HW #1

Solution 1:

(a)

$$C(s) = G_3(s)M(s) = G_3(s)[G_1(s)E(s) + G_2(s)R(s)]$$
$$= G_3(s)G_1(s)[R(s) - H(s)C(s)] + G_2(s)G_3(s)R(s)$$
$$[1 + G_1(s)G_3(s)H(s)]C(s) = [G_1(s) + G_2(s)]G_3(s)R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{[G_1(s) + G_2(s)]G_3(s)}{1 + G_1(s)G_3(s)H(s)}$$

(b)

$$C(s) = G_2(s)M(s) = G_1(s)G_2(s)E(s) = G_1(s)G_2(s)[R(s) - H_2(s)M(s) - H_1(s)C(s)]$$
and $M(s) = C(s)/G_2(s)$

$$\therefore \left[1 + \frac{G_1G_2H_2}{G_2} + G_1G_2H_1\right]C(s) = G_1G_2R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$$

(c)

$$C(s) = G_p(s)M(s) = G_c(s)G_p(s)E(s) = G_c(s)G_p(s)[R(s) - H(s)C(s)]$$
$$[1 + G_c(s)G_p(s)H(s)]C(s) = G_c(s)G_p(s)R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)H(s)}$$

Solution 2:

(a)

$$\Theta_a(s) = \frac{1}{100}\Theta_m(s) \quad \frac{1}{100s}\dot{\Theta}_m(s) = \frac{2}{s(0.5s+1)}E_a(s) = \frac{2K}{s(0.5s+1)}M(s)$$

$$\therefore \frac{\Theta_a(s)}{M(s)} = \frac{2K}{s(0.5s+1)};\ \frac{\Theta_a(s)}{E_a(s)} = \frac{2}{s(0.5s+1)}$$

(b)

$$\dot{\Theta}_a(s) = \frac{2}{0.5s+1}\times\frac{24}{s} = \frac{48}{s(0.5s+2)} = \frac{96}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$$

$$= \frac{48}{s} - \frac{48}{s+2} \Rightarrow \dot{\theta}_a(t) = 48 - 48\varepsilon^{-2t},\ t \geq 0$$

$$\therefore \dot{\theta}_{ass}(t) = 48^\circ/s \times \frac{60s}{1\text{min}} \times \frac{1\text{rev}}{360^\circ} = 8\text{rpm}$$

$$\therefore \dot{\theta}_{mss}(t) = 100\dot{\theta}_a(t) = 800\text{rpm}$$

(c) From (b):

$$\dot{\theta}_a(t) = 48^\circ/s$$

(d) From (b)

$$\dot{\theta}_a(t) = 48(1 - \varepsilon^{-2t}),\ t \geq 0$$

$$\therefore \tau = 0.5s$$

$$\varepsilon^{-2t_1} = 0.01 \Rightarrow 2t_1 = 4.60 \Rightarrow t_1 = 2.30s$$

(e)

$$K = 2.4 \Rightarrow e_a(t) = 24V,\ \text{rated voltage}$$