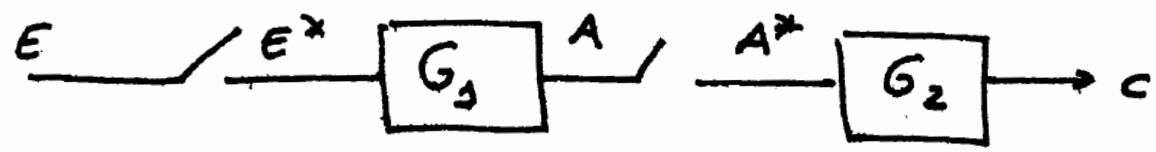
- Other configurations:

a) 

$$C(s) = G_2(s)\,A^*(s) \qquad A(s) = G_1(s)\,E^*(s) \Rightarrow A^* = (G_1 E^*)^* = G_1^* E^*$$

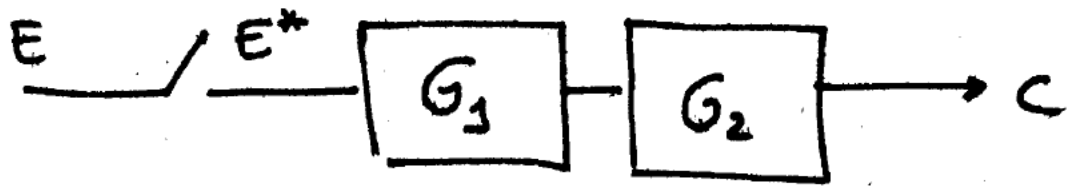
$$C^*(s) = G_2^* A^* = G_2^*(s)\,G_1^*(s)\,E^*(s)$$

$$\Downarrow$$

$$\boxed{C(z) = G_2(z)\,G_1(z)\,E(z)} \quad \text{or} \quad \boxed{G(z) = G_1(z)\cdot G_2(z)}$$

Key feature: we have an ideal sampler between G_1 & G_2

b)

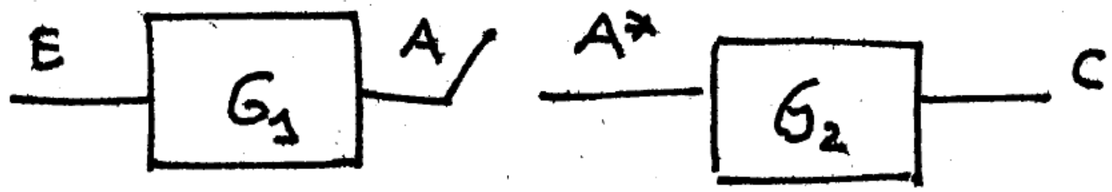


$$C(s) = G_2(s)\, G_1(s)\, E^*(s) \Rightarrow C^*(s) = \left[G_1 \cdot G_2\right]^* E^*(s) \quad \text{or}$$

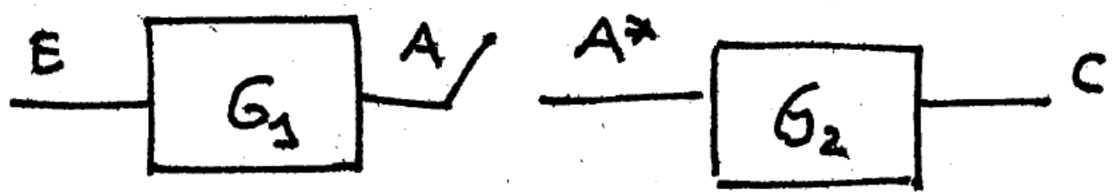
$$C(z) = \mathcal{Z}\left\{G_1 \cdot G_2\right\} \cdot E(z) \Rightarrow \boxed{G(z) = \mathcal{Z}\left[G_1 \cdot G_2\right]}$$

<u>Note</u>: $\mathcal{Z}\left[G_1 G_2\right] \neq G_1(z)\, G_2(z)$!!

c)



c)



$$C = G_2 A^* \Rightarrow C^* = G_2^* \left[G_1 E\right]^*$$

For this system a transfer function <u>cannot</u> be written!
The reason is that you can't factor $E(z)$ out of $[G_1 E](z)$

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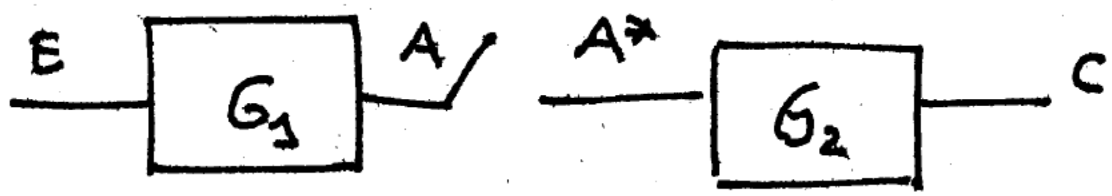

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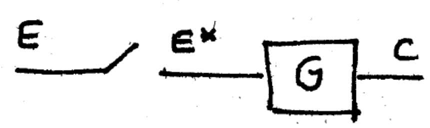
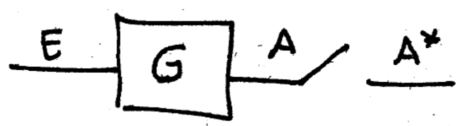
<u>Physical reason</u>:

When you have 

, E^* contains information about E <u>only</u> at the sampling instants. $\Rightarrow$ When you compute C^* you can factor E^* out.

On the other hand, if you have something like this:



$$A(s) = G(s)\,E(s)$$

$$a(t) = \int_0^t g(t-\tau)\,e(\tau)\,d\tau \Rightarrow$$

to compute A^* (and latter C^*) you need information about $e(t)$ at <u>all</u> times, not just the sampling instants.

EECE 5610 Digital Control Systems

Lecture 10

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Important consequence: If you want to have a discrete T.F you have to
<u>select as your unknowns the inputs to the sampler</u>

This variables will always "come free" after the equation sampling process and give a set of starred variables for which we can solve.

Important consequence: If you want to have a discrete T.F you have to select as your unknowns the inputs to the sampler

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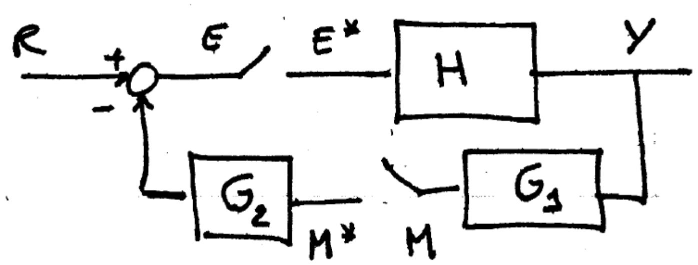
Example:



$$E(s) = R - G_2 M^*$$

$$\left.\begin{array}{l} E^* = R^* - [G_2 M^*]^* = R^* - G_2^* M^* \\ M = (G_1 H) E^* \Rightarrow M^* = (G_1 H)^* E^* \end{array}\right\} \quad E^* = R^* - G_2^* [G_1 H]^* E^*$$

$$\Rightarrow \boxed{E^* = \frac{R^*}{1 + (G_1 H)^* G_2^*}}$$

or: $\boxed{\dfrac{E(z)}{R(z)} = \dfrac{1}{1 + [G_1 H](z)\, G_2(z)}}$

Which one is correct?

A

$$\boxed{\frac{Y(z)}{R(z)} = \frac{H(z)}{1 + (HG_1)(z)\,G_2(z)}}$$

B

$$\boxed{\frac{Y(z)}{R(z)} = \frac{G_1^* G_2^*}{1 + G_1^* G_2^* + (G_2 H)^*}}$$

To obtain Y we can use the equation: $Y(s) = H E^* \Rightarrow Y^* = H^* E^*$

$$\Rightarrow \frac{Y^*}{R^*} = \frac{H^*}{1 + (G_1 H)^* G_2^*} \Leftrightarrow \boxed{\frac{Y(z)}{R(z)} = \frac{H(z)}{1 + (HG_1)(z)\, G_2(z)}}$$

Note that in this case we have a TF. This is because R goes right through a sampler before entering other blocks.

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- <u>Open-loop Systems with Digital Filters</u>

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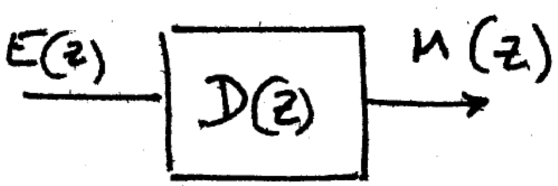
- <u>**Open-loop Systems with Digital Filters**</u>

We consider now the case where the sampled-data system contains a digital filter



$$M(z) = D(z)\, E(z)$$
$$M^{*}(s) = D^{*}(s)\, E^{*}(s)$$

• Open-loop Systems with Digital Filters

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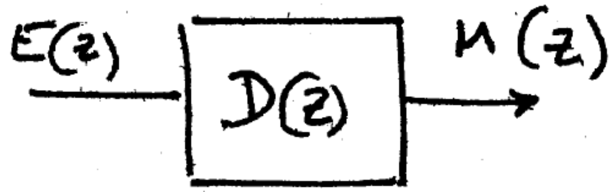
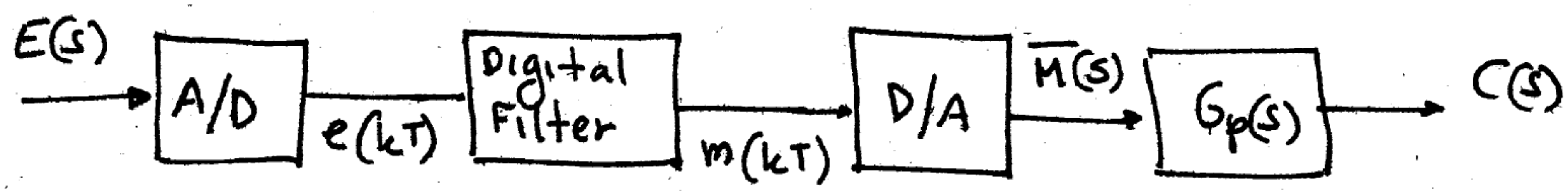


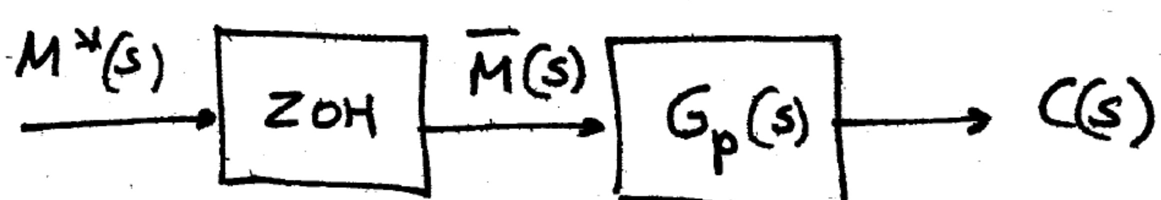
$$M(z) = D(z)\,E(z)$$
$$M^*(s) = D^*(s)\,E^*(s)$$

Physically:



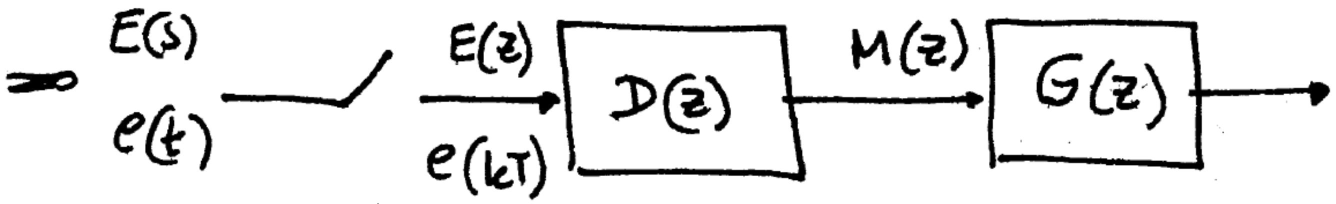
We will assume that the D/A can be represented as a zero order hold

$\Rightarrow$



(Recall that $G_{h_o} = \dfrac{1-e^{-sT}}{s}$)

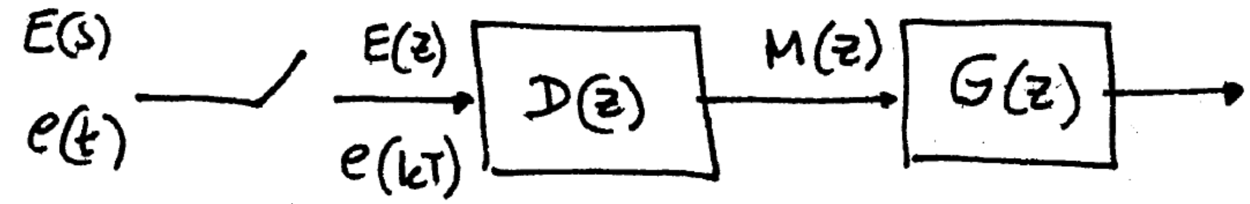
$\Rightarrow$



$$C(s) = \underbrace{G_{h_o}(s) \cdot G_p(s)}_{G(s)} \, M^*(s) \quad \Longrightarrow \quad C(z) = G(z)\,M(z) = G(z)\,D(z)\,E(z)$$

(Recall that $G_{h_0} = \dfrac{1-e^{-sT}}{s}$)

$\Rightarrow$



$$C(s) = \underbrace{G_{h_0}(s) \cdot G_p(s)}_{G(s)} M^*(s) \quad \Longrightarrow \quad C(z) = G(z) M(z) = G(z) D(z) E(z)$$

<u>Example</u>: Suppose that the digital filter is given by the following diff. eq.

$$m(kT) = 2e(kT) - e[(k-1)T]$$

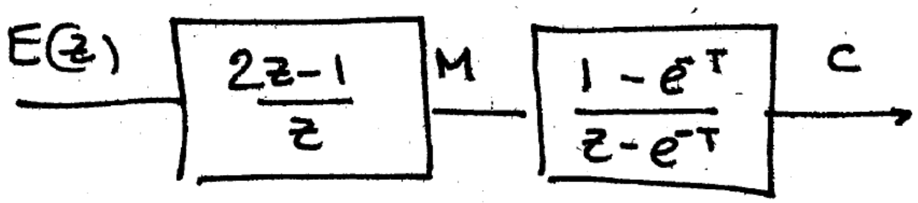
$$M(z) = 2E(z) - \frac{E(z)}{z} = \frac{2z-1}{z} E(z)$$

$$\Rightarrow D(z) = \frac{M}{E} = \boxed{\frac{2z-1}{z}}$$

$$G_p(s) = \frac{1}{(s+1)} \Rightarrow G(s) = G_{h0}\, G_p = \frac{1-e^{-sT}}{s(s+1)}$$

$$G(z) = \left(1-\frac{1}{z}\right) Z\left[\frac{1}{s(s+1)}\right] = \left(1-\frac{1}{z}\right)\left[\frac{1}{1-1/z} - \frac{1}{1-e^{-T}/z}\right]$$

$$= \frac{1-e^{-T}}{z-e^{-T}}$$



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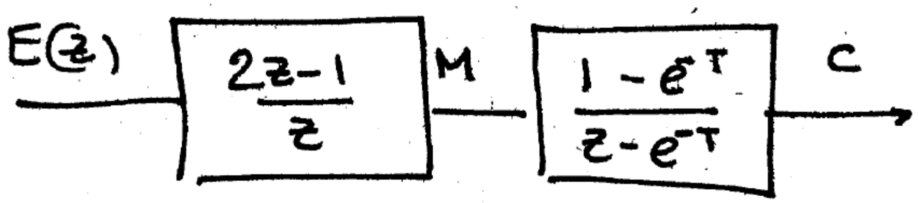

Assume that $E(z)$ is a step: $E(z) = \frac{z}{z-1} \Rightarrow$

$$C(z) = \left(\frac{1-e^{-T}}{z-e^{-T}}\right)\left(\frac{2z-1}{\cancel{z}}\right)\left(\frac{\cancel{z}}{z-1}\right)$$

From here we can get $c(kT)$ either by doing partial fraction expansion or using the residues formula.

Effect of T_s

Recall that $C^*(s)$ or $C(z)$ gives you information <u>only</u> on what happens at the sampling instants, but not in-between.

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Q: Is this a problem?

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Q: Is this a problem?

A: Depends on how fast the dynamics of your plant are, compared to the sampling rate

Example:

$$G_p(s) = \frac{25}{s^2 + 2s + 25} \Rightarrow \omega_n^2 = 25 \quad (\omega_n = 5)$$

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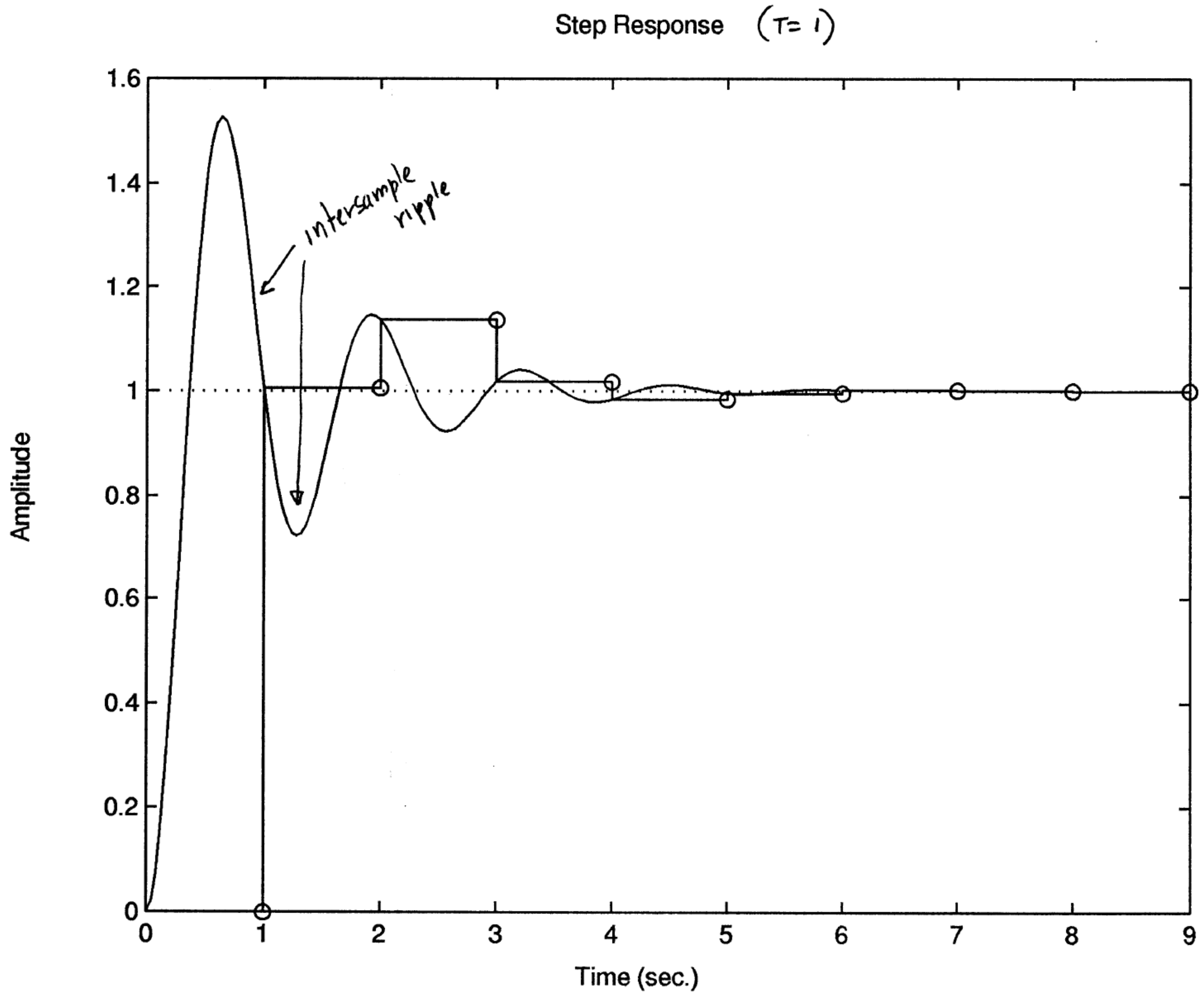
$$\zeta = 0.2 \quad \text{(underdamped)}$$

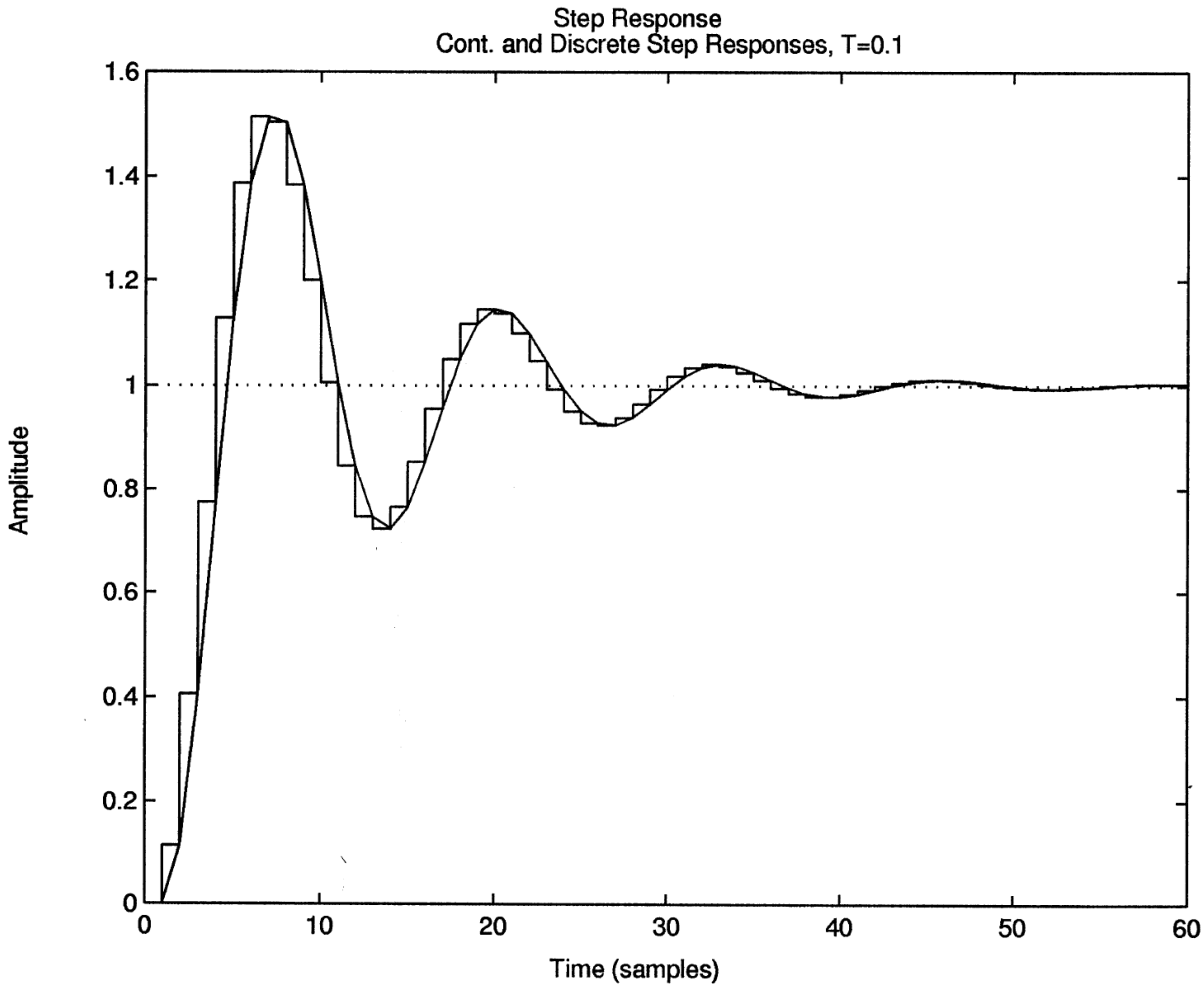
It can be shown that if we sample at $T=0.1$ and $T=1$ we get the following discrete time equivalents:

$$G(z) = \mathcal{Z}\left\{\left(\frac{1-e^{-sT}}{s}\right)\cdot G_p(s)\right\} = \left(1-\frac{1}{z}\right)\mathcal{Z}\left[\frac{25}{s(s^2+2s+25)}\right]$$

$$G_{T=1} = \frac{z-0.007}{z^2-0.1365z+0.1353}$$

$$G_{T=0.1} = \frac{0.115z-0.107}{z^2-1.6z+0.82}$$


Step Response (T= 1)
intersample ripple
Amplitude
Time (sec.)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9



Step Response
Cont. and Discrete Step Responses, T=0.1
Amplitude
Time (samples)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
10
20
30
40
50
60

Key assumptions: If we have <u>more than</u> one sampler then

(a) They work at the same rate (single rate sampling)
(b) They are <u>synchronized</u>

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Q: How do we deal with these cases

A: We need a new tool: the "modified" z-transform.

- <u>Modified z transform</u>

So far we have been considering systems with <u>integer number of sampling</u> periods delays and synchronous sampling.
In order to analyze systems with non-integer delays and/or nonsynchronous sampling we need a new tool: the "modified" z-transform

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The modified z-transform method is a modification of the regular z transform using fictitious time delays.

Consider a function $f(t)$ and delay it by an amount ΔT, $0 < \Delta \leq 1$

$$f(t) \xrightarrow{\text{delay}} f(t-\Delta T)\, u(t-\Delta T) \xrightarrow{\mathcal{L}} F(s)\, e^{-s\Delta T}$$

$$f(kT) \longrightarrow f(kT-\Delta T)$$

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delayed z transform

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$$\Rightarrow \mathcal{Z}\{f(t-\Delta T)\, u(t-\Delta T)\} = \sum_{n=1}^{\infty} f(nT-\Delta T)\, z^{-n}$$ delayed z transform

$$\boxed{F(z,\Delta) \triangleq \mathcal{Z}\left[f(t-\Delta T)\, u(t-\Delta T)\right] = \mathcal{Z}\left[F(s)\, e^{-s\Delta T}\right]}$$

Definition: Modified z-transform: $\boxed{F(z,m) = F(z,\Delta)\Big|_{\Delta = 1-m}}$

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$$\Rightarrow F(z,m) = \mathcal{Z}\left[F(s)\,e^{-s\Delta T}\Big|_{\Delta=1-m}\right] = f(mT)\cdot\frac{1}{z} + f\left[(1+m)T\right]\frac{1}{z^2} + \cdots +$$

$$= \boxed{\sum_{i=0}^{\infty} f\left[(m+i)T\right] z^{-(i+1)}}$$

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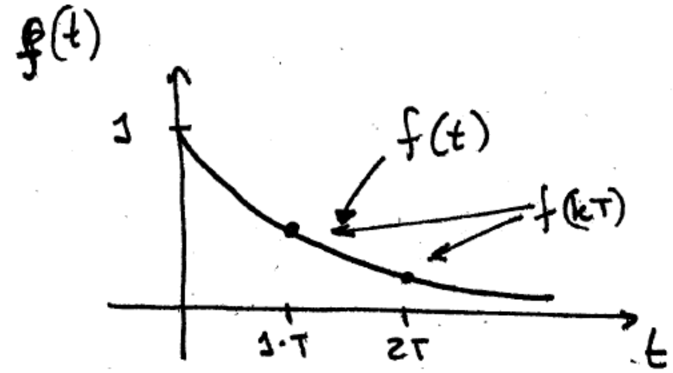
Example 1: Suppose that $f(t) = e^{-t} \Rightarrow f(kT) = e^{-kT}$

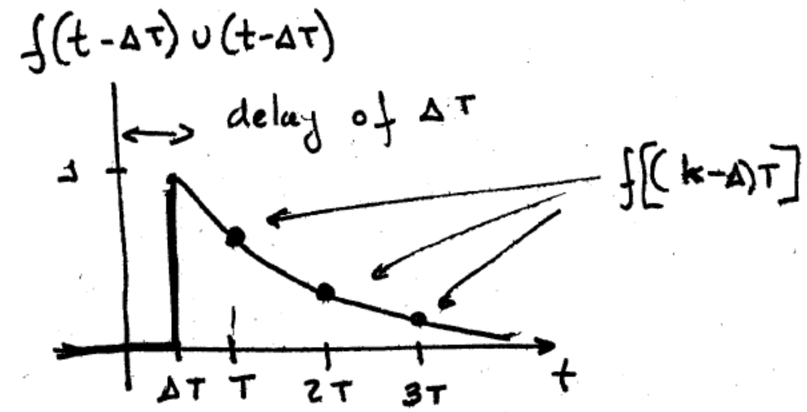
and that we want to find out the z transform of the signal delayed by ΔT

$$F(z,m) = \sum_{i=0}^{\infty} f[(m+i)T]\, z^{-(i+1)} = \sum_{0}^{\infty} e^{-(k+m)T} z^{-(k+1)} = \frac{e^{-mT}}{z} \cdot \frac{1}{1-\frac{1}{z\cdot e^{T}}}$$

$$F(z,m) = \sum_{i=0}^{\infty} f\left[(m+i)T\right] z^{-(i+1)} = \sum_{0}^{\infty} e^{-(k+m)T} z^{-(k+1)} = \frac{e^{-mT}}{z} \cdot \frac{1}{1 - \frac{1}{z \cdot e^{T}}}$$

$$\boxed{F(z,m) = \frac{e^{-mT}}{z - e^{-T}}}$$





$$F(z,m) = \sum_{i=0}^{\infty} f\left[(m+i)T\right] z^{-(i+1)} = \sum_{0}^{\infty} e^{-(k+m)T} z^{-(k+1)} = \frac{e^{-mT}}{z} \cdot \frac{1}{1 - \frac{1}{z \cdot e^{T}}}$$

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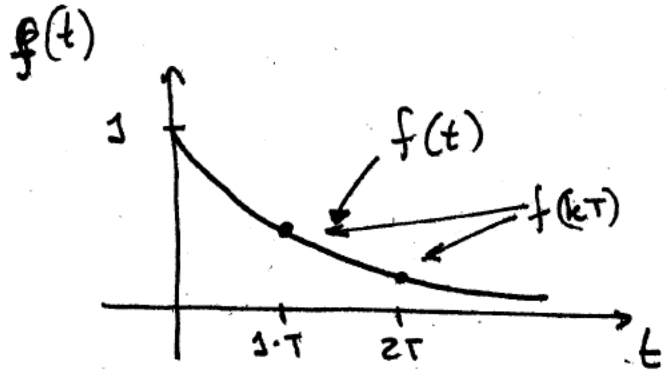
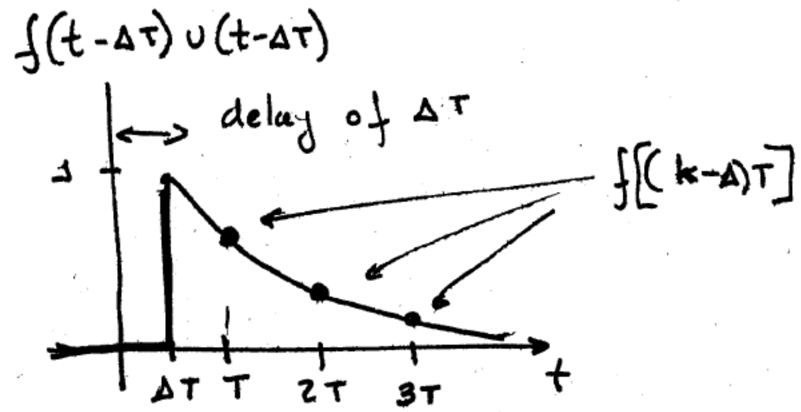
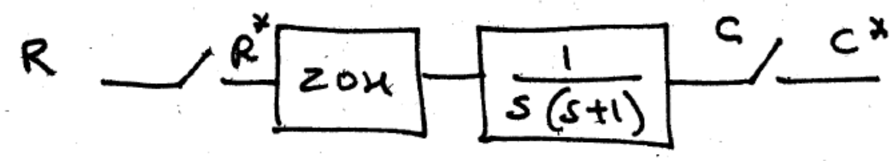




So we can use this technique to "peek" at the signal in between sampling instants (provided that we can handle that extra $\Delta \cdot T$ delay at the begining)

Example 2:



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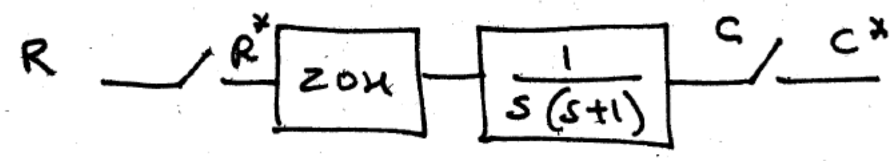
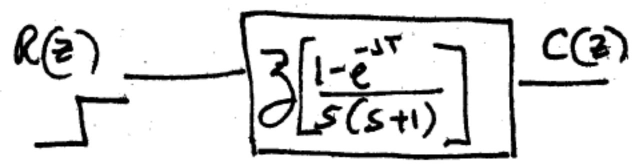


We may want to look at $c(kT-\Delta T)$ to make sure that there is no intersample ripple. In this case we have



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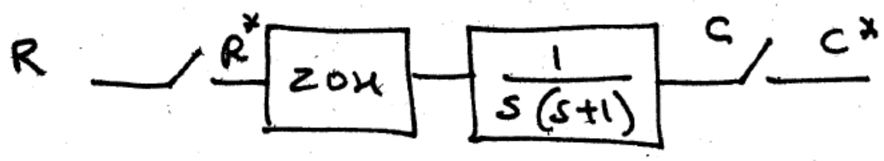
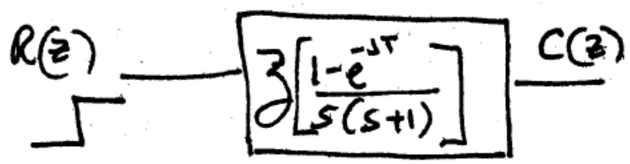


We may want to look at $c(kT - \Delta T)$ to make sure that there is no intersample ripple. In this case we have



From previous lectures we know that $\mathcal{Z}\left[\dfrac{1-e^{sT}}{s(s+1)}\right] = \dfrac{1-e^{-T}}{z-e^{-T}}$

and that $C(z) = \left(\dfrac{1-e^{-T}}{z-e^{-T}}\right) R(z) = \left(\dfrac{1-e^{-T}}{z-e^{-T}}\right)\dfrac{z}{z-1} = \dfrac{z}{z-1} - \dfrac{z}{z-e^{-T}}$

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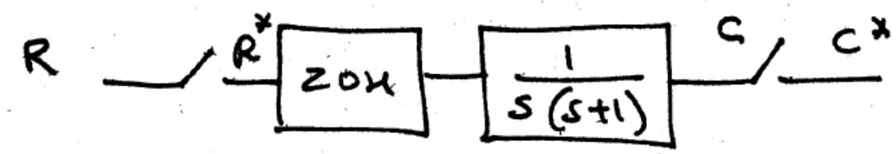

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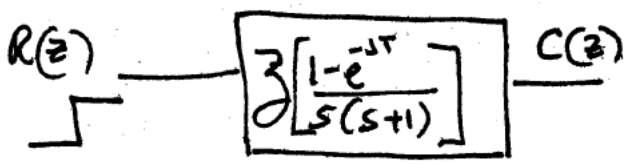

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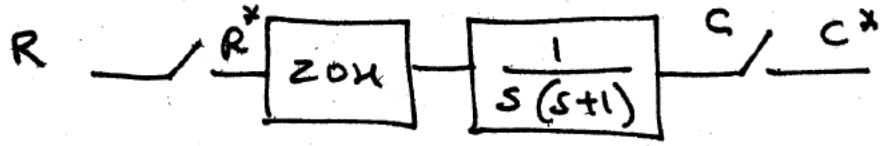

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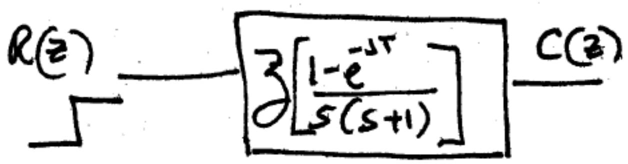

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What about the modified z transform?

$$G(s) = \frac{1-e^{-sT}}{s(s+1)} = \underbrace{\frac{1}{s(s+1)}}_{C_1(s)} - \underbrace{\frac{e^{-sT}}{s(s+1)}}_{C_2(s)} =$$

$\Rightarrow$ We can find out $C_1(z,m)$ proceeding as before.

Q: What about $c_2(z, m)$?

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$$\boxed{\mathcal{Z}_m\left[e^{-kT_s}E(s)\right] = z^{-k}\mathcal{Z}_m\left[E(s)\right]}$$

Proof:

$$\mathcal{Z}_m\left[e^{-kT_s}E(s)\right] = \mathcal{Z}\left[e^{-kT_s}\cdot e^{-\Delta T_s}E(s)\right] = z^{-k}\mathcal{Z}\left[e^{-\Delta T\cdot s}E(s)\right] = z^{-k}\mathcal{Z}_m\left[E(s)\right]$$

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(3) Additional properties:

$$E(z,1) = E(z,m)\big|_{m=1} = e(1)z^{-1} + \dots \qquad = E(z) - e(0)$$
$$E(z,0) = E(z,m)\big|_{m=0} = e(0)z^{-1} + \dots \qquad = \frac{1}{z}E(z)$$

- <u>How to compute the modified z transform</u>

(Recall that usual tables do not apply)

$$E(z,m) = \mathcal{Z}\left[E(s)\, e^{-(1-m)Ts}\right] = \mathcal{Z}\left[E(s)\, e^{mTs} \cdot e^{-Ts}\right]$$

Let $E_1(s) = E(s)e^{mTs} \Rightarrow E(z,m) = \mathcal{Z}\left[e^{-Ts} E_1(s)\right] = z^{-1}\mathcal{Z}\left[E_1(s)\right]$

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$$E(z,m) = z^{-1}\left[\sum_{\substack{\text{poles}\\ E_1(\lambda)}} \text{residues } E_1(\lambda)\,\frac{1}{1-\frac{e^{\lambda T}}{z}}\right] = z^{-1}\left[\sum_{\substack{\text{poles}\\ E(\lambda)}} \text{residues } E(\lambda)\, e^{mT\lambda}\,\frac{1}{1-\frac{e^{\lambda T}}{z}}\right]$$

(similarly: $E^*(s,m) = \frac{1}{T}\sum_{n=-\infty}^{n=+\infty} E(s+jn\omega_s)\, e^{-(1-m)(s+jn\omega_s)T}$

Example: modified z transform of $e(t) = e^{-t} \Leftrightarrow E(s) = \frac{1}{s+1}$

$$E(z,m) = \frac{1}{z} \sum_{\substack{\text{poles}\\ E(\lambda)}} \left\{ \text{res } E(\lambda) e^{mT\lambda} \frac{1}{1 - \frac{e^{\lambda T}}{z}} \right\} = \frac{1}{z} \frac{\cancel{(1+\lambda)}}{\cancel{(1+\lambda)}} e^{mT\lambda} \frac{1}{\left(1 - \frac{e^{\lambda T}}{z}\right)} \Bigg|_{\lambda=-1}$$

$$= \frac{1}{z} e^{-mT} \frac{1}{1 - \frac{e^{-T}}{z}} = \boxed{\frac{e^{-mT}}{z - e^{-T}}} \quad \# \quad \text{(same as before)}$$

Now we will see how to use the modified z transform to deal with

(a) systems with time delays

(b) non-synchronous sampling

(c) multirate sampling

Summary of Chapter 4 :

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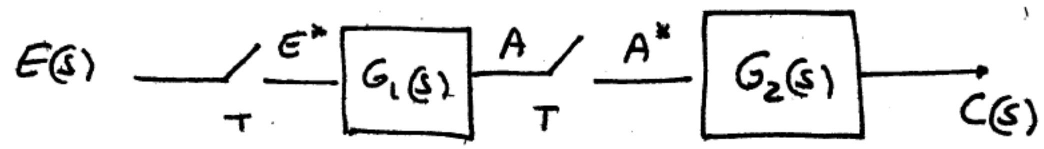
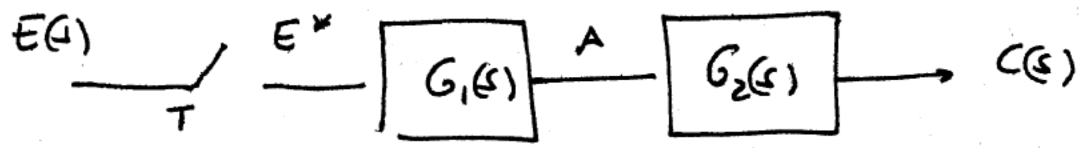
(a)



$$C(s) = G_2(s)\,A^* = G_2(s)\,G_1^*(s)\,E^*$$
$$C^*(s) = G_2^*\,G_1^*\,E^*$$

$$C(z) = G_2(z)\,G_1(z)\,E(z)$$

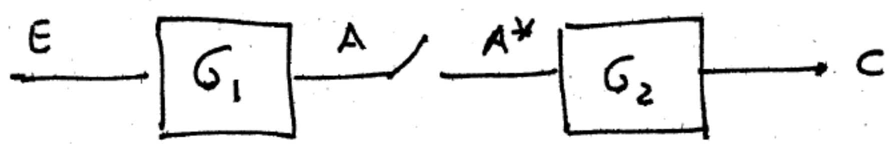
(b)



$$C^*(s) = \left[G_2(s)\,G_1(s)\right]^* E^*(s)$$

$$C(z) = \mathcal{Z}\left[G_1 \cdot G_2\right] E(z)$$

(c)



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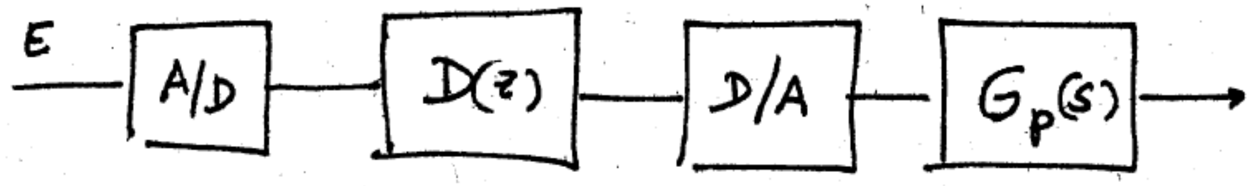
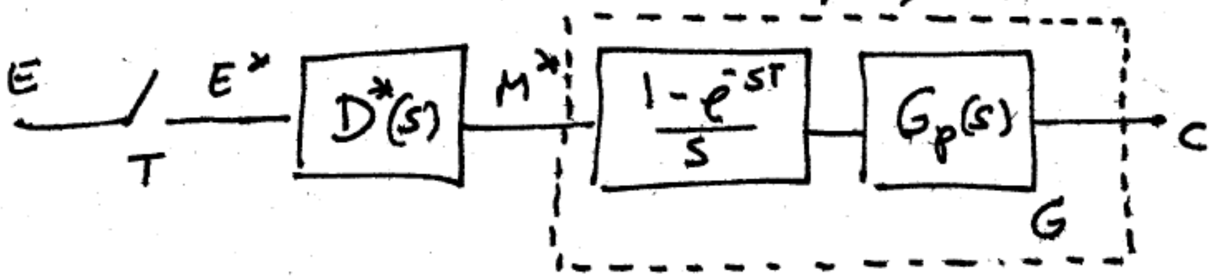
$$C^*(s) = G_2^*\left[G_1 E\right]^* \Rightarrow C(z) = G_2(z)\,\mathcal{Z}\left[G_1 \cdot E\right]$$

In this case a transfer function <u>does not</u> exist: $E(z)$ can't be factored out

(d) Systems with digital filters:



model the A/D as an ideal sampler, D/A as a data hold:



$$G(z) = \mathcal{Z}\left[\frac{1-e^{-sT}}{s} \cdot G_p(s)\right] = \frac{z-1}{z}\,\mathcal{Z}\left[\frac{G_p(s)}{s}\right]$$

$$C(z) = G(z) \cdot D(z) \cdot E(z)$$

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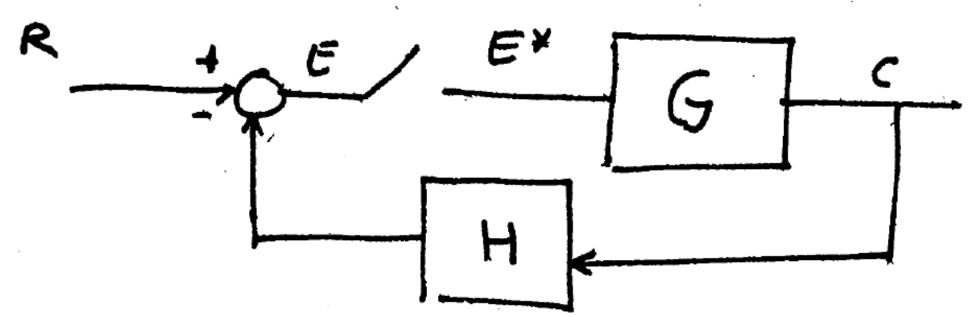

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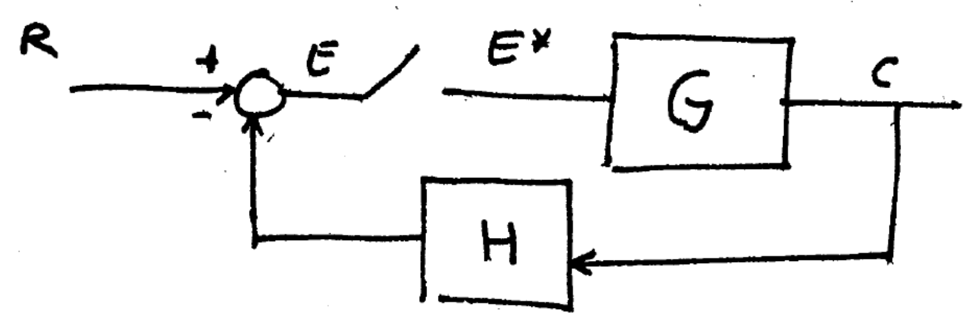

Want to find closed loop transfer function: $\Rightarrow$

$$E = R - HC = R - (HG)E^* \Rightarrow E^* = R^* - (HG)^* E^* \Rightarrow$$

$$E^* = \frac{R^*}{1+[HG]^*} \quad \text{and} \quad C = \frac{GR^*}{1+(HG)^*}$$

However, if we had selected C instead of E as variable we get

$$C = GE^*$$

$$E = R - HC \Rightarrow E^* = R^* - [HC]^* \Rightarrow C = GR^* - G[HC]^*$$

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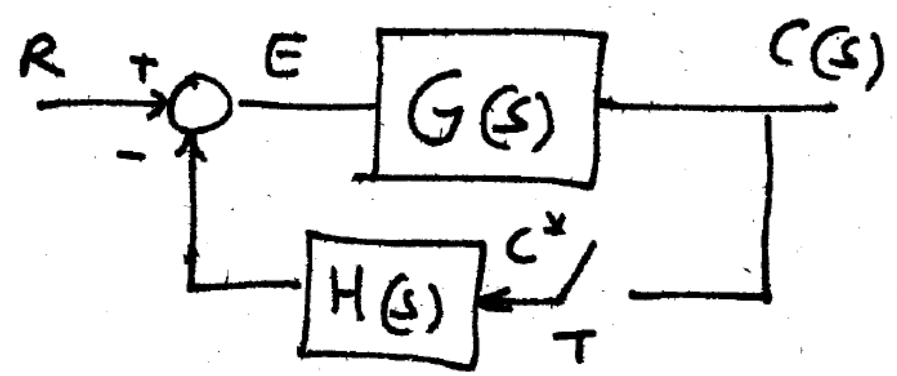

$$E = R - HC^*$$
$$C = GR - GHC^*$$
$$C^* = (GR)^* - (GH)^* C^*$$