

CSCE 222: Discrete Structures for Computing
Section YOUR SECTION NUMBER HERE
Fall 2020

YOUR NAME HERE

Assigned 25 August 2020

Problem Set 1

Due: 6 September 2020 (Sunday) before 11:59 p.m. on eCampus (ecampus.tamu.edu).
You must show your work in order to receive credit.

Problem 1. (20 points)

Exercise 62 of Section 1.4 (page 60). Let $P(x)$, $Q(x)$, and $R(x)$ be the statements " x is a clear explanation", " x is satisfactory", and " x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, and $R(x)$.

- a) All clear explanations are satisfactory.
- b) Some excuses are unsatisfactory.
- c) Some excuses are not clear explanations.
- d) Does (c) follow from (a) and (b)?

Problem 2. (20 points)

Exercise 36 of Section 1.5 (page 71). Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

- a) No one has lost more than one thousand dollars playing the lottery.
- b) There is a student in this class who has chatted with exactly one other student.
- c) No student in this class has sent e-mail to exactly two other students in this class.
- d) Some student has solved every problem in this book.
- e) No student has solved at least one exercise in every section of this book.

Problem 3. (20 points)

Exercise 34 of Section 1.6 (page 84). The Logic Problem, taken from *WFF"N PROOF, The Game of Logic*, has these two assumptions:

- 1. "Logic is difficult or not many students like logic."
- 2. "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

- a) That mathematics is not easy, if many students like logic.
- b) That not many students like logic, if mathematics is not easy.
- c) That mathematics is not easy or logic is difficult.
- d) That logic is not difficult or mathematics is not easy,

e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

Problem 4. (20 points)

Exercises 20 and 32 of Section 1.7 (pages 95 & 96).

20) Prove that if n is an integer and $3n + 2$ is even, then n is even using

a) a proof by contraposition.

b) a proof by contradiction.

32) Show that these three statements are equivalent: (i) a is less than b , (ii) the average of a and b is greater than a , and (iii) the average of a and b is less than b .

Problem 5. (20 points)

Exercises 4, 6, 8, and 10 of Section 1.8 (page 113).

4) Prove that there are no positive perfect cubes less than 1000 that are the sum of the cubes of two positive integers.

6) Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever a , b , and c are real numbers.

8) Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.

10) Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or non-constructive?

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist:

1. Did you abide by the Aggie Honor Code?
2. Did you solve all problems and start a new page for each?
3. Did you show your work clearly?
4. Did you submit the PDF to eCampus?