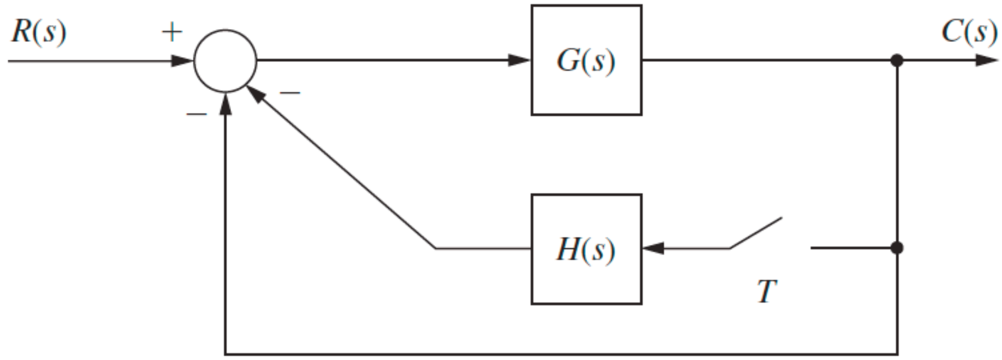
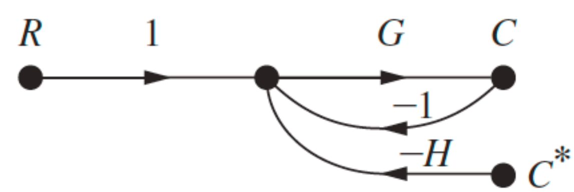
(1) – 10 points

For the following system, express C(z) as a function of the input and the transfer functions.



Solution:

(d)



$$C = \frac{GR}{1+G} - \frac{GH}{1+G} C^*$$

$$C(z) = \left[\frac{GR}{1+G}\right](z) - \left[\frac{GH}{1+G}\right](z)\ C(z)$$

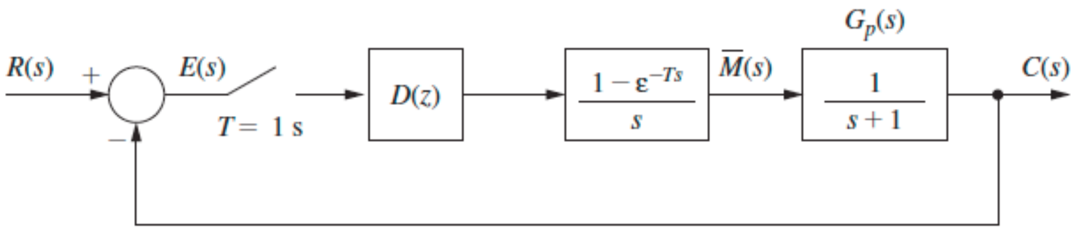
$$\therefore C(z) = \frac{\left[\frac{GR}{1+G}\right](z)}{1+\left[\frac{GH}{1+G}\right](z)}$$

(2) -- 30 points

Consider the following system with T = 1 s. Let the digital controller be a variable gain K such that D(z) = K.

2.1 (15 pts) Write the closed-loop system characteristic equation. (Hint you need to find the denominator of the closed-loop transfer function)

2.2 (15 pts) Determine the range of K for which the system is stable. (Hint you need to find a range of K such that for all the poles are inside the unite disk--|z|<1)



Solution:

$$. \text{(a) } G(z) = \frac{z-1}{z} / \left[\frac{1}{s(s+1)} \right] = \frac{z-1}{z} \frac{(1-\varepsilon^{-T})z}{(z-1)(z-\varepsilon^{-T})} \Rightarrow \frac{0.6321}{z-0.3678}$$

$$\therefore 1 + KG(z) = 0 = z - 0.3679 + 0.6321\,K$$

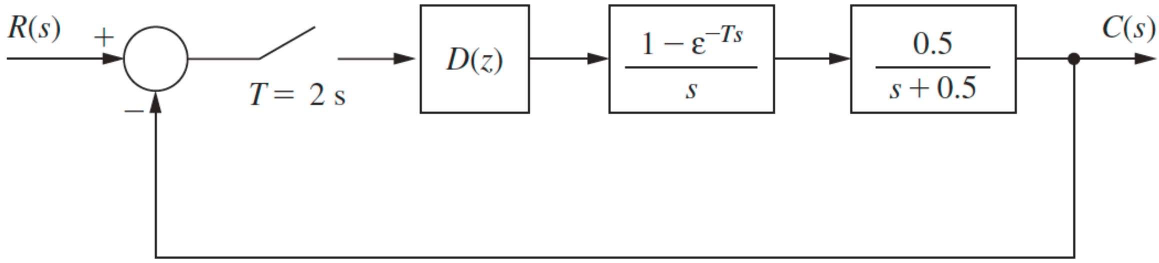
$$\text{(b)} \quad z = 0.3679 - 0.6321K > -1 \Rightarrow K < \frac{1.3679}{0.6321} = 2.164$$

$$z = 0.3679 - 0.6321K < 1 \Rightarrow K > -1$$

$$\therefore -1 < K < 2.164$$

(3) – 40 points

Consider the following closed-loop system



3.1 (10pts) Calculate and plot the unit-step response at the sampling instants, for the case that D(z) = 1 .

3.2 (10pts) Calculate the system unit-step response of the analog system, that is, with the sampler, digital controller, and data hold removed.

3.3 (10pts) Let assume that D(z) = 1 and **T = 0.4 s** . Calculate the system unit-step response at the sampling instants (with the sampler, digital controller, and data hold).

3.4 (10pts) Use the system dc gains to calculate the steady-state responses for each of the systems of parts (3.1), (3.2), and (3.3). Why are these gains equal?

Solution:

(a) $$G(z) = \frac{z-1}{z}\mathfrak{z}\left[\frac{0.5}{s(s+0.5)}\right] = \frac{z-1}{z}\,\frac{(1-\varepsilon^{-1})}{(z-1)(z-\varepsilon^{-1})} = \frac{0.6321}{z-0.3679}$$

$$\frac{G(z)}{1+G(z)} = \frac{0.6321}{z-0.2642}$$

$$\frac{C(z)}{z} = \frac{0.6321}{(z-1)(z+0.2642)} = \frac{0.5}{z-1} + \frac{-0.5}{z+0.2642}$$

$$\therefore c(kT) = 0.5[1-(-0.2642)^k]$$

(b) $$\frac{G_p(s)}{1+G_p(s)} = \frac{0.5}{s+1}$$

$$C(s) = \frac{0.5}{s(s+1)} = \frac{0.5}{s} - \frac{0.5}{s+1} \Rightarrow c(t) = 0.5(1-\varepsilon^{-t})$$

(c) $$G(z) = \frac{0.1813}{z-0.8187},\ \frac{G(z)}{1+G(z)} = \frac{0.1813}{z-0.6374}$$

$$\therefore \frac{C(z)}{z} = \frac{0.1813}{(z-1)(z-0.6374)} = \frac{0.5}{z-1} + \frac{0.5}{z-0.6374}$$

$$c(kT) = 0.5[1-(0.6374)^k]$$

In (b), $t = 0.4k$, and $c(t)\big|_{t=0.4k} = 0.5[1-\varepsilon^{-0.4k}] = 0.5[1-(0.6703)^k]$

(d) (a) $T(1) = 0.5$ (c) $T(1) = 0.5$

(b) $T(0) = 0.5$

dc gain is a function of only $G_p(0)$ and is independent of T.