C h a p t e r T e n

Noise and Nonlinear Distortion

The effect of noise is critical to the performance of most RF and microwave communications, radar, and remote sensing systems because noise ultimately determines the threshold for the minimum signal that can be reliably detected by a receiver. Noise power in a receiver will be introduced from the external environment through the receiving antenna, as well as generated internally by the receiver circuitry. Here we will study the sources of noise in RF and microwave systems, and the characterization of components in terms of noise temperature and noise figure, including the effect of impedance mismatch. The additional noise-related topics of transistor amplifier noise figure, oscillator phase noise, and antenna noise temperature will be discussed in later chapters.

We will also discuss the related topics of compression, harmonic distortion, intermodulation distortion, and dynamic range. These can have important limiting effects when large signal levels are present in mixers, amplifiers, and other components that use nonlinear devices such as diodes and transistors.

10.1 NOISE IN MICROWAVE CIRCUITS

Noise power is a result of random processes such as the flow of charges or holes in an electron tube or solid-state device, propagation through the ionosphere or other ionized gas, or, most basic of all, the thermal vibrations in any component at a temperature above absolute zero. Noise can be passed into a microwave system from external sources, or generated within the system itself. In either case the noise level of a system sets the lower limit on the strength of a signal that can be detected in the presence of the noise. Thus, it is generally desired to minimize the residual noise level of a radar or communications receiver to achieve the best performance. In some cases, such as radiometers or radio astronomy systems, the desired signal is actually the noise power received by an antenna, and it is necessary to distinguish between the received noise power and the undesired noise generated by the receiver system itself.

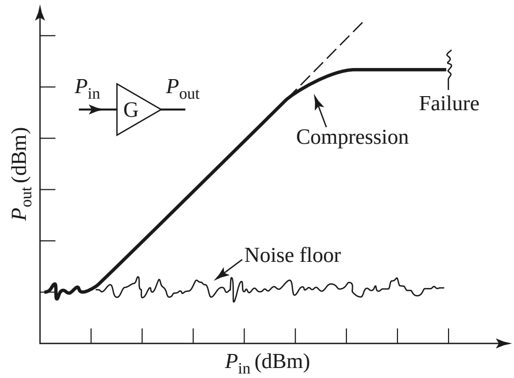

FIGURE 10.1 Illustrating the dynamic range of a realistic amplifier.

Dynamic Range and Sources of Noise

In previous chapters we have implicitly assumed that all components were *linear* (meaning that the output signal level is directly proportional to the input signal level), and *deterministic* (meaning that the output signal is predictable from the input signal). In reality no component can perform in this way over an unlimited range of input/output signal levels. In practice, however, there is usually a range of signal levels over which such assumptions are approximately valid; this range is called the *dynamic range* of the component.

As an example, consider a realistic microwave transistor amplifier having a power gain G, as shown in Figure 10.1. If the amplifier were ideal, the output power would be related to the input power as $P_{out} = GP_{in}$, and this relation would hold true for any value of P_{in}. Thus, if $P_{in} = 0$, we would have $P_{out} = 0$, and if $P_{in} = 10^6$ W and $G = 10$ dB, we would have $P_{out} = 10^7$ W. Neither of these results would actually occur in practice, however. Because of noise generated by the amplifier itself, some nonzero noise power will always be delivered by the amplifier, even when the input power is zero. At the other extreme, very high input power will cause the amplifier to fail. Thus, the actual relation between the output and input power will be as shown in Figure 10.1. At very low input power levels, the output will be dominated by the noise generated by the amplifier. This level is often called the *noise floor* of the component or system; typical values may range from -80 to -140 dBm over the bandwidth of the system, with the lowest values being obtained with thermally cooled components. Above the noise floor, the amplifier will have a range of input power for which $P_{out} = GP_{in}$ is closely approximated. This is the usable *dynamic range* of the component. At the upper end of this range, the output will begin to saturate, meaning that the output power no longer increases linearly as the input power increases. Excessive input power will lead to failure of the amplifier.

Noise that is generated internally in a device or component is usually caused by random motions of charges or charge carriers in devices and materials. Such motions may be due to any of several mechanisms, leading to various types of noise:

- *Thermal noise* is the most basic type of noise, being caused by thermal vibration of bound charges. It is also known as *Johnson* or *Nyquist* noise.
- *Shot noise* is due to random fluctuations of charge carriers in an electron tube or solid-state device.
- *Flicker noise* occurs in solid-state components and vacuum tubes. Flicker noise power varies inversely with frequency, and so is often called $1/f$-noise.

- *Plasma noise* is caused by random motion of charges in an ionized gas, such as a plasma, the ionosphere, or sparking electrical contacts.
- *Quantum noise* results from the quantized nature of charge carriers and photons; it is often insignificant relative to other noise sources.

External noise may be introduced into a system either by a receiving antenna or by electromagnetic coupling. Some sources of external RF noise include the following:

- Thermal noise from the ground
- Cosmic background noise from the sky
- Noise from stars (including the sun)
- Lightning
- Gas discharge lamps
- Radio, TV, and cellular stations
- Wireless devices
- Microwave ovens
- Deliberate jamming devices

The characterization of noise effects in RF and microwave systems in terms of noise temperature and noise figure will apply to all types of noise, regardless of the source, as long as the spectrum of the noise is relatively flat over the bandwidth of the system. Noise with a flat frequency spectrum is called *white noise*.

Noise Power and Equivalent Noise Temperature

Consider a resistor at a physical temperature of T degrees kelvin (K), as depicted in Figure 10.2. The electrons in the resistor are in random motion, with a kinetic energy that is proportional to the temperature. These random motions produce small, random voltage fluctuations at the resistor terminals, as illustrated in Figure 10.2. This voltage has a zero average value but a nonzero root mean square (rms) value given by Planck's blackbody radiation law,

$$V_n = \sqrt{\frac{4hf\ BR}{e^{hf/kT} - 1}}, \tag{10.1}$$

where

$h = 6.626 \times 10^{-34}$ J-sec is Planck's constant.
$k = 1.380 \times 10^{-23}$ J/K is Boltzmann's constant.
$T =$ the temperature in degrees kelvin (K).
$B =$ the bandwidth of the system in Hz.
$f =$ the center frequency of the bandwidth in Hz.
$R =$ the resistance in Ω.

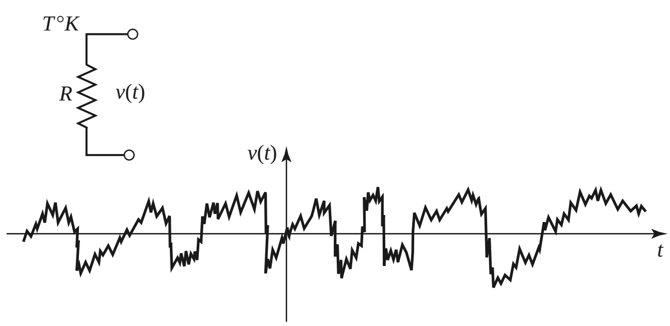

FIGURE 10.2 A random voltage generated by a noisy resistor.

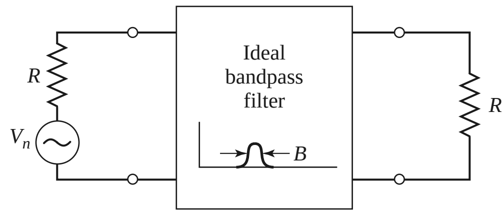

FIGURE 10.3 Equivalent circuit of a noisy resistor delivering maximum power to a load resistor through an ideal bandpass filter.

This result comes from quantum mechanical considerations, and is valid for any frequency f. At microwave frequencies the above result can be simplified by making use of the fact that $hf \ll kT$. (As a worst-case example, let $f = 100$ GHz and $T = 100$ K. Then $hf = 6.6 \times 10^{-23} \ll kT = 1.4 \times 10^{-21}$.) Using the first two terms of a Taylor series expansion for the exponential in (10.1) gives

$$e^{hf/kt} - 1 \simeq \frac{hf}{kT},$$

so that (10.1) reduces to

$$V_n = \sqrt{4kTBR}. \tag{10.2}$$

This is the *Rayleigh–Jeans approximation,* and is the result that is most commonly used in microwave work [1]. For very high frequencies or very low temperatures, however, this approximation may be invalid, in which case (10.1) should be used.

The noisy resistor of Figure 10.2 can be replaced with a Thevenin equivalent circuit consisting of a noiseless resistor and a generator with a voltage given by (10.2), as shown in Figure 10.3. Connecting a load resistor R results in maximum power transfer from the noisy resistor, with the result that power delivered to the load in a bandwidth B is

$$P_n = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R} = kTB, \tag{10.3}$$

since V_n is an rms voltage. This important result gives the maximum available noise power from the noisy resistor at temperature T. Note that this noise power is independent of frequency; such a noise source has a power spectral density that is constant with frequency, and is an example of a white noise source. The noise power is directly proportional to the bandwidth, which in practice is usually limited by the passband of the RF or microwave system. Independent white noise sources can be treated as Gaussian-distributed random variables, so the noise powers (variances) of independent noise sources are additive.

The following trends can be observed from (10.3):

- As $B \to 0$, $P_n \to 0$. This means that systems with smaller bandwidths collect less noise power.
- As $T \to 0$, $P_n \to 0$. This means that cooler devices and components generate less noise power.
- As $B \to \infty$, $P_n \to \infty$. This is the so-called *ultraviolet catastrophe,* which does not occur in reality because (10.2)–(10.3) are not valid as f (or B) $\to \infty$; (10.1) must be used in this case.

If an arbitrary source of noise (thermal or nonthermal) is "white," so that the noise power is not a strong function of frequency over the bandwidth of interest, it can be modeled as an equivalent thermal noise source, and characterized with an *equivalent noise*

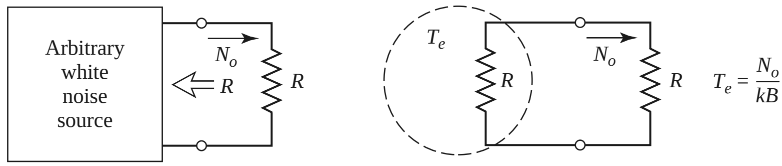

FIGURE 10.4 The equivalent noise temperature, T_e, of an arbitrary white noise source.

temperature. Thus, consider the arbitrary white noise source of Figure 10.4, which has a driving-point impedance of R and delivers a noise power N_o to a load resistor R. This noise source can be replaced by a noisy resistor of value R at temperature T_e, where T_e is an equivalent temperature selected so that the same noise power is delivered to the load. That is,

$$T_e = \frac{N_o}{kB}. \tag{10.4}$$

Components and systems can then be characterized by saying that they have an equivalent noise temperature T_e; this implies some fixed bandwidth B, which is generally the operational bandwidth of the component or system.

For example, consider a noisy amplifier with a bandwidth B and gain G. Let the amplifier be matched to noiseless source and load resistors, as shown in Figure 10.5. If the source resistor is at a (hypothetical) temperature of $T_s = 0\,K$, then the input power to the amplifier will be $N_i = 0$, and the output noise power N_o will be due only to the noise generated by the amplifier itself. We can obtain the same load noise power by driving an ideal noiseless amplifier with a resistor at the temperature

$$T_e = \frac{N_o}{GkB}, \tag{10.5}$$

so that the output power in both cases is $N_o = GkT_eB$. Then T_e is the equivalent noise temperature of the amplifier.

It is sometimes useful for measurement purposes to have a calibrated noise source. A passive noise source may simply consist of a resistor held at a constant temperature, either

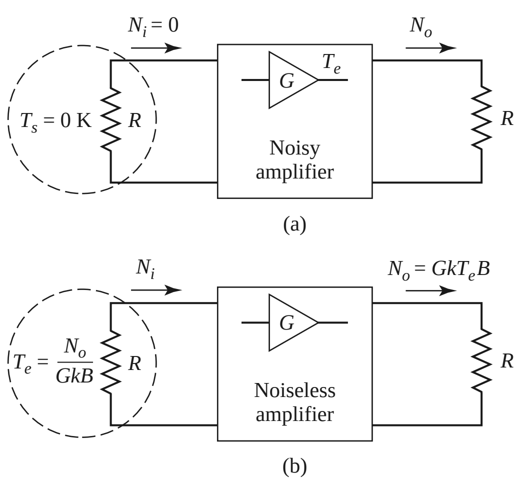

FIGURE 10.5 Defining the equivalent noise temperature of a noisy amplifier. (a) Noisy amplifier. (b) Noiseless amplifier.

in a temperature-controlled oven, or in a cryogenic flask. Active noise sources may use a diode, transistor, or tube to provide a calibrated noise power output. Noise generators can be characterized by an equivalent noise temperature, but a more common measure of noise power for such components is the *excess noise ratio* (ENR), defined as

$$\text{ENR (dB)} = 10 \log \frac{N_g - N_o}{N_o} = 10 \log \frac{T_g - T_0}{T_0}, \tag{10.6}$$

where N_g and T_g are the noise power and equivalent noise temperature of the generator, and N_o and T_0 are the noise power and temperature associated with a room-temperature ($T_0 = 290$ K) passive source (a matched load). Solid-state noise generators typically have ENRs ranging from 20 to 40 dB.

Measurement of Noise Temperature

In principle, the equivalent noise temperature of a component can be determined by measuring the output power when a matched load at 0 K is connected at the input of the component. In practice, of course, a 0 K source temperature cannot be obtained, so a different method must be used. If two matched loads at significantly different temperatures are available, then the *Y-factor method* can be applied.

This technique is illustrated in Figure 10.6, where the amplifier (or other component) under test is connected to one of two matched loads at different temperatures, and the output power is measured for each case. Let T_1 be the temperature of the hot load and T_2 the temperature of the cold load ($T_1 > T_2$), and let P_1 and P_2 be the respective powers measured at the amplifier output. The output noise power consists of noise power generated by the amplifier as well as noise power from the source resistor. Thus we have

$$N_1 = GkT_1B + GkT_eB, \tag{10.7a}$$

$$N_2 = GkT_2B + GkT_eB, \tag{10.7b}$$

which are two equations for the two unknowns, T_e and GB (the gain–bandwidth product of the amplifier). Define the Y-factor as

$$Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1, \tag{10.8}$$

which is determined as the ratio of the output power measurements. Then (10.7) can be solved for the equivalent noise temperature of the device under test as

$$T_e = \frac{T_1 - YT_2}{Y - 1}, \tag{10.9}$$

in terms of the load temperatures and the Y-factor.

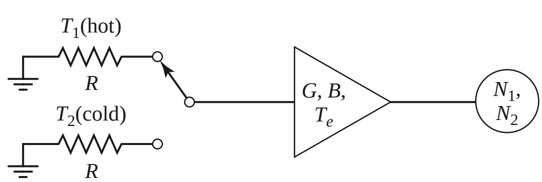

FIGURE 10.6 The Y-factor method for measuring the equivalent noise temperature of an amplifier.

Note that to obtain accurate results from this method, the two source temperatures must not be too close together. If they are, N_1 will be close to N_2, Y will be close to unity, and the evaluation of (10.9) will involve the subtractions of numbers close to each other, resulting in a loss of accuracy. In practice, one noise source is usually a load resistor at room temperature ($T_0 = 290$ K), while the other noise source is either "hotter" or "colder," depending on whether T_e is greater or less than T_0. An active noise generator can be used as a "hot" source, while a "cold" source can be obtained by immersing a load resistor in liquid nitrogen ($T = 77$ K) or liquid helium ($T = 4$ K).

EXAMPLE 10.1 NOISE TEMPERATURE MEASUREMENT

An X-band amplifier has a gain of 20 dB and a 1 GHz bandwidth. Its equivalent noise temperature is to be measured via the Y-factor method. The following data are obtained:

$$\text{For } T_1 = 290 \text{ K}, \qquad N_1 = -62.0 \text{ dBm}.$$
$$\text{For } T_2 = 77 \text{ K}, \qquad N_2 = -64.7 \text{ dBm}.$$

Determine the equivalent noise temperature of the amplifier. If the amplifier is used with a source having an equivalent noise temperature of $T_s = 450$ K, what is the output noise power from the amplifier, in dBm?

Solution
From (10.8), the Y-factor in dB is

$$Y = (N_1 - N_2) \text{ dB} = (-62.0) - (-64.7) = 2.7 \text{ dB},$$

which is a numeric value of $Y = 1.86$. Using (10.9) gives the equivalent noise temperature as

$$T_e = \frac{T_1 - YT_2}{Y - 1} = \frac{290 - (1.86)(77)}{1.86 - 1} = 170 \text{ K}.$$

If a source with an equivalent noise temperature of $T_s = 450$ K drives the amplifier, the noise power into the amplifier will be kT_sB. The total noise power out of the amplifier will be

$$\begin{aligned} N_o &= GkT_sB + GkT_eB = 100(1.38 \times 10^{-23})(10^9)(450 + 170) \\ &= 8.56 \times 10^{-10} \text{ W} = -60.7 \text{ dBm}. \end{aligned}$$

■

10.2 NOISE FIGURE

Definitio of Noise Figure

We have seen that a noisy microwave component can be characterized by an equivalent noise temperature. An alternative characterization is the *noise figure* of the component, which is a measure of the degradation in the signal-to-noise ratio between the input and output of the component. The *signal-to-noise ratio* is the ratio of desired signal power to undesired noise power, and so is dependent on the signal power. When noise and a desired signal are applied to the input of a noiseless network, both noise and signal will be attenuated or amplified by the same factor, so that the signal-to-noise ratio will be unchanged. However, if the network is noisy, the output noise power will be increased more than the

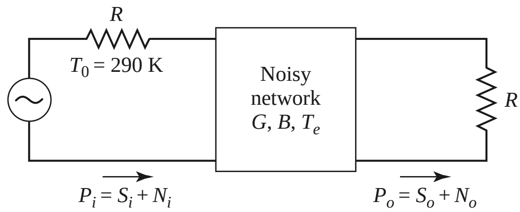

FIGURE 10.7 Determining the noise figure of a noisy network.

output signal power, so that the output signal-to-noise ratio will be reduced. The noise figure, F, is a measure of this reduction in signal-to-noise ratio, and is defined as

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1, \tag{10.10}$$

where S_i, N_i are the input signal and noise powers, and S_o, N_o are the output signal and noise powers. By definition, the input noise power is assumed to be the noise power resulting from a matched resistor at $T_0 = 290$ K; that is, $N_i = kT_0B$.

Consider Figure 10.7, which shows noise power N_i and signal power S_i being fed into a noisy two-port network. The network is characterized by a gain, G, a bandwidth, B, and an equivalent noise temperature, T_e. The input noise power is $N_i = kT_0B$, and the output noise power is a sum of the amplified input noise and the internally generated noise: $N_o = kGB(T_0 + T_e)$. The output signal power is $S_o = GS_i$. Using these results in (10.10) gives the noise figure as

$$F = \frac{S_i}{kT_0B}\frac{kGB(T_0 + T_e)}{GS_i} = 1 + \frac{T_e}{T_0} \geq 1. \tag{10.11}$$

In dB, $F = 10\log(1 + T_e/T_0)$ dB ≥ 0. If the network were noiseless, T_e would be zero, giving $F = 1$, or 0 dB. Solving (10.11) for T_e gives

$$T_e = (F - 1)T_0. \tag{10.12}$$

It is important to keep in mind two things concerning the definition of noise figure: noise figure is defined for a matched input source, and for a noise source equivalent to a matched load at temperature $T_0 = 290$ K. Noise figure and equivalent noise temperatures are interchangeable characterizations of the noise properties of a component.

An important special case occurs in practice for a two-port network consisting of a passive, lossy component, such as an attenuator or lossy transmission line, held at a physical temperature T. Consider such a network with a matched source resistor that is also at temperature T, as shown in Figure 10.8. The power gain, G, of a lossy network is less than unity; the loss factor, L, can be defined as $L = 1/G > 1$. Because the entire system is in thermal equilibrium at the temperature T, and has a driving point impedance of R, the output noise power must be $N_o = kTB$. However, we can also think of this power as coming from the source resistor (attenuated by the lossy line), and from the noise generated by the line itself. Thus we also have that

$$N_o = kTB = GkTB + GN_{\text{added}}, \tag{10.13}$$

where N_{added} is the noise generated by the line, as if it appeared at the input terminals of the line. Solving (10.13) for this power gives

$$N_{\text{added}} = \frac{1 - G}{G}kTB = (L - 1)kTB. \tag{10.14}$$

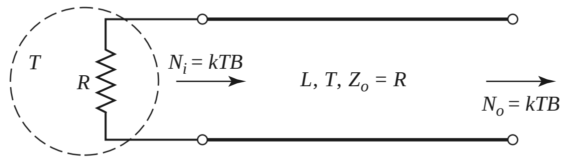

FIGURE 10.8 Determining the noise figure of a lossy line or attenuator with loss L and temperature T.

Then (10.4) shows that the lossy line has an equivalent noise temperature (referred to the input) given by

$$T_e = \frac{1-G}{G}T = (L-1)T. \tag{10.15}$$

From (10.11) the noise figure is

$$F = 1 + (L-1)\frac{T}{T_0}. \tag{10.16}$$

If the line is at temperature T_0, then $F = L$. For instance, a 6 dB attenuator at room temperature has a noise figure of $F = 6$ dB.

Noise Figure of a Cascaded System

In a typical microwave system the input signal travels through a cascade of many different components, each of which may degrade the signal-to-noise ratio to some degree. If we know the noise figure (or noise temperature) of the individual stages, we can determine the noise figure (or noise temperature) of the cascade connection of stages. We will see that the noise performance of the first stage is usually the most critical, an interesting result that is very important in practice.

Consider the cascade of two components, having gains G_1, G_2, noise figures F_1, F_2, and equivalent noise temperatures T_{e1}, T_{e2}, as shown in Figure 10.9. We wish to find the overall noise figure and equivalent noise temperature of the cascade, as if it were a single component. The overall gain of the cascade is G_1G_2.

Using noise temperatures, we can write the noise power at the output of the first stage as

$$N_1 = G_1kT_0B + G_1kT_{e1}B, \tag{10.17}$$

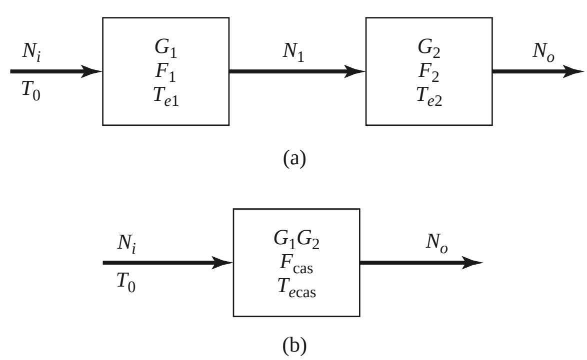

FIGURE 10.9 Noise figure and equivalent noise temperature of a cascaded system. (a) Two cascaded networks. (b) Equivalent network.

since $N_i = kT_0B$ for noise figure calculations. The noise power at the output of the second stage is

$$\begin{aligned} N_o &= G_2 N_1 + G_2 k T_{e2} B \\ &= G_1 G_2 k B \left(T_0 + T_{e1} + \frac{1}{G_1} T_{e2} \right). \end{aligned} \tag{10.18}$$

For the equivalent system we have

$$N_o = G_1 G_2 k B (T_{\text{cas}} + T_0), \tag{10.19}$$

so comparison with (10.18) gives the noise temperature of the cascade system as

$$T_{\text{cas}} = T_{e1} + \frac{1}{G_1} T_{e2}. \tag{10.20}$$

Using (10.12) to convert the temperatures in (10.20) to noise figures yields the noise figure of the cascade system as

$$F_{\text{cas}} = F_1 + \frac{1}{G_1}(F_2 - 1). \tag{10.21}$$

Equations (10.20) and (10.21) show that the noise characteristics of a cascaded system are dominated by the characteristics of the first stage since the effect of the second stage is reduced by the gain of the first (assuming $G_1 > 1$). Thus, for the best overall system noise performance, the first stage should have a low noise figure and at least moderate gain. Expense and effort should be devoted primarily to the first stage, as opposed to later stages, since later stages have a diminished impact on the overall noise performance.

Equations (10.20) and (10.21) can be generalized to an arbitrary number of stages, as follows:

$$T_{\text{cas}} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots, \tag{10.22}$$

$$F_{\text{cas}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots. \tag{10.23}$$

EXAMPLE 10.2 NOISE ANALYSIS OF A WIRELESS RECEIVER

The block diagram of a wireless receiver front-end is shown in Figure 10.10. Compute the overall noise figure of this subsystem. If the input noise power from a feeding antenna is $N_i = kT_AB$, where $T_A = 150$ K, find the output noise power in dBm. If we require a minimum signal-to-noise ratio (SNR) of 20 dB at the output of the receiver, what is the minimum signal voltage that should be applied

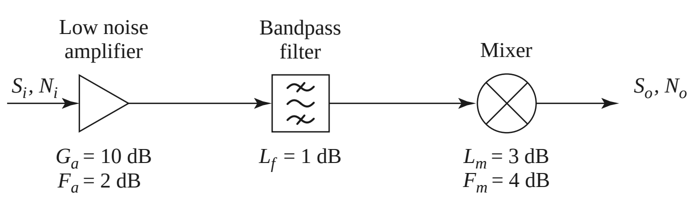

FIGURE 10.10 Block diagram of a wireless receiver front-end for Example 10.2.

at the receiver input? Assume the system is at temperature T_0, with a characteristic impedance of 50 Ω, and an IF bandwidth of 10 MHz.

Solution
We first perform the required conversions from dB to numerical values:

$$G_a = 10 \text{ dB} = 10 \qquad G_f = -1.0 \text{ dB} = 0.79 \qquad G_m = -3.0 \text{ dB} = 0.5$$
$$F_a = 2 \text{ dB} = 1.58 \qquad F_f = 1 \text{ dB} = 1.26 \qquad F_m = 4 \text{ dB} = 2.51$$

Next, use (10.23) to find the overall noise figure of the system:

$$F = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f} = 1.58 + \frac{(1.26 - 1)}{10} + \frac{(2.51 - 1)}{(10)(0.79)}$$
$$= 1.80 = 2.55 \text{ dB}.$$

The best way to compute the output noise power is to use noise temperatures. From (10.12), the equivalent noise temperature of the overall system is

$$T_e = (F - 1)T_0 = (1.80 - 1)(290) = 232 \text{ K}.$$

The overall gain of the system is $G = (10)(0.79)(0.5) = 3.95$. Then we can find the output noise power as

$$N_o = k(T_A + T_e)BG = (1.38 \times 10^{-23})(150 + 232)(10 \times 10^6)(3.95)$$
$$= 2.08 \times 10^{-13} \text{ W} = -96.8 \text{ dBm}.$$

For an output SNR of 20 dB = 100, the input signal power must be

$$S_i = \frac{S_o}{G} = \frac{S_o}{N_o}\frac{N_o}{G} = 100\frac{2.08 \times 10^{-13}}{3.95} = 5.27 \times 10^{-12} \text{ W} = -82.8 \text{ dBm}.$$

For a 50 Ω system impedance, this corresponds to an input signal voltage of

$$V_i = \sqrt{Z_o S_i} = \sqrt{(50)(5.27 \times 10^{-12})} = 1.62 \times 10^{-5} \text{ V} = 16.2\ \mu\text{V (rms)}.$$

Note: It may be tempting to compute the output noise power from the definition of the noise figure, as

$$N_o = N_i F \left(\frac{S_o}{S_i}\right) = N_i F G = kT_A BFG$$
$$= (1.38 \times 10^{-23})(150)(10 \times 10^6)(1.8)(3.95) = 1.47 \times 10^{-13} \text{ W}.$$

This is an *incorrect* result! The reason for the disparity with the earlier result is that the definition of noise figure assumes an input noise level of kT_0B, while this problem involves an input noise of kT_AB, with $T_A = 150 \text{ K} \neq T_0$. This is a common error, and suggests that when computing absolute noise power it is often safer to use noise temperatures to avoid this confusion. ■

Noise Figure of a Passive Two-Port Network

We previously derived the noise figure for a matched lossy line or attenuator by using a thermodynamic argument. Here we generalize that technique to evaluate the noise figure of general passive networks (networks that do not contain active devices such as diodes or transistors, which generate nonthermal noise). In addition, this method will account for the change in noise figure that occurs when a component is impedance mismatched at either its

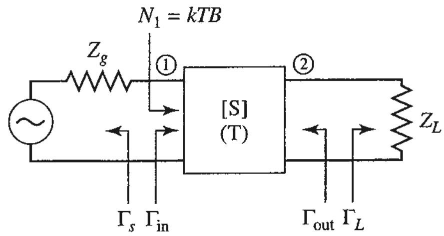

FIGURE 10.11 A passive two-port network with impedance mismatches. The network is at physical temperature T.

input or output port. Generally it is easier and more accurate to find the noise characteristics of an active device, such as a diode or transistor, by direct measurement than by calculation from first principles.

Figure 10.11 shows an arbitrary passive two-port network, with a generator at port 1 and a load at port 2. The network is characterized by its scattering matrix, $[S]$. In the general case, impedance mismatches may exist at each port, and we define these mismatches in terms of the following reflection coefficients:

$$\begin{aligned}
\Gamma_s &= \text{reflection coefficient looking toward generator,}\\
\Gamma_{\text{in}} &= \text{reflection coefficient looking toward port 1 of network,}\\
\Gamma_{\text{out}} &= \text{reflection coefficient looking toward port 2 of network,}\\
\Gamma_L &= \text{reflection coefficient looking toward load.}
\end{aligned}$$

If we assume the network is at temperature T, and that an available input noise power of $N_1 = kTB$ is applied to the input of the network, we can write the available output noise power at port 2 as

$$N_2 = G_{21}kTB + G_{21}N_{\text{added}}, \tag{10.24}$$

where N_{added} is the noise power generated internally by the network (referenced to port 1), and G_{21} is the *available power gain* of the network from port 1 to port 2. The available power gain can be expressed in terms of the scattering parameters of the network and the port mismatches as (also see Section 12.1),

$$G_{21} = \frac{\text{power available from network}}{\text{power available from source}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{\text{out}}|^2)}. \tag{10.25}$$

As derived in Example 4.7, the output port mismatch is given by

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}. \tag{10.26}$$

Observe that when the network is matched to its external circuitry, so that $\Gamma_s = 0$ and $S_{22} = 0$, we have $\Gamma_{\text{out}} = 0$ and $G_{21} = |S_{21}|^2$, which is the gain of the network when it is matched. Also observe that the available gain of the network does not depend on the load mismatch, Γ_{L}. This is because available gain is defined in terms of the maximum power that is available from the network, which occurs when the load impedance is conjugately matched to the output impedance of the network.

Since the input noise power is kTB, and the network is passive and at temperature T, the network is in thermodynamic equilibrium, and so the available output noise power must be $N_2 = kTB$. Then we can solve for N_{added} from (10.24) to give

$$N_{\text{added}} = \frac{1 - G_{21}}{G_{21}} kTB. \tag{10.27}$$

Then the equivalent noise temperature of the network is

$$T_e = \frac{N_{\text{added}}}{kB} = \frac{1 - G_{21}}{G_{21}} T, \tag{10.28}$$

and the noise figure of the network is

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{1 - G_{21}}{G_{21}} \frac{T}{T_0}. \tag{10.29}$$

Note the similarity of (10.27)–(10.29) to the results in (10.14)–(10.16) for the lossy line—the essential difference is that here we are using the available gain of the network, which accounts for impedance mismatches between the network and the external circuit. We can illustrate the use of this result with some applications to problems of practical interest.

Noise Figure of a Mismatched Lossy Line

Earlier we found the noise figure of a lossy transmission line under the assumption that it was matched to its input and output circuits. Now we consider the case where the line is mismatched to its input circuit. Figure 10.12 shows a transmission line of length ℓ at temperature T, with a power loss factor $L = 1/G$, and an impedance mismatch between the line and the generator. Thus, $Z_g \neq Z_0$, and the reflection coefficient looking toward the generator is

$$\Gamma_s = \frac{Z_g - Z_0}{Z_g + Z_0} \neq 0.$$

The scattering matrix of the lossy line of characteristic impedance Z_0 can be written as

$$[S] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{e^{-j\beta\ell}}{\sqrt{L}}, \tag{10.30}$$

where β is the propagation constant of the line. Using the elements of (10.30) in (10.26) gives the reflection coefficient looking into port 2 of the line as

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = \frac{\Gamma_s}{L} e^{-2j\beta\ell}. \tag{10.31}$$

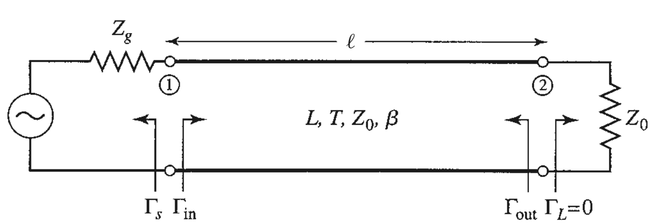

FIGURE 10.12 A lossy transmission line at temperature T with an impedance mismatch at its input port.

Then the available gain, from (10.25), is

$$G_{21} = \frac{\frac{1}{L}(1 - |\Gamma_s|^2)}{1 - |\Gamma_{\text{out}}|^2} = \frac{L(1 - |\Gamma_s|^2)}{L^2 - |\Gamma_s|^2}. \tag{10.32}$$

We can verify two limiting cases of (10.32): when $L = 1$ we have $G_{21} = 1$, and when $\Gamma_s = 0$ we have $G_{21} = 1/L$. Using (10.32) in (10.28) gives the equivalent noise temperature of the mismatched lossy line as

$$T_e = \frac{1 - G_{21}}{G_{21}} T = \frac{(L - 1)(L + |\Gamma_s|^2)}{L(1 - |\Gamma_s|^2)} T. \tag{10.33}$$

The corresponding noise figure can then be evaluated using (10.11). Observe that when the line is matched, $\Gamma_s = 0$, and (10.33) reduces to $T_e = (L - 1)T$, in agreement with the result for the matched lossy line given by (10.15). If the line is lossless, then $L = 1$, and (10.33) reduces to $T_e = 0$ regardless of mismatch, as expected. However, when the line is lossy and mismatched, so that $L > 1$ and $|\Gamma_s| > 0$, then the noise temperature given by (10.33) is greater than $T_e = (L - 1)T$, the noise temperature of the matched lossy line. The reason for this increase is that the lossy line actually delivers noise power out of both its ports, but when the input port is mismatched some of the available noise power at port 1 is reflected from the source back into port 1 and appears at port 2. When the generator is matched to port 1, none of the available power from port 1 is reflected back into the line, so the noise power available at port 2 is a minimum. This result implies that impedance matching is important in minimizing noise temperature and noise figure.

EXAMPLE 10.3 APPLICATION TO A WILKINSON POWER DIVIDER

Find the noise figure of a Wilkinson power divider when one of the output ports is terminated in a matched load. Assume an insertion loss factor of L from the input to either output port.

Solution

From Chapter 7 the scattering matrix of a Wilkinson divider is given as

$$[S] = \frac{-j}{\sqrt{2L}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

where the factor $L \geq 1$ accounts for the dissipative loss from port 1 to port 2 or 3 (note that dissipative loss is distinct from the -3 dB power division ratio). To evaluate the noise figure of the Wilkinson divider, we first terminate port 3 with a matched load; this converts the three-port device into a two-port device. If we assume a matched source at port 1, we have $\Gamma_s = 0$. Equation (10.26) then gives $\Gamma_{\text{out}} = S_{22} = 0$, and so the available gain can be calculated from (10.25) as

$$G_{21} = |S_{21}|^2 = \frac{1}{2L}.$$

The equivalent noise temperature of the Wilkinson divider is, from (10.28),

$$T_e = \frac{1 - G_{21}}{G_{21}} T = (2L - 1)T,$$

where T is the physical temperature of the divider. Using (10.11) gives the noise figure as

$$F = 1 + \frac{T_e}{T_0} = 1 + (2L - 1)\frac{T}{T_0}.$$

Observe that if the divider is at room temperature, then $T = T_0$ and the above reduces to $F = 2L$. If the divider is at room temperature and lossless, this reduces to $F = 2 = 3$ dB. In this case the source of the noise power is the isolation resistor contained in the Wilkinson divider circuit.

Because the network is matched at its input and output, it is easy to obtain these same results using the thermodynamic argument directly. Thus, if we apply an input noise power of kTB to port 1 of the matched divider at temperature T, the system will be in thermal equilibrium and the output noise power must be kTB. We can also express the output noise power as the sum of the input power times the gain of the divider, and N_{added}, the noise power added by the divider itself (referenced to the input to the divider):

$$kTB = \frac{kTB}{2L} + \frac{N_{added}}{2L}.$$

Solving for N_{added} gives $N_{added} = kTB(2L - 1)$, so the equivalent noise temperature is

$$T_e = \frac{N_{added}}{kB} = (2L - 1)T,$$

in agreement with the above. ■

Noise Figure of a Mismatched Amplifie

Finally, consider the effect of an input impedance mismatch on the noise figure of an amplifier. As shown in Figure 10.13, the amplifier, when matched, has a gain G, a noise figure F, and a bandwidth B. The amplifier output is matched, but there is an impedance mismatch at the input represented by the reflection coefficient, Γ. Our previous results involving the effect of mismatch on noise figure made use of (10.29), but that was derived for a passive network and so cannot be directly used in this case. Instead we will use noise temperatures.

Since we are dealing with noise figure, let the input noise power to the amplifier be $N_i = kT_0B$. Then the output noise power from the amplifier (referenced to the input) is given by

$$N_o = kT_0GB\left(1 - |\Gamma|^2\right) + kT_0\,(F - 1)\,GB \tag{10.34}$$

where the first term is due to the input noise power, decreased by the reflection at the input, and the second term is the noise power due to the amplifier itself, based on the equivalent noise temperature as given by (10.12). For an applied signal power S_i, the output signal

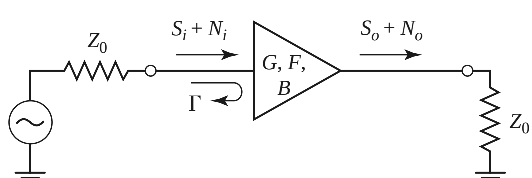

FIGURE 10.13 A noisy amplifier with an impedance mismatch at its input.

power is

$$S_o = G(1 - |\Gamma|^2)S_i. \tag{10.35}$$

The overall noise figure, F_m, of the mismatched amplifier can be found from (10.10) as

$$F_m = \frac{S_i N_o}{S_o N_i} = 1 + \frac{F - 1}{1 - |\Gamma|^2}. \tag{10.36}$$

Observe from (10.36) the limiting case that $F_m = F$ when $|\Gamma| = 0$ (no mismatch), and that this is the minimum noise figure that can be achieved since F_m increases as the mismatch increases. This result demonstrates that good noise figure requires good impedance matching. This problem would be more complicated if a mismatch also existed at the output of the amplifier, particularly if the amplifier is not unilateral.

10.3 NONLINEAR DISTORTION

We have seen that thermal noise is generated by any lossy component. Since all realistic components have at least a small loss, the ideal linear component does not exist in practice because all realistic devices are nonlinear at very low signal levels due to noise effects. In addition, practical components may also become nonlinear at high signal levels. In the case of active devices, such as diodes and transistors, this may be due to effects such as gain compression or the generation of spurious frequency components due to device nonlinearities, but all devices ultimately fail at very high power levels. In either case, these effects set a minimum and maximum realistic power range, or *dynamic range*, over which a given component or network will operate as desired. In this section we will study the response of nonlinear devices in general, and two definitions of dynamic range. These results will be useful for our later discussions of amplifiers (Chapter 12), mixers (Chapter 13), and wireless receivers (Chapter 14).

Devices such as diodes and transistors have nonlinear characteristics, and it is this nonlinearity that is of great utility for desirable functions such as amplification, detection, and frequency conversion [2]. Nonlinear device characteristics, however, can also lead to undesirable effects such as gain compression and the generation of spurious frequency components. These effects may lead to increased losses, signal distortion, and possible interference with other radio channels or services. Some of the many possible effects of nonlinearity in RF and microwave circuits are listed below [3]:

- Harmonic generation (multiples of a fundamental signal)
- Saturation (gain reduction in an amplifier)
- Intermodulation distortion (products of a two-tone input signal)
- Cross-modulation (modulation transfer from one signal to another)
- AM-PM conversion (amplitude variation causes phase shift)
- Spectral regrowth (intermodulation with many closely spaced signals)

Figure 10.14 shows a general nonlinear network, having an input voltage v_i and an output voltage v_o. In the most general sense, the output response of a nonlinear circuit can

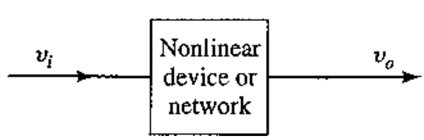

FIGURE 10.14 A general nonlinear device or network.