Nyquist Stability Criterion

- Derive Nyquist plots from Transfer Functions $G(s)$ where $s = \sigma + j\omega$

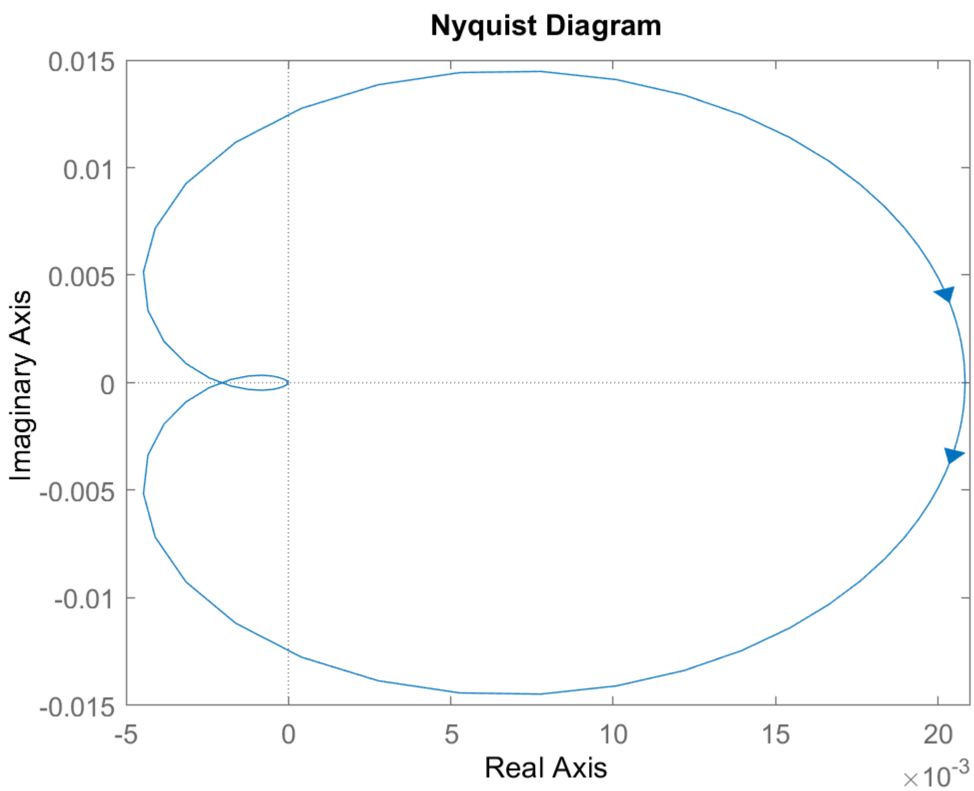
```
s = tf('s');
G = 1/((s+2)*(s+4)*(s+6))
```

```
G =

            1
  ------------------------
  s^3 + 12 s^2 + 44 s + 48

Continuous-time transfer function.
```

```
figure, nyquist(G), xlim([-0.005 .021])
```




```
syms s w
G = 1/((s+2)*(s+4)*(s+6))
```

G =

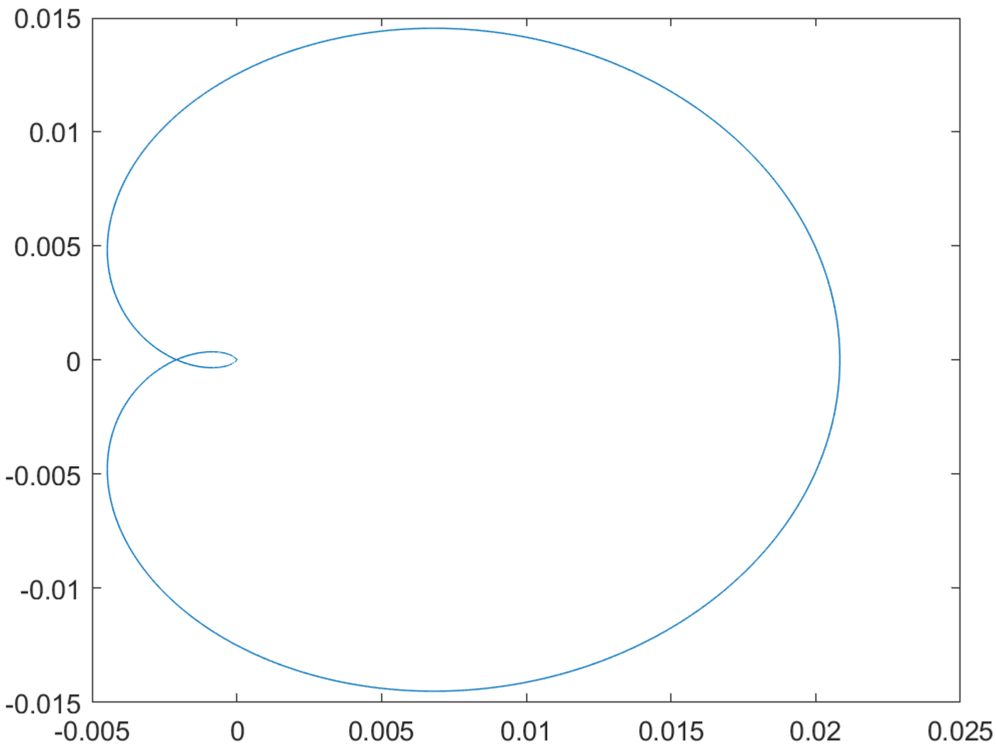
$$\frac{1}{(s+2)\ (s+4)\ (s+6)}$$

```
G_w = subs(G,s,j*w)
```

G_w =

$$\frac{1}{(2+w\,\mathrm{i})\,(4+w\,\mathrm{i})\,(6+w\,\mathrm{i})}$$

```
W = -100:.01:100;
Nyq = eval(subs(G_w,w,W));
x = real(Nyq);
y = imag(Nyq);
figure, plot(x,y)
```



Now we can solve for the Gain margin:

Want to find the points when our imaginary values change sign

```
tmp = sign(y);
d = diff(tmp);
indexes = find(d ~= 0)
```

```
indexes = 1×4
        9337       10000       10001       10664
```

```
points = W(indexes), a = points(4)
```

```
points = 1×4
   -6.6400   -0.0100         0    6.6300
a = 6.6300
```

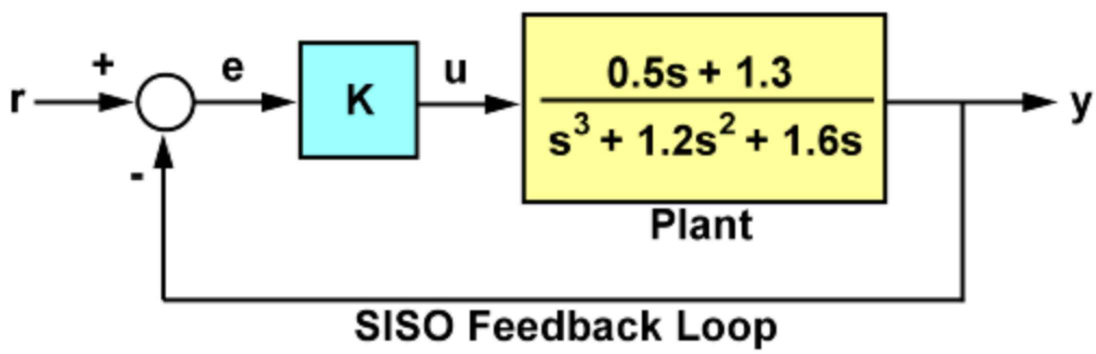
```
GainFactor = 1/abs(x(indexes(4)))
```

```
GainFactor = 479.4830
```

Thus our Gain Factor for this transfer function is about 480

Assessing Gain and Phase Margins

```
f = imread('StabilityExample.png'); imshow(f)
```




```
G = tf([0.5 1.3],[1 1.2 1.6 0])
```

```
G =

      0.5 s + 1.3
  ---------------------
  s^3 + 1.2 s^2 + 1.6 s

Continuous-time transfer function.
```

```
T = feedback(G,1)
```

```
T =

         0.5 s + 1.3
  ---------------------------
  s^3 + 1.2 s^2 + 2.1 s + 1.3

Continuous-time transfer function.
```

```
p = pole(T)
```

```
p = 3×1 complex
  -0.2305 + 1.3062i
  -0.2305 - 1.3062i
  -0.7389 + 0.0000i
```

```
p1mag = abs(p(1)), p1pha = angle(p(1))*(180/pi)
```

```
p1mag = 1.3264
p1pha = 100.0092
```

```
p2mag = abs(p(2)), p2pha = angle(p(2))*(180/pi)
```

```
p2mag = 1.3264
p2pha = -100.0092
```

```
p3mag = abs(p(3)), p3pha = angle(p(3))*(180/pi)
```

```
p3mag = 0.7389
p3pha = 180
```

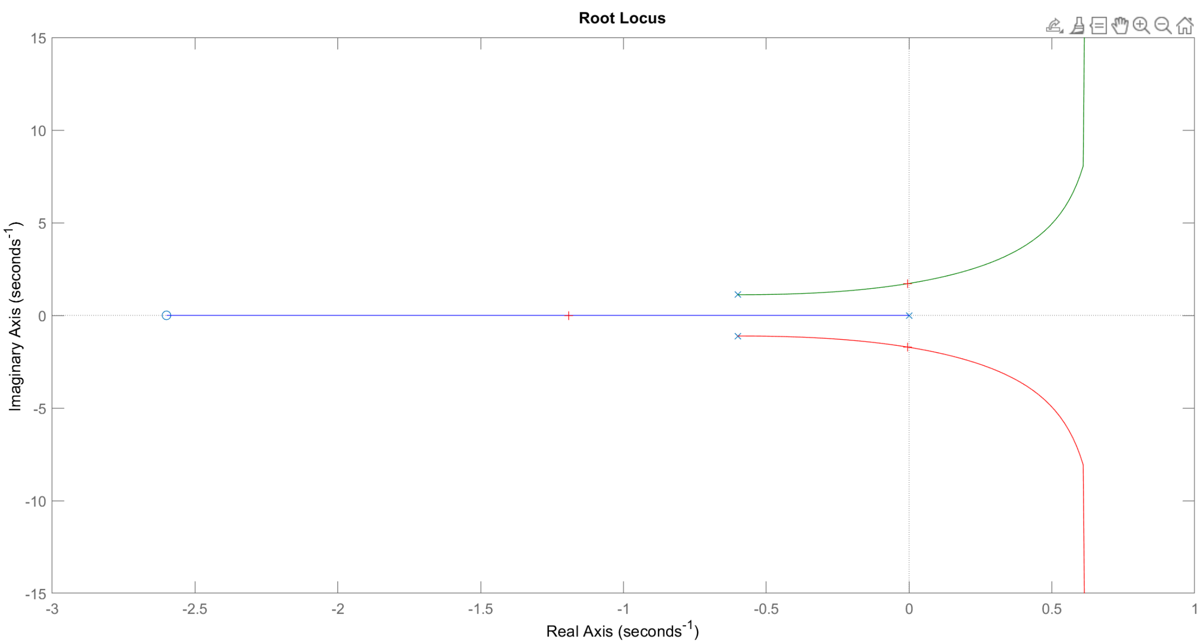
Root Locus of G:

```
rlocus(G)
```

Using the Root Locus, we want to assess how much the gain can change before stability to the system is lost

```
[k,poles] = rlocfind(G)
```

```
Select a point in the graphics window
```




```
selected_point = 0.0005 + 1.7105i
k = 2.6869
```

```
poles = 3×1 complex
  -0.0045 + 1.7125i
  -0.0045 - 1.7125i
  -1.1910 + 0.0000i
```

Clicking on the imaginary axis of the graph we see that when $k = 2.7$ is when the system becomes unstable.

Thus our gain can be within the range of $0 < k < 2.7$

Gain and Phase Margins

The phase margin measures how much phase variation is needed at the gain crossover frequency to lose stability. Similarly, the gain margin measures what relative gain variation is needed at the gain crossover frequency to lose stability. Together, these two numbers give an estimate of the "safety margin" for closed-loop stability. The smaller the stability margins, the more fragile stability is

```
bode(G), grid, margin(G)
```