HW #2

Solution 1:

(**a**)

$$e(t) = \varepsilon^{-at} \Rightarrow E(z) = 1 + \varepsilon^{-aT} z^{-1} + \varepsilon^{-2aT} z^{-2} + \cdots = \frac{z}{z - \varepsilon^{-aT}}$$

(**b**)

$$e(t) = \varepsilon^{-(t-T)} u(t - T)$$

$$E(z) = z^{-1} + \varepsilon^{-T} z^{-2} + \varepsilon^{-2T} z^{-3} + \cdots = z^{-1}\left[\frac{z}{z - \varepsilon^{-T}}\right] = \frac{1}{z - \varepsilon^{-T}}$$

(**c**)

$$e(t) = \varepsilon^{-(t-5T)} u(t - 5T)$$

$$E(z) = z^{-5} + \varepsilon^{-T} z^{-6} + \varepsilon^{-2T} z^{-7} + \cdots = z^{-5}\left[\frac{z}{z - \varepsilon^{-T}}\right] = \frac{1}{z^{4}(z - \varepsilon^{-T})}$$

Solution 2:

By complex translation

$$\mathfrak{z}\left[\varepsilon^{akT}e\left(kT\right)\right] = E(z\varepsilon^{-aT}) = \frac{z\varepsilon^{-aT} - b}{z^{3}\varepsilon^{-3aT} - cz^{2}\varepsilon^{-2aT} + d}$$

Solution 3:

(**a**)

$$e(\infty) = \lim_{z \to 1} (z-1)\frac{z}{(z-1)^2}, \quad \therefore \text{unbounded}$$

(**b**)

$$E(z) = \frac{z}{(z+1)^2}$$

(**c**) For both parts, $e(\infty)$ does not exist. However, if we use FVT for part (b) we get

$$e(\infty) = \lim_{z \to 1} (z-1)E(z) = \left.\frac{z(z-1)}{(z+1)^2}\right|_{z=1} = 0$$

But based on the definition of $e(k)$, we know that $\lim_{k \to \infty} e(k)$ does not exist (because $e(\infty)$ is unbounded).