CSCE 222: Discrete Structures for Computing
Section 502 & 503
Fall 2020

Dr. McGuire

September 7, 2020

Problem Set 1

Due: 13 September 2020 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`).

Problem 1. (20 points)
You have n coins, exactly one of which is counterfeit. You know counterfeit coins weigh more than authentic coins. Devise an algorithm for finding the counterfeit coin using a balance scale[1]. Express your algorithm in pseudocode. For $n = 12$, how many weighings does your algorithm use?

Problem 2. (20 points)
Devise an algorithm that takes as input a list of n integers in unsorted order, where the integers are not necessarily distinct, and outputs the location (index of first element) and length of the longest contiguous non-decreasing subsequence in the list. If there is a tie, it outputs the location of the first such subsequence. Express your algorithm in pseudocode. For the list $9, 7, 9, 4, 5, 8, 1, 0, 5, 9$, what is the algorithm's output? How many comparison operations between elements of the list are used?

Problem 3. (20 points)
Arrange the following functions in order such that each function is big-O of the next function: $2 \cdot 3^n$, $3n!$, $2019 \log n$, $\frac{n^3}{10^6}$, $n \log n$, $\sqrt{n}$, $3 \cdot 2^n$. Prove your answer is correct by giving the witnesses for each pair of consecutive functions.

Problem 4. (20 points)
For each of the following functions, give a big-O estimate, including witnesses, using a simple function $g(n)$ of the smallest order:

1. $(n^2 + 8)(n + 1)$
2. $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$
3. $n^{2^n} + n^{n^2}$
4. $\frac{n^4 + 5 \log n}{n^3 + 1}$
5. $2n^4 + 7n^3 + 5n + 3$

[1] A balance scale compares the weight of objects placed on it. The result of the comparison is either left side heavier, right side heavier, or both sides equal.

Problem 5. Attached is a Java program which will sort an array of doubles into non-decreasing order. The data comes from the attached file `numbers.txt` Modify the code so that every time a comparison of two doubles is performed, the static variable `count` is incremented. Do **not** count comparisons of integer values, only doubles.

a. Run the program with input sizes of 10, 100, 1000, and 10000. Use this data to create a table showing number of comparisons for the the best case, worst case, and average case for each of the input sizes.
b. For the best case, give a function $b(n)$ which gives the number of comparisons made as a function of the input size.
c. For the worst case, give a function $w(n)$ giving the number of comparisons as a function of the input size.
d. How many comparisons would be necessary in the best and worst cases for an input size of 1,000,000?

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist:
1. Did you abide by the Aggie Honor Code?
2. Did you solve all problems and start a new page for each?
3. Did you submit the PDF to eCampus?