MIS 64018 - Assignment 3

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The Transportation Model

Say the decision variables are as follows - Xpw is the total number of AEDs shipped from plant (p) to warehouse (w)

So, (p = A,B) and (w = 1,2,3)

1 - Objective function

Z = 622(XA1) +614(XA2) +630(XA3) +641(XB1) +645(XB2) +649(XB3)

```
And the constraints are
```

XA1+XA2+XA3 <= 100 XB1+XB2+XB3 <= 120 XA1+XB1 = 80 XA2+XB2 = 60 XA3+XB3= 70 and Xpw >= 0

Use the lpSolveAPI to formulate this problem

```
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.2.1

library(lpSolveAPI)

## Warning: package 'lpSolveAPI' was built under R version 4.2.1
```

This linear programming problem has 5 constraints, 6 decision variables and the goal is to minimize the function

```
lprec <- make.lp(5,6)

# Set the objective function for the problem
set.objfn(lprec, c(622,614,630,641,645,649))

# set the direction towards minimum
lp.control(lprec, sense = "min")

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
```

```
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"       "dynamic"      "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##       epsb       epsd      epsel     epsint epsperturb   epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##    1e-11    1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"    "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"   "equilibrate" "integers"
```

```
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual"   "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Now make sure you add all the constraints to solve the problem

```
# Adding the constraint values to both the plants

# Set the production capacity constraints
set.row(lprec, 1, c(1,1,1), indices = c(1,2,3))
set.row(lprec, 2, c(1,1,1), indices = c(4,5,6))

# Set the warehouse demand constraints
set.row(lprec, 3, c(1,1), indices = c(1,4))
set.row(lprec, 4, c(1,1), indices = c(2,5))
set.row(lprec, 5, c(1,1), indices = c(3,6))

# Set the right hand side values ro the function
rhs <- c(100,120,80,60,70)
set.rhs(lprec, rhs)

# Set the constraint type
set.constr.type(lprec, c("<=","<=","=","=","="))
```

Make sure all the values are greater than 0 to minimize the problem correctly

```
# Add the boundary conditions to the decision variables
set.bounds(lprec, lower = rep(0, 6))

# Naming all the rows (constraints) and columns (decision variables) for the
problem

lp.rownames <- c("Plant A Capacity", "Plant B Capacity", "W1 Demand", "W2 Dem
and", "W3 Demand")
lp.colnames <- c("PlantA to W1", "PlantA to W2", "PlantA to W3", "PlantB to W
1", "PlantB to W2", "PlantB to W3")

dimnames(lprec) <- list(lp.rownames, lp.colnames)

# Now re-check the values of the problem
lprec
```

```
## Model name:
##                    PlantA to W1  PlantA to W2  PlantA to W3  PlantB to W1
PlantB to W2  PlantB to W3
## Minimize                    622           614           630           641
645           649
## Plant A Capacity              1             1             1             0
0             0  <=  100
## Plant B Capacity              0             0             0             1
1             1  <=  120
## W1 Demand                     1             0             0             1
0             0   =   80
## W2 Demand                     0             1             0             0
1             0   =   60
## W3 Demand                     0             0             1             0
0             1   =   70
## Kind                        Std           Std           Std           Std
Std           Std
## Type                       Real          Real          Real          Real
Real          Real
## Upper                       Inf           Inf           Inf           Inf
Inf           Inf
## Lower                         0             0             0             0
0             0
```

Now formulate the linear programming problem to find the optimal solution for both the plants A ans B. If the result says 0, then it the optimal solution.

```
# Solve the linear program
solve(lprec)

## [1] 0
```

The model returned a “0”, so it has found an optimal solution to the problem.

Now we fix a minimum value to the objective function

```
# The objective function value is
get.objective(lprec)

## [1] 132790
```

The minimum shipping and production costs is $132,790 with all the constraints fixed

Now we add the decision variables to find the number of units that can be produced and shipped from both plants A and B.

```r
# Get the optimum decision variable values
get.variables(lprec)

## [1]  0 60 40 80  0 30
```

Results

Plant A ships 0 units to Warehouse 1, Plant A ships 60 units to Warehouse 2, Plant A ships 40 units to Warehouse 3, Plant B ships 80 units to Warehouse 1, Plant B ships 0 units to Warehouse 2, Plant B ships 30 units to Warehouse 3.

This distribution can help minimize cost and maximize production of all the 210 units out of both the plants A and B.

2 - Formulating the dual for this problem

VA = Pi^d - Pi^0 Max VA = (80p1^d +60p2^{d+7op3}d)-(100p1$^{0-120p2}$0) Say for Plant A p1^d -p1^0 >=22 p2^d -p1^0 >=14 p3^d -p1^0 >=30

For Plant B p1^d -p2^o >=16 p2^d -p2^0 >=20 p3^d -p2^0 >=24

And for all non-negative variables we need pw^p >=0

3 - Economic interpretation

```r
# Switch the matrix to calculate the dual
costs <- matrix(c(622,614,630,0,
 641,645,649,0) , ncol=4 , byrow=TRUE)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)



lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
lptrans$duals

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

The economic interpretation of the dual problem is there is no solution. Here, we see that the shadow price is primal. The dual solution of this transportation model indicates that all the prices for the primal problem is “0”. This says that there might be no scope of increasing the profit or decreasing the cost of reallocating the resources in plant A and b.

Therefore, the feasible solution is that the marginal cost is the same as the marginal revenue.

```
MR = MC
And, the shadow price = 0 : No reallocation of resources.
```