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ENGIN 435 – Antenna Design

Project #1 – Moon Bounce

For this first project of the semester, we as engineer students were tasked with finding what the minimum size would be for a receive antenna to observe a 20 dB peak SNR after the signal has been transmitted and bounced back from the illuminated surface of the Moon. With only this little bit of information, my job was to use my intuition, mathematical approaches, and find legitimate research sources to help backup my estimates and calculations. The first evidence of a successful man-made moon-bounce reception by the US was given to us in a declassified CIA article [1]. The article outlines a Soviet antenna that the Russians were working on in secret and the US attempting to pool together their own antennas to classify this new Soviet technology. The rest of this paper will step through my findings from this CIA article, as well as others I found during my research and estimations for what I believe should be the minimal size for a receive antenna to meet the 20 dB SNR requirement.

$$\frac{P_r}{P_t} = \frac{\sigma \cdot G_r \cdot G_t}{4\pi}\left(\frac{\lambda}{4\cdot\pi\cdot R_1\cdot R_2}\right)^2 \qquad \text{(Eq. 1)}$$

The key component to this entire project all points to the Radar Range Equation. This equation (Eq. 1) is used in terms of a bistatic communication link, where the transmission signal will reflect or, in our case, "bounce" off a target at some distance then travel another distance to be received by another antenna. This type of communication forced me to rule out the Friis Transmission Equation, which would be more suited if the signal was intended for a more direct path from transmitter to receiver without reflecting off any targets. The Radar Range Equation accounts for parameters such as: the power transmitted by one antenna, the power received by the other antenna, the corresponding transmit and receive gains for each, the wavelength of the signal, the distance the signal travels, and the radar cross section of the target the signal reflects off. My approach for this project was to research for as much physical evidence I could provide to this equation, and then estimate or calculate for the terms remaining. To start, I know from previous antenna research I've done, radio frequencies below 100MHz can not successfully escape Earth's Ionosphere and will reflect towards the Earth [7]. For the Soviet antenna of interest, I knew that it's transmitted signal must be above this frequency limit to propagate to the Moon and back. From an article about an Eastern Siberia radar, I found a potential operating frequency range to be 154Mhz to 162Hz [4]. This would mean my potential wavelengths of interest would be between about 1.95m to 1.85m. The same source gives dimensions for an antenna that operated within those frequencies and had a length of 244m and height of 20m [4]. These dimensions could help calculate an estimate of the transmit gain that would be close to the Soviet antenna.

The CIA article I mentioned before gave me two important pieces of information that factor into the Radar Range Equation and the SNR requirement for the project. The greatest observation that the CIA article expressed was that the transmit power for the Soviet Union antenna was 25 Megawatts, which made it the one of the most powerful antennas at the time [1]. The US was also able to capture traces of the pulses on an oscillograph, shown in (Figure 1) below.

Figure 1 - Oscillograph trace of an 800-microsecond pulse observed by US, which has similar shape to original transmitted pulse. [1]

This pulse width also leads to an estimate of the operating bandwidth of the signal. By using a mathematical process called the Fourier Transform, this pulse width in the time-domain becomes a sinc function in the frequency-domain. The bandwidth of the signal can then be estimated by examining the bounds of which the main envelope of the sinc function are mostly non-zero. Using MATLAB, I was able to create two plots (Fig 2-1 & Fig 2-2), shown below.

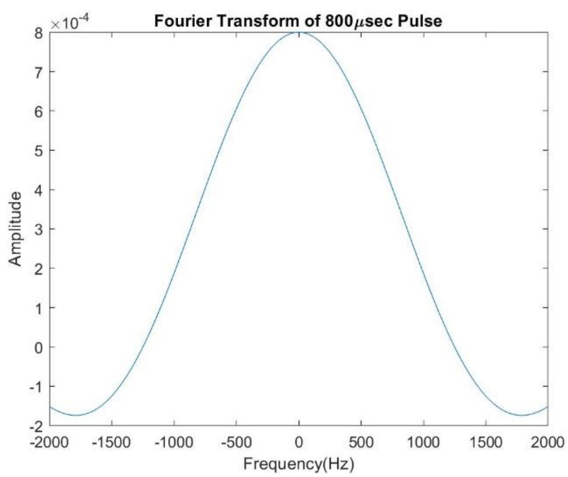

Figure 2-1 - Fourier Transform of 800-microsec Pulse Width observed from CIA as function of Amplitude vs Frequency.

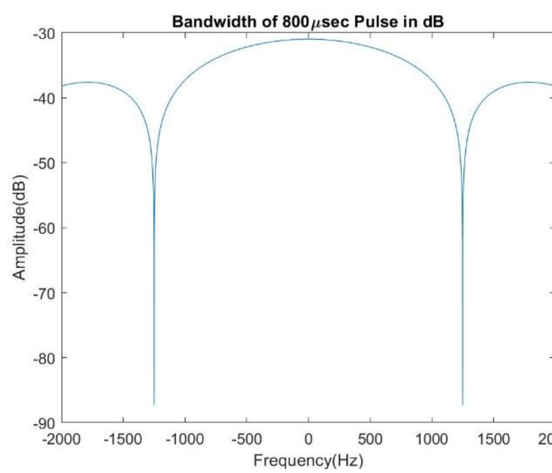

Figure 2-2 - Fourier Transform of Pulse width in Amplitude (dB) vs Frequency (Hz). Nulls in Plot occur at +1250Hz and -1250Hz.

These plots imply that the bandwidth of the pulse observed by the CIA have a bandwidth of about 1250Hz. This value for the bandwidth will factor into the equation to calculate the noise power of the receive antenna that meets the 20 dB specification.

In terms of the Moon, I needed to find the radar cross section value of σ and the distances R_1 and R_2 in the Radar Range Equation. In order to do so, I found two articles that gave me some rough estimates of these physical dimensions. The equation for the radar cross section I found was:

$$\sigma = (0.086) \cdot \pi \cdot a^2 \qquad \text{(Eq. 2)}$$

which, accounts for radius (a) of the Moon being about 1737km and the reflection coefficient from the relative permittivity of the lunar surface, which is comprised of dust-like granular rock [2]. Another article I found during my research states that, "the moon subtends an angle of almost exactly ½ degree or 0.0087 radians, the precise amount depending upon the exact position of the moon in its orbit [3]." This means that any angle dependence in the distance between the moon and earth can be treat as negligible, and the distances R_1 and R_2 can be equated to one another. The article continues and gives a value for the distance to be about $4*10^8$ meters for the maximum distance between earth and the moon [3]. With these pieces of info, the only remaining factors left in the Radar Range Equation are the gains of the transmitter and receiver.

The gains of the antennas can be calculated by either the diameters or by using the effective apertures of the antennas. The equation I used to calculate the gain of the transmitter (Eq. 3) is shown below:

$$G = \frac{4\pi A_{phys} e_a}{\lambda^2} \qquad \text{(Eq. 3)}$$

where, A_{phys} is the physical area of the antenna and e_a is the aperture efficiency. The aperture efficiency ranges between $0 \leq e_a \leq 1$ [6], so I chose to use an average value of about 50% efficiency. Referencing back to the East Siberian article, they gave dimensions of 244m by 20m area for the antenna panel, which gave a value of 4880m^2 for physical aperture and calculating the lowest possible operating frequency of 154MHz gave me a lambda value of 1.95m for wavelength. Plugging all these values in for the transmit antenna, I got a gain value of about 8100 or about 39 dB. For a sanity check, I also calculated the 150 ft Palo Alto dish antenna that was able intercept the Soviet transmission from the CIA article [1]. Converting the units and using the same aperture efficiency and wavelength, I found a reference receive gain to be about 3800 or about 34.4 dB. Once I was able to calculate the noise power of the receive antenna, my objective was to rework the Radar Range Equation to find what the receive gain would be for the minimum size needed to satisfy the 20 dB requirement.

Estimating the noise would prove to be the most complicated measurement out of the whole project and gave me the most trouble. Since the signal is being received after reflecting off the moon, some galactic noise, as well as some man-made noise, needed to be considered when calculating the noise power. The graph below (Figure 3) shows a plot of Noise Factor (dB) vs Frequency (Hz) and includes temperature values on the right-hand side of the y-axis [5].

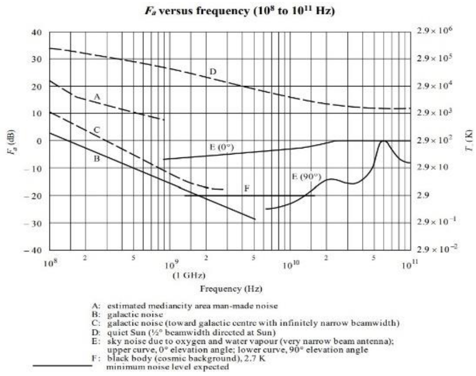

Figure 3 - Noise Factors (dB) and Temperatures (Kelvin) vs Frequency (Hz). The graph is labeled A-F for different noises to consider for the noise factor [5].

For my estimate of the noise temperature that attributed to the noise power, I assumed my receive antenna would pickup galactic noise and man-made noise from the area surrounding my hypothetical receive antenna. Using the chart, I found a noise temperature of about 524088 K around 150MHz on the x-axis, which would factor into (Eq. 4) below to calculate my noise power.

$$P_n = f_a \cdot k \cdot T_a \cdot B \qquad \text{(Eq. 4)}$$

Here, f_a is the noise factor from line C in the chart converted from dB to numerical, which was about 6.3, k is Boltzmann's constant ($1.38*10^{-23}$), T_a is the temperature I estimated above, and B is the bandwidth which I found was 1250Hz from the Fourier Transform of the pulse width observed in the CIA article. Solving for the noise power, I calculated a value of about $5.7*10^{-14}$ watts. Then, since the SNR should be a value of 20 dB, which corresponds to a receive power that's one hundred times the noise power, so $P_r = 5.7*10^{-12}$. Reworking the receive antenna gain term in the Radar Range Equation, I found a value of about 463.7 or about 26.7 dB for the receive gain. Then using this value to solve for the physical aperture area in (Eq. 3) I found a receive antenna area of about $280m^2$, which translates to a diameter of about 13 meters.

For a sanity check, I compared my receive antenna dimensions to the Palo Alto antenna described in the CIA article. The Palo Alto had a diameter of about 46m and physical aperture of $831m^2$, while my hypothetical receive antenna had a diameter of about 13m and physical aperture of $280m^2$. My hypothetical antenna seems to be a reasonable size dish antenna (about 43 ft dish) and is nearly ¼ the size of the Palo Alto antenna. As an overview, I feel as though my values calculated for every term in the Radar Range equation seem reasonable, however I have a slight inclination that my noise temperature value is either too high or my bandwidth value may be too small. I initially was only considering a noise temp of 290K, which gave me a diameter of about 0.8m. This value seemed far to low to be able to receive a signal reflected off the moon. I felt it was appropriate to consider the galactic and man-made noise values from (Fig 3), but those values also don't seem to be within the realm of reasonability. I felt as though my approach in researching each term necessary to complete the Radar Range Equation was the correct process overall.

Citations

[1] “Moon Bounce Elint,” *Central Intelligence Agency*, 04-Aug-2011. [Online]. Available: https://www.cia.gov/library/center-for-the-study-of-intelligence/kent-csi/vol11no2/html/v11i2a05p_0001.htm. [Accessed: 01-Oct-2020].

[2] D. F. Winter, “A theory of radar reflections from a rough moon,” *Journal of Research of the National Bureau of Standards, Section D: Radio Propagation*, vol. 66D, no. 3, pp. 215–221, 1962.

[3] J. Dewitt and E. Stodola, “Detection of Radio Signals Reflected from the Moon,” *Proceedings of the IRE*, vol. 37, no. 3, pp. 229–242, 1949.

[4] “Incoherent Scatter Radar,” *East Siberian Center for the Earth's Ionosphere Research. Incoherent Scatter Radar*. [Online]. Available: http://rp.iszf.irk.ru/esceir/isr/isradaren.htm. [Accessed: 01-Oct-2020].

[5] “International Telecommunication Union (ITU-R),” *Radio Noise*, vol. P.372, no. 13, pp. 1–77, Sep. 2016.

[6] W. Stutzman and G. Thiele, *Antenna Theory and Design*, 3rd ed., New York: Wiley & Sons, 2013.

[7] W. Silver and M. J. Wilson, *The ARRL General Class License Manuel for Ham Radio: All You need to Pass your General Class Exam*. Newington, CT: ARRL, 2011.

MATLAB Code

```
%% This code calculates the Fourier Transform of a Pulse Width and plots results

a = 400*10^-6;
omega = -50000:50000;
freq = omega/(2*pi);
s = 2*a*(sin(a*omega)./(a*omega));

figure(1)
plot(freq, s)
xlim([-2000 2000])
xlabel('Frequency(Hz)')
ylabel('Amplitude')
title('Fourier Transform of 800\musec Pulse')

figure(2)
s_dB = 10*log10(s);
plot(freq, s_dB)
xlim([-2000 2000])
xlabel('Frequency(Hz)')
ylabel('Amplitude(dB)')
title('Bandwidth of 800\musec Pulse in dB')
```

```matlab
%% Calculations for Moon Bounce Project
clear all

T0 = 524088;              % Room temperature in Kelvin
R = 4*10^8;               % Distance from Earth to Moon in meters
f = 154*10^6;             % Frequency array for 154MHz-162GHz range
c = 3*10^8;               % Speed of Light
lambda = c/f;             % Wavelength array for 3m to 3cm range


a = 1.737*10^6;           % radius of the moon in meters
sigma = (0.086)*pi*a^2;  % RCS of Moon in squared meters
k = 1.38*10^-23;          % Boltzmann's Constant in J/K
BW = 1250;              % Bandwidth of Pulsewidth-CIA article in Hz

%Rx Antenna Parameters
ea = .5;                  % Aperture efficiency of antenna
rxDiameter = 46;          % Rx Antenna of 150ft in Dish in Palo Alto in meters
rxRadius = rxDiameter/2; % Radius of 150ft dish in meters
rxAperture = (pi*rxRadius^2)*(ea);
rxGain = ((4*pi*rxAperture)/lambda^2); % Equation for Rx Gain
rxGain_dB = 10*log10(rxGain);

%Tx Antenna Parameters
Pt = 25*10^6;              % Pt is peak power from CIA article
txAperture = (244*20)*ea; % Aperture of Tx Antenna - East Siberia Article
txGain = (4*pi*txAperture)/(lambda^2);
txGain_dB = 10*log10(txGain);

% Radar Range Equation to solve for Pr
variable = ((lambda)/(4*pi*R^2))^2;
Pr = (Pt*sigma*rxGain*txGain*variable)/(4*pi);

%Noise Power of the Rx Antenna
fa = 10^0.8;
Pn = k*T0*BW*fa;
SNR = Pr/Pn;
SNR_dB = 10*log10(SNR);

%If Pr is 100*Pn
Fa = 8;
B = 10*log10(BW);
noisePower = Fa + B - 204;
noisePower = 10^(noisePower/10);


Pr1 = 100*Pn;
Gr = ((Pr1*4*pi)/(sigma*Pt*txGain*variable));
Gr_dB = 10*log10(Gr);
D = (sqrt(Gr)*lambda)/pi
Aphys = (Gr*lambda^2)/(4*pi*ea)
```