$$\boxed{A_e = \varepsilon_{ap} A_p} \qquad 0 \le \varepsilon_{ap} \le 1 \tag{9-79}$$

Aperture efficiency ε_{ap} is a measure of how efficiently the antenna physical area is utilized. If ε_{ap} is known, it is a simple matter to calculate the gain of an aperture antenna of aperture area A_p using (9-78).

There are several contributions to the overall aperture efficiency. The following form shows the factors separately and is appropriate for general use:

$$\boxed{\varepsilon_{ap} = e_r \varepsilon_t \varepsilon_s \varepsilon_a} \tag{9-80}$$

All these factors have values from zero to unity. We discussed radiation efficiency e_r in Sec. 2.5; it represents all forms of dissipation on the antenna structure such as conductor losses. In most aperture antennas, these losses are very low, so $e_r \approx 1$ and

$$G \approx D \qquad \textit{most aperture antennas} \tag{9-81}$$

This may not hold if one of the following situations applies: The antenna size is less than a wavelength, a lossy transmission line or device is considered to be part of the antenna, or lossy materials are an integral part of the antenna such as a dielectric lens.

Aperture taper efficiency ε_t represents gain loss strictly due to the aperture amplitude distribution. Often, the amplitude is tapered from the center to the edges of an aperture intentionally to reduce sidelobes. ε_t is the ratio of directivity computed with only the amplitude taper present, D_t, to the directivity of the same aperture uniformly illuminated, D_u:

$$\varepsilon_t = \frac{D_t}{D_u} \quad \text{or} \quad D_t = \varepsilon_t D_u \tag{9-82}$$

Examples for line sources are given in Table 5-2. Also, in Example 9-4 we found $\varepsilon_t = 0.81$ for an open-ended waveguide.

Antennas that have a secondary radiating aperture illuminated by a primary (feed) antenna, such as a parabolic reflector, experience spillover loss due to power from the feed missing the radiating aperture. This *spillover efficiency* ε_s and aperture taper efficiency are the main sources of gain loss in most aperture antennas. The product $\varepsilon_t\varepsilon_s$ is called the *illumination efficiency* ε_i.

The remaining factor in (9-80), ε_a, is *achievement efficiency* and can include many subefficiencies. More subefficiencies will be treated with reflector antennas in Sec. 9.5, but the following two are usually dominant:

$$\varepsilon_a \approx \varepsilon_{cr}\varepsilon_{ph} \tag{9-83}$$

Cross-polarization efficiency, ε_{cr}, represents loss due to power being radiated in a polarization state orthogonal to the intended polarization. *Phase efficiency*, ε_{ph}, represents loss due to nonuniform phase across the aperture.

Any of the efficiency factors can be expressed as a gain factor in decibels as

$$\varepsilon_n(\text{dB}) = 10 \log \varepsilon_n \tag{9-84}$$

Gain "loss" is negative of this. For example, the aperture taper efficiency for Example 9-4 is $\varepsilon_t = 0.81$, so $\varepsilon_t(\text{dB}) = -0.91$ dB and the gain loss is $+0.91$ dB. This is the only source of loss in this case. In general, (9-78) and (9-80) can be written in dB form as

$$G(\text{dB}) = 10 \log\left(\frac{4\pi}{\lambda^2}A_p\right) + e_r(\text{dB}) + \varepsilon_t(\text{dB}) + \varepsilon_s(\text{dB}) + \varepsilon_a(\text{dB}) \tag{9-85}$$

Recall that polarization mismatch factor p and impedance mismatch factor q are not included in aperture efficiency nor gain, but they play a role similar to the efficiency factors (as discussed in Sec. 4.4).

9.3.3 Simple Directivity Formulas

It is often necessary to estimate the gain of an antenna, especially in system calculations. If the gain cannot be measured, simple gain equations can be used. The most direct and simplest approach is to use (9-78). The operating wavelength and physical aperture area are easily obtained. Aperture efficiency can sometimes be determined by using a theoretical model, as will be discussed for horns and reflectors later in this chapter. In many cases, it can be estimated. In general ε_{ap} ranges from 30% to 80% with 50% being a good overall value. Optimum gain pyramidal horns have an aperture efficiency near 50%. Parabolic reflector antennas have an efficiency of 55% or greater. Gain can be found by estimating the aperture efficiency. For example, a 30-dB gain antenna with an actual efficiency of 55% will have a gain error of 0.38 dB when an estimated efficiency of 60% is used.

It is very useful to have an approximate directivity expression that depends only on the half-power beamwidths of the principal plane patterns. This is expected to yield good results since we know that directivity varies inversely with the beam solid angle ($D = 4\pi/\Omega_A$) and the beam solid angle is primarily controlled by the main beam. Thus, we expect to find that $D \propto (\mathrm{HP}_E \mathrm{HP}_H)^{-1}$, where the product of the principal plane beamwidths approximates the beam solid angle. We now derive such relations.

The topic of directivity and gain estimation was introduced in Sec. 4.5.3, and the full development along with references is found in [3]. The directivity of a rectangular aperture with a separable distribution given by (9-74) for broadside operation ($\theta_o = 0$) is

$$D = \pi D_x D_y \tag{9-86}$$

where D_x and D_y are the directivities of a line source (or linear array) associated with the x and y aperture distribution variations. But we know from studying several linear current distributions that these directivities are related to the aperture extents as

$$D_x = c_x \frac{2L_x}{\lambda}, \qquad D_y = c_y \frac{2L_y}{\lambda} \tag{9-87}$$

where directivity factors c_x and c_y are constants that vary slightly with the distributions $E_{a1}(x)$ and $E_{a2}(y)$. For uniform line sources, $c_x = c_y = 1$; see (5-19). Using (9-87) in (9-86) and rearranging give

$$D = \pi c_x \frac{2L_x}{\lambda} c_y \frac{2L_y}{\lambda} = \frac{4\pi c_x c_y k_x k_y}{\left(k_x \frac{\lambda}{L_x}\right)\left(k_y \frac{\lambda}{L_y}\right)} = \frac{4\pi c_x c_y k_x k_y}{\mathrm{HP}_x \mathrm{HP}_y} \tag{9-88}$$

The beamwidth factors k_x and k_y are constants associated with the following beamwidth formulas that we have used frequently (see Table 5-2):

$$\mathrm{HP}_x = k_x \frac{\lambda}{L_x}, \qquad \mathrm{HP}_y = k_y \frac{\lambda}{L_y} \tag{9-89}$$

For uniform line sources, $k_x = k_y = 0.886$. The numerator in (9-88) is the *directivity-beamwidth product*:

$$\mathrm{DB} = 4\pi c_x c_y k_x k_y \cdot \left(\frac{180}{\pi}\right)^2 \quad [\deg^2] \tag{9-90}$$