Digital Control Systems - Chapter 6 Notes

System Time-Response Characteristics

6.1 Introduction

In this chapter we consider 5 important topics

- Time Response of a discrete-time system is investigated
- Regions in the s-plane are mapped into regions in the z-plane
- Using correlation between regions in the two planes, the effect of the closed-loop z-plane poles on the system transient response
- Effects of the system transfer characteristics on the steady-state system error
- Simulations of analog and discrete-time systems are introduced

6.2 System Time Response

In this section the time response of discrete-time systems is introduced via examples. In these examples some of the techniques of determining the system time response are illustrated.

Example 6.1

The unit-step response will be found for the first-order system in Fig. 6-1(a). Since the plant of a temperature control system is often modeled as a first-order system, this system might then be the model of a temperature control system (see Section 1.6). Using the techniques developed in Chapter 5, we can express the system output as

$$C(z) = \frac{G(z)}{1 + G(z)} R(z)$$

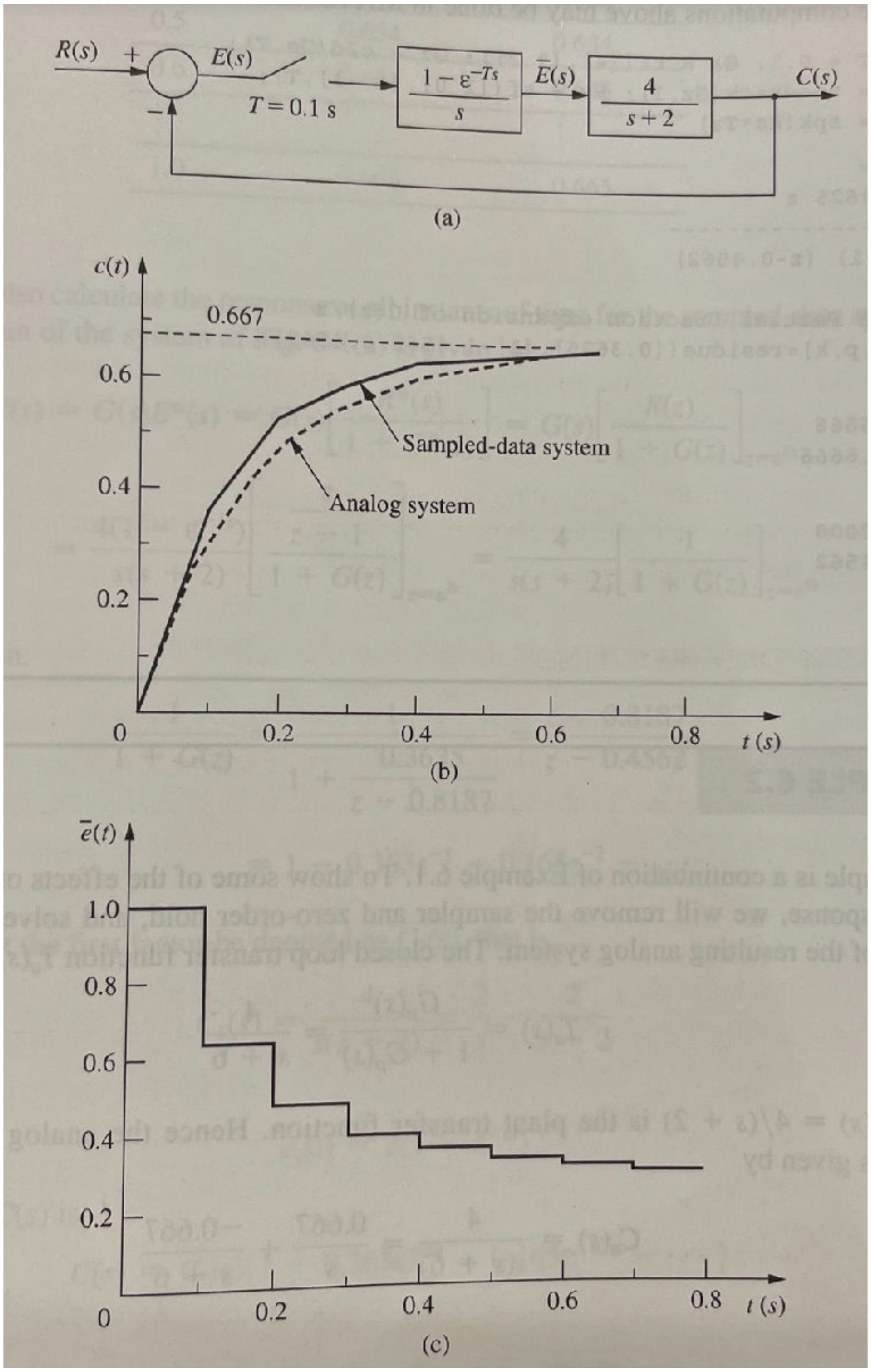
where $G(z)$ is defined as

$$G(z) = z\left[\frac{1 - \varepsilon^{Ts}}{s} \frac{4}{s+2}\right] = \frac{z-1}{z} z\left[\frac{4}{s(s+2)}\right]$$

$$= \frac{z-1}{z} \frac{2(1-\varepsilon^{-2T})z}{(z-1)(z-\varepsilon^{-2T})} = \frac{0.3625}{z - 0.8187},\ T = 0.1 \text{ secs}$$

from the transform table in Appendix VI. Thus the closed-loop transfer function $T(z)$ is given by

$$T(z) = \frac{G(z)}{1 + G(z)} = \frac{0.3625}{z - 0.4562}$$



Since $R(z) = z\{1/s\} = z/(z-1)$

$$C(z) = \frac{0.3625z}{(z-1)(z-0.4562)} = \frac{0.667z}{z-1} - \frac{0.667z}{z-0.4562}$$

The inverse z-transform of this function yields the system time response at the sampling instants. Thus

$$c(kT) = 0.667[1 - (0.4562)^k]$$

This response is listen in Table 6-1. It is seen that the response reaches a steady-state value of 0.667. The computations above may be done in MATLAB:

```
T = 0.1; Gs = tf(4, [1 2]); Gz = c2d(Gs,T);
Tz = feedback(Gz,1); Rz = tf([1 0],[1 -1],T);
Cz = zpk(Rz*Tz)
```

```
Cz =

     0.36254 z
  ----------------
  (z-1) (z-0.4562)

Sample time: 0.1 seconds
Discrete-time zero/pole/gain model.
```

```
% Partial Fraction Expansion of C(z)/z
[R,P,K] = residue(0.3625,[1 -1.4562 0.4562])
```

```
R = 2×1
    0.6666
   -0.6666
P = 2×1
    1.0000
    0.4562
K =

     []
```

Example 6.2

This example is a continuation of Example 6.1. To show some of the effects of sampling on the system response, we will remove the sampler and zero-order hold, and solve for the unit-step response of the resulting analog system. The closed loop transfer function $T_a(s)$ is given by

$$T_a(s) = \frac{G_p(s)}{1 + G_p(s)} = \frac{4}{s + 6}$$

where $G_p(s) = 4/(s+2)$ is the plant transfer function. Hence the analog system unit-step response is given by

$$C_a(s) = \frac{4}{s(s+6)} = \frac{0.667}{s} - \frac{0.667}{s+6}$$

and

$$c_a(t) = 0.667(1 - \varepsilon^{-6t})$$

This response is also listed in Table 6-1. Both step responses are plotted in Fig. 6-1(b).

TABLE 6-1 Responses for Example 6.1

kT	$c(kT)$	$c_a(t)$
0	0	0
0.1	0.363	0.300
0.2	0.528	0.466
0.3	0.603	0.557
0.4	0.639	0.606
0.5	0.654	0.634
0.6	0.661	0.648
⋮	⋮	⋮
1.0	0.666	0.665

We may also calculate the response at all instants of time for the sampled-data system. The continuous output of the system of Fig. 6-1(a) is given by

$$C(s) = G(s)E^*(s) = G(s)\left[\frac{R^*(s)}{1+G^*(s)}\right] = G(s)\left[\frac{R(z)}{1+G(z)}\right]_{z=\varepsilon^{Ts}}$$

$$= \frac{4(1-\varepsilon^{-Ts})}{s(s+2)}\left[\frac{z/z-1}{1+G(z)}\right]_{z=\varepsilon^{Ts}} = \frac{4}{s(s+2)}\left[\frac{1}{1+G(z)}\right]_{z=\varepsilon^{Ts}}$$

In this expression,

$$\frac{1}{1+G(z)} = \frac{1}{1+\dfrac{0.3625}{z-0.8187}} = \frac{z-0.8187}{z-0.4562}$$

$$= 1 - 0.363z^{-1} - 0.165z^{-2} - \dots$$

In $C(s)$ above, let the first factor be denoted as $C_1(s)$, that is,

$$C_1(s) = \frac{4}{s(s+2)} = \frac{2}{s} - \frac{2}{s+2}$$

Then

$$c_1(t) = 2(1-\varepsilon^{-2t})$$

Thus the output $C(s)$ is

$$C(s) = C_1(s)[1 - 0.363\varepsilon^{-\mathrm{Ts}} - 0.165\varepsilon^{-2\mathrm{Ts}} - \ldots]$$

and hence

$$c(t) = 2(1 - \varepsilon^{-2t}) - 0.363(2)(1 - \varepsilon^{-2(t-T)})u(t - T) - 0.165(2)(1 - \varepsilon^{-2(t-2T)})u(t - 2T) - \ldots$$

For example, since $T = 0.1$ secs,

$$c(3T) = c(0.3) = 2(1 - \varepsilon^{-0.6}) - 0.363(2)(1 - \varepsilon^{-0.4}) - 0.165(2)(1 - \varepsilon^{-0.2}) = 0.603$$

This value checks that calculated by the z-transform approach and listed in Table 6-1. We see then the reason for the unusual shape of $c(t)$ in Fig. 6-1(b). This response is the superposition of a number of delayed step responses of the open-loop system. The steps appear as a result of the sampler and zero-order hold. For example, for $0 \leq t \leq 0.1\ s$,

$$c(t) = 2(1 - \varepsilon^{-2t})$$

Note that the time response of a sampled-data system of the configuration of that in Fir. 6-1(a) is always the superposition of a number of step responses, independent of the form of the system input function. The steps in the input to the plant are also shown in the plot of the zero-order hold output, $\bar{e}(t)$, in Fig. 6-1(c).

6.3 System Characteristic Equation

6.4 Mapping the *s*-Plane into the *z*-Plane

6.5 Steady-State Accuracy

6.6 Simulation

6.7 Control Software

6.8 Summary