EECE 5610 Digital Control Systems

1. Introduction

Milad Siami
Assistant Professor of ECE
Email: m.siami@northeastern.edu

Northeastern University
College *of* Engineering

General info

Milad Siami, Assistant Professor of ECE

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- Lectures: **Tuesdays 11:45 am - 1:25 pm** and **Thursdays 2:50 pm - 4:30** pm, RY 275

- Office hours: Students are encouraged to utilize the class meeting time and Canvas to ask their questions, so the entire class benefits from them. <u>Office hours will be scheduled on-demand and held online</u>

<u>Tentative Time: Wednesdays 3:00pm-5:00pm on Teams</u>

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- Bibliography: The course textbook is

  - C.L. Phillips, H.T. Nagle Jr., and A. Chakrabortty, Digital Control Systems Analysis and Design, 4 th edition, Pearson, 2015, Chapters 1-7 and (part of 8). Occasionally we may also use a few additional book chapters and research papers.

- Course website: https://canvas.northeastern.edu

Course Name: EECE5610 13494 Digital Control Systems SEC 01 Fall 2022 [BOS-2-TR]

Prerequisites

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- **Courses**: EECE 5580 or equivalent; general mathematical maturity; or permission of instructor.

- Mathematical maturity
  - Proofs and deductive reasoning
  - Basic calculus and linear algebra

- Programming
  - We will use Matlab/python for homework and final project

Canvas is Here!

The university is transitioning fully to Canvas, and no courses will be available in Blackboard this fall. To learn more about Northeastern's Hybrid NUflex model for the fall 2020 semester, visit nuflex.northeastern.edu.

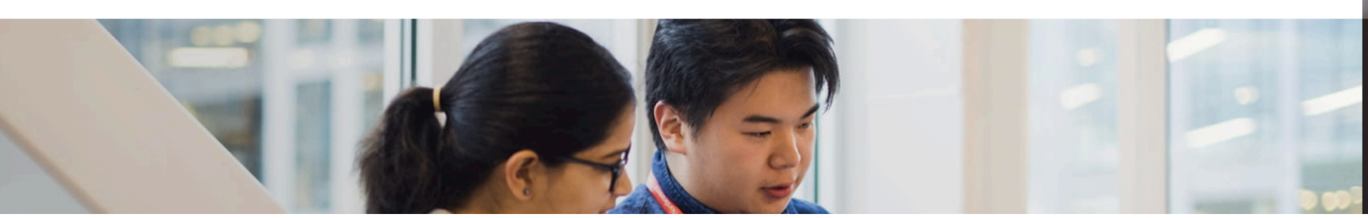
FACULTY RESOURCES STUDENT RESOURCES

Canvas Training and Support Resources

Northeastern has made many options available for faculty to schedule training or support sessions.

Academic Technologies, The Center for Advancing Teaching and Learning Through Research (CATLR), and the Office of the Provost are all supporting faculty to learn more about this transformation by offering a variety of training and support options.

EXPLORE SUPPORT OPTIONS



EECE5610.14733.202110 > Modules

202110_2 Fall 2020 Semest...

Home
Announcements
Discussions
Grades
People
Pages
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Syllabus
Quizzes
Modules
Collaborations
New Analytics
Leganto - Reading List
Zoom Meetings

Account
Dashboard
Courses
Calendar
Inbox
Studio
Help

Collapse All | Export Course Content

EECE 5610 Digital Control Systems

- About the course:
- Syllabus-2020.pdf

First Week -- Introduction

- General Info for Week#1

View Course Stream
New Analytics
View Course Calendar
View Course Notifications

To Do

Welcome to the EECE 561...
Sep 1 at 1:41pm |

Zoom Link for Remote Stu...
Sep 6 at 1:40pm |

General Info for Week#1

Week#1

Instructor: Milad Siami, Assistant Professor of ECE

m.siami@northeastern.edu

http://miladsiami.com

Learning Objectives for Week#1

In the first week of class, we will:

- Introduce the course, including its content, organization, expectations and grading policy.
- Provide an overview of Digital Control System

Week#1: Lectures 1:

- Objectives: Effectively communicate the week#1 specifics and provide resources for future reference (course webpage, canvas, etc.). Motivate the topic of Digital Control in general. Given an overview of which specific topics, the course will be touching upon.
- General motivation:
  - Understand the importance of feedback control theory
  - Understand why we are interested in digital control
  - Learn the advantages/disadvantages of digital control

Readings: Ch. 1 [text book: Digital Control System Analysis & Design]

Module Activities

In this module, you should complete the following tasks:

- Complete the lessons in the order they appear.
- Complete the Time Zone Survey, so we can correspondingly schedule the assignments due date.
- Participate in the discussion.

Questions?

Ask questions about this module in the Q&A Discussion Board.

Homework, midterm, and final project

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- Grading: Homework: 20%; 2 Midterms: 23 % each; Final Exam: 30%; Quiz: 4%; Final Project: (optional) extra 5%.

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  - Some additional materials (will be uploaded in Canvas during the semester)

Course Syllabus

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- Course Overview, Introduction to Digital Control

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Learning outcomes

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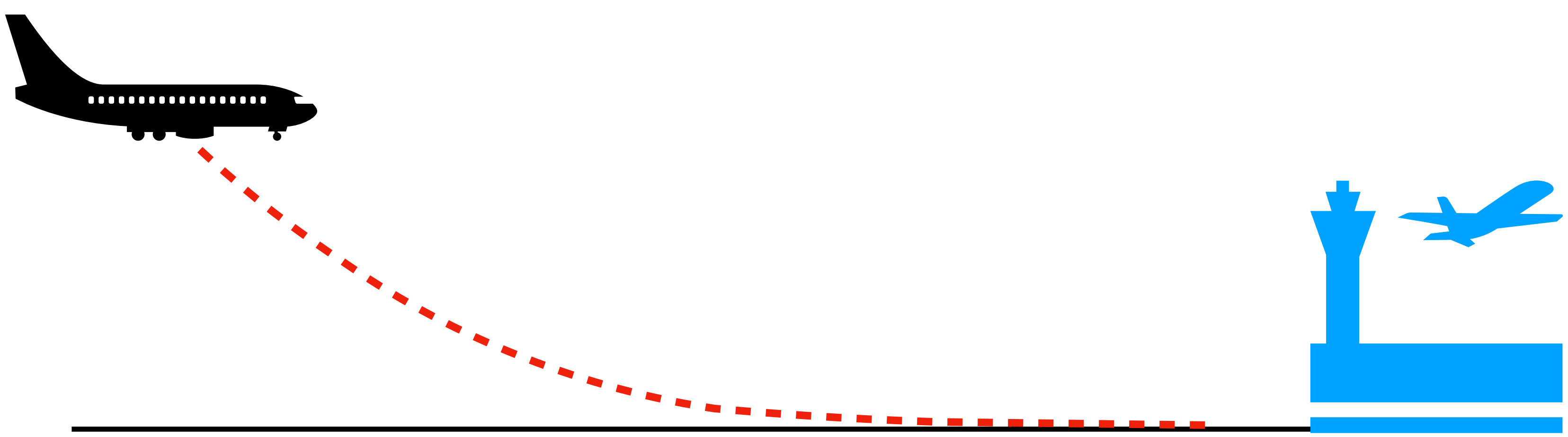
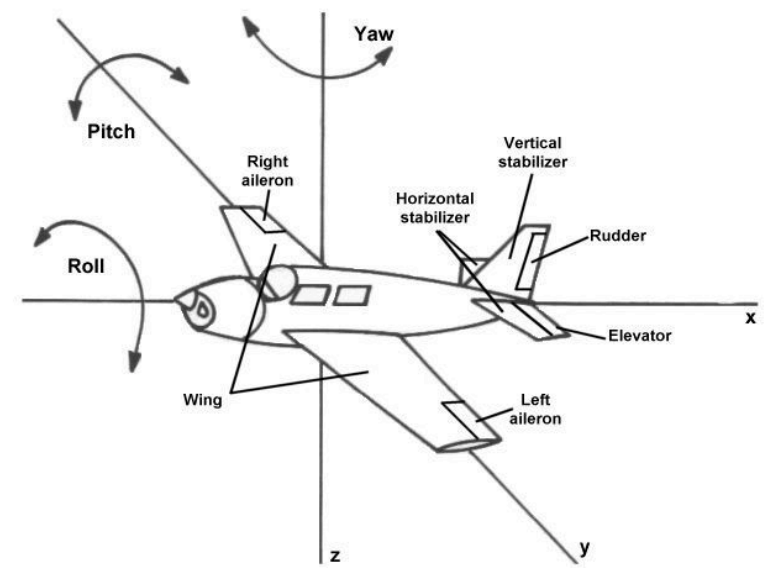
- Understand the importance of feedback control theory
- Understand why we are interested in digital control
- Learn the advantages/disadvantages of digital control
- Understand the implications when converting continuous-time models into discrete-time models

Basic Concepts

- **Control System:** A system designed to cause the output of a dynamic system (“the plant”) achieves a desirable behavior by manipulating some inputs (the “control” inputs).





Aircraft control surfaces and axes of motion

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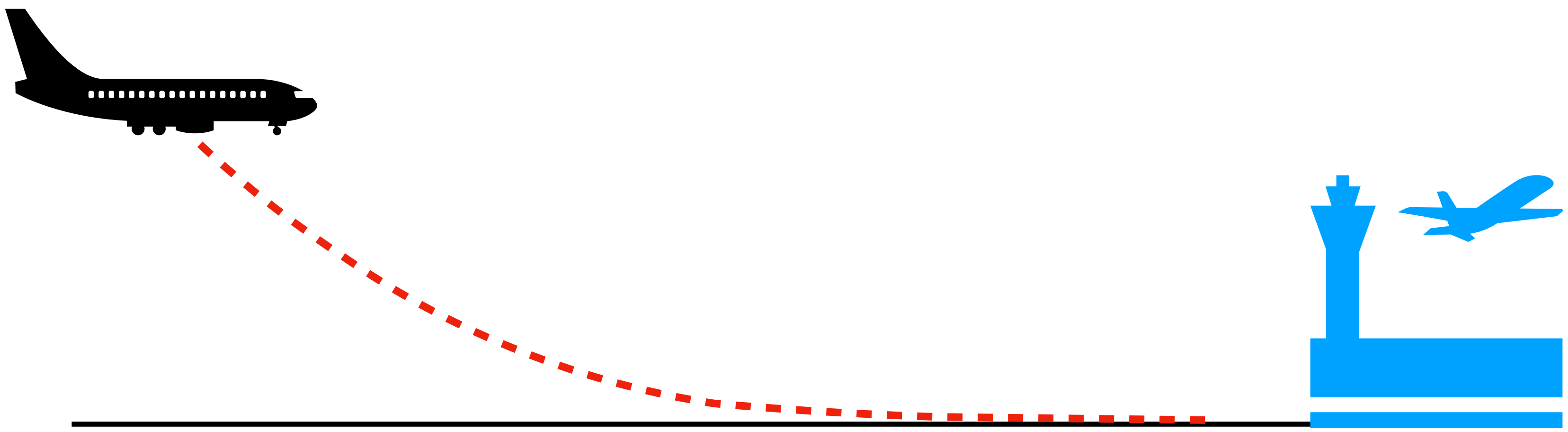

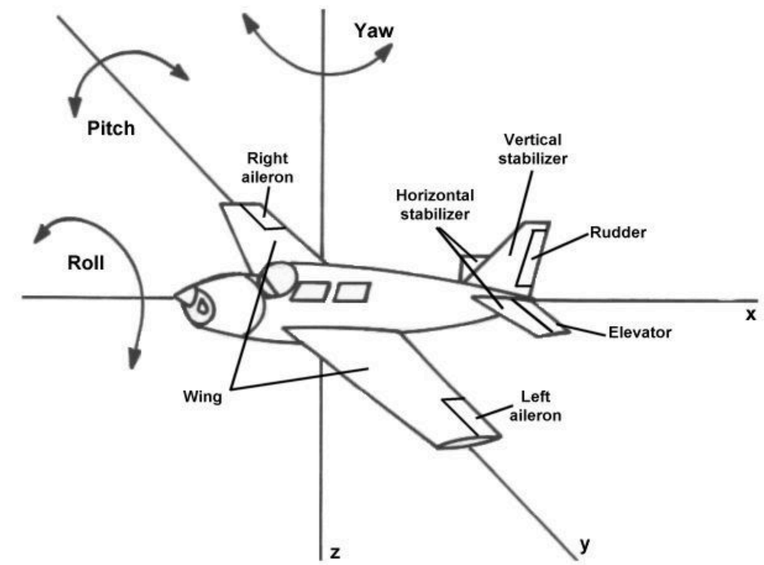

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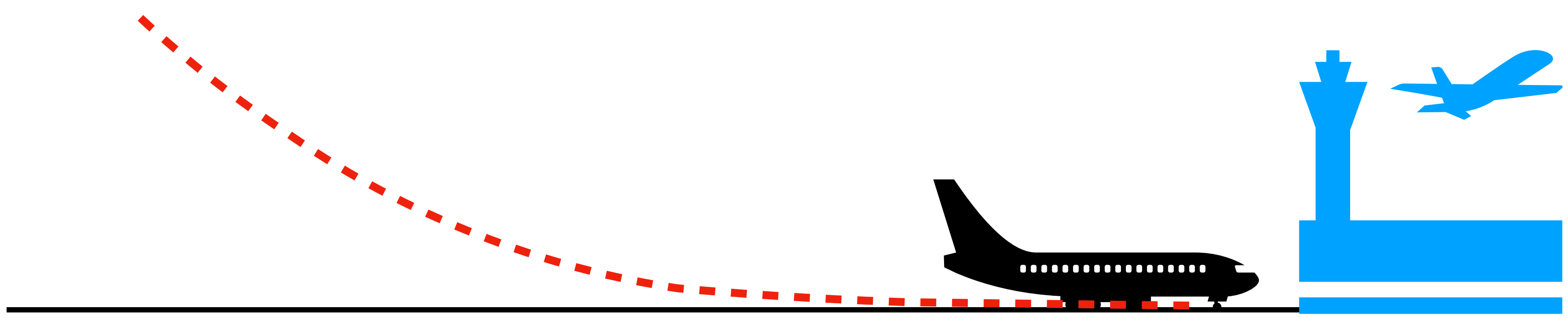

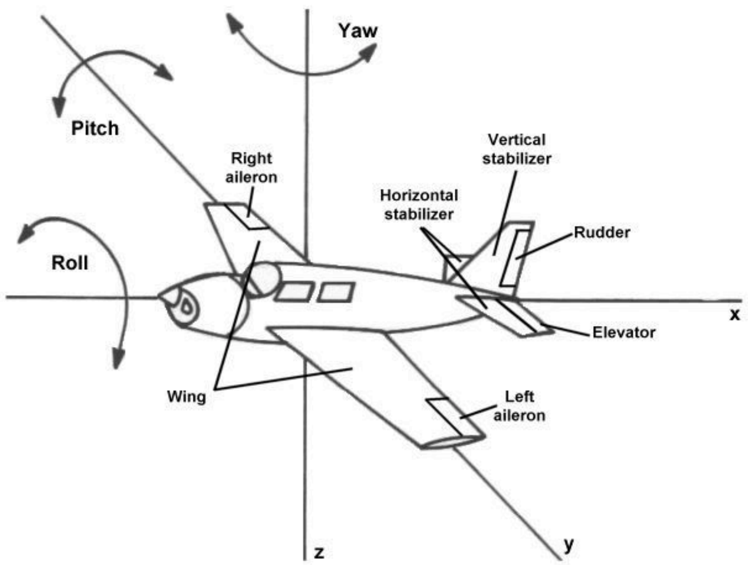

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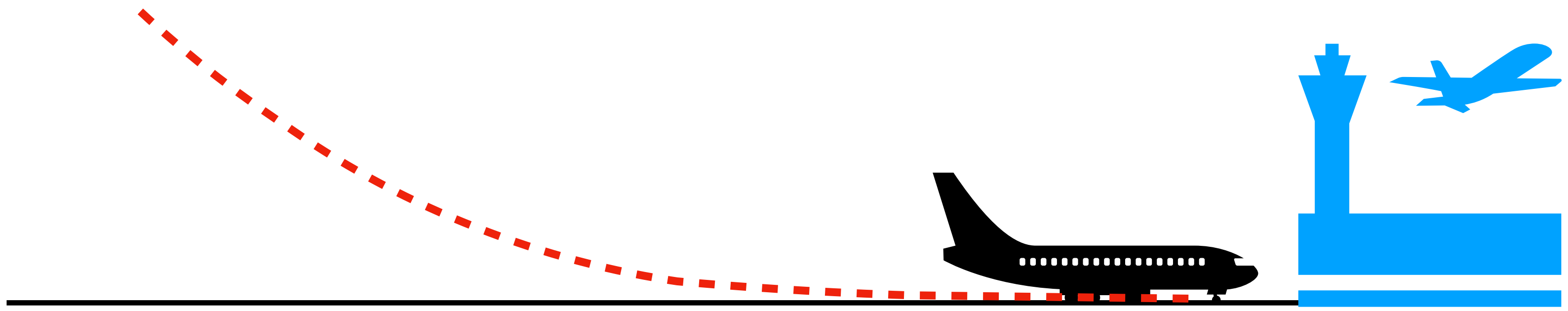

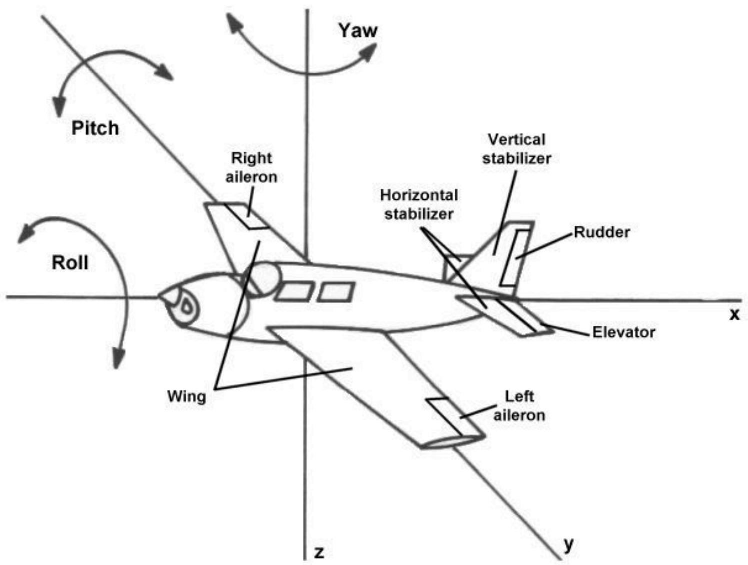

Aircraft control surfaces and axes of motion

- **Example 2:** A toaster: Here the controller (a timer) decides the amount of time that the toaster stays on, based upon the "darkness setting"

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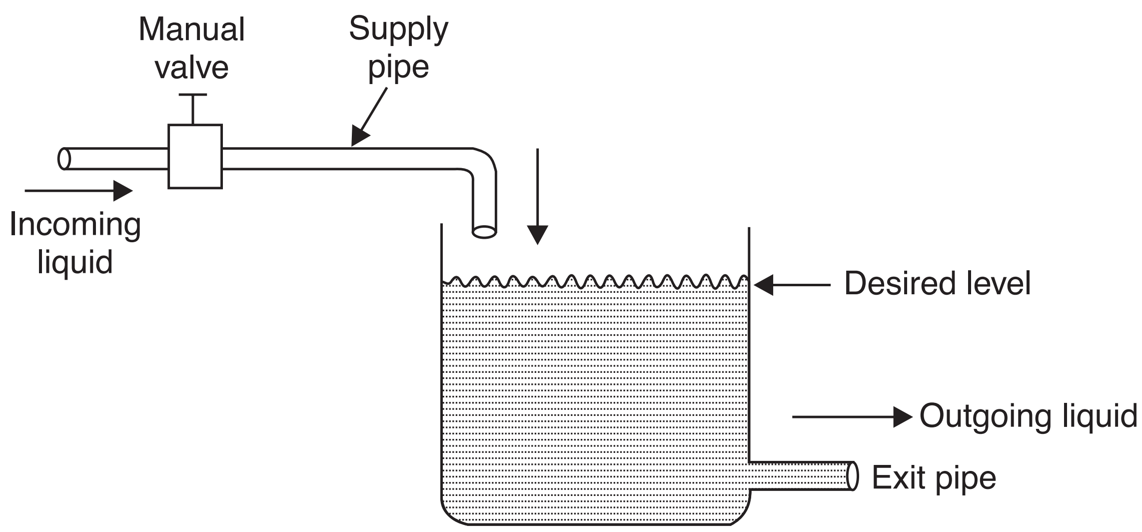
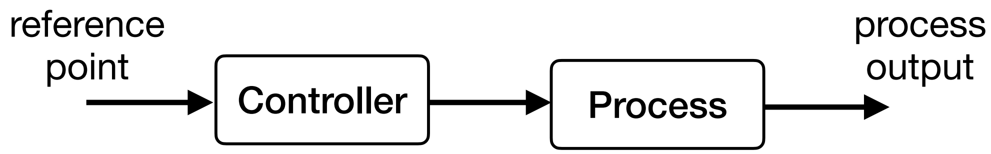
We can have two different configurations for control systems:

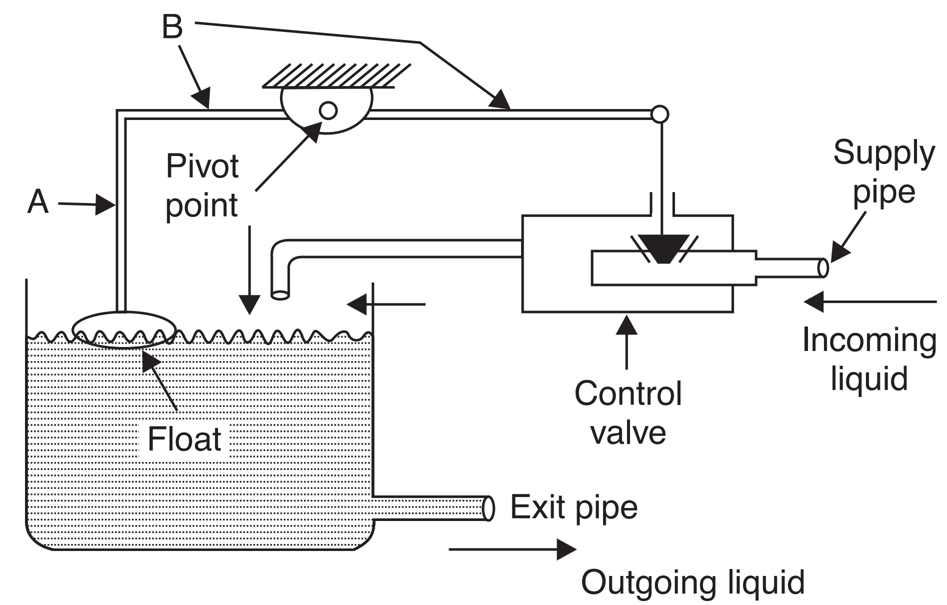
Open-loop system

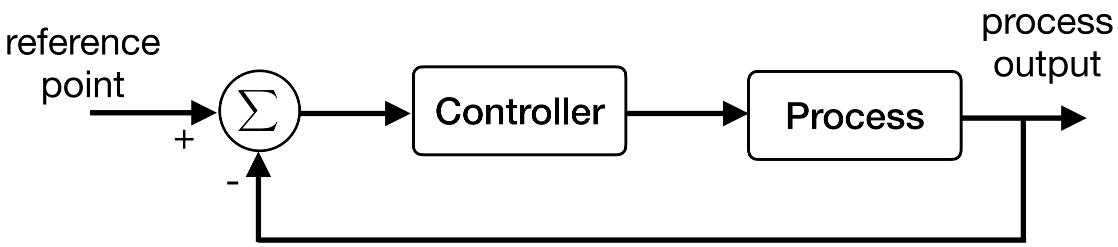




- No need for sensors
- A controller can be adjusted to give the right output
- Works when there is *no disturbance*!

Closed-loop system





Why do we need feedback control systems?

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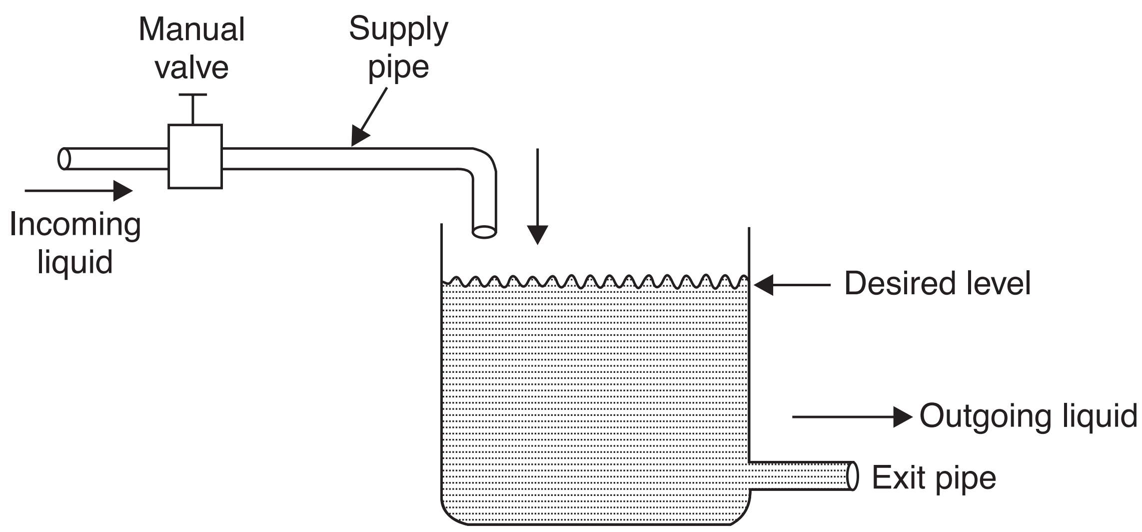

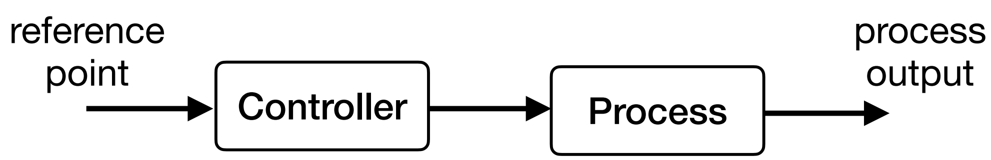

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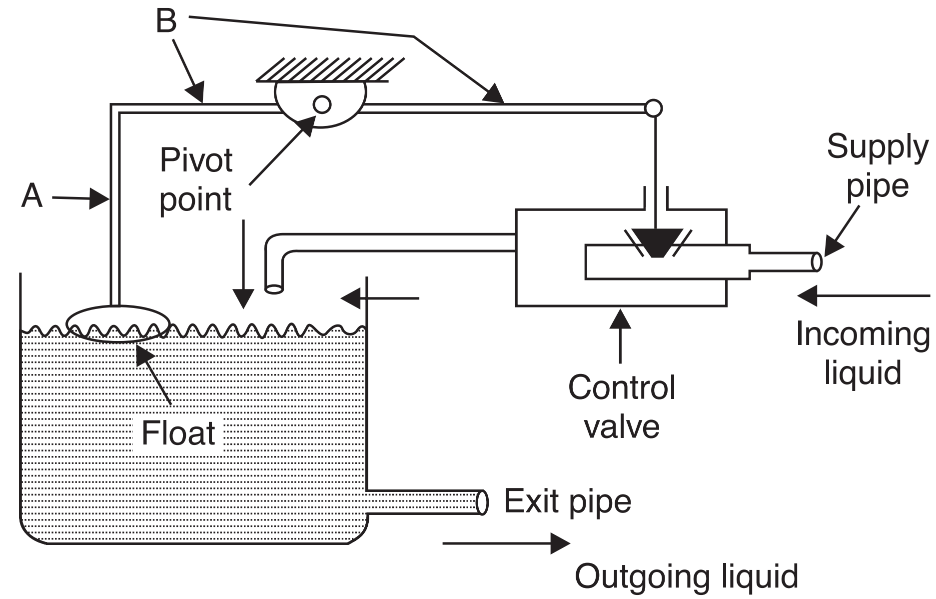

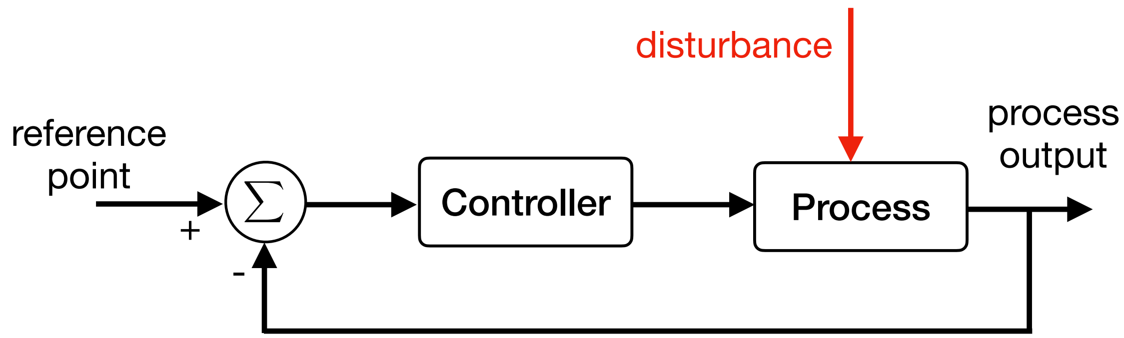

- Deviations from the reference point are corrected *automatically*

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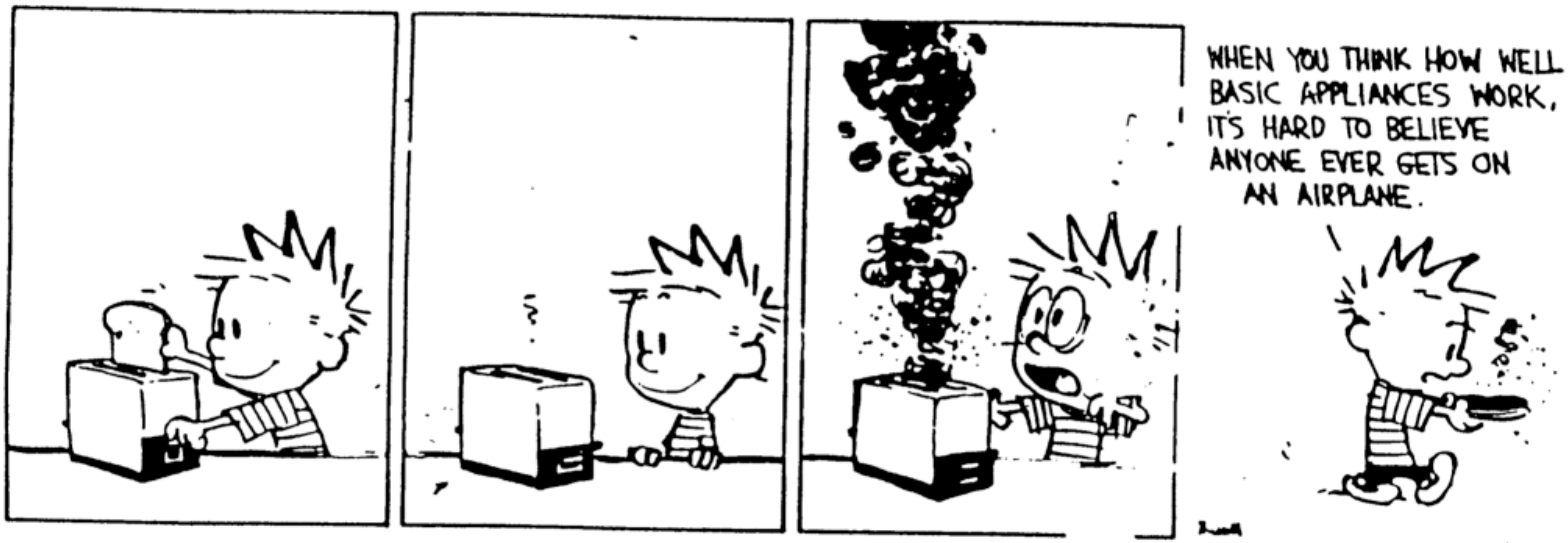
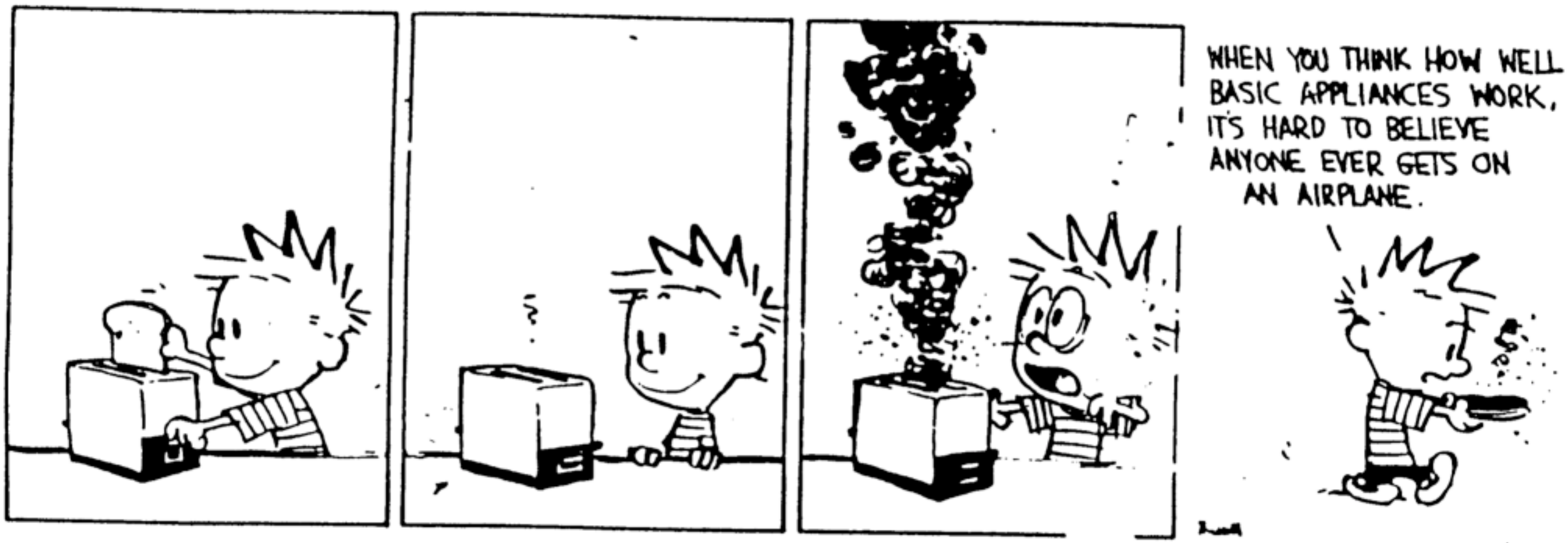
- Why feedback?



- <u>Why feedback?</u>



Any Questions so far?

Application areas of control engineering

- Automotive, Aeronautics & aerospace engineering
- Robotics
- Manufacturing
- Process control (chemical, pharmaceutical,…)
- Power electronics, power networks
- Telecommunications
- Environmental systems
- Financial engineering
- Supply chains
- …

Applications of feedback control systems

- **Domestic applications**

  Regulated voltage and frequency of electric power, thermostat control of refrigerators and electric iron, temperature and pressure control of hot water supply in cold countries, pressure of fuel gas, automatic focusing of digital cameras, …

- **Industrial applications**

  Process regulators, process and oven regulators, steam and air pressure regulators, gasoline and steam engine governors, motor speed regulators, …

- **Transportation systems**

  Speed control of the airplane engines with governors, control of engine pressure, instruments in the pilot's cabin contain feedback loops, instrument-landing system, …

- **Automobiles**

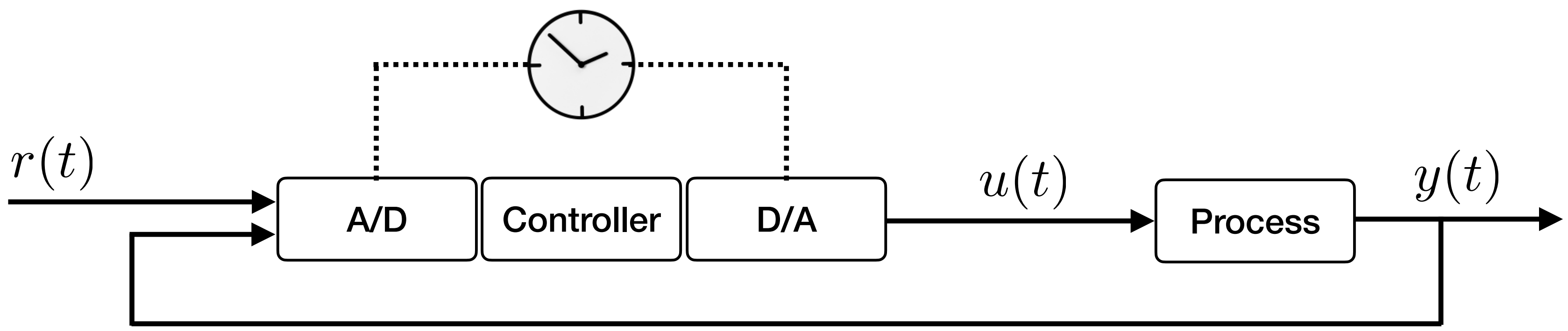
  Thermostatic cooling system, steering mechanisms, the gasoline gauge, and collision avoidance, idle speed control, antiskid braking, …

- **Scientific applications**

  Measuring instruments, analog computers, electron microscope, cyclotron, x-ray machine, space ships, moon-landing systems, remote tracking of satellites, …

Introduction to Digital Control Systems

- *The course is concerned with the analysis and design of control systems the incorporate digital elements (usually a computer or microprocessor) in the feedback loop. These systems involve both continuous and discrete time signals of the form:*



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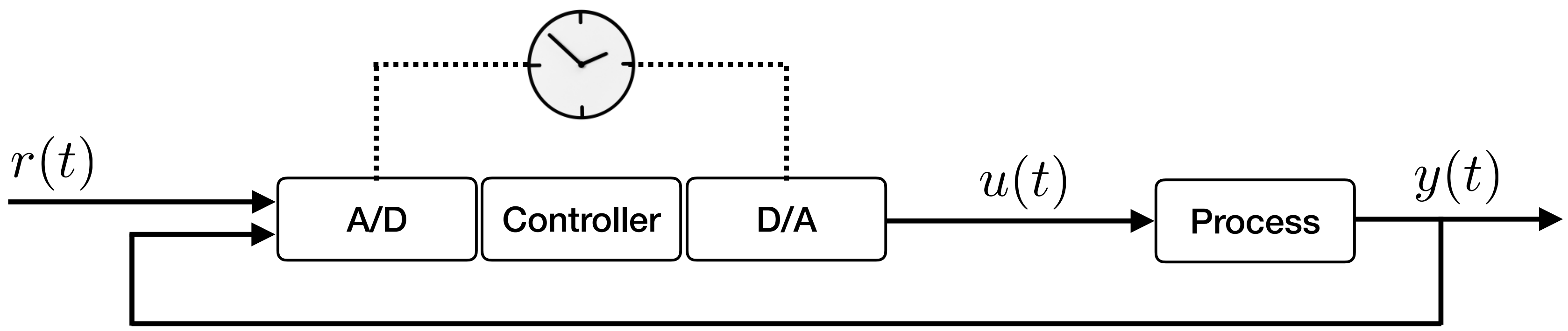

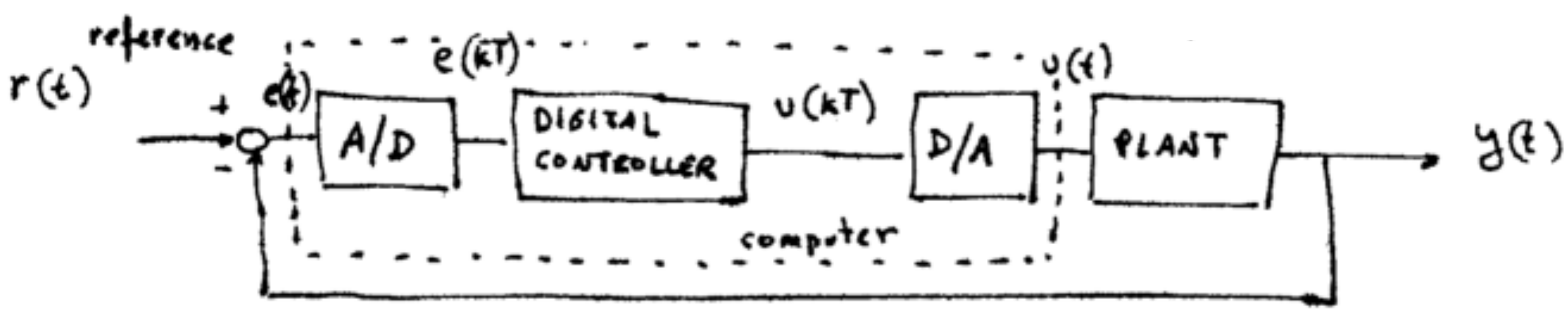

Introduction to Digital Control Systems

Example: An automatic landing system

Goal: keep the airplane centered on the runway (lateral control system) and following a pre-specified trajectory

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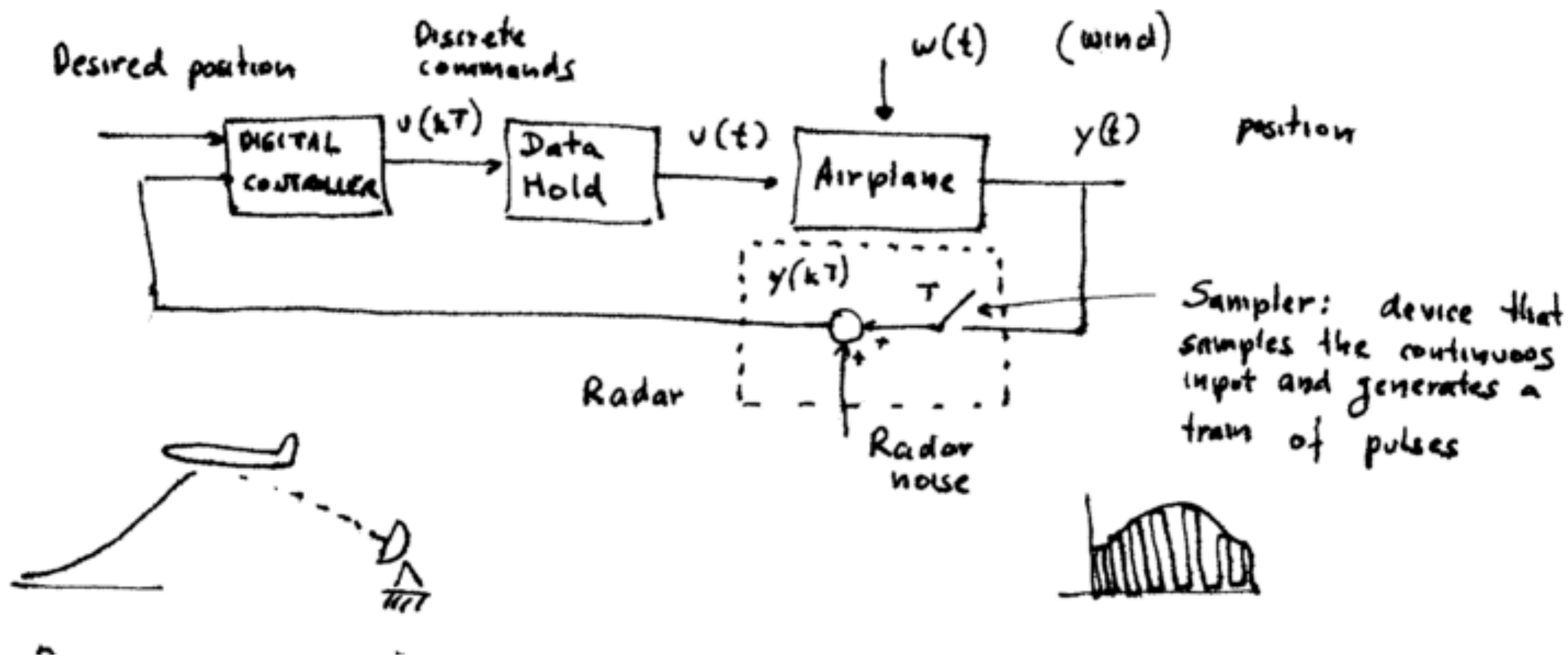

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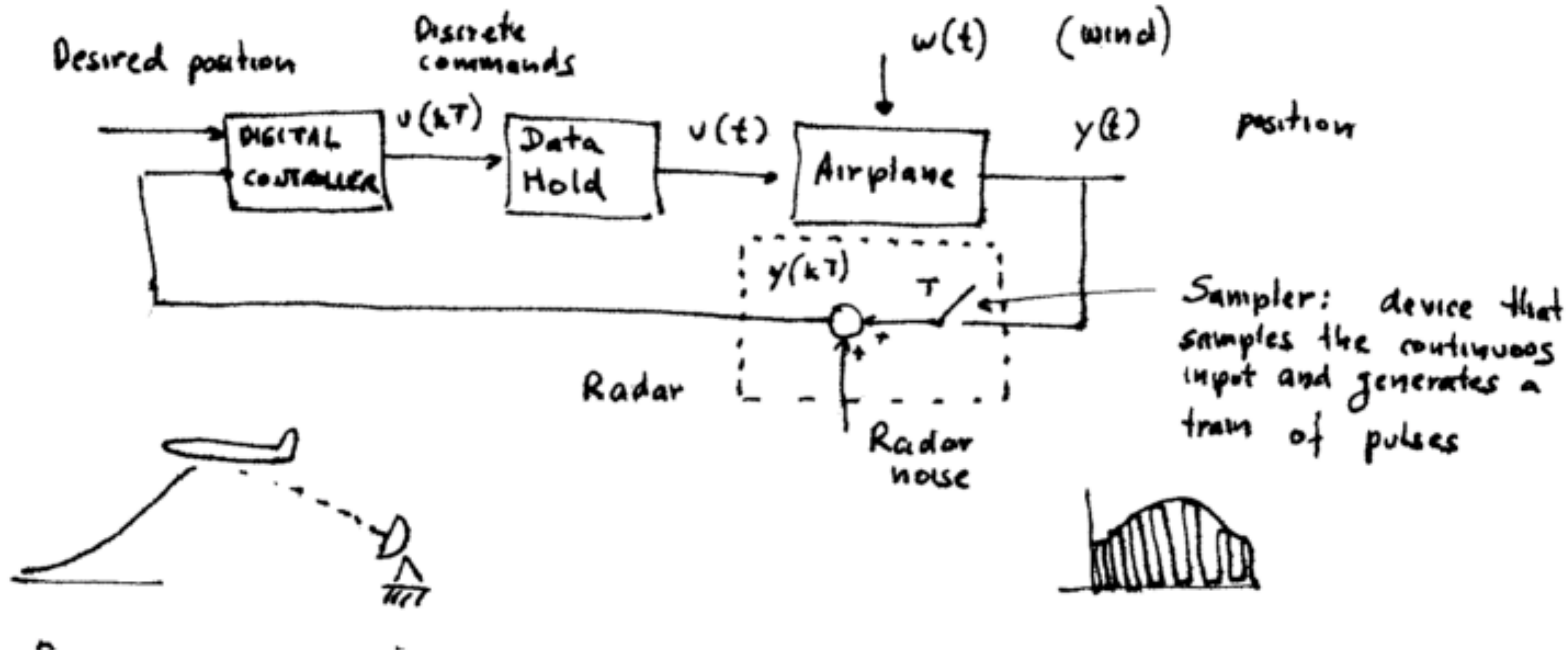


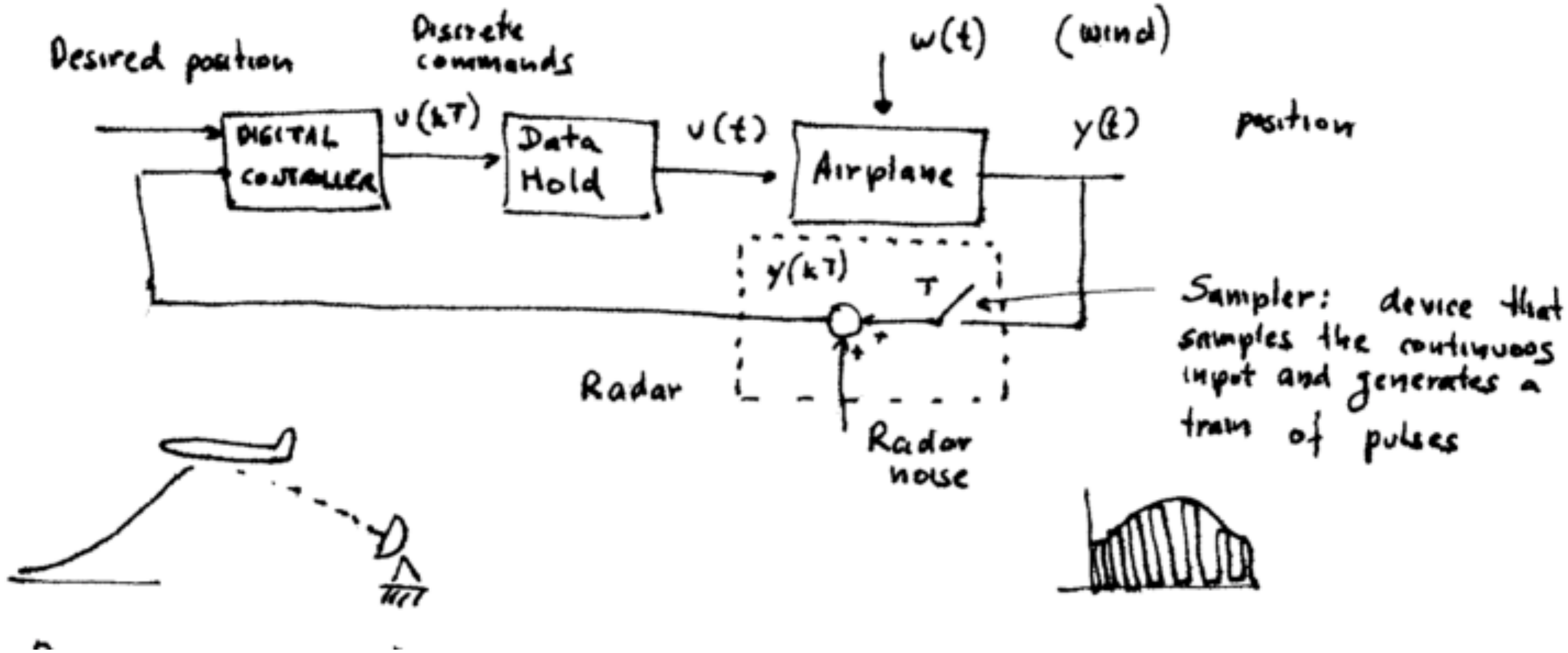
Radar measures the position *y(t)* every *T* seconds

Date hold clamps the value of *u(kT)* until the next command is received

Note that this is intrinsically a sampled-data system since the radar uses a pulse every *T* seconds

Assumption: All the numbers arrive at the digital controller at the same time and with the same fixed period *T* (the sampling period)

Observation:



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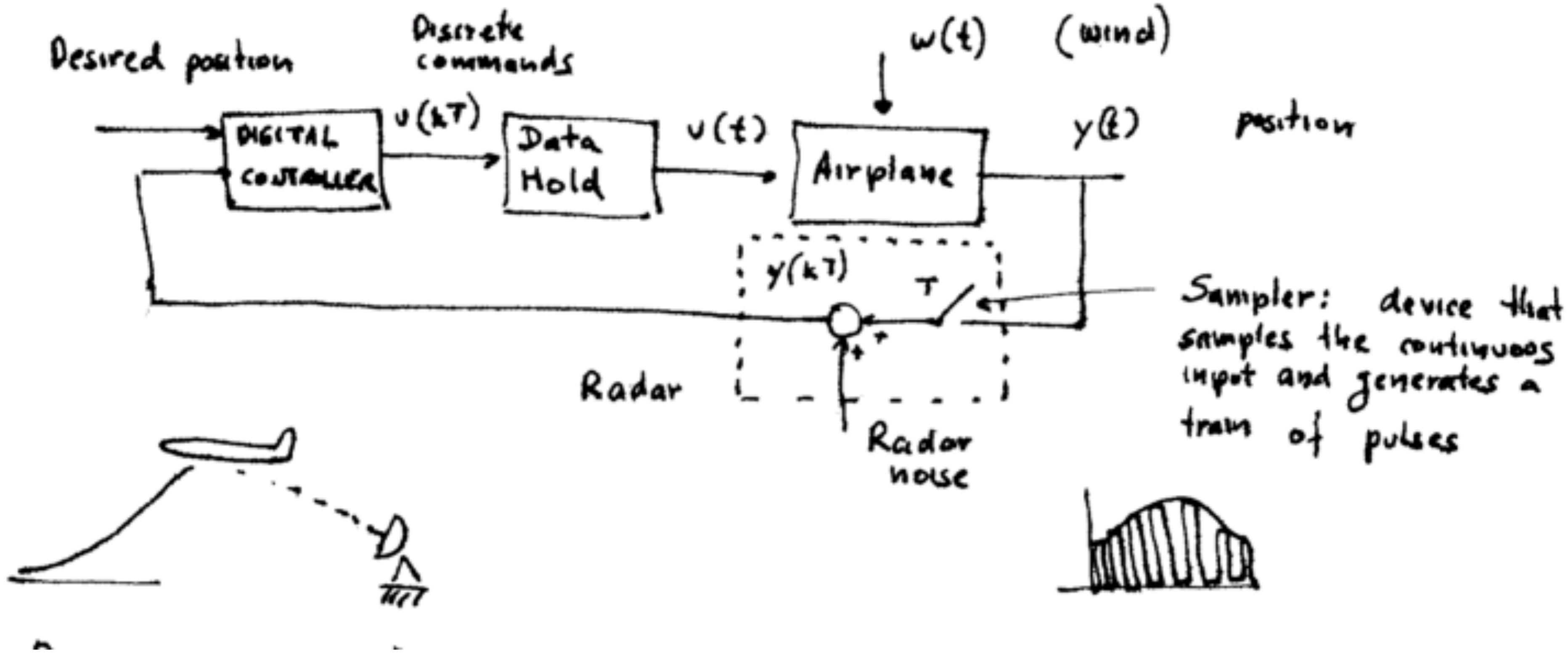

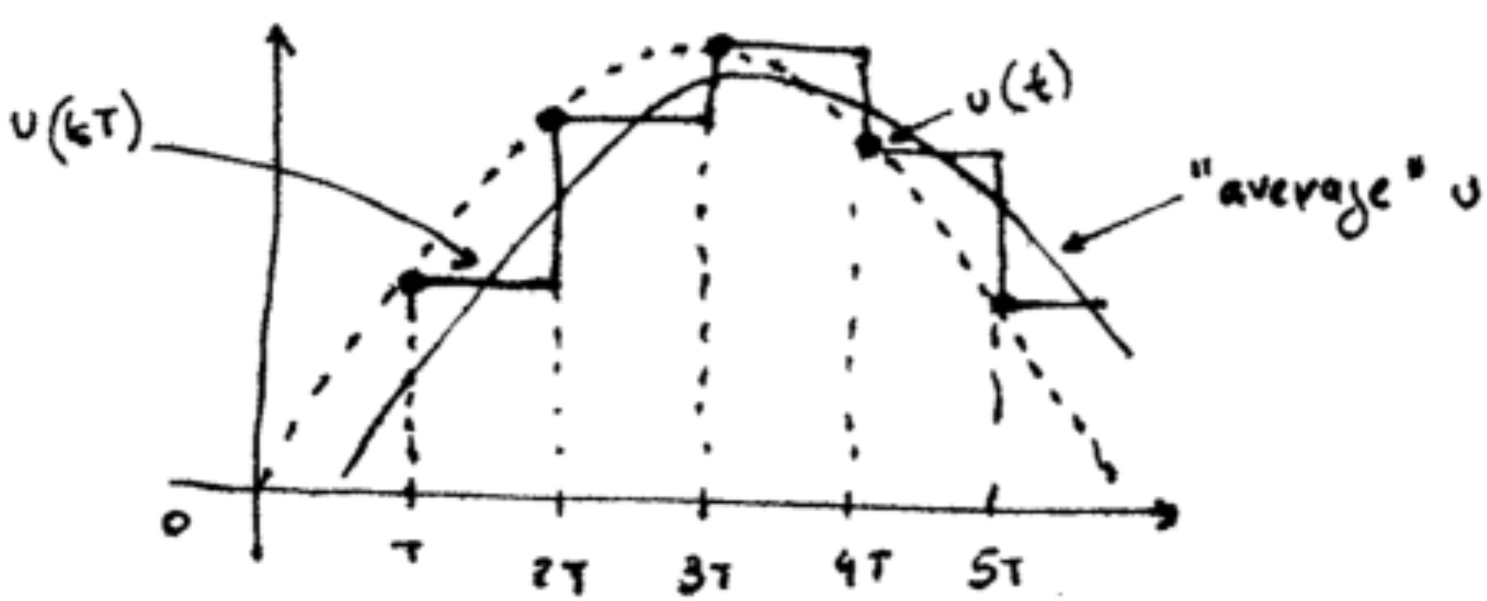

Note that the hold introduces a <u>delay</u>

(more on this later)

Why are we interested in digital control?

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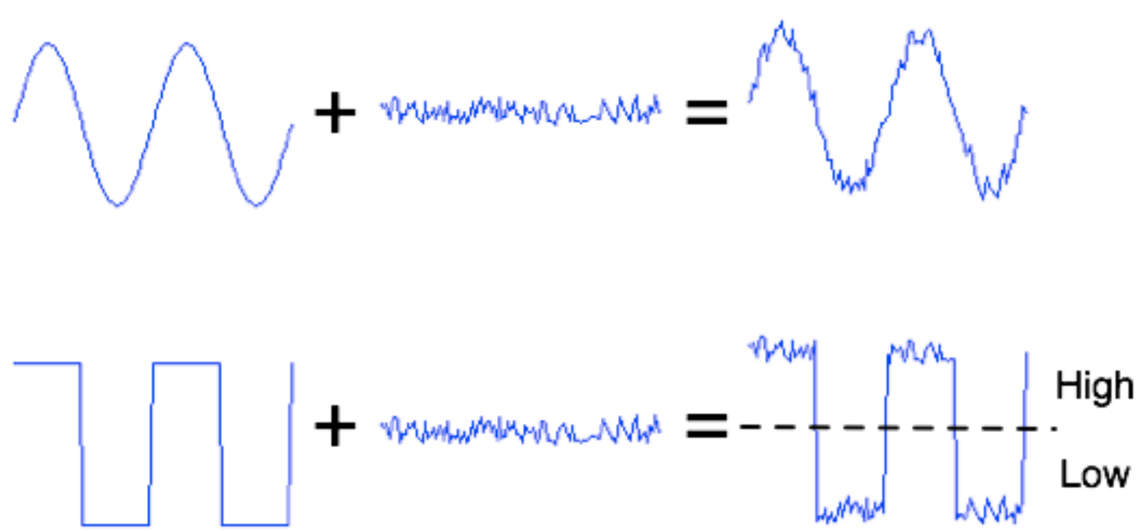
      - Internal combustion engines

Why are we interested in digital control?

- The use of digital computer as a controller & reduced complexity of operations - tremendous amount of **flexibility and versatility in the design approach**

- For some control system application, **better system performance** may be achieved by a digital control system design

- Digital control uses **digital communication**:
  - ✓ more reliable due to its improved noise immunity
  - ✓ better resolution when representing numbers
  - ✓ time sharing of a communication channel for transmitting digital signals for more than one control system



Why are we interested in digital control?

- Why digital control?

  - Increased flexibility — analog systems are changed by rewiring computer controlled systems can be reprogrammed. Also, you can add decision making capabilities to the loop

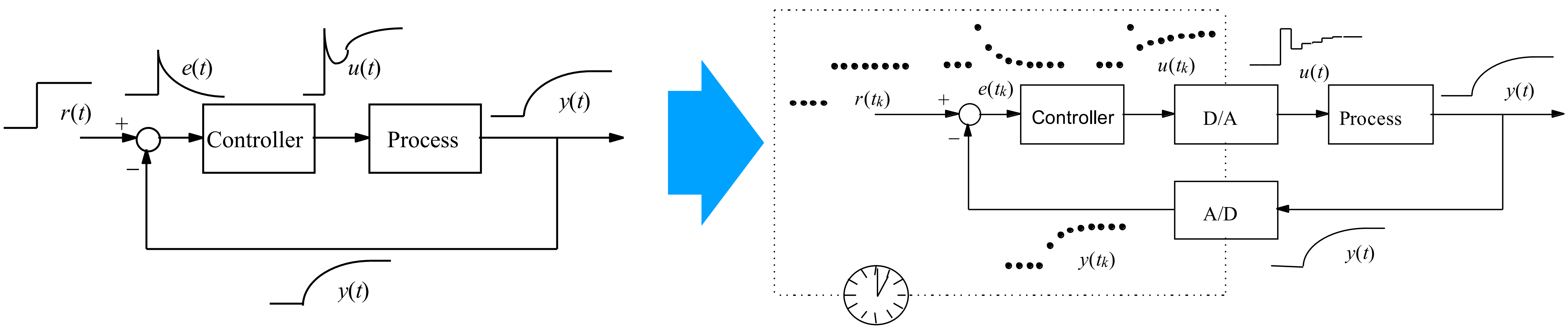
  - Economic reasons: Initial cost of a computer based controller may be high (although not usually the case now) but the cost of adding more loops is low. Moreover you get more compact & light weight controllers.

  - Better characteristics: No drift, better reliability, better noise rejection characteristics.

  - Many systems are inherently sampled:

Disadvantages/problems of digital control?

- Complicated controllers implemented in software may have **software errors**
- Most of the processes are analog in nature:
  - Signal *from* the digital controller → converted to analog (D/A converter) - *hold*
  - Signal *to* the digital controller → converted to digital (A/D converter) - *sampling*
  - Reconstruction of digital to analog signal (and vice versa) is only an *approximation* of the actual signal → some **signal information might be lost in the process**



- **A/D and D/A converters introduce some time-delay** → performance objectives may be difficult to achieve
- Mathematical analysis - sometimes more complex

Recap: Digital control systems are:

- ⊕ More flexible
- ⊕ Cheaper
- ⊕ More reliable (high MTBF, low drift)

than analog counterparts

However:

- ⊖ Less stable
- ⊖ Less effective
- ⊖ Less robust

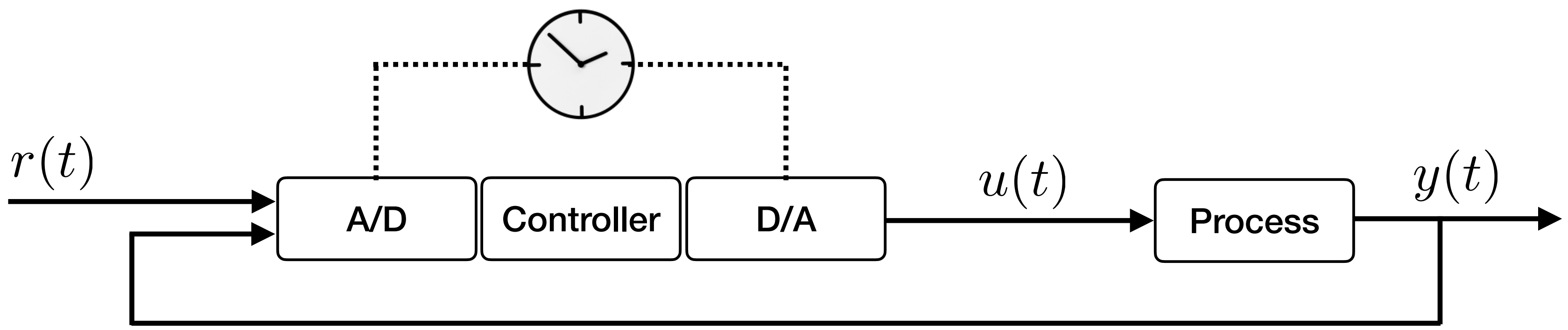
Unless T is chosen properly and adequate design techniques are used

(rather than "lets discretize an analog design" approach)

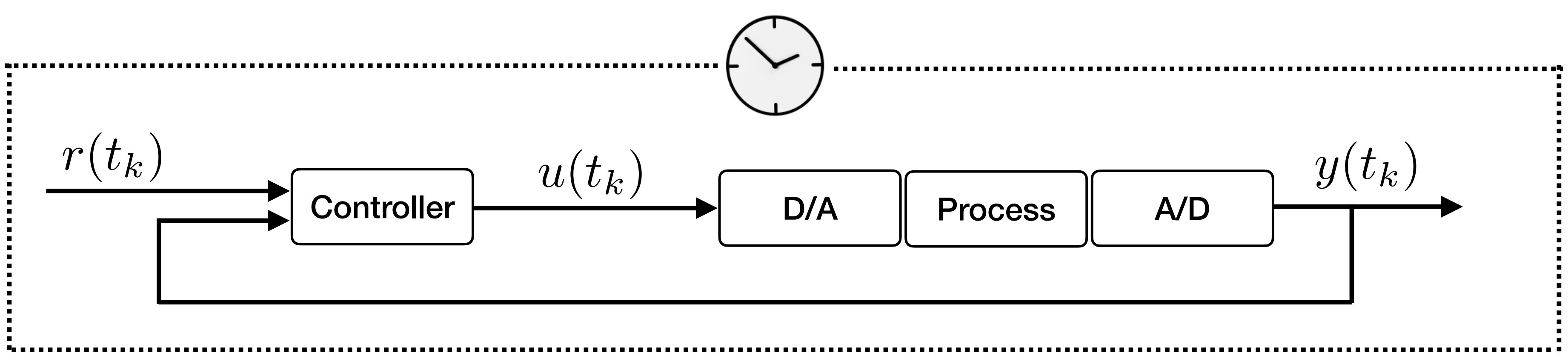
Discrete-time controller design

- **There are 2 main design approaches:**

(a) Discretize the analog controller



(b) Discretize the process and do the design totally in discrete time



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- How do you design digital controllers? What should be taken into account in implementation?

- Is it so that a digital controller only imitates the corresponding analog controller and the result is somewhat worse than the analog one (due to losing information in discretization)?

- Do discrete-time systems have properties that the corresponding analog systems do not have?

Some models that we will use later on:

- **Mechanical Translational Systems:**

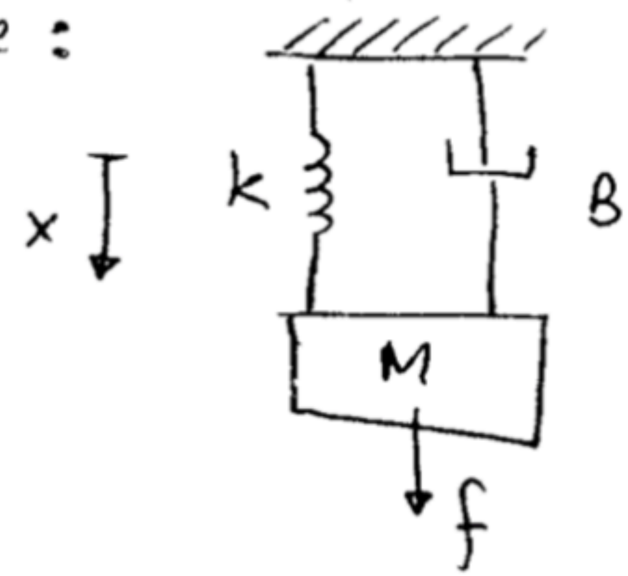
Basic Law: Newton's second law: $M\ddot{x} = \Sigma F$

Elements:

a) Spring $F_k = -kx$

b) Viscous damping and friction $F_b = -b\dot{x}$ (always opposes motion)

- **Example:**

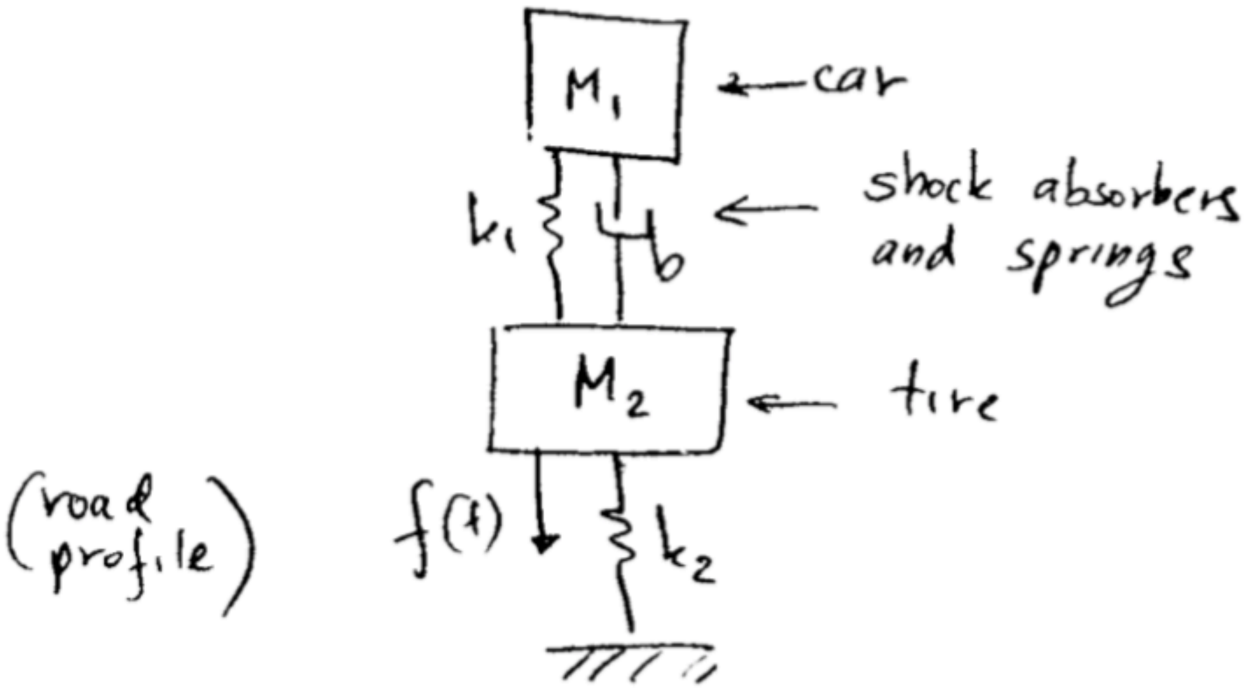


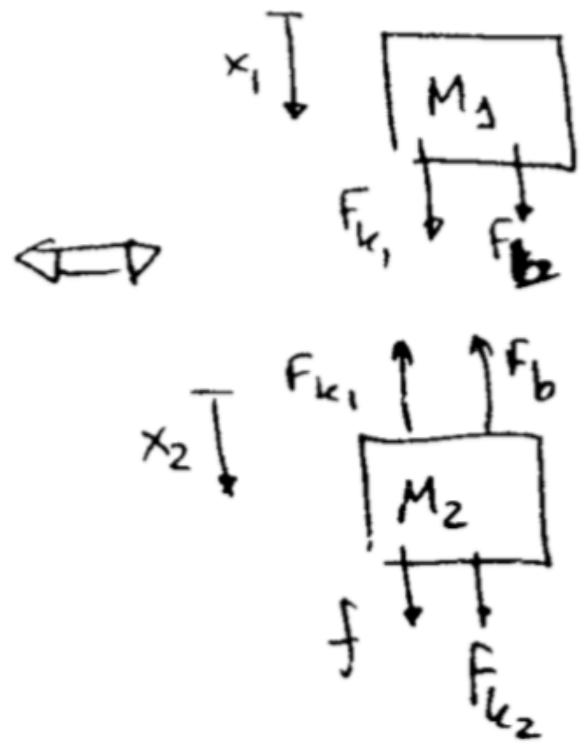
$$M\ddot{x} = -kx - b\dot{x}$$

$$\Rightarrow \boxed{M\ddot{x} + b\dot{x} + kx = 0}$$

Some models that we will use later on:

- Example 2: Simplified model of an automobile suspension:





$$M_1\ddot{x}_1 = -k_1(x_1-x_2) - b(\dot{x}_1-\dot{x}_2)$$

$$M_2\ddot{x}_2 = -k_1(x_2-x_1) - b(\dot{x}_2-\dot{x}_1) - k_2x_2 + f$$

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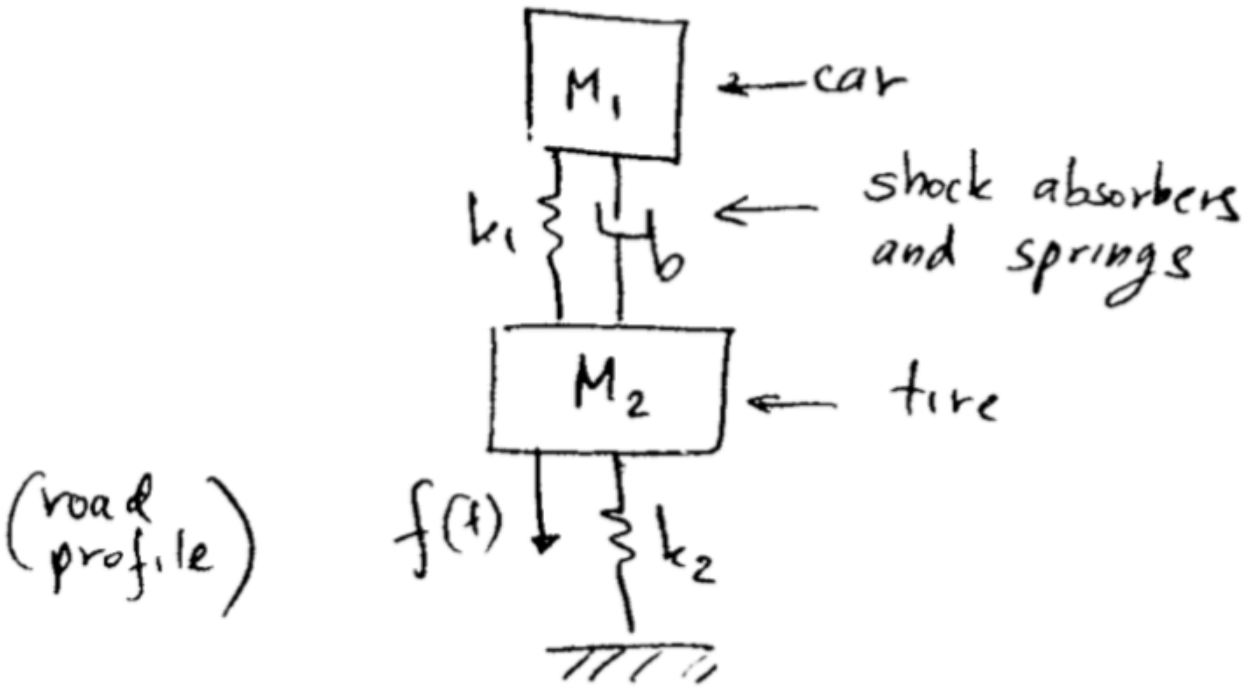
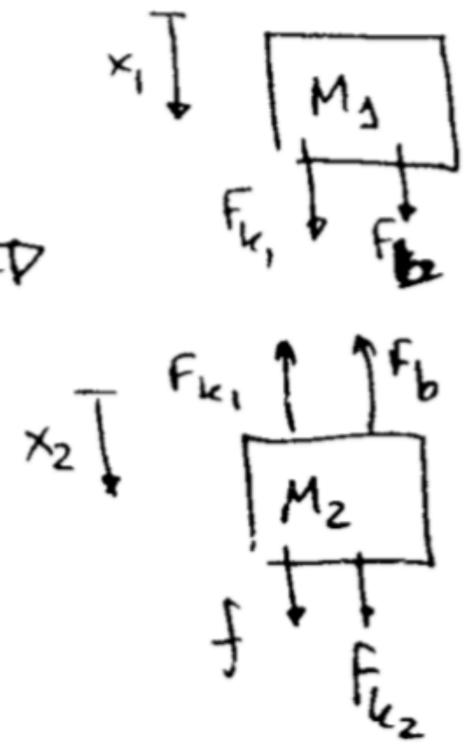


$\Longleftrightarrow$



$$M_1\ddot{x}_1 = -k_1(x_1-x_2) - b(\dot{x}_1-\dot{x}_2)$$

$$M_2\ddot{x}_2 = -k_1(x_2-x_1) - b(\dot{x}_2-\dot{x}_1) - k_2x_2 + f$$

Taking Laplace transforms yields:

$$(M_1 s^2 + b_1 s + k_1)x_1 - (bs+k_1)x_2 = 0$$

$$-(bs+k_2)x_2 + (M_2 s^2 + b_1 s + k_1 + k_2)x_2 = F(s)$$

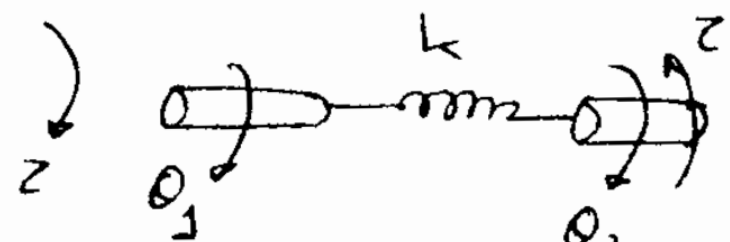
By solving these we can get the transfer functions $G_1(s) = \dfrac{X_1(s)}{F(s)}$ and $G_2(s) = \dfrac{X_2(s)}{F(s)}$

Some models that we will use later on:

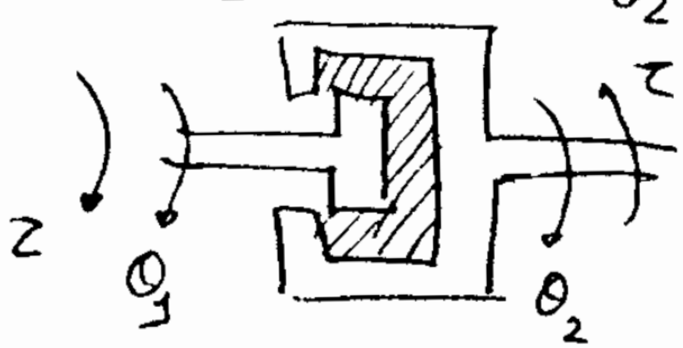
- <u>Mechanical Rotational Systems</u>

<u>Basic Law</u>: Newton's equation for rotational systems: $J\ddot{\theta} = \Sigma$ Torques

<u>Elements</u>: moment of inertia (similar to mass)
friction
torsion

Torsion spring: 

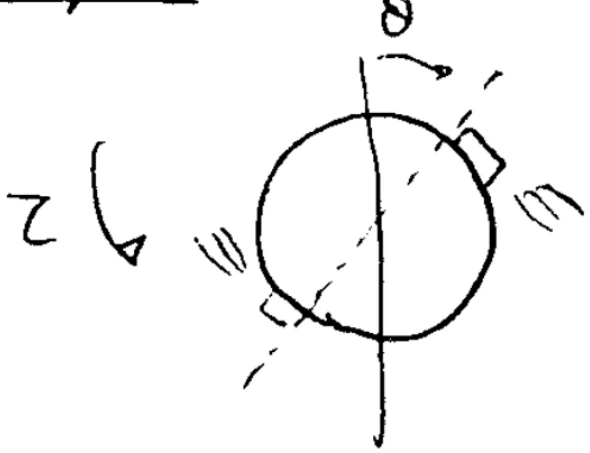
$\tau = k(\theta_2 - \theta_1)$

Viscous friction: 

$\tau = B(\dot{\theta}_2 - \dot{\theta}_1)$

Some models that we will use later on:

- <u>Example</u> Satellite attitude control: (with torque applied by 2 thrusters)



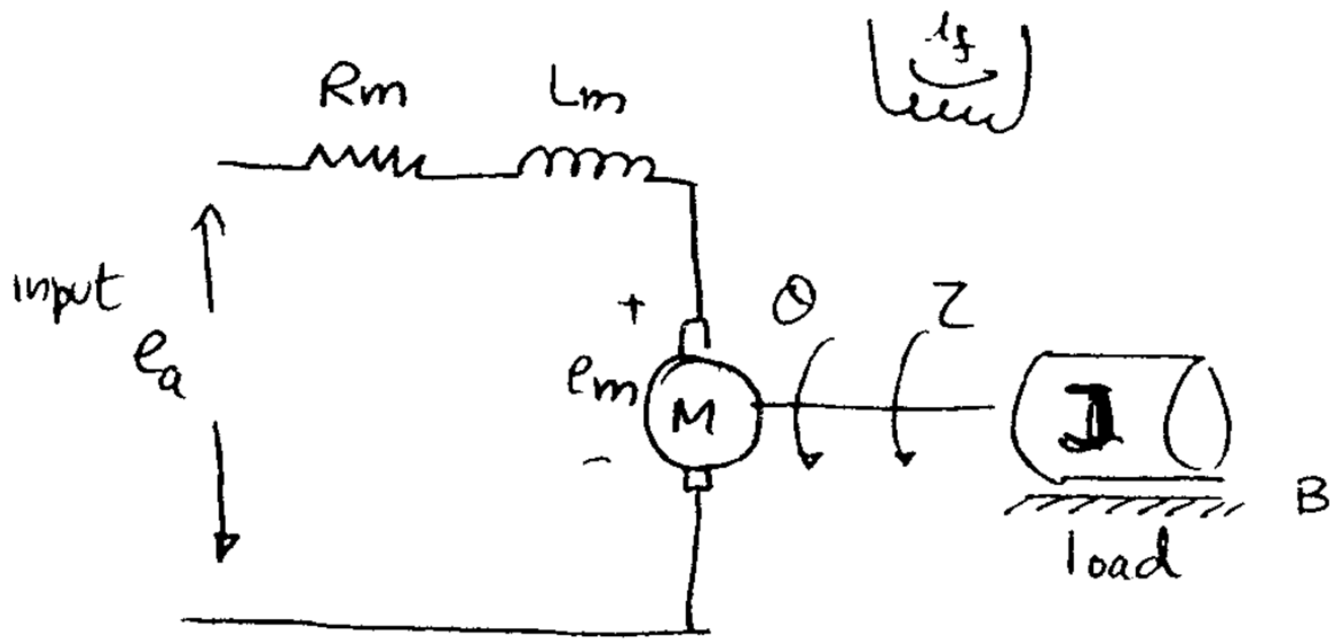
$$J\frac{d^2\theta}{dt^2} = \tau$$

(or, in Laplace domain: $Js^2\Theta = \tau \Rightarrow \Theta = \frac{1}{Js^2}\tau$:
essentially a double integrator)

Some models that we will use later on:

- Electromechanical Systems:

DC motor with independent excitation:



Some models that we will use later on:

1) Electrical equation: $e_a = R_m i_a + L_m \frac{d i_a}{dt} + e_m \Longleftrightarrow E_a(s) = (s L_m + R_m) I_a(s) + E_m(s)$

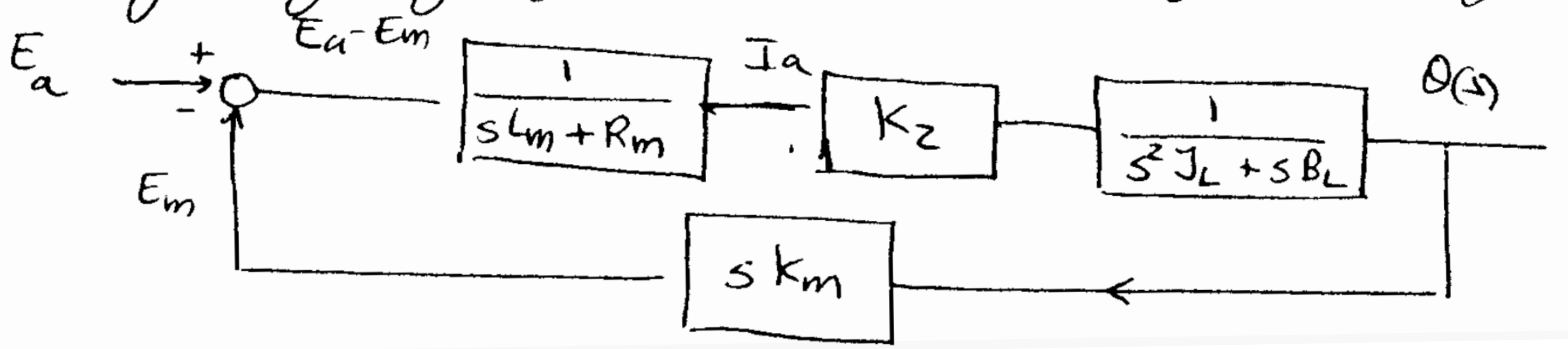
e_m: back emf

2) Back emf: $e_m = k_m \dot{\theta} \Longleftrightarrow E_m(s) = s k_m \theta(s)$

3) Mechanical equation: $\tau = k_\tau i_a$

4) Newton's second equation: $J_L \frac{d\ddot{\theta}}{dt^2} + B_L \dot{\theta} = \tau \Longleftrightarrow (s^2 J_L + s B_L)\,\theta = \tau(s)$

Putting everything together yields the following block diagram:



Some models that we will use later on:

Surprise! The system has built-in feedback (through the back emf)

- Q: How do we find the transfer function from $E_a(s)$ to $\theta(s)$?
- A: We could try solving the 4 simultaneous equations (messy) or applying Mason's formula to the loop above. The latter approach yields:

$$G(s) = \frac{G_1(s)}{1 + s k_m G_1(s)} \qquad \text{where} \qquad G_1(s) = \frac{K_2}{(sL_m + R_m)(s^2 J_L + s B_L)}$$

so in principle we get a third order system.

Some models that we will use later on:

<u>Common simplifying assumption</u>: neglect L_m $(sL_m \cong 0) \Rightarrow$

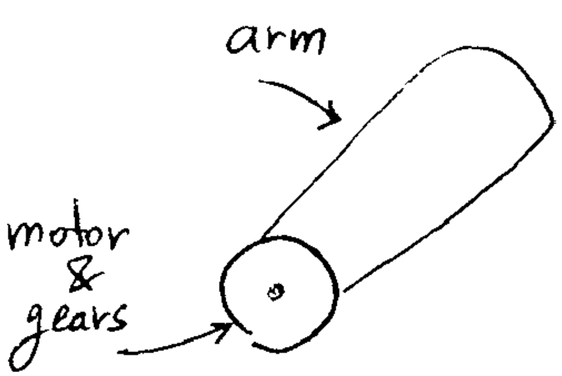
$$G(s) = \frac{\Theta(s)}{E_a(s)} = \frac{\dfrac{k_T}{R_m s (sJ_L + B_L)}}{1 + \dfrac{k_T k_m \cancel{s}}{R_m \cancel{s}(sJ_L + B_L)}} = \frac{k_T}{\cancel{R_m} s \cancel{(sJ_L + B_L)}} \cdot \frac{1}{\left(\dfrac{R_m(sJ_L + B_L) + k_T k_m}{\cancel{R_m}\cancel{(sJ_L + B_L)}}\right)}$$

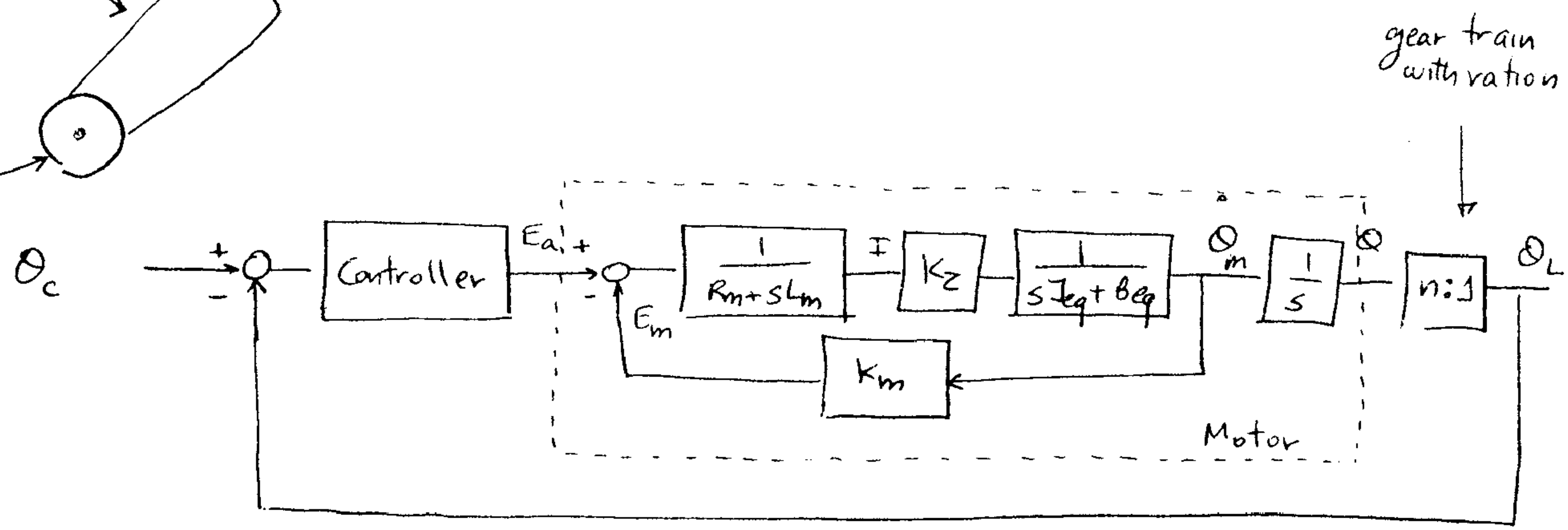
$$G(s) = \left(\frac{k_T}{R_m J_L}\right) \cdot \frac{1}{s\left(s + \dfrac{k_T k_m + B_L}{J_L R_m}\right)} = \boxed{\frac{K}{s(s+a)}}$$

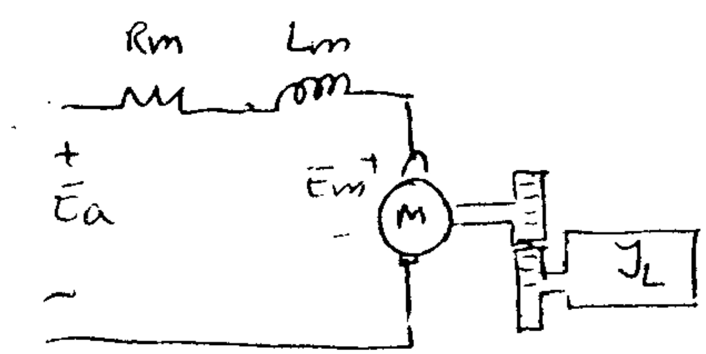
(looks like the cascade of a pure integrator and a first order lag)

Some models that we will use later on:

- Example of use: Position control of a single link, single joint, rigid robotic arm. or of the robotic head in room





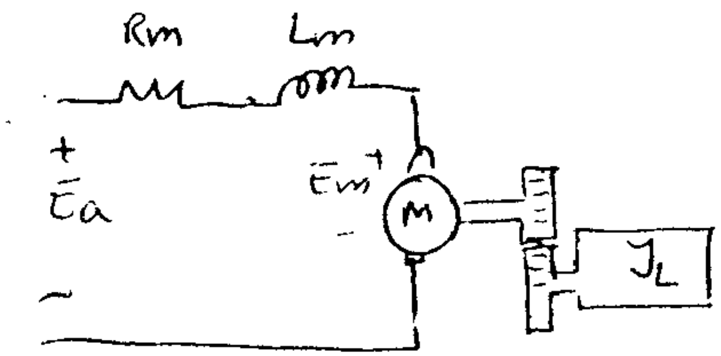


Here J_{eq} = DC motor inertia + (arm · inertia)$\cdot n^2$

$= J_m + J_{arm} \cdot n^2$

$B_{eq} = B_m + B_{arm} \cdot n^2$

Some models that we will use later on:



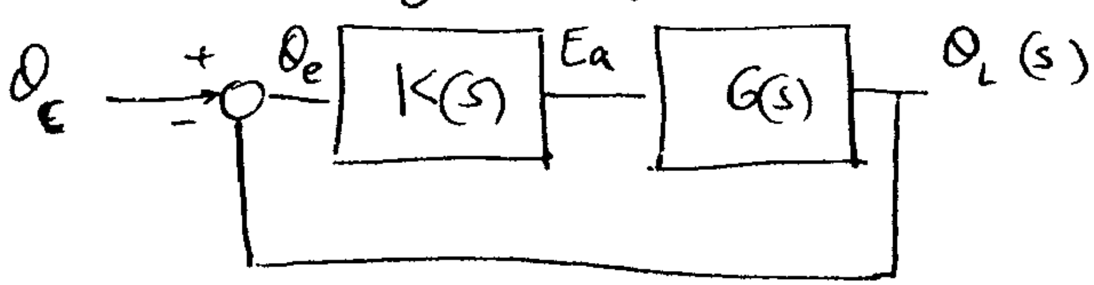
Here $J_{eq} =$ DC motor inertia + (arm. inertia)$\cdot n^2$

$$= J_m + J_{arm} \cdot n^2$$

$$B_{eq} = B_m + B_{arm} \cdot n^2$$

Again, you get a third order system unless you neglect L_m

The block diagram of the closed-loop system is given by:



where $K(s)$ is the transfer function of the controller and $G(s)$ is the T. F. of the arm (including reduction gears)

To find the closed-loop transfer function $\frac{\theta_L}{\theta_c}$ we could, for instance write down the equations:

$$\theta_e = \theta_c - \theta_L, \qquad \theta_L = G(s)\,K(s)\,\theta_e$$

Some models that we will use later on:

Eliminating θ_e yields: $G \cdot K(\theta_c - \theta_L) = \theta_L$ // $GK\,\theta_c = (1+GK)\,\theta_L$

$$\Rightarrow \boxed{\frac{\theta_L}{\theta_c} = \frac{GK}{1+GK}}$$

This is a special case of Mason's formula:

Signal Flow Diagrams and Mason's Formula

They provide an alternative representation of Transfer Function relationships and an alternative (often simpler) to Cramer's rule or block diagram manipulations for computing T. F.

Signal Flow Diagrams and Mason's Formula

Rules:

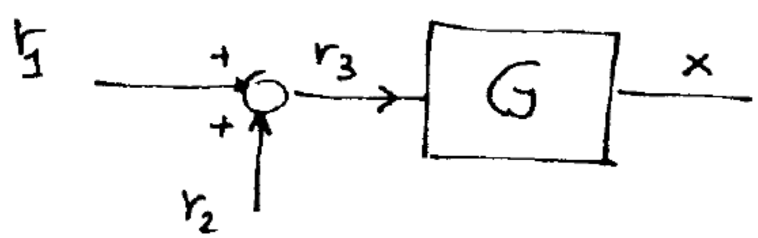
- Each signal is represented by a node
- Each transfer function is represented by a branch (arrow)

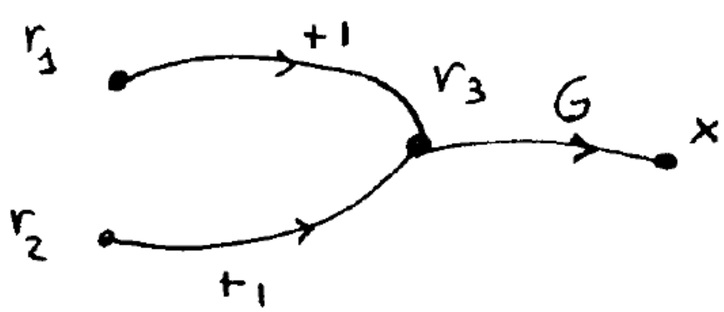
Block Diagram :

Signal flow :

$x(s) = G(s)\, r(s)$

- Summing junctions are represented implicitly: all the inputs converging to a node are added together:



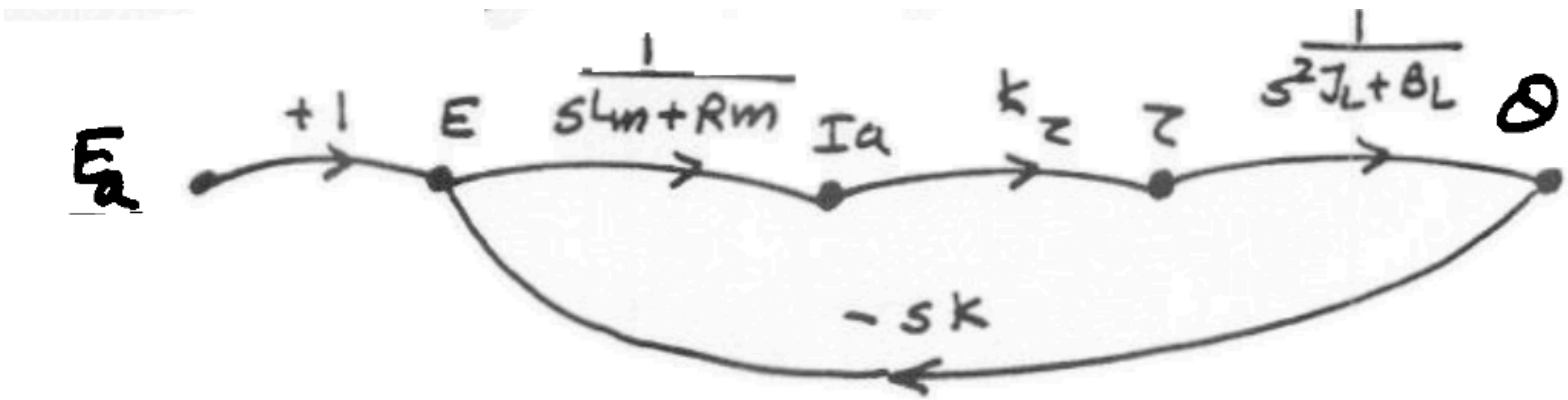


$r_3 = (+1)\cdot r_1 + (+1)\cdot r_2$

$x = G\, r_3$

Signal Flow Diagrams and Mason’s Formula

Question: Find the signal flow graph representation of the DC motor:



Signal Flow Diagrams and Mason's Formula

Some Terminology:

source node: A node that has all signals flowing away from it.

sink node: A node with incoming signals only

Path: Continuous connection of branches between 2 nodes (directed)

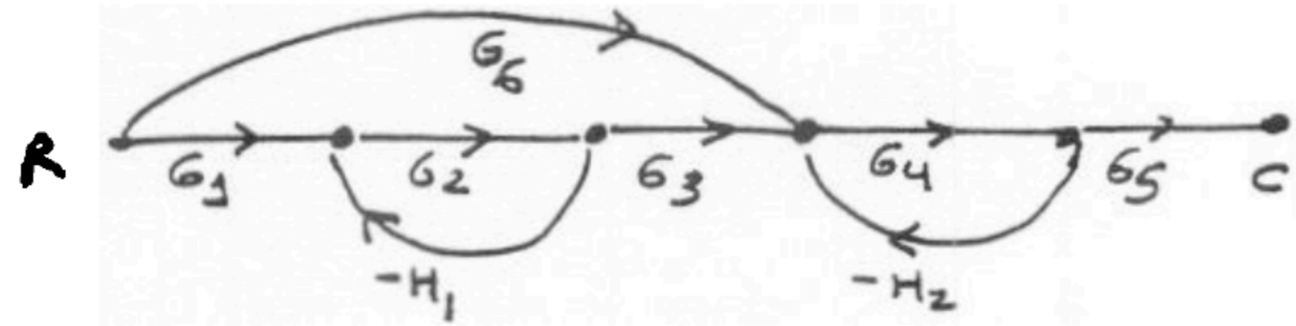
Loop: Closed path (i.e starting node = finishing node)

Path (loop) Gain: Product of all T.F. of all the branches in the path (loop)

Non Touching loops (paths): Loops that do not have any **nodes** in common.

Signal Flow Diagrams and Mason's Formula

Example:



2 loops: $-G_2 H_1$ (L_1)
$-G_4 H_2$ (L_2)

Path $G_6 G_4 G_5$ does not touch L_1
Path $G_1 G_2 G_3 G_4 G_5$ touches both L_1 and L_2

- Mason's Formula (section 2.4) Provides an alternative to Cramer's rule or elimination for finding Transfer Functions

$$T_{CR} = \frac{1}{\Delta} \sum_{k=1}^{P} M_k \Delta_k = \frac{1}{\Delta} \left(M_1 \Delta_1 + \quad + M_P \Delta_P \right)$$

Signal Flow Diagrams and Mason's Formula

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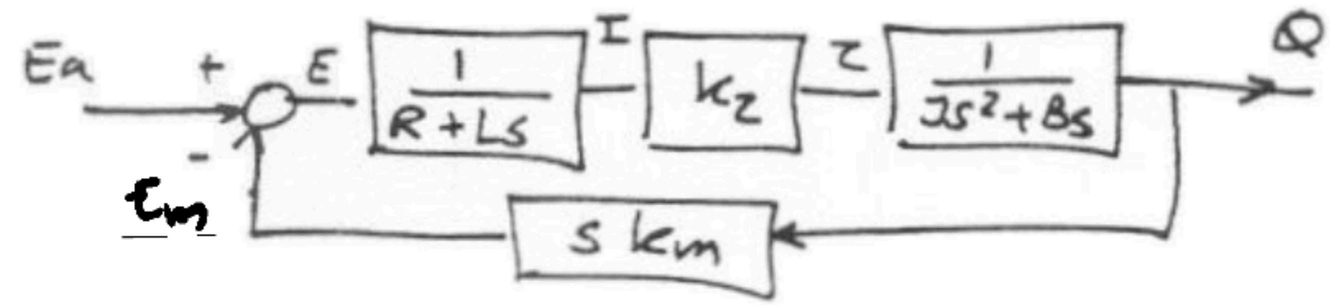
$$T_{CR} = \frac{1}{\Delta} \sum_{k=1}^{P} M_k \Delta_k = \frac{1}{\Delta}\left(M_1 \Delta_1 + \cdots + M_P \Delta_P\right)$$

Where:

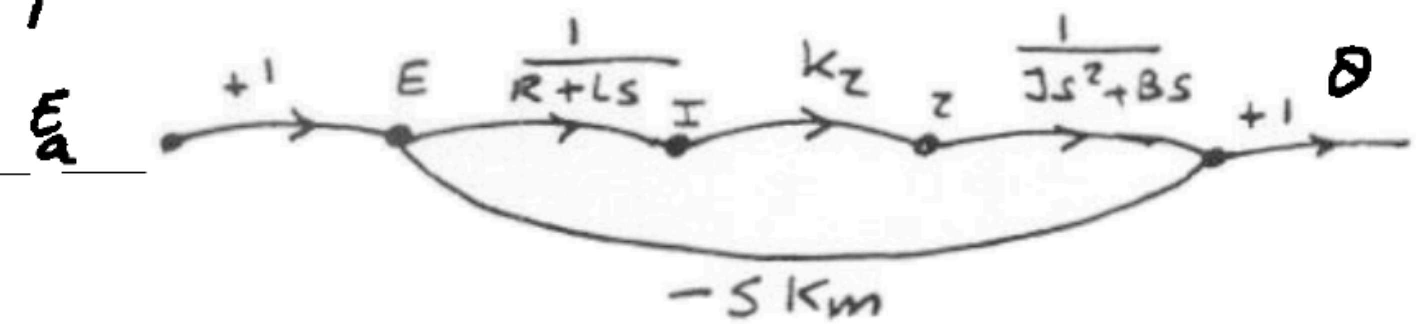
- $\Delta = 1 - \left(\sum \text{gains individual loops}\right) + \sum\left(\text{products of non-touching pairs of loops}\right)$
  $- \sum\left(\text{products of triplets of non-touching loops}\right)$
  $+ \cdots$

- $M_k =$ Gain of the k^{th} path between R and C
- $\Delta_k =$ Value of Δ when the nodes in the path M_k are removed from the graph

Example 1 : DC motor:



Signal flow graph:



1 loop: $L_1 = \dfrac{-k_z k_m s}{(R+Ls)(Js+B)s} \Rightarrow \Delta = 1 + \dfrac{k_z k_m s}{(R+Ls)(Js+B)s}$

Only 1 path from E_a to Θ: $M_1 = \dfrac{k_z}{(R+Ls)(Js+B)s}$

$\Delta_1 = 1$

$$T_{\Theta E_a} = \frac{1}{\Delta} \cdot M_1 \Delta_1 =$$

$$= \frac{\dfrac{k_z}{(R+Ls)(Js+B)s}}{1 + \dfrac{k_z k_m s}{(R+Ls)(Js+B)s}} = \frac{K_z}{(R+Ls)(Js+B)s + k_m k_z s} \quad \#$$