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ENGIN 435 – Antenna Design

Project #3 – Arrays

For the third project of the semester, we students were tasked with designing an electrically steered antenna array for an airborne radar application at X-band. The design requirements specified the antenna array needed to consist of half-wavelength dipole antenna elements and these elements must be operating at a frequency of 9.5GHz, which is within the X-band frequency range. This antenna array's radiation pattern must be oriented in a pencil beam, so both principal planes consist of the same beamwidth, and this radiation pattern must meet a 40dBi directivity over the bandwidth when the beam is at broadside ($\theta_o = 90°$). This requirement means that the radiation pattern of the array should be 40dB greater than an isotropic radiator, which has a directivity of 1 or 0dB. Since this an electrically steered antenna array, the radiation pattern should also still meet these requirements when the beam scan angle is up to $45°$ off broadside. Additionally, a bonus requirement was presented such that the maximum sidelobe level (SLL) be 30dB below the main beam of the array. For this project, I chose to design a 10x10 antenna array that consisted of a Dolph-Tschebyscheff amplitude tapering to meet the -30dB SLL and my design process is laid out in the following paragraphs.

To start of the project, I used an electromagnetic design and simulating software called FEKO to establish the single element antenna structure of my half-wavelength dipole. Since we are operating the antenna array at 9.5GHz, the dipole needed to be built at a length equal to $\frac{\lambda}{2} = 0.0158m$. I was able to construct the single element antenna and export its directivity data from FEKO into MATLAB, where I was able to plot the radiation pattern of the half-wavelength dipole. The plot for the directivity of the dipole can be seen Figure 1 below.

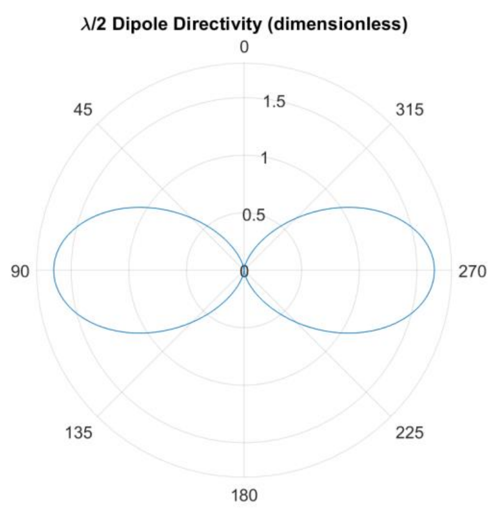

Figure 1 - Directivity of Half-Wavelength Dipole at 9.5GHz.

The expected value for the directivity of a half-wavelength dipole is about 1.643 or 2.156dB [1].

The plot of Figure 1 shows the max directivity at 90° and is about 1.65. MATLAB was giving me some trouble converting this plot into decibels, so I only managed to plot the dimensionless values for the directivity of the dipole. Luckily, the values I got from the exported FEKO simulation data lined up with expected values for a half-wavelength dipole. It should be noted that the design asks for a pencil beam radiation pattern, which is a property of a 2D array (planar array), of which would consist of all half-wavelength dipoles. If the antenna is an array of identical elements, the total field can be obtained by pattern multiplication similar to a linear array [1]. This is because of the duality of the Fourier transform, where in the time domain the antenna's radiation is convolving with the array factor (AF), but in the frequency domain, the radiation pattern is being multiplied by this AF pattern. Thus, we can derive the total field of the 2D array by the expression:

$$\boldsymbol{E(total) = [E(single\ element\ at\ reference\ point)] * [array\ factor]} \quad \text{(Eq 6-5)}$$

where the array factor is a function of the geometry of the array and the excitation phases [1]. Since I decided to strive for a Dolph-Tschebyscheff array, the amplitude excitation coefficients for the array factor would not be uniformly distributed. My process for deriving the amplitude coefficients and the array factor for the 10x10 planar array is explained in the next paragraph.

The process for deriving the array factor with a Dolph-Tschebyscheff amplitude tapering is similar to designing a broadside Dolph-Tschebyscheff array of 10 elements. We are striving for a major-to-minor lobe ratio of 30dB with a spacing *d* between the half-wavelength elements. Referring to table 6.7 in the textbook, the spacing for a nonuniform Dolph-Tschebyscheff linear array is

$$dmax \le \frac{\lambda}{\pi} cos^{-1}\left(-\frac{1}{z_0}\right)$$

where z_o is the value where the nth-order Tschebyscheff polynomial will equal the desired SLL of Ro, that is $T_n(z_o) = R_o$ [1]. To solve for these values, I determined that the 9^{th}-order Tschebyscheff polynomial was needed to solve for the R_o value of 30dB. Converting the voltage ratio, R_o, to a dimensionless value, I got a value of 31.6228. I then used this along with the nth-order polynomial value to solve for z_o. Using the equation (6-73) from the textbook, I calculated a z_o value of 1.108 [1]. The remainder of the process involved a substitution of equation (6-61b), which shows the AF for an even element array, so that $cos(u) = \frac{z}{z_0}$ into my AF [1]. So, my 10-element array factor expression looked like,

$$(AF)_{10} = a_1\, cos(u) + a_2\, cos(u) + a_3\, cos(u) + a_4\, cos(u) + a_5\, cos(u)$$

where the next step of the process after the cos(u) substitution required me to expand each cosine term according to equation (6-69) and then equate each order of the polynomial to the 9^{th} order Tschebyscheff polynomial [1]. Doing so, I acquired the coefficient values of $a_1 = 9.776$, $a_2 = 8.584$, $a_3 = 6.542$, $a_4 = 4.303$, and $a_5 = 2.518$. Since I am using a planar array which consists of a linear Dolph-Tschebyscheff array in the x and y-directions, the amplitudes for the 2D array would simply be the multiplication of each row vs column location in the grid of antennas. One of the features of the Dolph-Tschebyscheff array is that the amplitude coefficients in center of the array have the values of $2a_1$ and thus my values in each x and y array were arranged as: $a_5, a_4, a_3, a_2, a_1, a_1, a_2, a_3, a_4, a_5$. This would distribute the nonuniform excitations from the lowest values at the edges with the center values having the highest. For the spacing of the array, I solved for the d_{max} value after obtaining my z_o value and calculated a spacing around 6λ/7 which is about 27.1mm between each element in the array. Now that I had my spacing of the elements and my amplitude coefficients, I then calculated the array factors for the x and y arrays using the equation:

$$AF = S_{xm}S_{yn} \qquad \text{(Eq 6-88)}$$

where,

$$S_{xm} = \sum_{n=1}^{10} Ime^{j(m-1)kdx\,sin\,\theta\,cos\,\phi+\beta_x} \qquad \text{(Eq 6-88a)}$$

$$S_{yn} = \sum_{n=1}^{10} Ine^{j(n-1)kd_y\,sin\,\theta\,cos\,\phi+\beta_y} \qquad \text{(Eq 6-88b)}$$

which the array factor for the planar array can be expressed as. Here, the constants within the summation for each iteration would then be my amplitude coefficients solved from before. To achieve the pencil beam requirement for the project, it is required that the conical main beams of S_{xm} and S_{yn} intersect and their maxima be directed toward the same direction [1]. To only have one main beam directed along $\theta = \theta_0$ and $\phi = \phi_0$, the progressive phase shifts, $\beta_x\ and\ \beta_y$, between the elements in the x and y-directions must be equal to

$$\beta x = -kd_x\,sin\,\theta_0\,cos\,\phi_0 \qquad \text{(Eq 6-93a)}$$

$$\beta_y = -kd_y\,sin\,\theta_0\,sin\,\phi_0 \qquad \text{(Eq 6-93b)}$$

and these phase excitations would achieve the pencil beam pattern for the project design requirement [1]. So with all this information, I started to formulate all the equations and calculations needed to plot the array factor with the Dolph-Tschebyscheff amplitude tapering and was able to plot the AF and the total field directivities after the pattern multiplication, which can be seen in Figure 2 below.

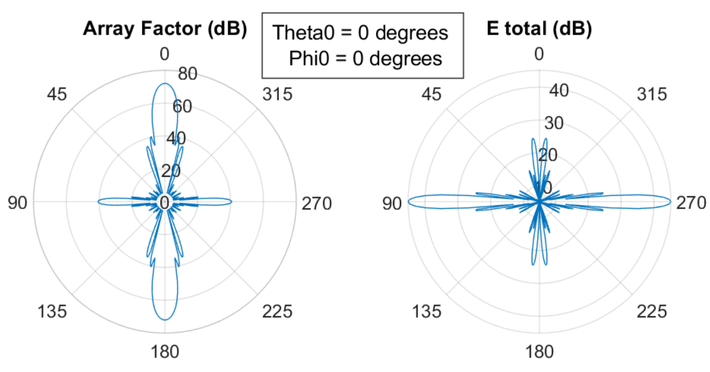

Figure 2 - Array Factor and Total field patterns for the half-wavelength dipole centered at 9.5GHz

With $\theta_0 = 0°$ and $\phi_0 = 0°$, the total field pattern can be seen to show a value of 45dBi off broadside with a SLL about 20dB below the main beam. I then plotted the array factor and total field when the beam scan angle was equal to broadside, so $\theta_0 = 90°$ and $\phi_0 = 90°$, which can be seen in Figure 3 below.

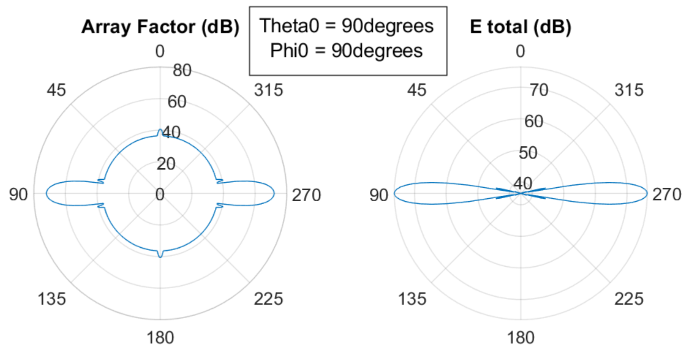

Figure 3 - Array factor and Total Field pattern for half-wavelength dipole at 9.5GHz with beam scan set to broadside

Here with the beam scan angle set to broadside for both phase excitations, the main beam has a value of 75dBi and SLL of 44dBi, which is 30dB below the main beam. This satisfies the 40dBi at broadside requirement and meets the SLL requirement for the Dolph-Tschebyscheff array. When allowing the beam scan angle to scan 45° off broadside, the total field pattern showed a similar pattern, Figures 4a and 4b below show the beam scan angles off broadside.

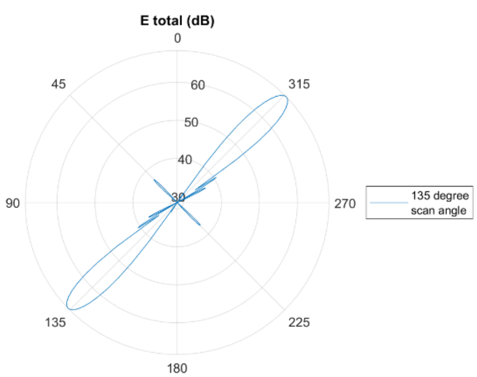

Figure 4a - Total Field Directivity pattern with beam scan angle 45 degrees above broadside

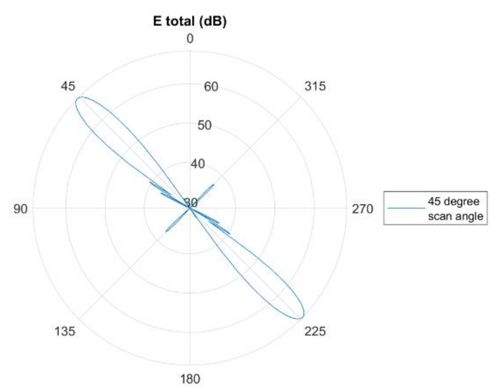

Figure 4b - Total Field pattern with beam scan angle 45 degrees below broadside

With the beam scan angle 45° above and below broadside, the main beam has a directivity of 68dBi, with the sidelobe at 36dBi, which still meets the 30dB SLL requirement. To verify that this total directivity radiation pattern is in fact a pencil beam, the beamwidths in both principal planes should be equivalent.

This means that the half power beamwidths Θ_h and ψ_h, which represents the elevation plane half-power beamwidth and the half-power beamwidth in the plane this is perpendicular to the $\phi = \phi_0$, should be the same value. Solving for Θ_h required looking at Fig 6-12 in the textbook and finding the HPBW for a linear broadside array at an array length of 8.57, based on the number of elements and spacing I chose [1]. I acquired a HPBW of about 6.5° and needed to multiply this by the beam broadening factor. Using (Eq 6-78) and referencing Fig 6-25a from the textbook, I derived a beam broadening factor of about 1.14, thus resulting in a value of $7.436°$ for the HPBW[1]. Since I am using a square array, ψ_h was then equal to this calculated value [1]. To solve for Θ_h, I used (Eq 6-98) and acquired a value of $7.436°$ whenever $\theta_0 = \phi_0$. Thus, whatever value the beam scan angle was set to, both the half power beamwidths Θ_h and ψ_h were equal, meaning the total radiation pattern was a pencil beam. The grating lobes of the AF are also impacted by the bandwidth. So, I plotted the AF at 10% above and below the center frequency of 9.5GHz in addition to beam steering and can be seen in Figures 5a and 5b below.

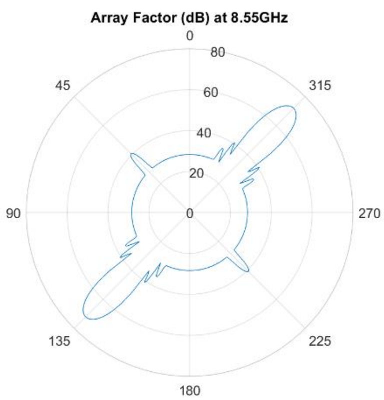

Figure 5a - AF directivity at 10% below center frequency with beam scan angle of 45 degrees below broadside

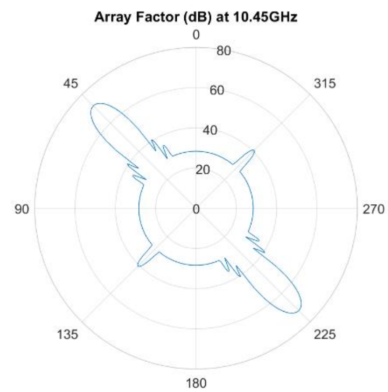

Figure 5b - AF directivity at 10% above center frequency with beam scan angle 45 degrees above broadside.

Checking how the grating lobes of the AF were affected at the edges of the bandwidth, I found that the sidelobes were still 30dB below the main lobe, which was about 72dBi.

Lastly, the aperture size for the antenna array needed to be calculated. Since this is a grid of antennas, the aperture value would then be described as the area over which the elements are distributed within the grid of the 2D array. Since I used M = 10 elements in the x-direction and N = 10 elements in the y-direction, and the spacing between elements was 6λ/7, the aperture could then be calculated by the multiplication of each total length in the x-direction and y-direction. So, $L_x = Md_x$ and $L_y = Nd_y$, which equaled to 0.3043m for each length in the x-direction and y-direction. The total area was then calculated as $(0.3043\text{m})^2$, which came out to be an area of 0.093m^2 or about 1.001ft^2. So, the total area for this planar array with Dolph-Tschebyscheff amplitude tapering was about 1 square foot. This concludes the design process for an electrically steered antenna array radiating at X-band. All the requirements for the project were met because of the design process. I feel as though I could have extended the length of the report since deriving the amplitude coefficients for a Dolph-Tschebyscheff required a couple more steps that I chose to quickly sum up due to length restrictions. However, my design process and calculations can also be examined in more detail by referencing the MATLAB code in the next section.

References

[1] Balanis, C. A. (2016). *Antenna theory: Analysis and design* (pp. 176-360). Hoboken, NJ: Wiley.

MATLAB Code

```
%% 10x10 Dolph-Tshcebyscheff planar array
% specify # of elements, spacing, order of polynomial, Ro, aperture size, and
% amplitude coefficients
clear all
close all

M = 10;                                 %# of elements in x-direction
N = 10;                                 %# of elements in y-direction
c = 3e8;                                %speed of light
f = 9.5e9;                              %center frequency
lambda = c/f;                           %wavelength
k = (2*pi)/lambda;                      %wavenumber
dx = lambda/(8/7);                      %spacing in x-direction
dy = lambda/(8/7);                      %spacing in y-direction
theta = (-180:180)*pi/180;              %full range of theta values
phi = (-180:180)*pi/180;                %full range of phi values
theta0 = 0*(pi/180);                    %direction of theta
phi0 = 0*(pi/180);                      %direction of phi
Bx = -k*dx.*sin(theta0).*cos(phi0);     %x-directed phase excitation
By = -k*dy.*sin(theta0).*sin(phi0);     %y-directed phase excitation
Psi_x = (k*dx.*sin(theta).*cos(phi) + Bx);
Psi_y = (k*dy.*sin(theta).*sin(phi) + By);
BW = .1*f;                              %bandwidth
FBW = BW/f;                             %fractional bandwidth
Q = 1/FBW;                              %Q-factor
Lx = (M-1)*dx;                          %Length of x-array
Ly = (N-1)*dy;                          %Length of y-array
arrayLx = (Lx + dx)/lambda;             %Array x length in wavelengths
arrayLy = (Ly + dy)/lambda;             %Array y length in wavelengths
Aperture = Lx*Ly;

Ro = 10^(30/20); %Voltage ratio Ro for SLL = -30dB
P = N - 1; % P = # of elements - 1 for P order of Tschebyscheff polynomial
zo = (1/2)*((Ro + sqrt((Ro)^2 - 1))^(1/P) + (Ro - sqrt((Ro)^2 - 1))^(1/P));
dmax = (lambda/pi)*acos(-1/zo);         %dmax for non uniform Dolph-Tscheby
% Solve for amplitude coefficients of both x and y coefficients
a5 = zo^9;
a4 = ((-576*zo^7) + 576*a5)/64;
a3 = ((432*zo^5) - 432*a5 + 112*a4)/16;
a2 = ((-120*zo^3) + 120*a5 - 56*a4 + 20*a3)/4;
a1 = 9*zo - 9*a5 + 7*a4 - 5*a3 + 3*a2;

Im = [a5, a4, a3, a2, a1, a1, a2, a3, a4, a5];
In = [a5, a4, a3, a2, a1, a1, a2, a3, a4, a5];

fields_x = zeros(N, length(theta));     % x-fields
fields_y = zeros(N, length(theta));     % y-fields
fields_x = exp(1i*((1:M).').*Psi_x);    % x-fields uniform
fields_y = exp(1i*((1:N).').*Psi_y);    % y-fields uniform
fields_X = Im'.*fields_x;               % x-fields with DT amplitude tapering
fields_Y = In'.*fields_y;               % y-fields with DT amplitude tapering
Sxm = sum(fields_X,1);                  % sum x-fields into one array
Syn = sum(fields_Y,1);                  % sum y-fields into one array
AF = Sxm.*Syn;

%Determine beamwidth in both principal planes
Broadfactor = 1 + 0.636*((2/Ro)*cosh(sqrt((acosh(Ro))^2 - pi^2)))^2;
thetaXo = 6.5;                          % Determined from Fig 6.12 in CH6
thetaXo = thetaXo*Broadfactor;          % For DT array, need to multiple HPBW by broad factor
```

```
thetaYo = thetaXo;                      % For squqre array, thetaXo = thetaYo
u = (1/2)*(1+cos(2*theta0));            % trig identity for cos^2(theta0)
w = (1/2)*(1+cos(2*phi0));              % trig identity for cos^2(phi0)
y = (1/2)*(1-cos(2*phi0));              % trig identity for sin^2(phi0)
z = (1/2)*(1-cos(2*theta0));            % trig identity for sin^2(theta0)
Theta_HPBW = sqrt(1/((u*((thetaXo^-2)*w + (thetaYo^-2)*y))));   % Compute theta HPBW for DT array
Psi_HPBW = sqrt(1/((thetaXo^-2)*y + (thetaYo^-2)*w));           % Compute psi HPBW for DT array
beamSolidAngle = Theta_HPBW*Psi_HPBW;                            % Beam solid angle for Pencil
Beam

%Import HalfWave Dipole Field info and mulitply by Dolph-Tschebyscheff AF
degree2rad = (pi/180);

dipoleData = importdata('Half_Wave_Dipole_Directivity.dat');
dipoleData = dipoleData.data;
theta1 = dipoleData(:,1)';
theta1 = theta1*degree2rad;
dipole = dipoleData(:,2)';

figure(1)
polarplot(theta1, dipole)
set(gca,'ThetaZeroLocation','top')
thetaticks([ 0 45 90 135 180 225 270 315])      %angle label locations - must be positive and
increasing
%thetaticklabels({ '0' '-45' '-90' '-135' '180' '135' '90' '45'})
title('\lambda/2 Dipole Directivity (dimensionless)')

E_total = dipole.*AF;
E_dB = 20*log10(abs(E_total));
AFdB = 20*log10(abs(AF));
pat_min = max(E_dB)-40;

figure(2)
polarplot(theta, max(AFdB, pat_min));
%rlim([pat_min, max(AFdB)])
set(gca,'ThetaZeroLocation','top')  %radius limits for plot
title('Array Factor (dB)')
thetaticks([ 0 45 90 135 180 225 270 315])      %angle label locations - must be positive and
increasing
%thetaticklabels({ '0' '-45' '-90' '-135' '180' '135' '90' '45'})

figure(3)
polarplot(theta, max(E_dB, pat_min));
rlim([pat_min, max(E_dB)])
set(gca,'ThetaZeroLocation','top')
thetaticks([ 0 45 90 135 180 225 270 315])      %angle label locations - must be positive and
increasing
%thetaticklabels({ '0' '-45' '-90' '-135' '180' '135' '90' '45'})
title('E total (dB)')
```