Ryden - Chapter 2 Solutions

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1   Exercise 1

We want to calculate the mean free path l of an arrow fired in a forest, where the trees have radius R and the average number of trees per unit area is Σ. Thinking about scaling, if the trees were more tightly packed, then l should decrease, so then l should be inversely proportional to Σ. Similarly, if the trees are larger the arrow will not travel as far, so l should be inversely proportional to R. Thus, the solution is $l = (R\Sigma)^{-1}$. Given that $R = 1$ m and $\Sigma = 0.005$ m^{-2}, this means $l = 200$ m. *Note* this answer isn't perfect, because the trees protrude forward toward the path of the arrow, but computing this correction is tricky because it involves an integral over the impact parameter b of the arrow with respect to the tree it hits. To a first approximation, the true answer is somewhere between 199 m and 200 m.

2   Exercise 2

This problem is identical to Exercise 1 but in three dimensions. The number density of stars is given by n_*, and the average stellar radius is R_*, meaning the cross-sectional area is $\sigma = 4\pi R_*^2$. The mean free path is again inversely proportional to n_* and σ, so $l = (\sigma n_*)^{-1}$. Given that $n_* = 10^9$ Mpc^{-3} and $R_* = 7 \times 10^8$ m, this means $l = 2.1 \times 10^{17}$ Mpc.

Similarly, if galaxies are spaced according to $n_g = 1$ Mpc^{-3} and $R_g = 2000$ pc, then the mean free path to hit a galaxy is $l_g = 20000$ Mpc. We see that a photon is fairly likely to pass through a galaxy, but not necessarily hit a star in that galaxy.