Astr 513 – Statistical Method in (Astro)physics – Spring 2016

Homework #1
Due Date: Friday February 12, 10am
by email (as pdf) to dpsaltis@email.arizona.edu or at my mailbox in SO

You are encouraged to work in groups of up to 3 people on this assignment. Please turn in only one set of answers with all names of people involved. If you received some help or ideas from other people (students in class, other faculty members, or colleagues) please credit their contribution appropriately.

1. Random Number Generators

In class, we talked about many variants of random number generators and explored their properties in terms of their *(i)* period, *(ii)* clustering, *(iii)* efficiency, and *(iv)* portability. In your computer language and/or algorithm library of choice, choose the random generator that you will be using for the rest of the class. Search the literature for articles that explore the above properties of your generator and summarize your findings in a couple of paragraphs. Make sure to include references and, if you find it necessary, figures.

2. Designing surveys

The Kepler mission has been observing a very large number of stars in a small patch in the sky and is making a very reliable measurement of the occurrence rate of planets around solar type stars (see Batalha, N. M. 2014, Proceedings of the National Academy of Science, 111, 12647; arXiv:1409.1904). For the purposes of this homework problem, we will assume that the occurrence rate of planets with radii between one and two Earth radii has been measured to a very high accuracy and it is equal to 10% (i.e., 10% of solar type stars harbor such planets; the actual rate is consistent with this number but has some considerable uncertainty).

You are designing a survey of solar type stars in a different patch of the sky to find the same type of planets. How many stars would you need to observe in order to have a 90% likelihood that you will find at least 30 planets with radii between one and two Earth radii? What if you want to have a 99% likelihood?

3. Blackbody distribution

The energy distribution of the number of photons that follow the blackbody distribution is given by

$$f(\epsilon;T)d\epsilon = C\frac{\epsilon^2 d\epsilon}{\exp(\epsilon/kT)-1}\,, \tag{1}$$

where ϵ is the photon energy, C is a normalization constant, T is the temperature of the distribution and k is the Boltzmann constant. (Note that this is the distribution of the number of photons and not of the radiation energy density, which is the expression that you are probably more familiar with). Our goal is to generate an ensemble of photons with energies drawn from this distribution and in a range (ϵ_1, ϵ_2), using the rejection method.

(i) Start by making a change of variables

$$\epsilon' = \frac{\epsilon}{kT} \tag{2}$$

in order to remove any parameters in the distribution.

(ii) Find the energy ϵ'_0 for which this distribution has a maximum. You will need to do this numerically.

(iii) Normalize your blackbody distribution such that its maximum value is equal to unity, i.e., consider the distribution

$$f'(\epsilon)d\epsilon = \frac{C}{f(\epsilon'_0)} \frac{\epsilon'^2 d\epsilon'}{\exp(\epsilon') - 1} . \tag{3}$$

(iv) Use your random number generator to draw a random number between $\epsilon'_1 = 0.1$ and $\epsilon'_2 = 5$. This will be the energy ϵ' of your photon.

(v) Use your random number generator to draw a second random number between 0 and 1. If this number is larger than $f'(\epsilon')$ then reject this photon, otherwise accept it and repeat the last two steps.

(vi) Plot the distribution you just generated and compare it to the blackbody distribution to verify your result. What fraction of your initial photon energies was acceptable (i.e., how efficient was your rejection algorithm)?.