Astr 513: Homework 3

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1 Linear models of data with correlated uncertainties

The definition for Δ_i will not change when $\rho_i \neq 0$, as it does not depend on the uncertainties. So plugging in for $\vec{u}^\top$, $\vec{Z}_i$, and $\cos\theta$, we have

$$\Delta_i = \frac{1}{\sqrt{1+b^2}} \left(\begin{bmatrix} -b & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} - a \right), \tag{1}$$

and simplifying and squaring gives

$$\Delta_i^2 = \frac{(y_i - a - bx_i)^2}{1+b^2}. \tag{2}$$

However, Σ_i^2 is defined with the covariance matrix, which will change when the uncertainties are correlated. We have that $\Sigma_i^2 = \vec{u}^\top \mathbf{S_i} \vec{u}$, and the covariance matrix for correlated uncertainties with $\sigma_{xyi} = \rho_i \sigma_{xi} \sigma_{yi} \neq 0$ is given as, for the data point (x_i, y_i),

$$\mathbf{S_i} = \begin{bmatrix} \sigma_{xi}^2 & \rho_i \sigma_{xi} \sigma_{yi} \\ \rho_i \sigma_{xi} \sigma_{yi} & \sigma_{yi}^2 \end{bmatrix}. \tag{3}$$

Thus, we find

$$\begin{aligned} \Sigma_i^2 &= \frac{1}{1+b^2} \begin{bmatrix} -b & 1 \end{bmatrix} \begin{bmatrix} \sigma_{xi}^2 & \rho_i \sigma_{xi} \sigma_{yi} \\ \rho_i \sigma_{xi} \sigma_{yi} & \sigma_{yi}^2 \end{bmatrix} \begin{bmatrix} -b \\ 1 \end{bmatrix} \\ &= \frac{1}{1+b^2} \begin{bmatrix} \sigma_{xi}\left(-b\sigma_{xi} + \rho_i \sigma_{yi}\right) & \sigma_{yi}\left(-b\rho_i \sigma_{xi} + \sigma_{yi}\right) \end{bmatrix} \begin{bmatrix} -b \\ 1 \end{bmatrix} \\ &= \frac{1}{1+b^2} \left(b^2 \sigma_{xi}^2 - b\rho_i \sigma_{xi}\sigma_{yi} - b\rho_i \sigma_{xi}\sigma_{yi} + \sigma_{yi}^2 \right) \\ \Rightarrow \Sigma_i^2 &= \frac{\sigma_{yi}^2 - 2b\rho_i \sigma_{xi}\sigma_{yi} + b^2 \sigma_{xi}^2}{1+b^2}. \end{aligned} \tag{4}$$

When we then take $\Delta_i^2 / 2\Sigma_i^2$, we will see the factor of $1+b^2$ cancel from both terms, and are left with

$$\boxed{\frac{\Delta_i^2}{2\Sigma_i^2} = \frac{(y_i - a - bx_i)^2}{2\left(\sigma_{yi}^2 - 2b\rho_i \sigma_{xi}\sigma_{yi} + b^2 \sigma_{xi}^2\right)}} \tag{5}$$

2 The Tully-Fisher relation

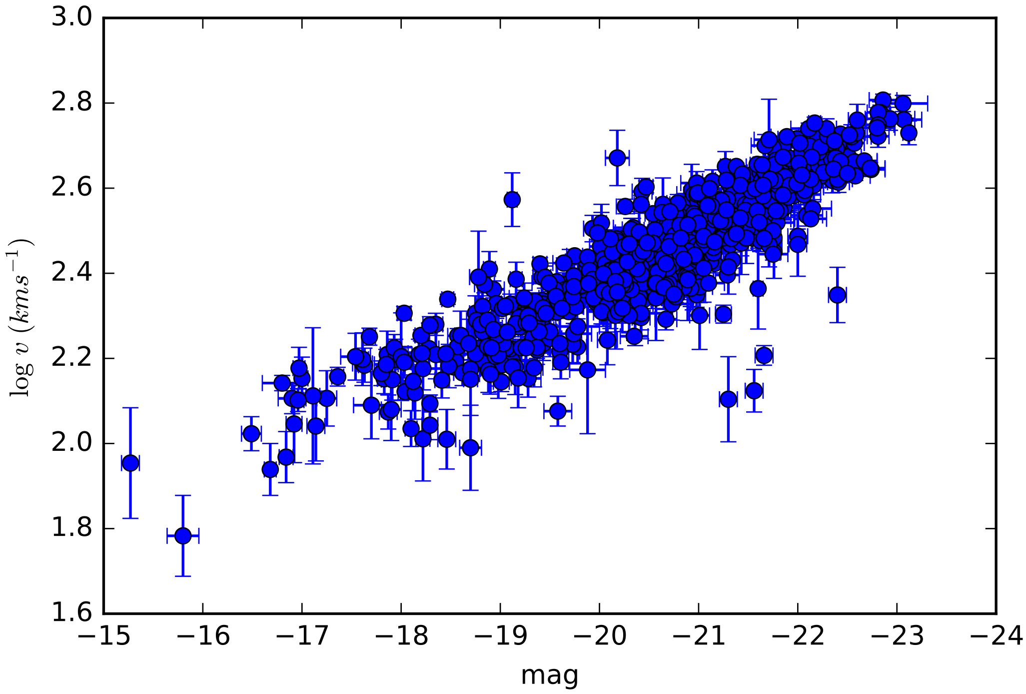

Figure 1: The magnitude and angular velocity form a power law.

We took our catalog and maximized the likelihood given in Equation 5. Our raw data is shown in Figure 1, and the best fit is shown in Figure 2.

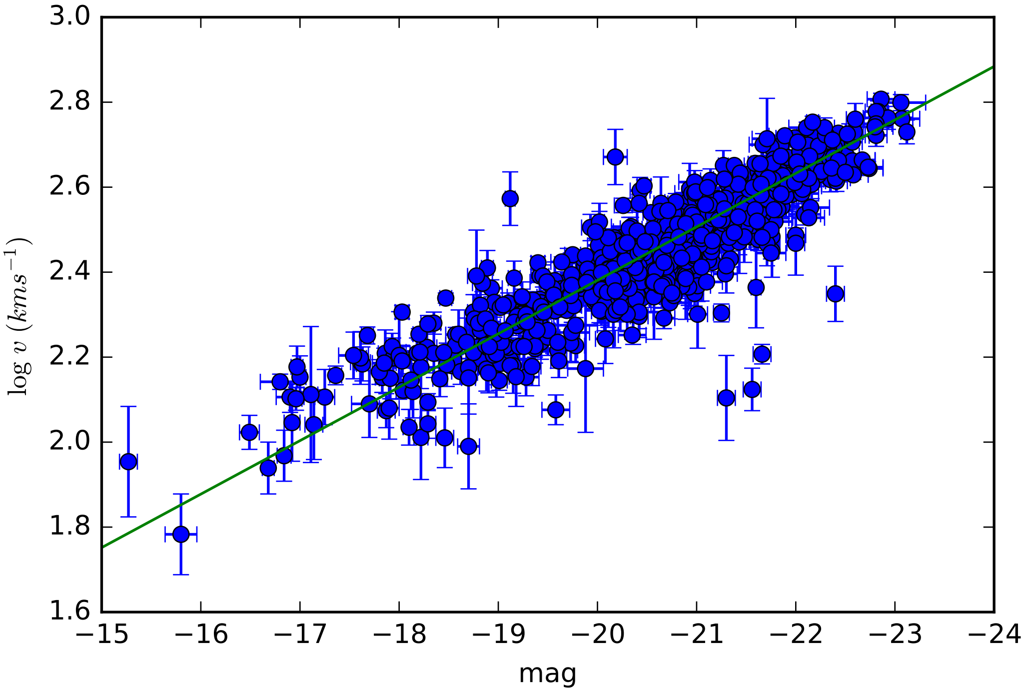

Figure 2: The magnitude and angular velocity form a power law. The best fit Tully-Fisher relation is overplotted with the points from the galaxy catalog.