Ryden - Chapter 2 Solutions

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1 Exercise 1

We want to calculate the mean free path l of an arrow fired in a forest, where the trees have radius R and the average number of trees per unit area is Σ. Thinking about scaling, if the trees were more tightly packed, then l should decrease, so then l should be inversely proportional to Σ. Similarly, if the trees are larger the arrow will not travel as far, so l should be inversely proportional to R. Thus, the solution is $l = (R\Sigma)^{-1}$. Given that $R = 1$ m and $\Sigma = 0.005$ m^{-2}, this means $l = 200$ m. *Note* this answer isn't perfect, because the trees protrude forward toward the path of the arrow, but computing this correction is tricky because it involves an integral over the impact parameter b of the arrow with respect to the tree it hits. To a first approximation, the true answer is somewhere between 199 m and 200 m.

2 Exercise 2

This problem is identical to Exercise 1 but in three dimensions. The number density of stars is given by n_*, and the average stellar radius is R_*, meaning the cross-sectional area is $\sigma = 4\pi R_*^2$. The mean free path is again inversely proportional to n_* and σ, so $l = (\sigma n_*)^{-1}$. Given that $n_* = 10^9$ Mpc^{-3} and $R_* = 7 \times 10^8$ m, this means $l = 2.1 \times 10^{17}$ Mpc.

Similarly, if galaxies are spaced according to $n_g = 1$ Mpc^{-3} and $R_g = 2000$ pc, then the mean free path to hit a galaxy is $l_g = 20000$ Mpc. We see that a photon is fairly likely to pass through a galaxy, but not necessarily hit a star in that galaxy.

3 Exercise 3

Let us pretend our bodies are entirely made of water and we absorb all CMB photons that pass through us. Let us further assume that we are a nice, round 100 kg, and also a nice, spherical 1 m radius sphere, so that our volume is $V = 4\pi/3$ m^3 and our surface area is $A = 4\pi$ m^2. We know from the text that the number density of CMB photons is $n_\gamma = 4.11 \times 10^8$ m^{-3}, all of which are traveling at the speed of light $c \approx 3 \times 10^8$ m/s. Since we are sitting in a photon bath, the number of photons that hit our spherical body per unit time is given by $N = cn_\gamma A = 1.55 \times 10^{18}$ s^{-1}. Each of these photons is carrying an energy of 6.34×10^{-4} eV, or about 1.016×10^{-22} J. This means the power delivered by the CMB to our bodies is $P = NE_\gamma = 1.57$ J/s. If our heat capacity is $C = 4200$ J kg^{-1} K^{-1}, then we can calculate the temperature change per second as $\mathrm{d}T/\mathrm{d}t = P/mC = 3.75 \times 10^{-10}$ K s^{-1}, meaning we would heat up by 1 nanoKelvin in about $t = 2.67$ s.

4 Exercise 4

Given differences between the square of the masses, Δm_{12}^2 and Δm_{23}^2, we want to find the values of m_1, m_2 and m_3 that minimize the sum $m_1 + m_2 + m_3$, where all masses have to be positive.

TM: Finish this.