

CSCE 222: Discrete Structures for Computing
Section 502 & 503
Fall 2020

YOUR NAME HERE

Homework 6

Due: 1 November (Sunday) before 11:59 p.m. on Gradescope.
You must show your work in order to receive credit.

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?

Problem 1.

Suppose that you have analyzed a benchmark that runs on your company's processor. This processor runs at 2.4 GHz and has the following characteristics:

Instruction Type	Frequency (%)	Cycles
Arithmetic and logical	40	1
Load and Store	30	8
Branches	20	2
Floating Point	10	4

- a. What is the average CPI (cycles per instruction) of this processor running this benchmark?
- b. Another (virtually meaningless, but common) measure of performance is MIPS (for "millions of instructions per second.") What is the MIPS rating of this processor on this benchmark?
- c. What is the MIPS rating of this processor on this benchmark?
- d. If a real-time function must be computed in 5 milliseconds or less, to how many instructions should you limit that function if the distribution of instructions is the same as in the above table?

Problem 2.

- a. You have a bag with 100 blue marbles and 100 red marbles. You close your eyes and grab five marbles at random. What is the probability that at least 3 of them will be blue?
- b. You have a bag with 100 blue marbles and 100 red marbles. One by one you draw a marble out of the bag, note its color, and put it back in the bag. You do this 5 times. What is the probability that at least 3 of the marbles are blue?
- c. You have two nonahedral (9-sided) dice. Let X be the random variable equal to the sum of the number that appear when a pair of these dice is rolled.
 - (a) How many outcomes are there for the event of rolling these two dice?
 - (b) List the range of the random variable X .
 - (c) What is the probability of each of the values of the random variable X (assuming that the dice are fair dice)?

Problem 3. Family Issues

Bonnie has three children, Michael, Brooke, and Meagan. Brooke has two daughters named Charlotte and Jaina, and Michael is the father of Zaid and Rose.

- a. Write down the relation $R = \{(a, b) \mid a \text{ is a parent of } b\}$ defined on the set P of the eight people, so that it reflects the family structure specified above. Use the set notation, the digraph notation, and the matrix notation.
- b. Now let $S = \{(a, b) \mid a \text{ is a sibling of } b\}$ Write down the relation S using the set notation, the digraph notation, and the matrix notation.
- c. For the relations R and S , indicate which of the following properties they hold: reflexive, symmetric, antisymmetric, asymmetric, transitive.

Problem 4. Equivalence Relations

- a. How many different equivalence relations can we define on the set $A = \{1, 2, 3, 4\}$?
- b. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

Problem 5.

Solve the following recurrence relations:

- a. $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$, where $a_0 = 2, a_1 = 7, a_2 = 21$
- b. $a_n = 4a_{n-1} - 5a_{n-2} + 2a_{n-3}$, where $a_0 = 4, a_1 = 4, a_2 = 5$
- c. $a_n = a_{n-1} + n$, where $a_0 = 0$