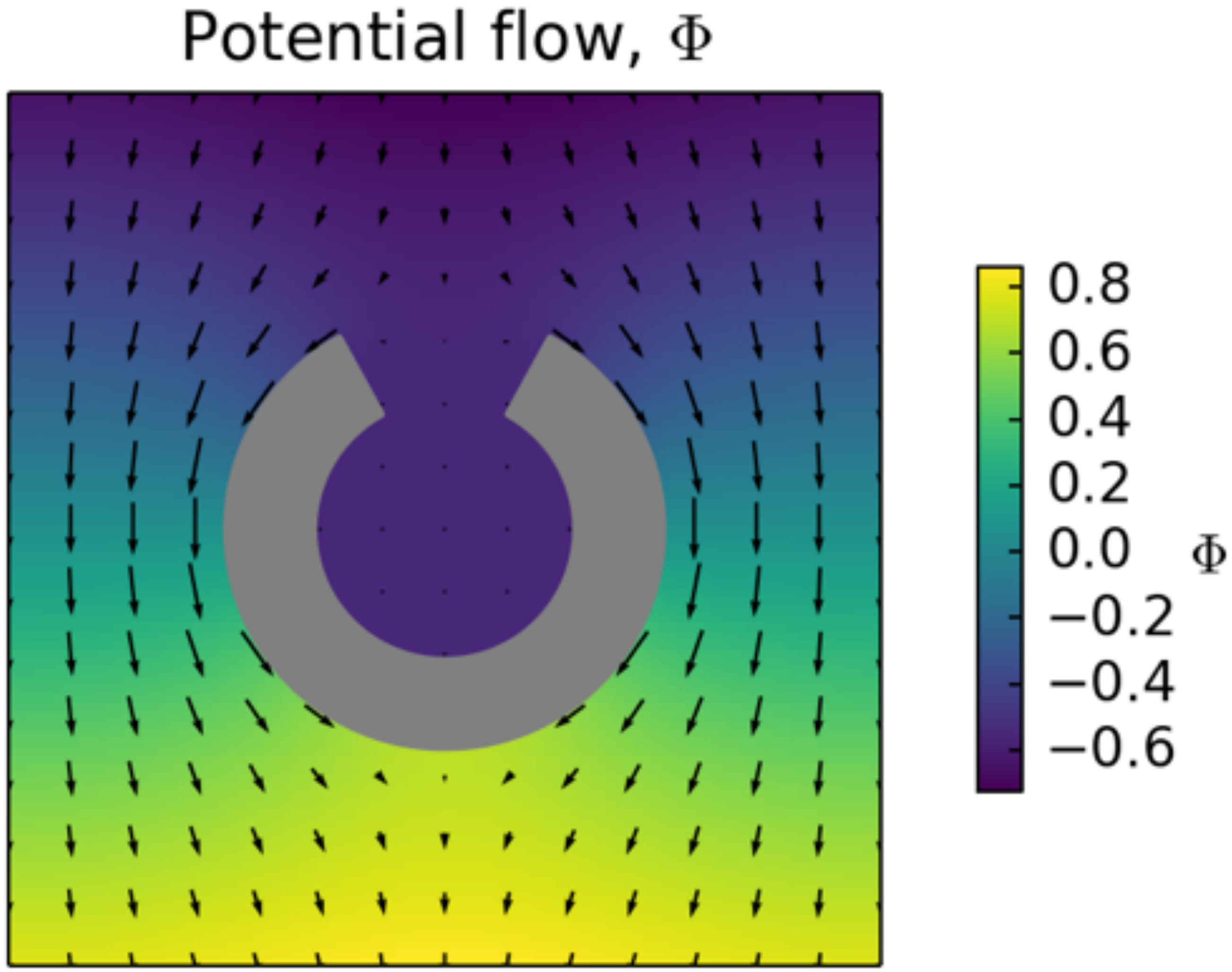












Can also solve for incompressible, irrotational case (potential flow)