Feedback Amplifier Design

This script shows the designing process for a non-inverting feedback amplifier using the Control Systems Toolbox.

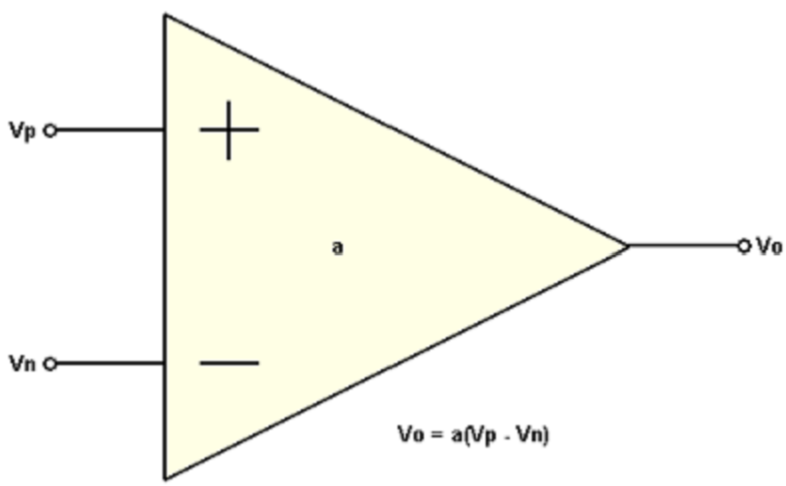
The design is based on the operational amplifier (op-amp), which is a standard buidling block in electrical circuits

- An operational amplifier is a modern implementation of Harold Black's feedback amplifier and is a universal component that is widely used for instrumentation, control, and communication
- Very versatile as any linear system can be implemented by combining operational amplifiers with resistors and capacitors, most notely, analog proportional-integral-derivative controllers

Op-Amp Description

```
clc; close all; clear;
f = imread('OpAmp.png'); imshow(f), title('Non-Inverting Op-Amp')
```

Non-Inverting Op-Amp



The Open-Loop Transfer Function can be described as:

$$A(s) = \frac{A_0}{(1 + s/\omega_1)(1 + s/\omega_2)}$$

The value of A_0 is ideally ∞ but typically within the range of $10^6 - 10^8$ in most practical Op Amp ICs

Assume Values and Create Transfer Function

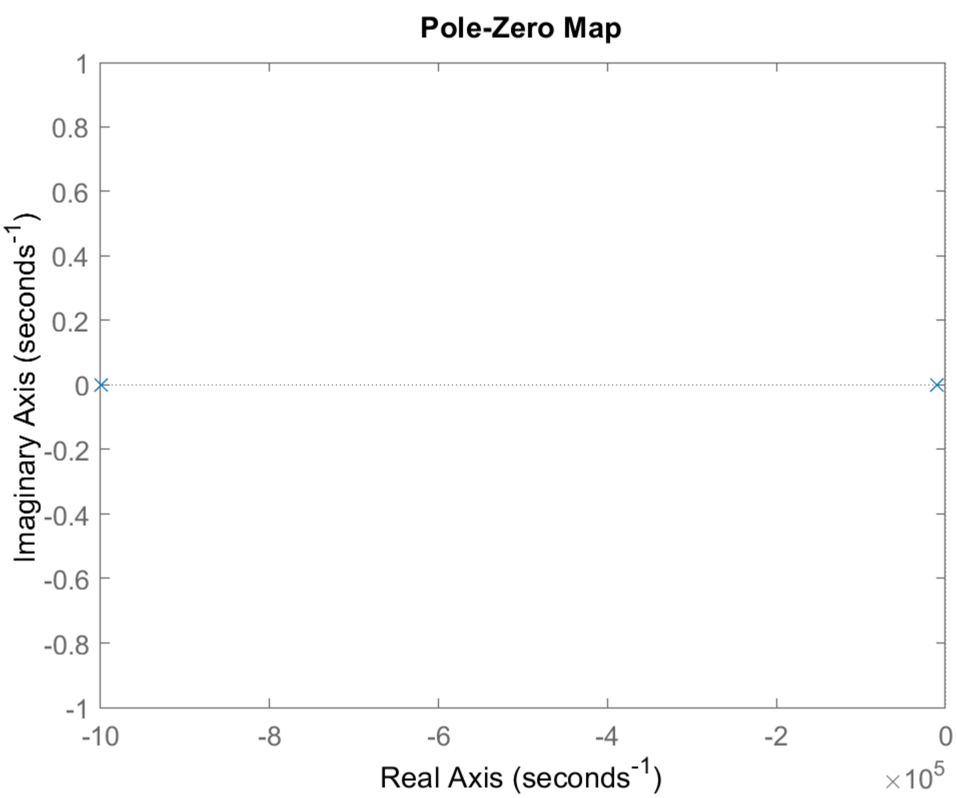
```
A0 = 1e5; w1 = 1e4; w2 = 1e6;
s = tf('s');
```

```
A_open = A0/(1 + s/w1)/(1 + s/w2)
```

```
A_open =

          1e15
  ----------------------
  s^2 + 1.01e06 s + 1e10

Continuous-time transfer function.
```

Plot Pole-Zero Map

```
figure, pzmap(A_open)
```

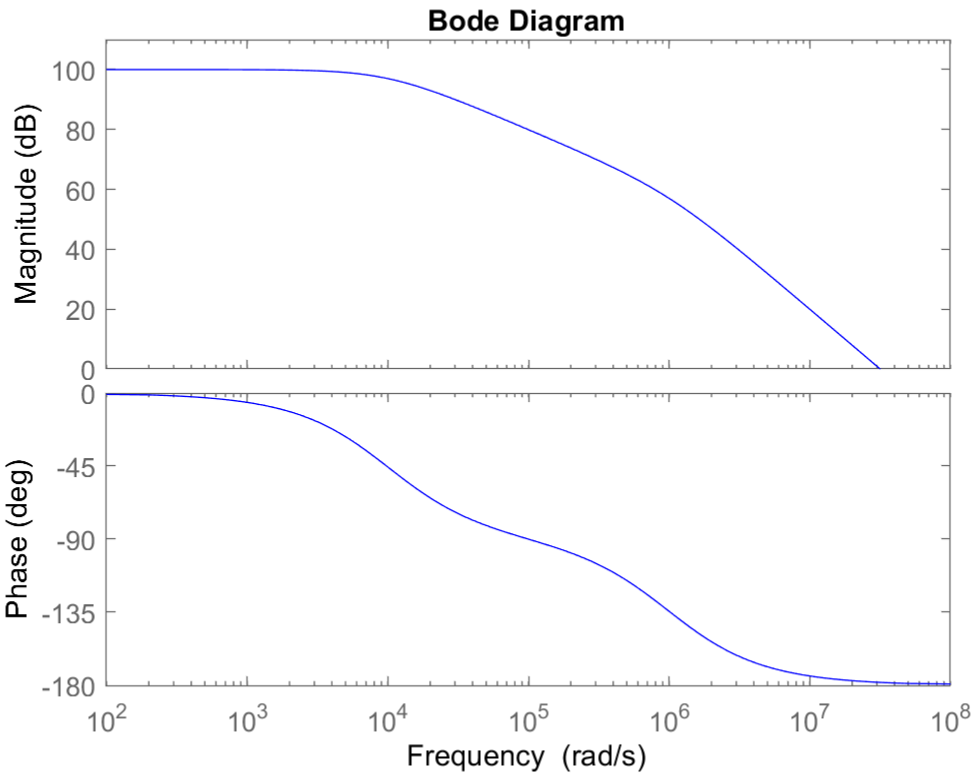


Transfer Function has two real poles at

$\omega_1 = 10,000$ and $\omega_2 = 1,000,000$

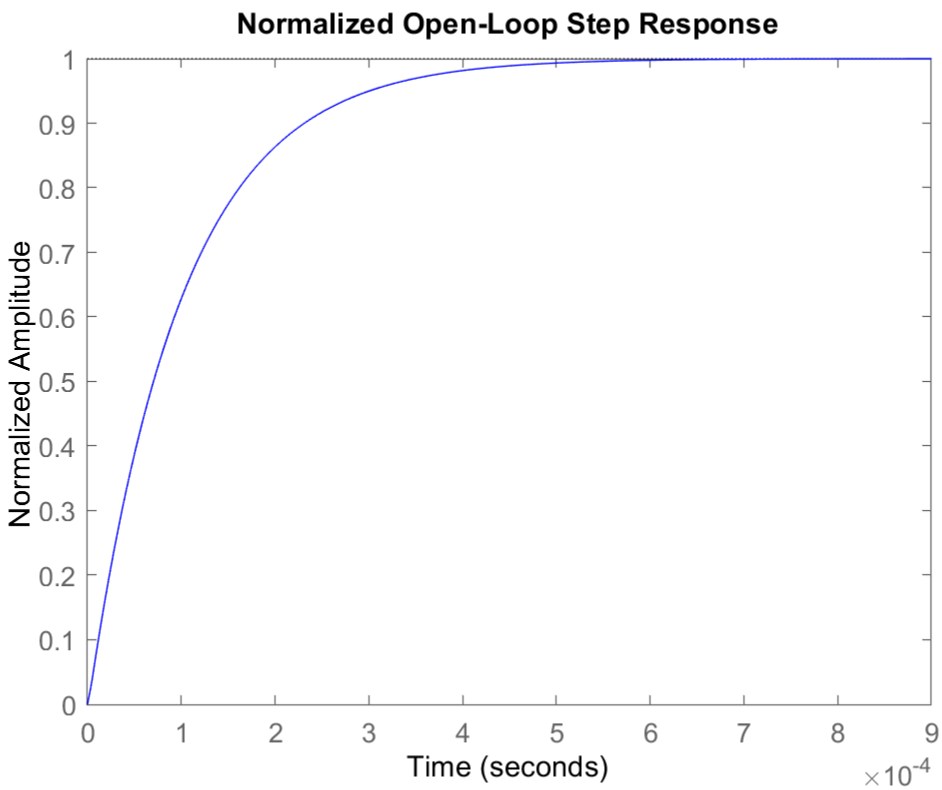
Display Bode Plot for Transfer Function

```
h = bodeplot(A_open,'b');
setoptions(h,'FreqUnits','rad/s','MagUnits','dB','PhaseUnits','deg',...
    'YLimMode','Manual','YLim',{[0,110],[-180,0]});
```



View the Normalized Open-Loop Step Response of System

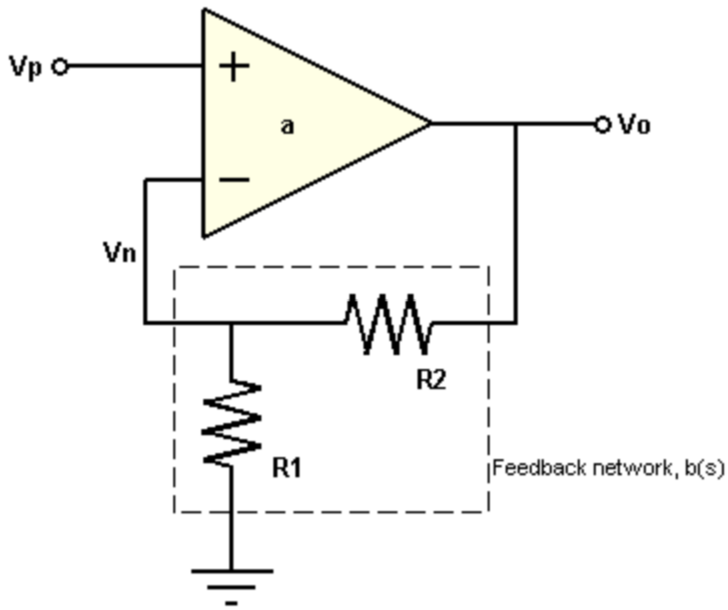
```
A_norm = A_open / dcgain(A_open);
stepplot(A_norm,'b')

title('Normalized Open-Loop Step Response');
ylabel('Normalized Amplitude');
```



Feedback Amplifier

```
f = imread('FeedbackAmplifier.png'); imshow(f), title('Feedback Amplifier')
```

Feedback Amplifier



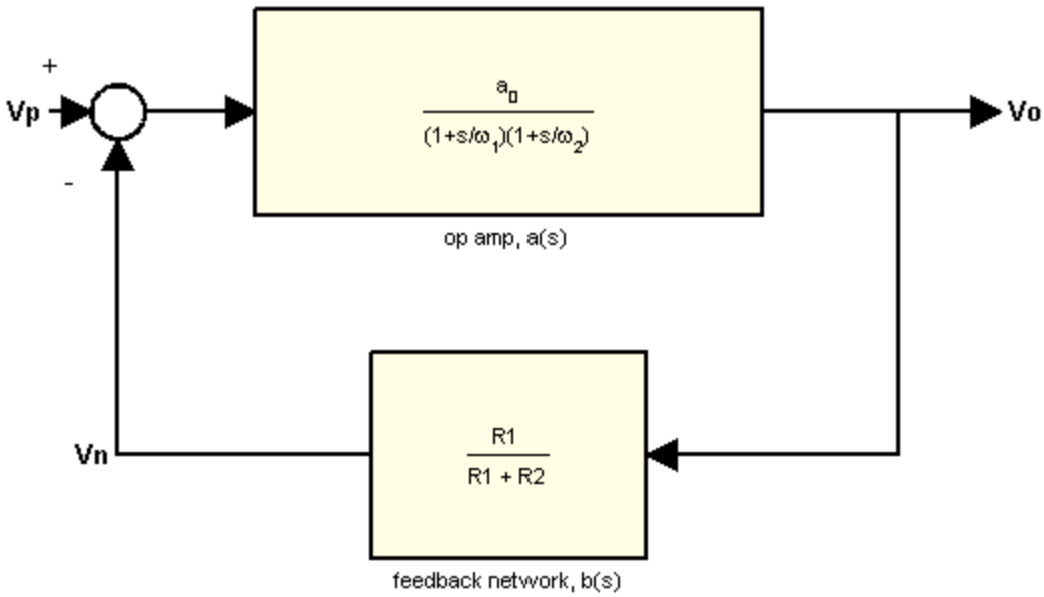
The Feedback network, $B(s)$ is voltage divider between input V_n and output V_o . Solving for this ratio gives the transfer function:

$$B = \frac{V_o}{V_n} = \frac{R_1}{R_1 + R_2}$$

The Closed-Loop Diagram:

```
f = imread('ClosedLoopGain.png'); imshow(f), title('Closed-Loop Block Diagram')
```

Closed-Loop Block Diagram



The ratio of the closed-loop gain would then be

$$A = \frac{V_o}{V_p} = \frac{A}{1 + AB}$$

However, if the product, AB, is extremely large, then the gain can be approximated too

$$A = \frac{1}{B}$$

Design Amplifier with DC Gain of 10 with $R_1 = 10k\Omega$. Solve for R_2 :

```
A0 = 10; B = 1/A0;
R1 = 1e4; R2 = R1 * (1/B - 1)
```

```
R2 = 90000
```

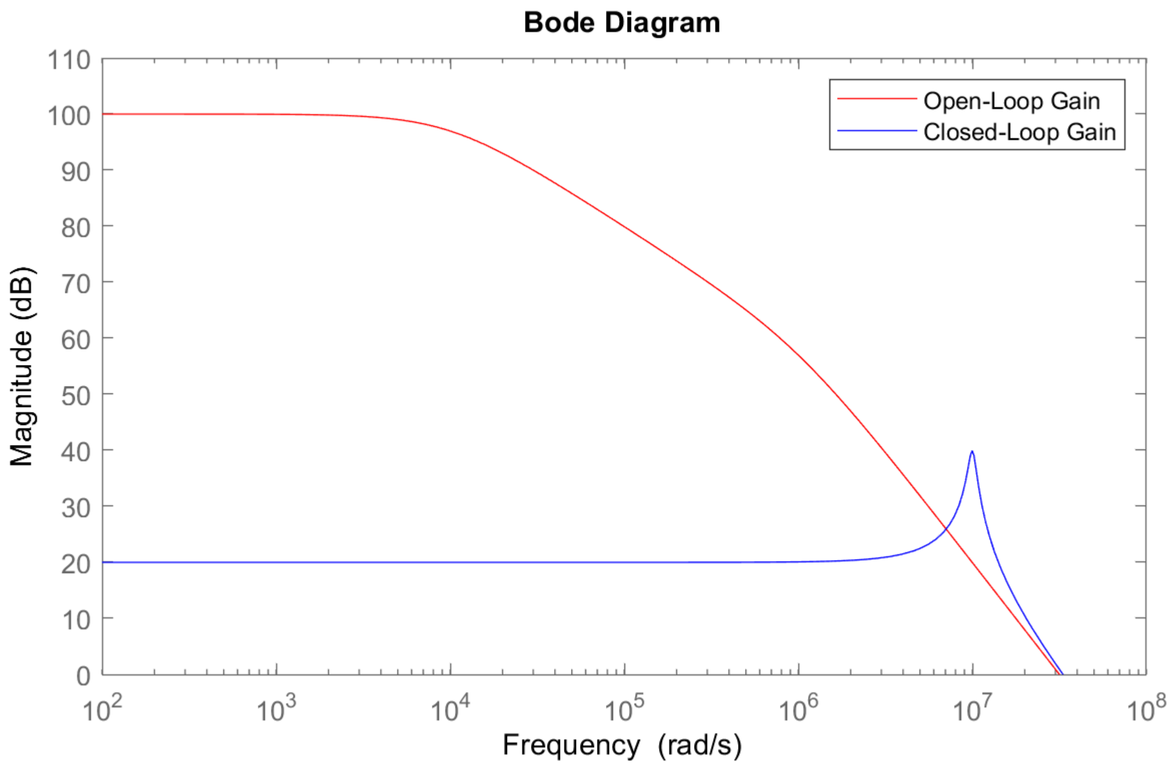
Thus, $R_2 = 90k\Omega$ would produce the desired Gain of $A = 10V/V$

Construct Closed-Loop System using Feedback() Function

```
A_closed = feedback(A_open,B)
```

```
A_closed =

           1e15
  ----------------------
  s^2 + 1.01e06 s + 1e14

Continuous-time transfer function.
```

```
bodemag(A_open,'r',A_closed,'b');
legend('Open-Loop Gain','Closed-Loop Gain')
ylim([0,110]);
```



Observations:

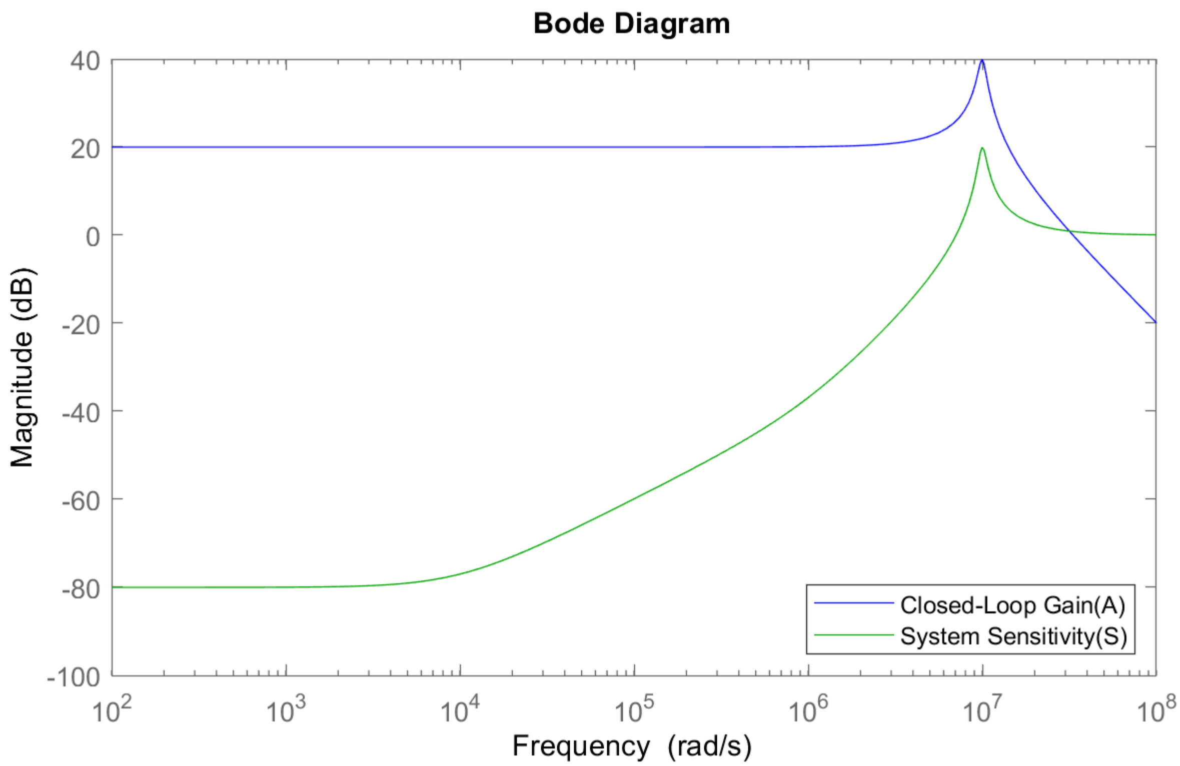
- We can see that the Closed-Loop Gain is much lower for Low Frequencies, but the bandwidth has increased as a trade-off
- This Gain vs Bandwidth trade-off presents a powerful tool when designing feedback circuits
- Presents a drastic reduction in sensitivity that allows to make precise systems from uncertain components
- Feedback can be used to trade high gain and low robustness for low gain and high robustness

Sensitivity of the Gain to Variations

```
% Define Loop gain - total gain a signal experiences traveling around the
% loop
L = A_open*B;
S = 1/(1 + L);
S = feedback(1,L)
```

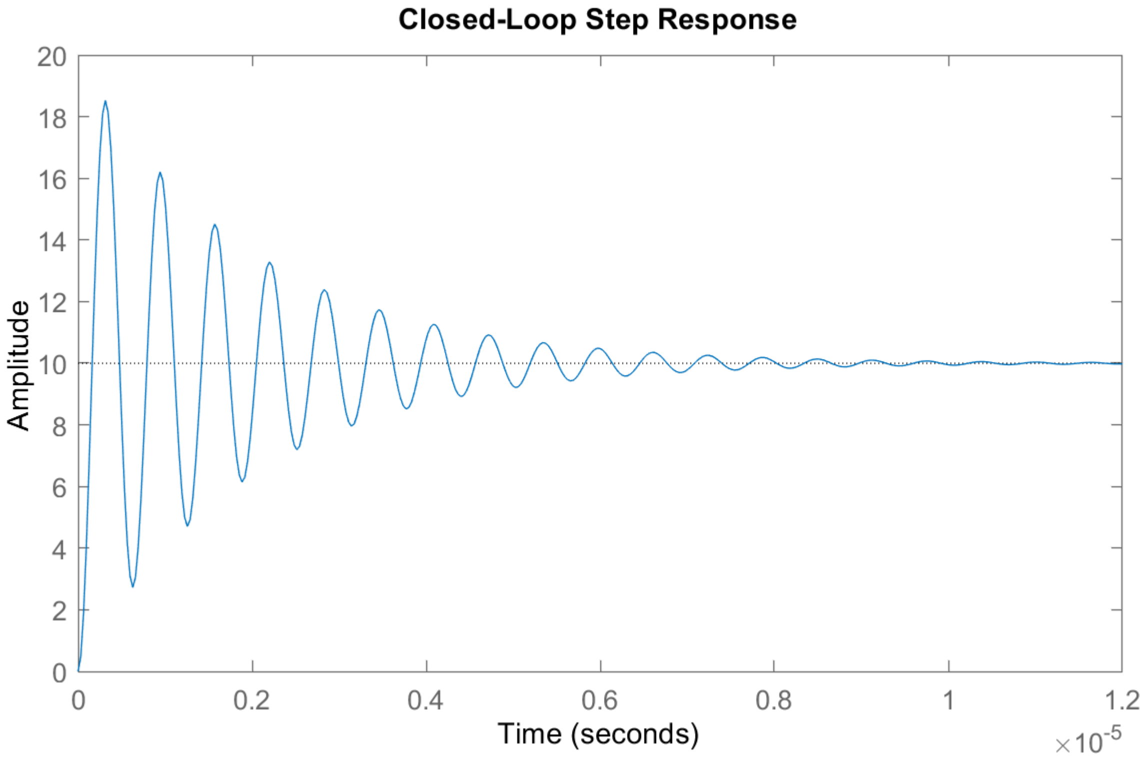
```
S =

  s^2 + 1.01e06 s + 1e10
  ----------------------
  s^2 + 1.01e06 s + 1e14

Continuous-time transfer function.
```

```
bodemag(A_closed,'b',S,'g')
legend('Closed-Loop Gain(A)', 'System Sensitivity(S)','Location','SouthEast')
```



The very small low-frequency sensitivity (-80 dB) indicates a design whose closed-loop gain suffers minimally from open-loop gain variation

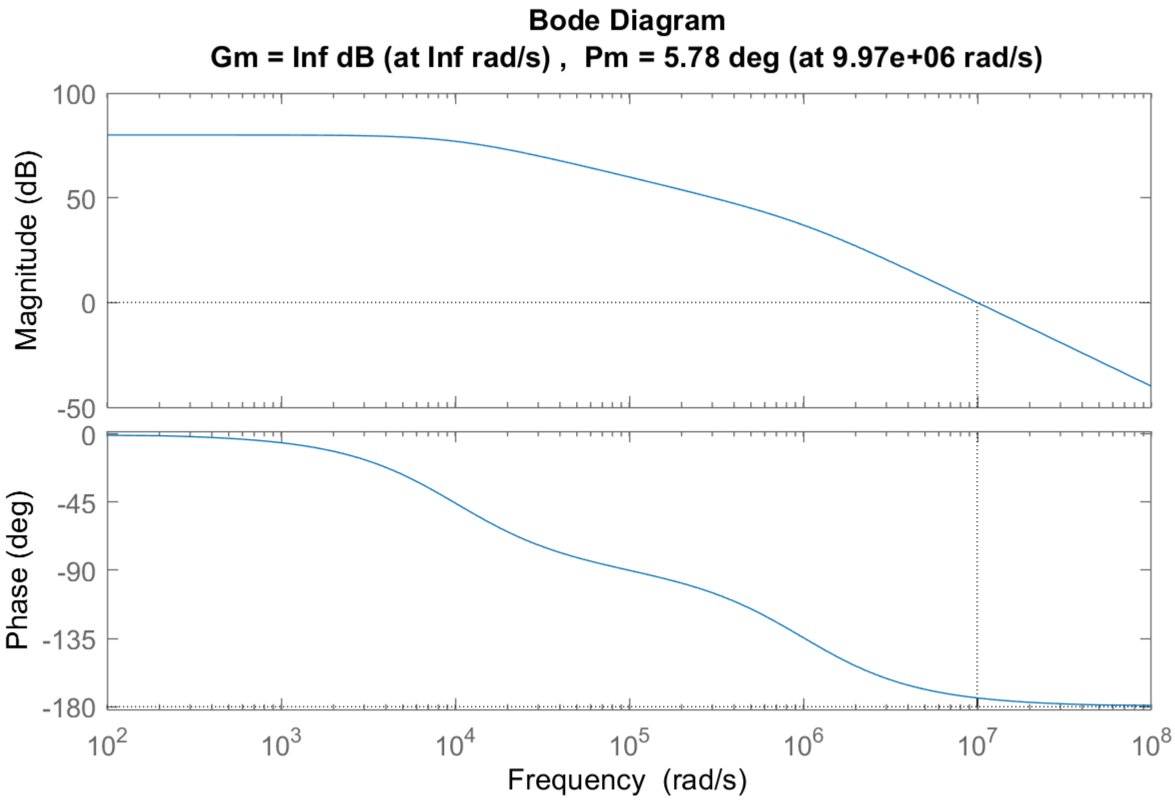
```
stepplot(A_closed), title('Closed-Loop Step Response')
```



Note that the use of feedback has greatly reduced the settling time (by about 98%). However, the step response now displays a large amount of ringing, indicating poor stability margin.

You can analyze the stability margin by plotting the loop gain, L(s)

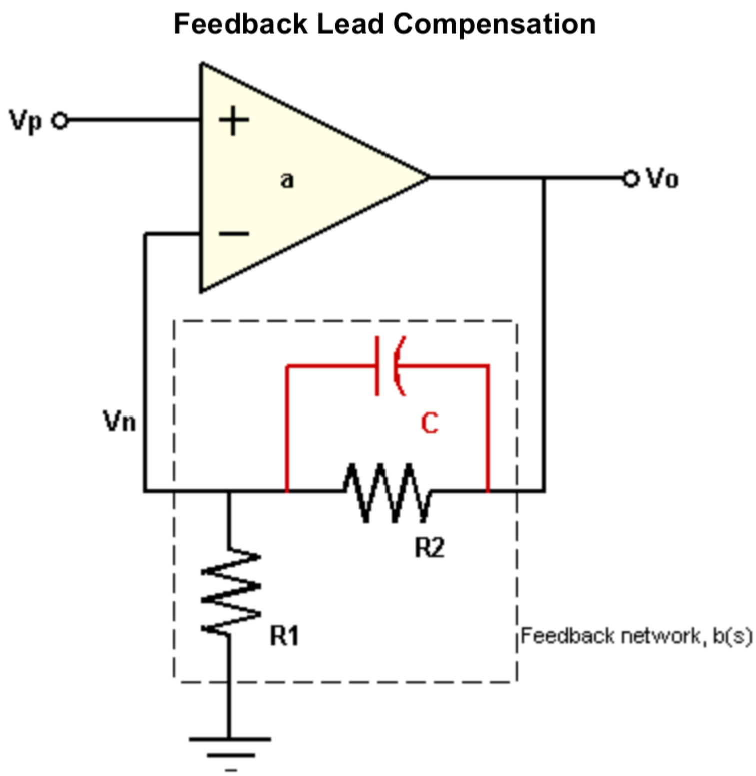
```
margin(L)
```



The resulting plot indicates a phase margin of less than 6 degrees. You will need to compensate this amplifier in order to raise the phase margin to an acceptable level (generally 45 deg or more), thus reducing excessive overshoot and ringing.

Feedback Lead Compensation

```
f = imread('FeedbackCompensation.png'); imshow(f), title('Feedback Lead Compensation')
```

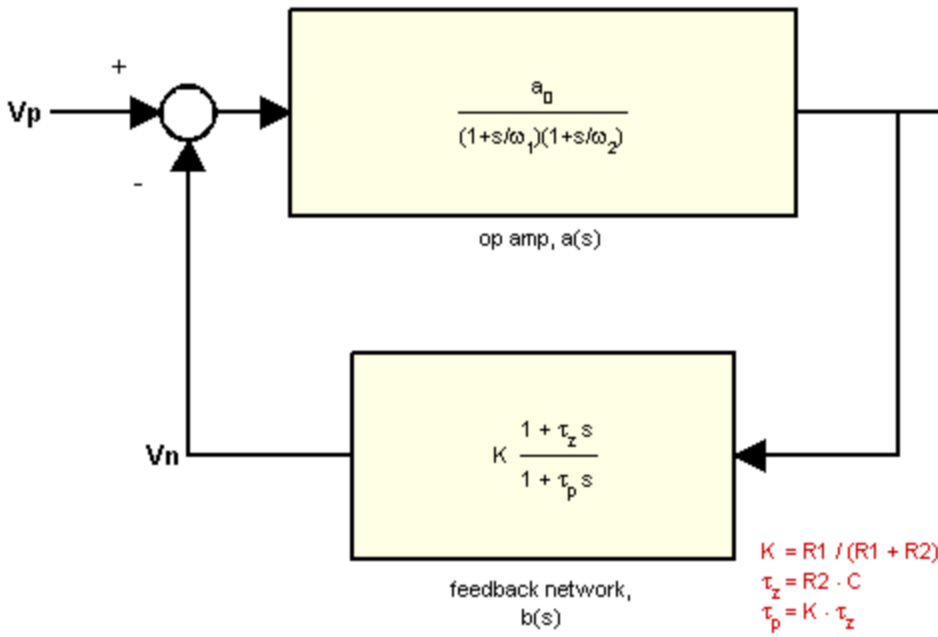


A commonly used method of compensation in this type of circuit is "feedback lead compensation". This technique modifies b(s) by adding a capacitor, C, in parallel with the feedback resistor, R2

The capacitor value is chosen so as to introduce a phase lead to b(s) near the crossover frequency, thus increasing the amplifier's phase margin.

```
f = imread('FeedbackCompensationBlockDiagram.png'); imshow(f), title('Closed-Loop Diagram')
```

Closed-Loop Diagram



You can approximate a value for C by placing the zero of b(s) at the 0dB crossover frequency of L(s):

```
[Gm,Pm,Wcg,Wcp] = margin(L);
C = 1/(R2*Wcp)
```

```
C = 1.1139e-12
```

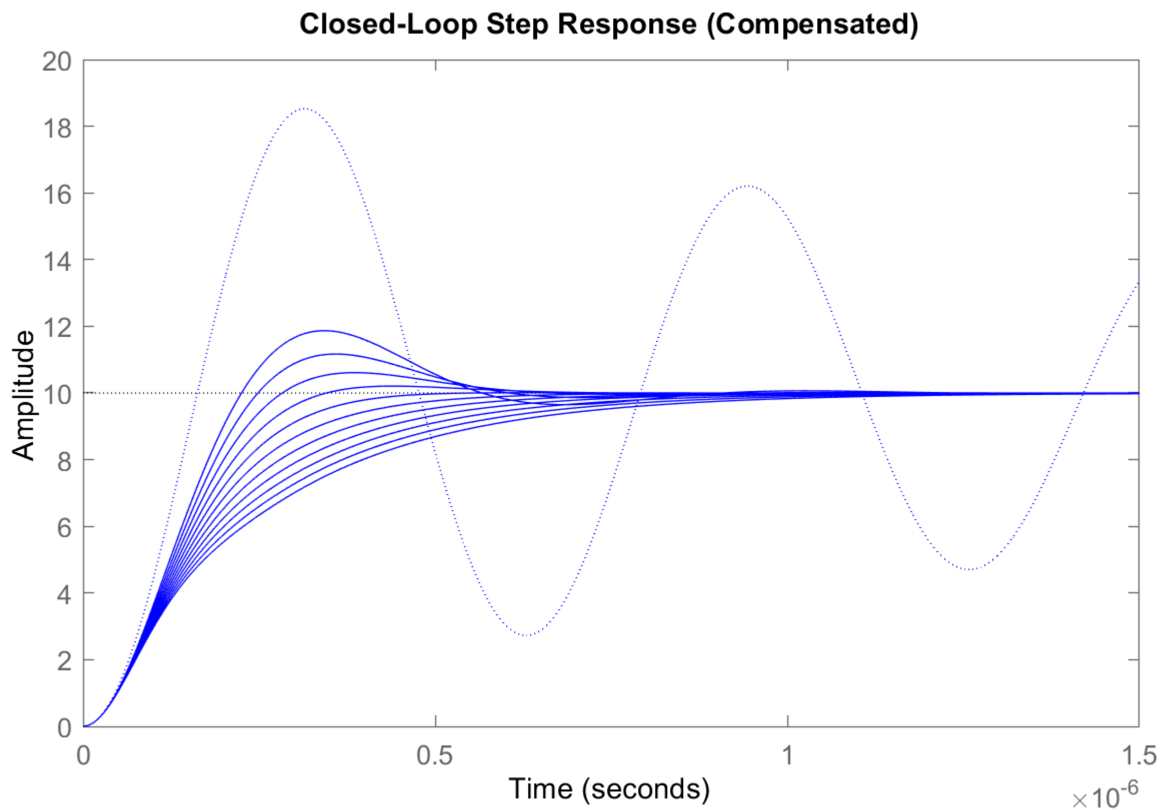
```
K = R1/(R1+R2);
C = [1:1/5:3]*1e-12;
for n = 1:length(C)
    B_array(:,:,n) = tf([K*R2*C(n) K],[K*R2*C(n) 1]);
end
```

Now you can create LTI arrays for A(s) and L(s):

```
A_array = feedback(A_open,B_array);
L_array = A_open*B_array;
```

Plot the step response for multiple values of the Capacitor:

```
stepplot(A_closed,'b:',A_array,'b',[0:.005:1]*1.5e-6);
title('Closed-Loop Step Response (Compensated)');
```

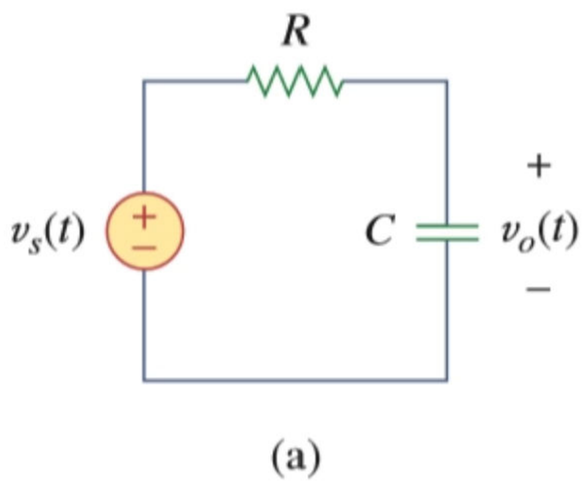


RC, RL, and RLC Circuits and their Transfer Functions

Simple RC Circuit

```
f = imread('simpleRC1.png'); figure, imshow(f), title('Simple RC Circuit')
```

Simple RC Circuit



Here, we examine the Transfer functions of different types of filters. We'll start with first-order circuits (RC or RL), then progress to second-order circuits (RLC) and examine their transfer functions

The Transfer function, $H(s)$ can be obtained from the RC circuit by:

$H(s) = \frac{V_o}{V_{in}} = \frac{1/sC}{R + sC}$ or in terms of $\omega \to H(\omega) = \frac{1/j\omega C}{R + j\omega C}$

Let us create a simple example where $R = 1k\Omega$ and $C = 3\mu F$

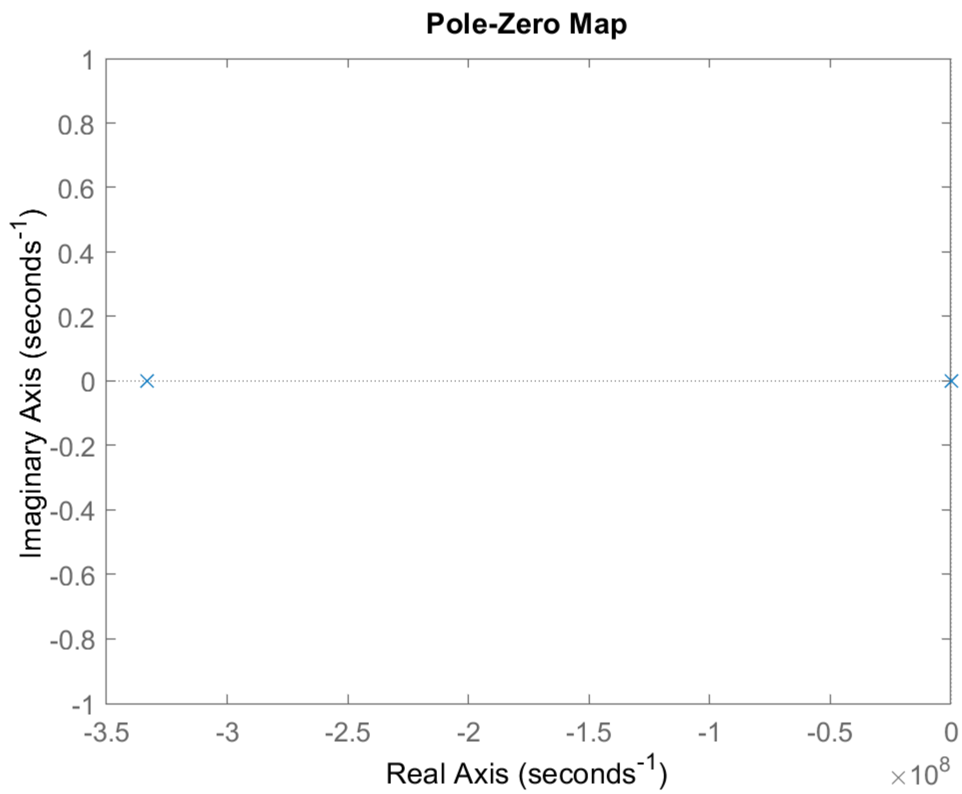
```
R = 1000; C = 3e-6; s = tf('s');
H_RC = (1/(s*C))/(R + s*C), p = roots([9e-12 0.003 0])'
```

```
H_RC =

           1
  -------------------
  9e-12 s^2 + 0.003 s

Continuous-time transfer function.
p = 1×2
10^8 ×
         0   -3.3333
```

This Transfer function has poles at $p_1 = 0 \text{ and } p_2 = -1,000,000,000$

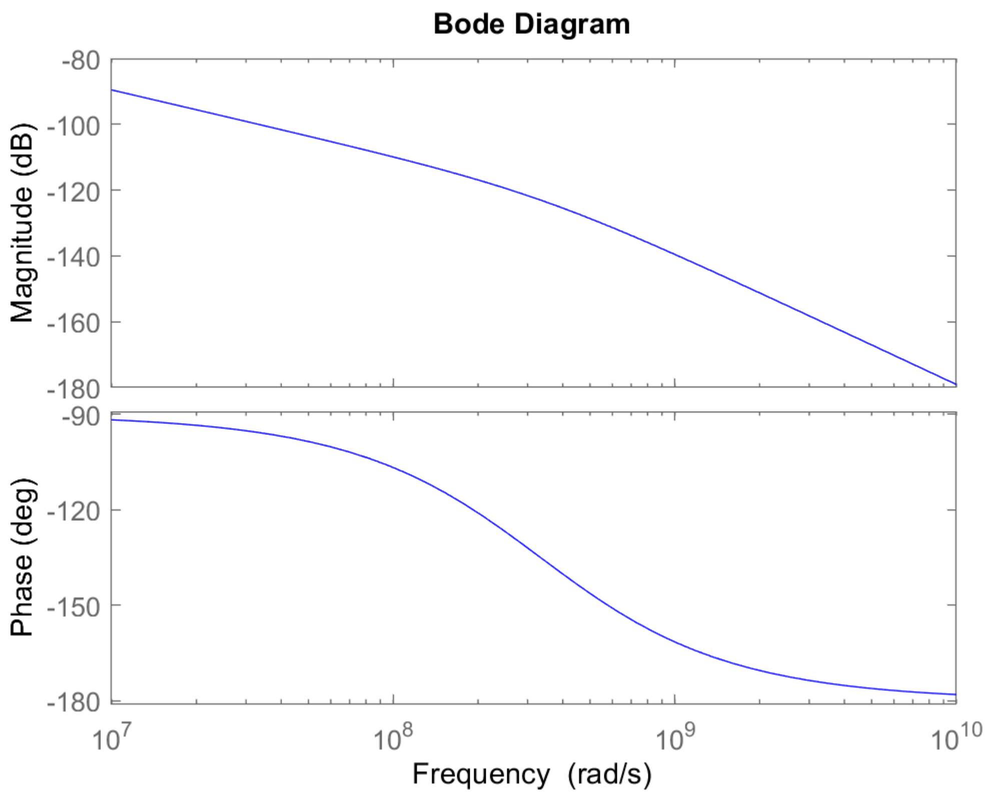
Pole-Zero Map

```
figure, pzmap(H_RC)
```



Bode Plots

```
figure, RC_bode = bodeplot(H_RC,'b');
```

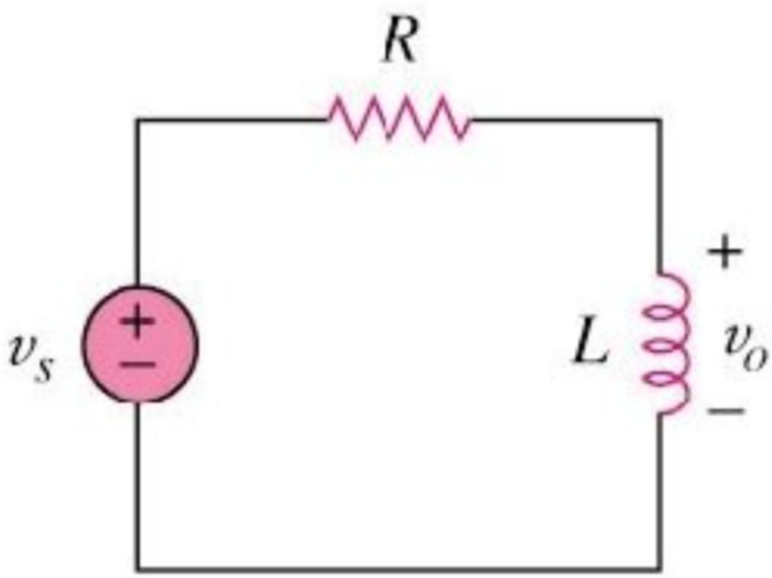


This Transfer Function resembles that of a Lowpass Filter

Simple RL Circuit

```
f = imread('simpleRL.png'); figure, imshow(f), title('Simple RL Circuit')
```

Simple RL Circuit



The Transfer function, $H(s)$ can be obtained from the RL circuit by:

$H(s) = \frac{V_o}{V_{in}} = \frac{sL}{R + sL}$ or in terms of $\omega \rightarrow H(\omega) = \frac{j\omega L}{R + j\omega L}$

Let us create a simple example where $R = 1k\Omega$ and $L = 3mH$

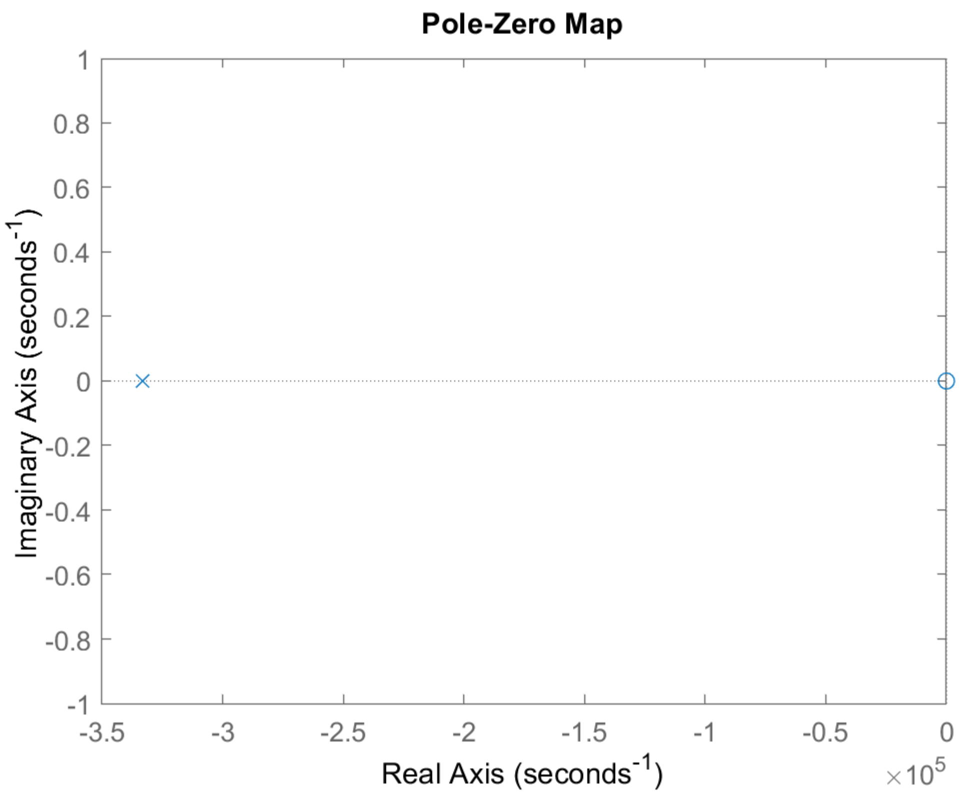
```
R = 1000; L = 0.003;
H_RL = (s*L)/(R + s*L), z = roots([0.003 0]), p = roots([0.003 1000])
```

```
H_RL =

      0.003 s
  --------------
  0.003 s + 1000

Continuous-time transfer function.
z = 0
p = -3.3333e+05
```

This Transfer function has a single pole at $p = -3.33e5$ and single zero at $z = 0$

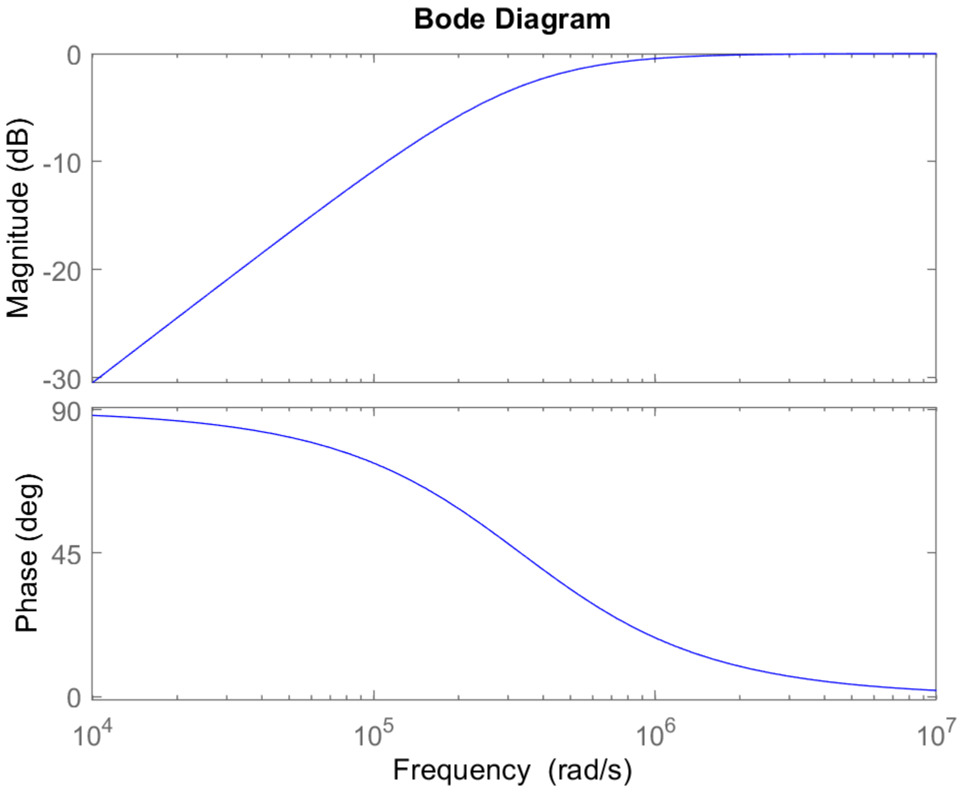
Pole-Zero Map

```
figure, pzmap(H_RL)
```



Bode Plots

```
figure, RL_bode = bodeplot(H_RL,'b');
```



This Transfer function resembles that of a Highpass Filter