HW #4

Solution 1:

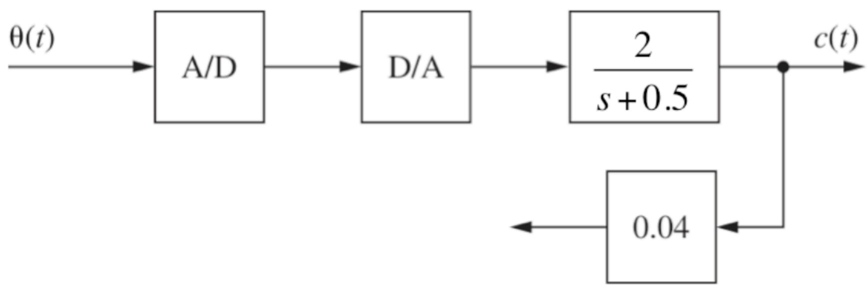
(d) $$E(s) = \frac{s+2}{s^2(s+1)} = \frac{2}{s^2} + \frac{-1}{s} + \frac{1}{s+1}, \text{ from Problem 4-3(d)}$$

$$E(z,m) = 2\left[\frac{mT}{z-1} + \frac{T}{(z-1)^2}\right] - \frac{1}{z-1} + \frac{\varepsilon^{-mT}}{z-\varepsilon^{-T}}$$

(f) $$E(s) = \frac{2}{(s+1)^2 + 2^2}; \qquad E(z,m) = \frac{2}{2}\left[\frac{\varepsilon^{-mT}[z\sin(2mT) + \varepsilon^{-T}\sin((1-m)T)]}{z^2 - 2z\varepsilon^{-T}\cos 2T + \varepsilon^{-2T}}\right]$$

Solution 2:

(a)



(b) $$G(z) = \frac{z-1}{z}\mathfrak{z}\left[\frac{2}{s(s+0.5)}\right] - \frac{z-1}{z}\frac{2}{0.5}\left[\frac{z(1-\varepsilon^{-0.5T})}{(z-1)(z-\varepsilon^{-0.5T})}\right]$$

$$= \frac{4(1-\varepsilon^{-0.5T})}{z-\varepsilon^{-0.5T}}$$

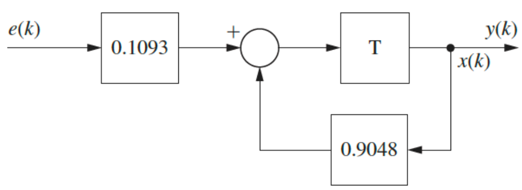
Solution 3:

(a) $(s+0.05)Y(s) = 0.1M(s), \;\; \therefore G_p(s) = \dfrac{0.1}{s+0.05}$

$$G(z) = \frac{Y(z)}{E(z)} = \frac{z-1}{z}\mathcal{Z}\left[\frac{0.1}{s(s+0.05)}\right]$$

$$= \frac{z-1}{z}\left(\frac{0.1}{0.05}\right)\frac{z(1-\varepsilon^{-(0.05)(2)})}{(z-1)(z-0.9048)} = \frac{0.1903}{z-0.9048}$$

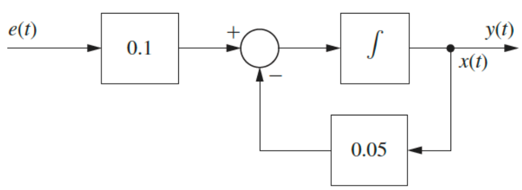
(b)



$$x(k+1) = 0.9048x(k) + 0.1093e(k)$$

$$y(k) = x(k)$$

(c)



$$\dot{x}(t) = -0.05x(t) + 0.1e(t)$$
$$y(t) = x(t)$$

(d) $(s\mathbf{I}-\mathbf{A})^{-1} = \dfrac{1}{s+0.05}, \;\therefore \mathbf{\Phi}(t) = \varepsilon^{-0.05t}$

$$\therefore \mathbf{A} = \mathbf{\Phi}(T) = 0.9048$$

$$\mathbf{B} = \mathbf{B}_c\int_0^2 \mathbf{\Phi}(\tau)d\tau = \mathbf{B}_c\int_0^2 \varepsilon^{-0.05\tau}d\tau$$

$$= \frac{0.1}{-0.05}\varepsilon^{-0.05\tau}\Big|_0^2 = 2[1-0.9048] = 0.1903$$

$$\therefore x(k+1) = 0.9048x(k) + 0.1093e(k)$$
$$y(k) = x(k)$$

(e) Same as (b).

(f) $G(z) = \dfrac{0.1093z^{-1}}{1-0.9048z^{-1}} = \dfrac{0.1093}{z-0.9048}$

MATLAB:

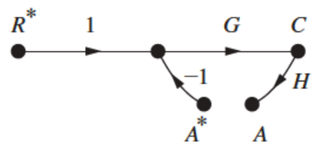
```
num=[0 0.1];

den=[1 0 .05];

[Ac,Bc,C,D]=tf2ss(num,den)

[A,B]=c2d(Ac,Bc,2)

[n,d]=ss2tf(A,B,C,D)

pause

Ac = –0.5; BC = 0.1;

[A,B]=c2d(Ac,Bc,2)
```

Solution 4:

(a) $C(s) = G(s)\left[R^*(s) - H^*(s)\, C^*(s)\right]$

$$\therefore C(z) = \frac{G(z)}{1 + G(z)H(z)} R(z)$$

(b)



$$A = GH\left[R^* - A^*\right]$$

$$\therefore A^* = \frac{\overline{GH}^* R^*}{1 + \overline{GH}^*}$$

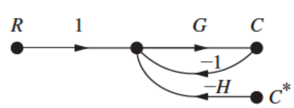
$$\therefore C = G\left[R^* - A^*\right] = G\left[\frac{R^*}{1 + \overline{GH}^*}\right]$$

$$\therefore C(z) = \frac{G(z)}{1 + \overline{GH}(z)} R(z)$$

(c) $E = R - GHE^* \Rightarrow E^* = R^* - \overline{GH}^* E^* \Rightarrow E(z) = \dfrac{R(z)}{1 + \overline{GH}(z)}$

$$C = GE^* \Rightarrow C(z) = \frac{G(z)}{1 + \overline{GH}(z)} R(z)$$

(d)

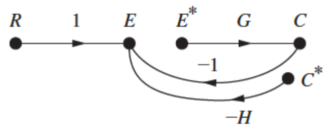


$$C = \frac{GR}{1+G} - \frac{GH}{1+G} C^*$$

$$C(z) = \left[\frac{GR}{1+G}\right](z) - \left[\frac{GH}{1+G}\right](z)\, C(z)$$

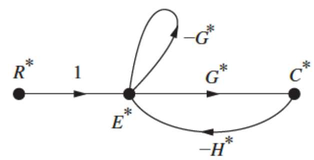
$$\therefore C(z) = \frac{\left[\dfrac{GR}{1+G}\right](z)}{1 + \left[\dfrac{GH}{1+G}\right](z)}$$

(e)



$$E = R - GE^* - HC^*$$
$$C = GE^*$$

$$\left.\begin{aligned} E^* &= R^* - G^*E^* - H^*C^* \\ C^* &= G^*E^* \end{aligned}\right\}$$



$$\therefore C(z) = \frac{G(z)}{1 + G(z) + G(z)H(z)} R(z)$$