$$A_0 = \int \vec{x} \times \vec{u}\, dV$$

$$= \underbrace{-\frac{1}{2}\int r^2 \vec{\omega}\, dV}_{\text{Lamb (1932)} \leftarrow \text{Angular impulse}} - \frac{1}{2}\int r^2 (\hat{n} \times \vec{u})\, dS$$

$$\|$$

$$= \overbrace{\underbrace{\frac{1}{3}\int \vec{x} \times (\vec{x} \times \vec{\omega})\, dV}_{\text{Batchelor (1967)} \nwarrow \text{Angular impulse}} + \underbrace{\frac{1}{6}\int r^2 \vec{x}\, (\vec{\omega} \cdot \hat{n})\, dS}_{A}} \underbrace{- \frac{1}{2}\int r^2 (\hat{n} \times \vec{u})\, dS}_{B}$$

Useful identity: $\int \hat{n} \times \vec{u}\, dS = \int \vec{\omega}\, dV = \int \vec{x}\,(\vec{\omega} \cdot \hat{n})\, dS$

In case of a spherical boundary ($r^2 = \text{const}$),

$$\therefore A_0 = \frac{1}{3}\int_V \vec{x} \times (\vec{x} \times \vec{\omega})\, dV - \frac{1}{3} r^2 \int \vec{x}\,(\vec{\omega} \cdot \hat{n})\, dS$$

$\therefore$ If V contains all vorticity, $\vec{\omega} \cdot \hat{n} = 0$ (on surface),

$$\vec{A}_0 = \underbrace{\int_V \vec{x} \times \vec{u}\, dV}_{\text{Angular momentum}} = \underbrace{\frac{1}{3}\int_V \vec{x} \times (\vec{x} \times \vec{\omega})\, dV}_{\text{Batchelor (1967)}} = \underbrace{-\frac{1}{2}\int_V r^2 \vec{\omega}\, dV}_{\text{Lamb (1932)}}$$

"these terms vanish at infinity" Saffman

- this isn't obvious as $u \sim \frac{|I|}{r^3}$; $\omega \sim \frac{|I|}{r^4}$!
- Only way this works is by requiring

① $\vec{\omega} \cdot \hat{n} = 0$

AND

② $\left(\hat{n} \times \vec{u} = 0 \text{ or } u \sim \frac{1}{r^6}\right)$

- For a spherical boundary and incompressibility,

$\sim_B = -\frac{1}{2}\int r^2 (\hat{n} \times \vec{u})\, dS = -\frac{1}{2}\int r^2 \vec{x}\,(\vec{\omega} \cdot \hat{n})\, dS \quad \because \int \hat{n} \times \vec{u}\, dS = \int \vec{\omega}\, dV = \int \vec{x}\,(\vec{\omega} \cdot \hat{n})\, dS$

$\Rightarrow \sim_A + \sim_B = -\frac{1}{3}\int r^2 \vec{x}\,(\vec{\omega} \cdot \hat{n})\, dS$