CSET Mathematics: Test II

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May 16, 2024

Contents

1 Plane Euclidean Geometry

"Candidates demonstrate an understanding of the foundations of geometry as outlined in the California Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of geometry and its underlying structures. They demonstrate an understanding of **axiomatic systems** and different forms of **logical arguments**. Candidates understand, apply, and **prove theorems** relating to a variety of topics in **two- and three-dimensional geometry**, including **coordinate**, **synthetic**, **non-Euclidean**, and **transformational geometry**."

1.1 Parallel Postulate

Apply the ***Parallel Postulate*** *and its* ***implications*** *and justify its* ***equivalents*** *(e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)*

Parallel Postulate

"If a line segment intersects two straight lines forming two interior angles on the same side that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles." Wikipedia

implications

The *Parallel Postulate* is Euclid's 5th postulate:

"And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side(of itself whose sum is) less than two right-angles, then the two (other) straight-lines, being produced to infinity, meet on that side (of the original straight-line) that the(sum of the internal angles) is less than two right-angles (and do not meet on the other side"

The following is based on this AI generated answer.

1. Non-Euclidian Geometries
2. Reimannian geometry
3. General relativity
4. Toplology
5. Computer Science
6. Philosophy
7. Mathematical Rigor

equivalents

The following is based on this AI generated answer.

1.2 Angles

Demonstrate knowledge of ***complementary***, ***supplementary***, *and* ***vertical*** *angles.*

"In Euclidean geometry, an angle is the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle." Wikipedia

complementary angles The sum of two **complementary** angles is 90°.

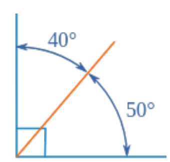


Figure 1: complementary angles

"The adjective complementary is from the Latin complementum, associated with the verb complere, "to fill up". An acute angle is "filled up" by its complement to form a right angle."

supplementary angles add up to 180°.

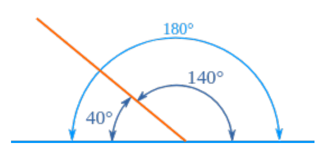


Figure 2: supplementary angles

vertical angles vertical anglesare formed by two intersecting lines.

1.3 Similarity and Congruence

*Prove theorems, justify steps, and solve problems involving **similarity** and **congruence**.*

1.3.1 Similarity

Geometric similarity is a relation between two objects. Objects that have the same **shape** are said to be similar.

triangle similarity Triangles are similar if:all their angles are equal and corresponding sides are in the same ratio. There are three ways to find if two triangles are similar:
AA, SAS and SSS:

AA AA stands for "angle, angle" and means that the triangles have two of their angles equal.

SAS SAS stands for "side, angle, side" and means that we have two triangles where the ratio between two sides is the same as the ratio between another two sides and we we also know the included angles are equal.

SSS SSS stands for "side, side, side" and means that we have two triangles with all three pairs of corresponding sides in the same ratio.

Similar Triangle Theorems

The following are several properties of similar triangles.

Side-splitter Theorem This theorem is about lines that split a triangle into proportional side lengths.

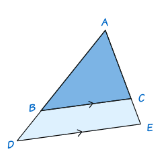


Figure 3: $BC \parallel DE \implies \frac{AB}{BD} = \frac{AC}{CE}$

Angle Bisector Theorem This theorem is about proportional sides and angle bisectors.

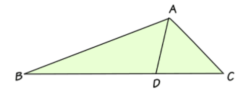


Figure 4: $\overline{AD} bisects \angle BAC \implies \frac{AB}{BD} = \frac{AC}{DC}$

1.3.2 Congruence

There are five ways to find if two triangles are congruent: SSS, SAS, ASA, AAS and HL.[1]

1.3.3 side, side, side(SSS)

SSS stands for "side, side, side" and means that we have two triangles with all three sides equal.

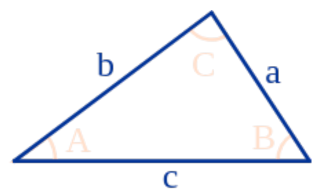


Figure 5: side, side, side

The **Law of Cosines** can be used to find angles.

1.4 properties of triangles

*Apply and justify properties of triangles (e.g., the **Exterior Angle Theorem**, **concurrence theorems**, **trigonometric ratios**, **triangle inequality**, **Law of Sines**, **Law of Cosines**, the **Pythagorean Theorem** and its **converse**)*

1.4.1 Exterior Angle Theorem

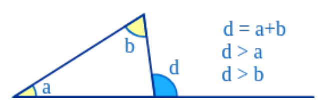


Figure 6: exterior angle theorem

[1] math is fun

1.4.2 concurrence theorems

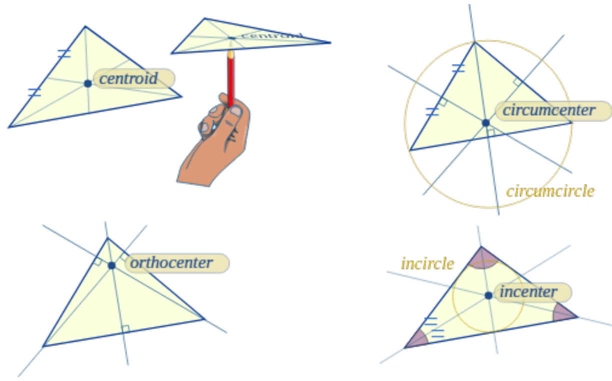
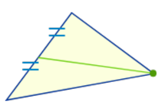
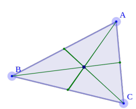


Figure 7: triangle centers

centroid Draw a line (called a "median") from each corner to the midpoint of the opposite side. Where all three lines intersect is the centroid, which is also the "center of mass":



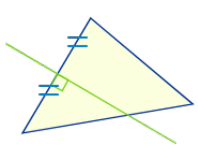
(a) median



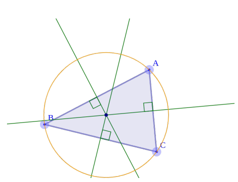
(b) centroid

Figure 8: median $\implies$ centroid

circumcenter Draw a line (called a "perpendicular bisector") at right angles to the midpoint of each side.Where all three lines intersect is the center of a triangle's "circumcircle", called the "circumcenter":



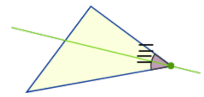
(a) perpendicular bisector



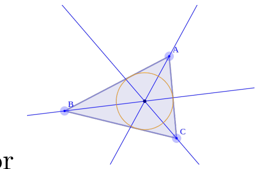
(b) circumcenter

Figure 9: perpendicular bisector $\implies$ circumcenter

incenter Draw a line (called the "angle bisector") from a corner so that it splits the angle in half Where all three lines intersect is the center of a triangle's "incircle", called the "incenter":



(a) angle bisector



(b) incenter

Figure 10: angle bisector $\implies$ incenter

1.4.3 trigonometric ratios

1.4.4 triangle inequality

1.4.5 Law of Sines

1.4.6 Law of Cosines

1.4.7 Pythagorean Theorem

1.4.8 Pythagorean Theorem (converse)

1.5 polygons and circles

Apply and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)

1.6 classical constructions

Identify and justify the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular polygons with 3, 4, 5, 6, and 8 sides)

[1]

2 Coordinate Geometry

2.1 Geometric Theorems

Use techniques in coordinate geometry to prove geometric theorems

[1](California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 7 [7.G]; Geometry, Grade 8; Congruence, High School [G-CO]; Similarity, Right Triangles, and Trigonometry, High School [G-SRT]; Circles, High School [G-C]; Geometric Measurement and Dimension, High School [G-GMD])

2.2 Real-World Problems

Model and solve mathematical and real-world problems by applying geometric concepts to two-dimensional figures

2.3 Conic Sections

Translate between the ***geometric description*** *and* ***the equation*** *for a conic section*

section	equation	geometric description
circle	$x^2 + y^2 = a^2$	the set of all point in a plane equidistant from a center point.
parabola	$y^2 = 4ax$	the locus of points in that plane that are equidistant from the directrix and the focus.
ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	for all points on the curve, the sum of the two distances to the focal points is a constant.
hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	for all points on the curve, the difference of the two distances to the focal points is a constant.

2.3.1 Cones

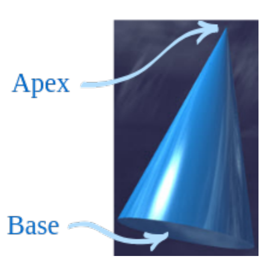
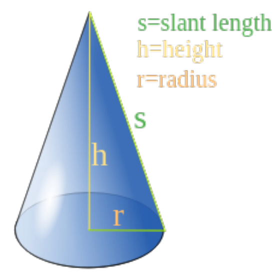


Figure 11: a cone has an apex and a base

1. a cone has a circle at one end and a point at the other
2. a cone can be made by rotating a triangle
3. surface area

$$SA_{cone} = \pi r(s + r)$$

$$s = \sqrt{h^2 + r^2}$$



2.3.2 Conic Sections

eccentricity The ratio of the distances from a curve to a focus and a curve to a directrix is called the **eccentricity**.

latus rectum The Latus Rectum is a line passing through the foci of a conic section and parallel to the directrix. I

2.3.3 Ellipse

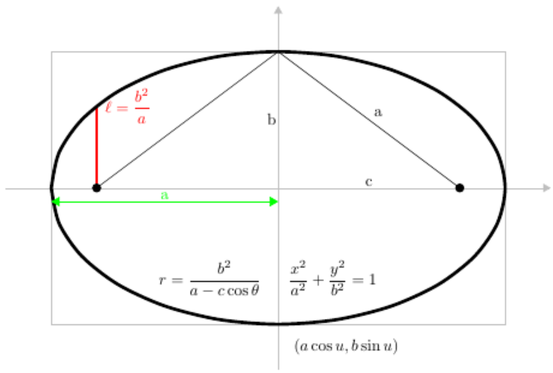
the set of all points (x,y) in a plane the **sum** of whose distances from two distinct points(the foci) is constant.

eccentricity

$$0 < eccentricity < 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

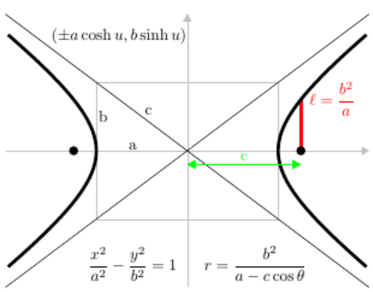
circle The circle is a special case of an ellipse. if $eccentricity = 0$, the ellipse is a circle.



2.3.4 Hyperbola

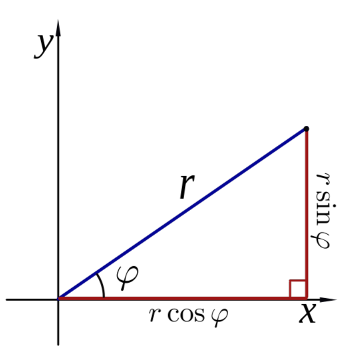
hyperbola - the set of all points (x,y) in a plane the **difference** of whose distances from two distinct points is constant

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



2.4 Polar Coordinates

*Translate between **rectangular** and **polar coordinates** and apply polar coordinates and vectors in the plane.*



$$x = r\cos\varphi y = r\sin\varphi$$

[2]

3 Three-Dimensional Geometry

3.1 Lines and Planes

Demonstrate knowledge of the relationships between lines and planes in three dimensions (e.g., parallel, perpendicular, skew, coplanar lines)

3.2 Formulae

Apply and justify properties of three-dimensional objects (e.g., the volume and surface area formulas for prisms, pyramids, cones, cylinders, spheres)

3.2.1 the volume and surface area formulas

prisms A **prism** is a solid object with identical ends, flat faces, and the same cross section all along its length. The ends of a prism are $\|$ and each end is called a base. The side faces of a prism are parallelograms (4-sided shapes with opposite sides parallel)

Surface Area The surface area of a prism encompasses the surface area of both bases and all the faces.

[2] (California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 8; Expressing Geometric Properties with Equations, High School [G-GPE]; GeometricMeasurement and Dimension, High School [G-GMD]; Modeling with Geometry, High School [G-MG]; Polar Coordinates and Curves, High School)

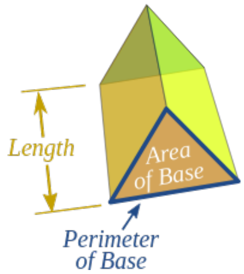


Figure 12: Surface Area = 2 × Base Area + Base Perimeter × Length

Volume The Volume of a prism is the area of one end times the length of the prism:

$$V_{prism} = Base_{area} \times h$$

pyramids A pyramid is made by connecting a base to an apex

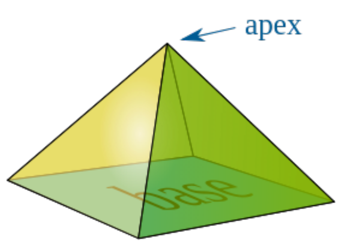


Figure 13: A pyramid is made by connecting a base to an apex

pyramid volume $V_{pyramid} = \frac{1}{3} Base_{area} \times height$

cones

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

cylinders A *cylinder* is not a polyhedron.A cylinder is like a prism with an infinite number of sides

volume of a cylinder $V_{cylinder} = \pi r^2 h$

spheres Of all the shapes, a sphere has the smallest surface area for a volume.

volume of a sphere $V_{sphere} = \frac{4}{3}\pi r^3$

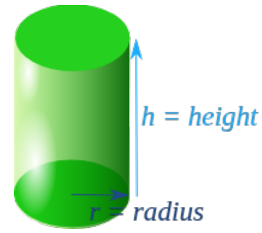


Figure 14: A pyramid is made by connecting a base to an apex

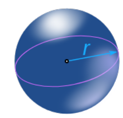


Figure 15: Of all the shapes, a sphere has the smallest surface area for a volume.

3.3 Real-World Problems

Model and solve mathematical and real-world problems by applying geometric concepts to three-dimensional figures

[3]

4 Transformational Geometry

4.1 Isometries

Demonstrate knowledge of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection), including their basic properties in relation to congruence

Isometries

an isometry is a rigid transformation.[2]

[3] (California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Congruence, High School [G-CO]; Similarity, Right Triangles, and Trigonometry, High School [G-SRT]; Geometric Measurement and Dimension, High School [G-GMD]; Modeling with Geometry, High School [G-MG])

[2] text

4.1.1 rotation

4.1.2 translation

4.1.3 reflection

4.2 Dilations

Demonstrate knowledge of dilations (e.g., similarity transformations or change in scale factor), including their basic properties in relation to similarity, volume, and area

notes[4]

5 Probability

Abstract

Probability is the branch of mathematics that studies the possible outcomes of given events together with the outcomes' relative likelihoods and distributions. In common usage, the word "probability" is used to mean the chance that a particular event (or set of events) will occur expressed on a linear scale from 0 (impossibility) to 1 (certainty), also expressed as a percentage between 0 and 100%. The analysis of events governed by probability is called statistics.

5.1 Basic Principles

Prove and apply basic principles of permutations and combinations

Permutations and Combinations

"A Permutation is an ordered Combination."

5.1.1 permutations

1. a selection of objects with regard for order

2. ${}_nP_k = \frac{n!}{(n-k)!}$

[4](California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Geometry, Grade 8; Congruence, High School [G-CO])

5.1.2 combinations

1. a selection of objects without regard for order

2. ${}_nC_k = \frac{n!}{k!(n-k)!} = \frac{{}_nP_k}{k!}$

	permutations	combinations
repeats	n^r	$\frac{(r+n-1)!}{r!(n-r)!}$

5.2 Finite Probability

Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles, sample space)

fundamental counting principle If there are p ways to do one thing and q ways to do another thing, then there are $p \times q$ ways to do both things. This is known as the *fundamental counting principle*(fcp). The fcp only works if the choices are independent of each other.

sample space All the possible outcomes of an experiment is known as the **sample space**.

5.3 Conditional Probability

Use and explain the concepts of conditional probability and independence

5.4 Uniform Probability

Compute and interpret the probability of an outcome, including the probabilities of compound events in a uniform probability model

5.4.1 uniform probability

A *uniform probability model* is a type of probability model where every outcome has an equal probability of occurring.

5.5 Distributions

Use normal, binomial, and exponential distributions to solve and interpret probability problems

5.6 Expected Values

Calculate expected values and use them to solve problems and evaluate outcomes of decisions

notes[5]

6 Statistics

6.1 Central Tendency

Compute and interpret the mean and median of both discrete and continuous distributions

"Discrete data can be counted,
Continuous data can be measured"

(Arithmetic) Mean

Median

Mode

6.2 Spread

Compute and interpret quartiles, range, interquartile range, and standard deviation of both discrete and continuous distributions

quartiles Quartiles are the values that divide a list of numbers into quarters.

$$Q2 = median$$

range The **range** is the difference between the lowest and highest values.

interquartile range

standard deviation The Standard Deviation is a measure of how spread out (distributed) numbers are. The standard deviation is the square-root of the **variance**.

[5](California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Statistics and Probability, Grade 7 [7.SP]; Conditional Probability and the Rules of Probability, High School [S-CP]; Using Probability to Make Decisions, High School [S-MD])

6.3 Sampling Methods

Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results

random

systematic This is where we follow some system of selection like "every *nth* person"

cluster We break the population into many groups, then randomly choose whole groups.

convenience sampling

6.4 Linear Regression

Apply the method of least squares to linear regression

6.5 chi-square test

Apply the chi-square test

6.6 scatter plots

f. Interpret scatter plots for bivariate data to investigate patterns of association between two quantities (e.g., correlation), including the use of linear models

6.7 Data

Interpret data on a single count or measurement variable presented in a variety of formats (e.g., dot plots, histograms, box plots)

6.8 Hypothesis Testing

Demonstrate knowledge of P-values and hypothesis testing

Khan

6.9 Confidence Intervals

Demonstrate knowledge of confidence intervals

notes[6]

[6](California Common Core Content Standards for Mathematics, including Standards for Mathematical Practice 1–8: Statistics and Probability, Grade 8; Interpreting Categorical and Quantitative Data, High School [S-ID])