Tyler McKean | DCS Final Exam

Problem 0 Code of Conduct

Tyler McKean ->

Problem 1

(a) Transfer function $G(z) = \Theta(z)/E(z)$

$$G(z) = \mathcal{Z}\left[1 - \varepsilon^{-Ts} \times \frac{K}{J} \times \frac{1}{s^3}\right]$$

transform pair -> $1/s^3 \xrightarrow{\mathcal{Z}} \dfrac{T^2 z(z+1)}{2(z-1)^3}$

(b) Find system's unit-step response

$$G(z) = \frac{K}{J}\left(\frac{\cancel{z-1}}{z}\right)\frac{T^2 \cancel{z}(z+1)}{2(z-1)^{\cancel{3}2}} = \boxed{G(z) = \frac{K}{J}\frac{T^2(z+1)}{2(z-1)^2}}$$

$$E(z) = \frac{z}{z-1} \Rightarrow \Theta(z) = \frac{K}{J}\frac{T^2(z+1)}{2(z-1)^2}\left(\frac{z}{z-1}\right)$$

$$\boxed{\Theta(z) = \frac{K}{J}\frac{T^2 z(z+1)}{2(z-1)^3}}$$

(c) Sketch ZOH output $m(t)$ in (b)

$e(t) = u(t) \Rightarrow E(s) =$

$$m(t) \Rightarrow M(s) = \frac{1 - \varepsilon^{-Ts}}{s} \Rightarrow \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{\varepsilon^{-Ts}}{s}\right] = u(t) - u(t-T)$$

$m(t) = u(t) - u(t-T)$

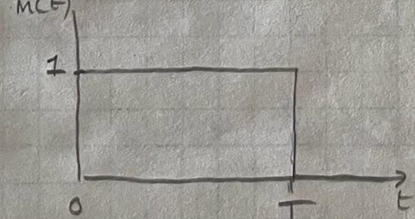


Figure 2

(d) $D(z) = K > 0$ and $T > 0$
find e_{ss} ~~to~~ step input

$$G(z) = K\, z\left[1 - \varepsilon^{-Ts} \times \frac{1}{s^3}\right] = K\left(\frac{\cancel{z-1}}{\cancel{z}}\right)\frac{T^2 z(z+1)}{2(z-1)^{\cancel{3}}2}$$

$$G(z) = \frac{KT^2(z+1)}{2(z-1)^2} \Rightarrow K_p = \lim_{z\to 1} G(z)$$

From $G(z) \Rightarrow \frac{KT^2(z+1)}{2(z-1)^2}$ $= \frac{KT^2(1+1)}{2(1-1)^2} = \boxed{\infty}$

System is <u>Type 2</u> because denom $\to (z-1)^{2}$

Because we inspect step input $\to$ system $\boxed{e_{ss} = 0}$

System is <u>marginally stable</u> with two poles at $z = 1$

(e) $$G_{cc} = \frac{G(z)}{1+G(z)} = \frac{\frac{KT^2(z+1)}{2(z-1)^2}}{1 + \frac{KT^2}{2}\frac{(z+1)}{(z-1)^2}}$$

Char - eq $\Rightarrow$ $$2(z-1)^2 + \frac{KT^2}{2}(z+1) = 0$$

$$4(z-1)^2 + KT^2(z+1) = 0$$

$$4z^2 + (-8 + KT^2)z + KT^2 + 4 = 0$$

z^0	z^1	z^2
KT^2+4	KT^2-8	4

$Q(1) > 0 \Rightarrow \cancel{4(1)^2} \cancel{-8} + KT^2 + KT^2 \cancel{+4} > 0 \Rightarrow 2KT^2 > 0$
$\boxed{K > 0}$

$(-1)^2 Q(-1) \Rightarrow 4(-1)^2 + 8 \cancel{- KT^2} \cancel{+ KT^2} + 4 > 0 \Rightarrow \boxed{16 > 0}$ untrue!

Second condition of Jury's stability Test failed, so the system is unstable, so no value of K can achieve performance specifications

$\boxed{\text{Problem 2}}$

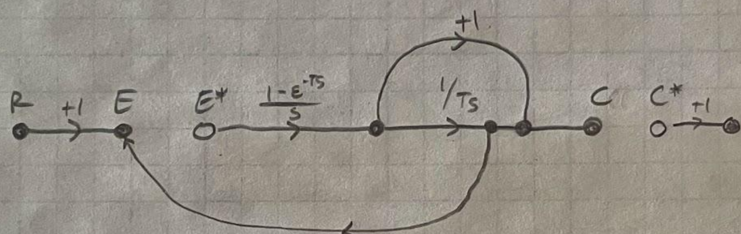
$$G_{foh}(s) = \frac{1+Ts}{T}\left[\frac{1-e^{-Ts}}{s}\right]^2 = \left(\frac{1+Ts}{T}\right)\left(\frac{1-e^{-Ts}}{s}\cdot\frac{1-e^{-Ts}}{s}\right)$$

$$= \frac{1+Ts}{T}\left(\frac{1-e^{-Ts}-e^{-Ts}+e^{-2Ts}}{s^2}\right)$$

$$G_{foh}(s) = \left(\frac{1}{s}+\frac{1}{Ts^2}\right)\left(1-2e^{-Ts}+e^{-2Ts}\right)$$

$$\mathcal{L}^{-1}\left\{\left(\frac{1}{s}+\frac{1}{Ts^2}\right)\left(1-2e^{-Ts}+e^{-2Ts}\right)\right\}$$

$$c(t) = u(t) + \frac{1}{T}t\,u(t) - 2u(t-T) - \frac{2}{T}(t-T)u(t-T)$$
$$+ u(t-2T) + \frac{1}{T}(t-2T)u(t-2T)$$



$$E = R - \left(\frac{1-\varepsilon^{-Ts}}{Ts^2}\right)E^{*}\,{}^{-1}$$

$$C = \left(\frac{1-\varepsilon^{-Ts}}{Ts^2}+\frac{1-\varepsilon^{-Ts}}{s}\right)E^*$$

$$E^* = R^* - \left[\frac{1-\varepsilon^{-Ts}}{Ts^2}\right]^* E^* \Rightarrow E^*\left(1+\left[\frac{1-\varepsilon^{-Ts}}{Ts^2}\right]^*\right) = R^*$$

$$E^* = \frac{R^*}{1+\left(\frac{1-\varepsilon^{-Ts}}{Ts^2}\right)^*} = \frac{E(z)}{R(z)} = \frac{1}{1+\left\{\frac{1-\varepsilon^{-Ts}}{Ts^2}\right\}(z)}$$

$$\frac{1}{s^2} \xleftrightarrow{z} \frac{Tz}{(z-1)^2} \qquad z\left[1-\varepsilon^{-Ts}\times\frac{1}{Ts^2}\right]$$

$$= \left(\frac{z-1}{\cancel{z}}\right)\left(\frac{\cancel{T}\cancel{z}}{\cancel{T}(z-1)^2}\right) = \frac{1}{z-1}$$

$$\frac{E(z)}{R(z)} = \frac{1}{1+\frac{1}{z-1}}$$

$$= \frac{z-1}{z-1+1} = \boxed{\frac{z-1}{z} = \frac{E(z)}{R(z)}}$$

(c) Find $\frac{C(z)}{R(z)}$

$$E^* = \frac{z-1}{z} R(z)$$

$$C^* = \left[\frac{1-\varepsilon^{-Ts}}{Ts^2} + \frac{1-\varepsilon^{-Ts}}{s} \right]^* E^*$$

$$= \left[\frac{1-\varepsilon^{-Ts}}{Ts^2} + \frac{1-\varepsilon^{-Ts}}{s} \right]^* \frac{R^*}{1 + \left[\frac{1-\varepsilon^{-Ts}}{s}\right]^*}$$

$$= \left[\left(\cancel{\frac{z-1}{z}}\right)\left(\frac{\cancel{Tz}}{\cancel{T}(z-1)^{\cancel{2}}}\right) + \left(\cancel{\frac{z-1}{z}}\right)\left(\frac{\cancel{z}}{\cancel{z-1}}\right) \right] \frac{R(z)}{1 + \frac{1}{z-1}}$$

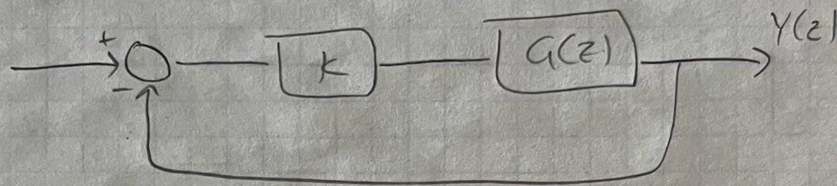
$$C(z) = \left[\frac{1}{(z-1)} + 1 \right] \frac{z-1}{z} R(z)$$

$$C(z) = \left[1 \frac{\cancel{z-1}}{\cancel{z-1}} + \frac{z-1}{z} \right] R(z)$$

$$\frac{C(z)}{R(z)} = \left[1 + \frac{z-1}{z} \right] = \frac{z}{z} + \frac{z-1}{z} = \boxed{\frac{2z-1}{z}}$$

Problem 3

$$G(z) = \frac{1}{z-2}$$



$$G_{op} = \frac{K}{z-2} \Rightarrow G_{CL} = \frac{G(z)}{1+G(z)} = \frac{\frac{K}{z-2}}{1+\frac{K}{z-2}}$$

$$G_{CL}(z) = \frac{K}{z-2+K}$$

Char - Eq $= z-2+K=0$

Stability need $|z|<1 \Rightarrow |-2+K|<1$

Jury's Stability Test $\qquad -1<-2+K<1$

$Q(1)>0 \qquad K<1+2 \Rightarrow K<3$

$(1)-2+K>0 \Rightarrow -1+K>0 \Rightarrow K>1$

$(-1)^1 Q(-1)>0$

$(-1)((-1)-2+K)>0 \Rightarrow 3-K>0 \Rightarrow K<3$

Thus, system is stable for $\boxed{1<K<3}$.

$|a_0|<a_n$

$|-2+K|<1 \Rightarrow -1<-2+K<1 \Rightarrow 1<K<3$