Solution
HW#6

Solution 1:

(a) $1+KG(z)H=0=z^2-1.8187z+0.8187+0.001311Kz+0.001226K$

$\therefore$ char.eq.: $Q(z)=z^2-(1.8187-0.001311K)z+0.8187+0.001226K=0$

(b) $z=\dfrac{1+\frac{T}{2}w}{1-\frac{T}{2}w}=\dfrac{1+0.05w}{1-0.05w}=\dfrac{20+w}{20-w}$

$\therefore Q(w)=(20+w)^2-(1.8187-0.001311K)(400-w^2)+(0.8187+0.001226K)(20-w)^2=0$

$\therefore (3.6374-0.000085K)w^2+(7.252-0.04904K)w+1.0148K=0$

(b)
$$\begin{array}{c|lll} w^2 & 3.6374-0.000085K & 1.0148K & \Rightarrow K<42,793 \\ w^1 & 7.252-0.04904K & & \Rightarrow K<147.9 \\ w^0 & 1.0148K & & \Rightarrow K>0 \end{array}$$

\ for stability, 0<K<147.9

(c) $Q(1)>0 \Rightarrow 1-1.8187+0.001311K+0.8187+0.001226K>0$

$\therefore K>0$

$(1)^2Q(-1)>0 \Rightarrow 1+1.8187-0.001311K+0.8187+0.001226K>0$

$\therefore K<42,793$

$|a_0|<a_2 \Rightarrow 0.8187+0.001226K<1, \therefore K<147.9$

For Stability: $0<K<147.9$

(d) $K=147.9$ From(a): char. eq. $z^2-1.6248z+1=0$

zeros: $z=1\angle\pm 35.67° = 1\angle\pm 0.6225$ rad.

From (b): char. eq. $3.6148w^2+150.09=0$

$\therefore W = \pm j6.437$

(e) $z - plane: 1\angle\omega T = 1\angle 0.6225 \qquad \therefore w = \dfrac{0.6225}{0.1} = 6.225\,\text{rad/s}$

w-plane: $\omega_w = 6.437$

(f) $(7-10)\ \omega_w = \dfrac{2}{T}\tan\left(\dfrac{\omega T}{2}\right) = \dfrac{2}{0.1}\tan\left(\dfrac{0.6225}{2}\right) = 6.433$

MATLAB:

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K = 147.9;

q = [1(–1.8187 + 0.001311*K) (0.8187 + 0.001226*K)]

roots(q)
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Problem 2

(a) $1+KG(z)H=0=z^2-1.7408z+0.7408+0.009072Kz+$

$\therefore$ char. eq.: $z^2+(0.009072K-1.7408)z+0.7408+0.008208K=0$

(b) $z=\dfrac{1\pm\frac{T}{2}w}{1-\frac{T}{2}w}=\dfrac{1+0.025w}{1-0.025w}=\dfrac{40+w}{40-w}$ yields

$$(40+w)^2+(0.009072K-1.7408(1600-w^2)+(0.7408+0.008208K)(40-w)^2=0$$

$$\therefore (3.4816-0.000864K)w^2+(20.736-0.65664K)w+27.65K=0$$

$$\begin{array}{l|lll} w^2 & 3.4816-0.000864K & 27.65K & \Rightarrow K<4{,}029.6 \\ w^1 & 20.736-0.65664K & & \Rightarrow K<31.58 \\ w^0 & 27.65K & & \Rightarrow K>0 \end{array}$$

\ for stability: $0<K<31.58$

(c) $Q(1)>0\Rightarrow 1+0.009072K-1.7408+0.7408+0.008208K>0\Rightarrow K>0$

$(-1)^2Q(-1)>0\Rightarrow 1-0.009072K+1.7408+0.7408+0.008208K>0\Rightarrow K<4029.6$

$|a_0|<a_2\Rightarrow 0.7408+0.008208K<1 \qquad \Rightarrow K<31.58$

\ for stability: $0<K<31.58$

(d) $K=31.58$, from (a): $z^2-1.4543z+1=0$

$\therefore z=1\angle\pm 43.36^\circ=1\angle\pm 0.7567\,\text{rad}$

from (b): $3.4543W^2+873.2=0\Rightarrow w=\pm j15.90=\pm j\omega_w$

(e) $z\text{-}plane: 1\angle\omega T=1\angle 0.7567\Rightarrow \omega=0.7567/0.05=15.13\,\text{rad/s}$

$w\text{-}plane: \omega_w=15.90$

(f) $(7-10)\ \omega_w = \frac{2}{T}\tan\left(\frac{\omega T}{2}\right) = 40\tan\left(\frac{0.7567}{2}\right) = 15.90$

MATLAB:

```
K = 31.58;
q = [1(.009072*K–1.7408) (0.7408+0.008208*K)]
roots(q)
```