ECE 5610 0, Fall 2020, Sample First Exam, 100 minutes

Name:______________________________ Signature: ______________________________

Instructions:

1. This is a closed–book test but **one** $8\frac{1}{2} \times 11$ ~~single~~–sided cheat–sheet is allowed.
2. Work as many problems as you can. Try not to spend too much time working on a single problem. If you get stuck, try working on a different question.
3. Show all your work, but try to be as concise as possible.
4. • **DO NOT LOOK** at the problems until told to do so.
5. • **STOP** working after the "time's up" announcement.
6. • **GOOD LUCK !**

(1) -- 10 point

Find $E^*(s)$ for

$$E(s) = \frac{1 - \varepsilon^{-Ts}}{s(s + 1)}$$

Solution:

$$E(s) = \frac{1-\varepsilon^{-Ts}}{s(s+1)}; \text{ define } E_1(s) = \frac{1}{s(s+1)}$$

$$\therefore E_1^*(s) = \left.\frac{1}{(\lambda+1)(1-\varepsilon^{-T(s-\lambda)})}\right|_{\lambda=0} + \left.\frac{1}{\lambda(1-\varepsilon^{-T(s-\lambda)})}\right|_{\lambda=-1}$$

$$= \frac{1}{1-\varepsilon^{-Ts}} - \frac{1}{1-\varepsilon^{-T(s+1)}}$$

$$E(s) = E_1(s) - E_1(s)\varepsilon^{-Ts}; \therefore E^*(s) = E_1^*(s) - E_1^*(s)\varepsilon^{-Ts}$$

$$= \left[\frac{1}{1-\varepsilon^{-Ts}} - \frac{1}{1-\varepsilon^{-T(s+1)}}\right](1-\varepsilon^{-Ts}) = \frac{\varepsilon^{-Ts}(1-\varepsilon^{-T})}{1-\varepsilon^{-T(s+1)}}$$

(2) – 30 point

Find $E^*(s)$ for each of the following functions. Express $E^*(s)$ in closed form.

(a) $e(t) = \varepsilon^{at}$ (b) $E(s) = \dfrac{\varepsilon^{-2Ts}}{s - a}$

(c) $e(t) = \varepsilon^{a(t-2T)}u(t - 2T)$ (d) $e(t) = \varepsilon^{a(t-T/2)}u(t - T/2)$

Solution:

(a) $E^*(s) = 1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \text{L} \;= 1 + \varepsilon^{(a-s)T} + [\varepsilon^{(a-s)T}]^2 + \text{L}$

$$= \frac{1}{1 - \varepsilon^{(a-s)T}}$$

(b) $e(t) = \varepsilon^{a(t-2T)}u(t - 2T)$

$$E^*(s) = \varepsilon^{-2Ts} + \varepsilon^{aT}\varepsilon^{-3Ts} + \varepsilon^{2aT}\varepsilon^{-4Ts} + \cdots$$

$$= \varepsilon^{-2Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots) = \frac{\varepsilon^{-2Ts}}{1 - \varepsilon^{(a-s)T}}$$

(c) From (b), $E^*(s) = \dfrac{\varepsilon^{-2Ts}}{1 - \varepsilon^{(a-s)T}}$

(d) $E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts} + \varepsilon^{3aT/2}\varepsilon^{-2Ts} + \varepsilon^{5aT/2}\varepsilon^{-3Ts} + \cdots$

$$E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots)$$

$$= \frac{\varepsilon^{aT/2}\varepsilon^{-Ts}}{1 - \varepsilon^{(a-s)T}}$$

(3) – 30 point

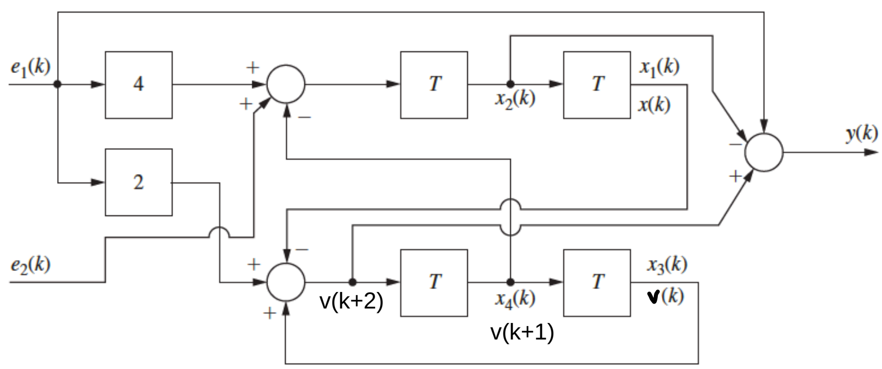
Find a state-variable formulation for the system described by the coupled second-order difference equations given. The system output is $y(k)$, and $e_1(k)$ and $e_2(k)$ are the system inputs. *Hint:* Draw a simulation diagram first.

$$x(k+2) + v(k+1) = 4e_1(k) + e_2(k)$$

$$v(k+2) - v(k) + x(k) = 2e_1(k)$$

$$y(k) = v(k+2) - x(k+1) + e_1(k)$$

Solution:



$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 & 0 \\ 4 & 1 \\ 0 & 0 \\ 2 & 0 \end{bmatrix} e(k)$$

$$y(k) = x_4(k+1) - x_2(k) + e_1(k) = -x_1(k) + x_3(k) - x_2(k) + e_1(k)$$

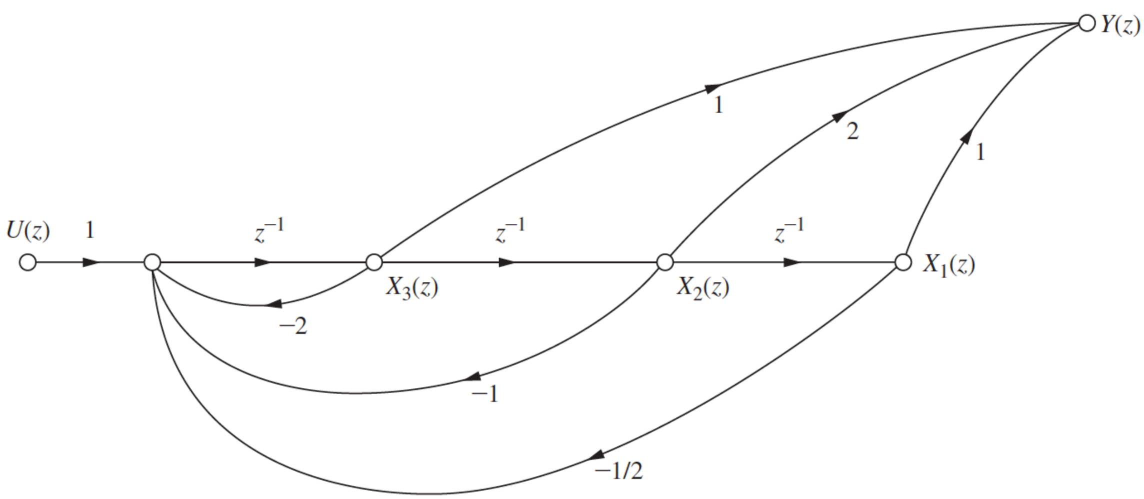
$$\therefore y(k) = \begin{bmatrix} -1 & -1 & 1 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 3 & 0 \end{bmatrix} e(k)$$

(4) – 30 point

Given the following function, find a signal flow graph and corresponding state space equations:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{z^2 + 2z + 1}{z^3 + 2z^2 + z + \frac{1}{2}}$$

Solution:



$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{2} & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 2 \quad 1] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

(5) – Look at the Mason’s formula examples from your lecture notes.

(6) – See examples and problems of the textbook.