EECE 5610 Digital Control Systems
NEU, Fall **2022**

Final Exam
December 9th, 2022

Name: ______________________________

NU Email: ______________________________

Instructions:

- This is a 100-minute exam, with an extra 20 + 20 minutes to download/upload/email your exam/solution.
- One double-sided exam cheat sheet is allowed.
- Work as many problems as you can. Try not to spend too much time on a single problem.
- No communication (online searching) is allowed during exam time.

Problem	Points	Score
Problem 0	5	
Problem 1	45	
Problem 2	30	
Problem 3	20	
Total	100	

Problem 0 (5 points): I affirm that I understand NEU's Code of Conduct and Academic Integrity expectations; I understand that all suspected violations (including cheating, collusion, and plagiarism) will be submitted to the Office of Student Conduct & Community Expectations; and I pledge that the work I submit will be my own.
(Write your full name as your signature.)

Problem 1 (45points)

The block diagram of a rigid-body satellite is given in Figure 1. The control signal is the voltage $e(t)$. The zero-order hold output $m(t)$ is converted into a torque $\tau(t)$ by an amplifier and the thrusters. The system output is the attitude angle $\Theta(t)$ of the satellite.

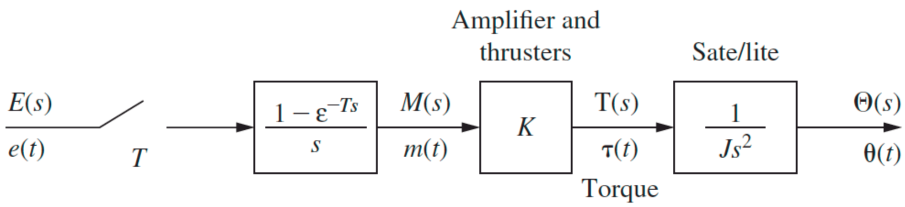

Figure 1: Block diagram for a satellite.

(a) Find the transfer function $G(z) = \Theta(z)/E(z)$.

(b) Use the results of part (a) to find the system's unit-step response, that is, the response with $e(t) = u(t)$. (You need to find $\Theta(z)$ and $\Theta(kT)$)

(c) Sketch the zero-order-hold output $m(t)$ in (b).

Figure 2 shows a simplified diagram of the attitude control of a satellite. Note that A/D is an ideal sampler with sampling time T>0, and D/A is ZOH.

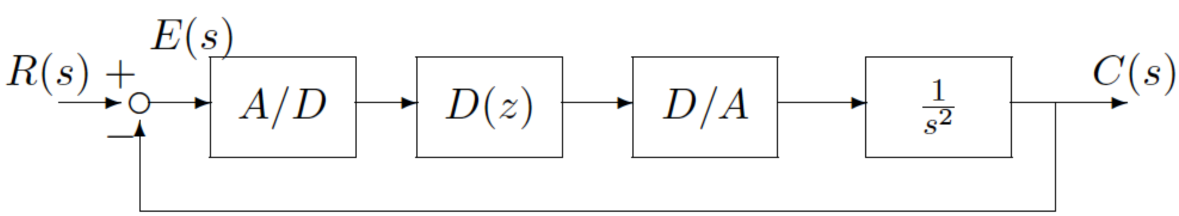

Figure 2: Simplified diagram of a satellite digital attitude control

(d) Assume that $D(z) = K > 0$ and the sampling time is $T > 0$. Find the steady state error to a step input as a function of K. (Hit: What is the type of system? Is it stable?)

(e) Can you find a value of K such that the following performance specifications are satisfied?
   a. Zero steady state error to a step input
   b. No overshoot in the step response

   (Hint: for checking stability you can use any methods you like!)

If so, give a suitable value for K, if not, fully justify you answer. **A simple yes or no answer will not give you partial credit.**

Problem 2 (30 points)

Consider the model of a first order hold shown in Figure 3

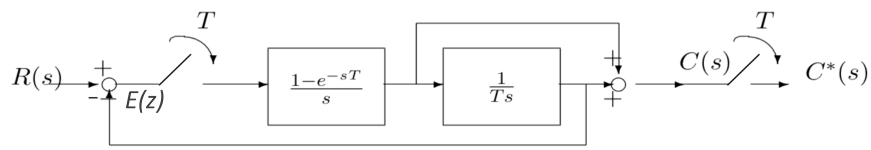

Figure 3: Block Diagram for Problem 2.

(a) Find an analytic expression for the impulse response of this system c(t). [Hint: The continuous-time transfer function of a first hold is given by:

$$G_{foh}(s) = \frac{1+Ts}{T}\left[\frac{1-e^{-Ts}}{s}\right]^2$$

Then you need to take inverse Laplace transform to obtain c(t)!]

(b) Find the discrete transfer function E(z)/R(z). (Hint: You need to use the general procedure! Do you remember it? ☺ 1)- find the original flow graph 2) assign variables to each sampler outputs 3) consider each sampler output as source. Find the sampler inputs and the system outputs in terms of each sampler output and the system input 4) take the starred transform)

(c) Find the discrete transfer function C(z)/R(z). Does your result make sense (Hint: Recall that this block diagram is supposed to implement a first-order hold!) **A simple yes or no answer will not give you partial credit.**

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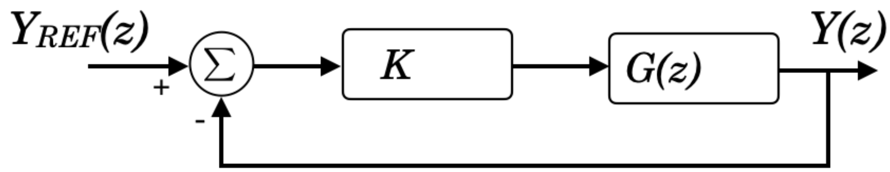
Problem 3 (20 points)

Find the values of K for which the following system is stable for $G(z) = 1/(z-2)$.



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Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z, m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\varepsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\varepsilon^{-amT}}{z-\varepsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ε^{-at}	$\frac{z}{z-\varepsilon^{-aT}}$	$\frac{\varepsilon^{-amT}}{z-\varepsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\varepsilon^{-at}$	$\frac{Tz\varepsilon^{-aT}}{(z-\varepsilon^{-aT})^2}$	$\frac{T\varepsilon^{-amT}[\varepsilon^{-aT}+m(z-\varepsilon^{-aT})]}{(z-\varepsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\varepsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\varepsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\varepsilon^{-amT}}{z-\varepsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\varepsilon^{-at}$	$\frac{z(1-\varepsilon^{-aT})}{(z-1)(z-\varepsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\varepsilon^{-amT}}{z-\varepsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\varepsilon^{-at}}{a}$	$\frac{z[(aT-1+\varepsilon^{-aT})z+(1-\varepsilon^{-aT}-aT\varepsilon^{-aT})]}{a(z-1)^2(z-\varepsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\varepsilon^{-amT}}{a(z-\varepsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\varepsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\varepsilon^{-aT}}-\frac{aT\varepsilon^{-aT}z}{(z-\varepsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\varepsilon^{-aT}}+\frac{aT\varepsilon^{-aT}}{(z-\varepsilon^{-aT})^2}\right]\varepsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\varepsilon^{-at}-\varepsilon^{-bt}$	$\frac{(\varepsilon^{-aT}-\varepsilon^{-bT})z}{(z-\varepsilon^{-aT})(z-\varepsilon^{-bT})}$	$\frac{\varepsilon^{-amT}}{z-\varepsilon^{-aT}}-\frac{\varepsilon^{-bmT}}{z-\varepsilon^{-bT}}$